Acknowledgment. I am indebted to Roy Jenne and the National Center for Atmospheric Research for their generosity in supplying data tapes.

REFERENCES

Svalgaard, L., 1974: *Solar Activity and the Weather, in Correlated Interplanetary and Magnetospheric Observations.* D. E. Page, Ed. Astrophys. Space Sci. Lib., Vol. 42, Reidel Publ. Co., 627–639.

Wilcox, J. M., P. H. Scherrer, L. Svalgaard, W. O. Roberts and R. H. Olson, 1973: Solar magnetic sector structure: Relation to circulation of the earth's atmosphere. *Science*, **180**, 185–186.

——, and R. L. Jenne, 1974: Influence of solar magnetic sector structure on terrestrial atmospheric vorticity. *J. Atmos. Sci.*, **31**, 581–588.

Comments on "Collision Efficiency of Water Drops in the Atmosphere"

JAMES D. KLETT[1]

Physics Department, New Mexico Institute of Mining and Technology, Socorro 87801

9 October 1975

In Fig. 7 of their recent paper, Lin and Lee (1975) compare their new theoretical values for the linear collision efficiency Y_c of water drops with the corresponding theoretical values of Klett and Davis (1973), and with experiment. Unfortunately, the Lin–Lee plot of the Klett–Davis results for Y_c is seriously in error, and a corrected version is offered below in Fig. 1. Data for the Klett–Davis curve comes directly from Fig. 4 of their paper, where $E=Y_c^2(1+r/R)^{-2}$ is plotted against the radius ratio r/R. From the latter figure it is easily seen that the effective collision cross section is nearly geometric for $R=70\ \mu$m and $0.3\leqslant r/R\leqslant 0.8$. This corresponds to $E\approx 1$ or $Y_c\approx 1+r/R$, which in the log-linear plot of Fig. 1 is the nearly straight dashed line shown intercepting the ordinate at $Y_c=1$, and near which the experimental data lie for intermediate values of r/R. It thus appears from the shape of the curves in Fig. 1 that Lin and Lee may have confused E with Y_c when they made their comparison.

Also, a relatively trivial error occurs in Lin and Lee's Fig. 8. There they give the Oseen drag coefficient as $C_D=(24/\text{Re})(1+3/16)$, whereas the correct expression is $C_D=(24/\text{Re})(1+3\ \text{Re}/16)$. Here Re is the Reynolds number based on drop diameter.

For large falling drops with Re$\gg$1, the strong nonlinearity of their hydrodynamic interaction makes the superposition method the only mathematically feasible approach presently available for estimating collision efficiencies. However, for Re$\lesssim$1, the realm where viscous forces dominate, it is known that superposition cannot provide an accurate description of the hydrodynamics for close drop separations, except for $r/R\ll 1$. Therefore, the indication from Fig. 1 that Lin–Lee's application of the superposition method apparently yields fairly good results even for Re$=O(1)$ is at the same time both encouraging and a little puzzling. They attribute their success to an improved representation of the flow field past a single sphere. But in view of the inherent limitations of superposition for small Re, it is

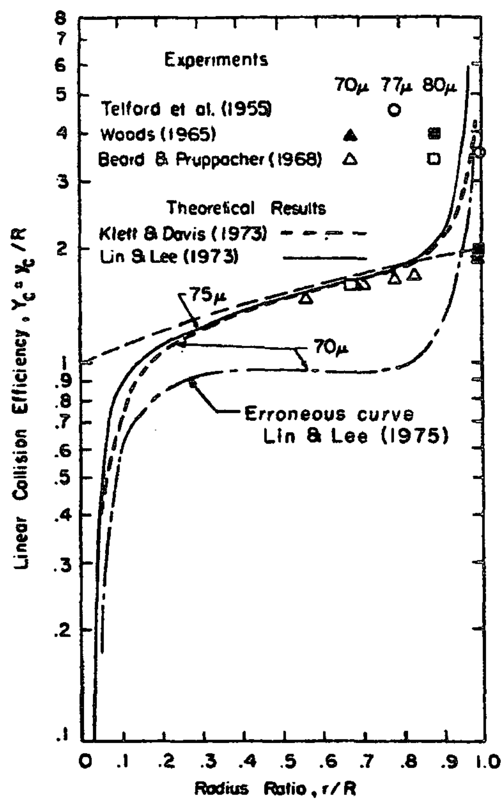

FIG. 1. Comparison between theory and experiment [from Fig. 7 of Lin and Lee (1975)].

[1] Present affiliation: Los Alamos Scientific Laboratory, Los Alamos, N. M.

not clear why even a perfect single sphere flow field should yield good results for Y_c.

A simple example suffices to illustrate the hazards associated with the superposition approach. Consider the motion of two identical spheres falling along their line of centers in the Stokes regime of vanishingly small Re. Stokes flow is simple enough so that a complete account of the viscous interaction is possible, i.e., one may satisfy rigorously the no-slip boundary condition on both sphere surfaces. Such an analysis yields the following result (Stimson and Jeffery, 1926): the hydrodynamic force on either of two identical touching spheres falling along their line of centers is approximately $0.645D_s$, where D_s is the corresponding Stokes drag for either of the spheres falling in isolation with the same speed. To the same accuracy the method of superposition, according to which each sphere is assumed to move in a flow field generated by the other sphere falling in isolation, predicts a force of $0.313D_s$, less than half the correct value. On the other hand, the approximate boundary-value analysis of Klett and Davis, being specifically tailored to provide a better description of the viscous interaction than the superposition method, as well as an approximate account of inertial forces, yields in the Stokes limit for the present configuration a force of $0.624D_s$. Finally, for the discussion which follows it is worth noting also the corresponding result of Hocking (1958), which is $0.256D_s$. (Though Hocking's computations were based on a complete boundary value formulation in Stokes flow, only approximate force expressions were extracted from the analysis.)

Now let us consider the consequences of these differences. Experience with the problem of integrating the equations of motion to determine droplet trajectories for $\mathrm{Re} \lesssim 1$ has shown that the component of general motion along the line of centers is by far the most important. Hence, one might expect to find a correlation between the force differences noted above and the curves of Y_c computed from the different models. Such a pattern is discernible in Fig. 2 (a reproduction of Lin and Lee's Fig. 3), which compares various determinations of Y_c. The $R=30\ \mu$m curves of Hocking (1958) and of Shafrir and Neiburger (1963), the latter calculated under the assumptions of Stokes flow and superposition, are seen to be quite similar, even though the mathematical approaches are very different. This is apparently a consequence of similar line-of-centers force components. Likewise, the curves of Klett–Davis in the Stokes limit (not shown in Fig. 2) and those based on the rigorous Stokesian formulation (Davis and Sartor, 1967; Hocking and Jonas, 1970), are also similar. Also, we may note the latter group of studies found no cutoff of zero Y_c for any size or size ratio, while the former predicted a cutoff size of 18 μm below which no collisions could occur.

All of this leads directly to the expectation that Lin–Lee should obtain results quantitatively similar to

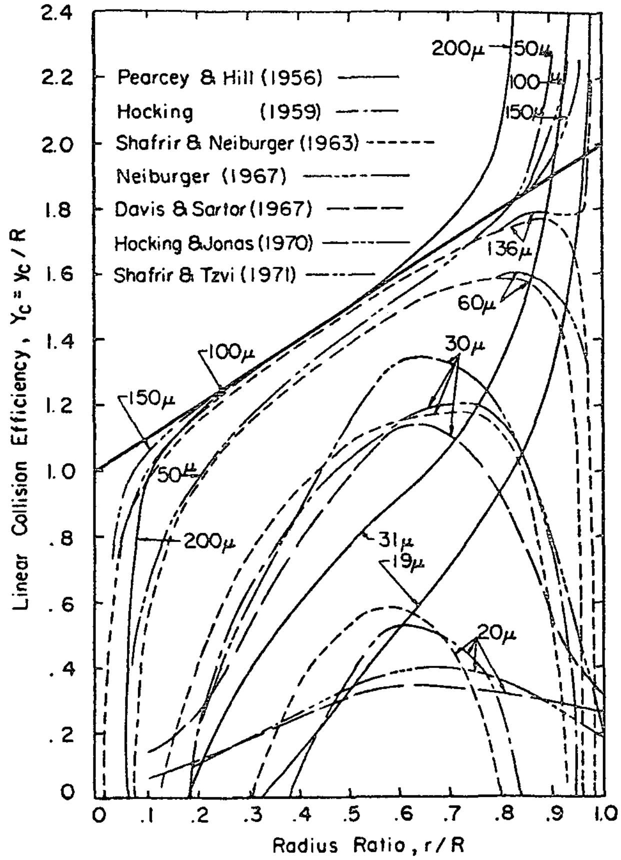

FIG. 2. Survey of collision efficiencies reported in the literature [Fig. 3 of Lin and Lee (1975)].

Hocking and Shafrir–Neiburger for $R \lesssim 30\ \mu$m and intermediate values of r/R. The relatively weak inertial forces for such drops are insignificant except for $r/R \gtrsim 0.7$. In particular, Lin–Lee's approach should also lead to a spurious cutoff size of about 18 μm. Lin and Lee do not say whether or not this happens, but it appears very unlikely in view of the results they do exhibit. This makes it appear probable that their numerical methods of integration have played a larger than expected role in their achievement of improved results under superposition. In fact, it appears that either Hocking and Shafrir–Neiburger independently selected numerical techniques which happened by chance to lead to similar erroneous results, or Lin and Lee have fortuitously selected a method of integration which to some extent leads to better results than could be expected on the basis of their physical-mathematical model.

It is appropriate to point out here that the collision efficiency problem is generally so complicated that it is difficult to be sure the results are not contaminated by numerical errors of one sort or another. For example, de Almeida (1975) has incorporated the Klett–Davis hydrodynamic forces in a larger interaction model which includes turbulence. For the case of no turbulence, his values of Y_c are significantly smaller than those of Klett–Davis for $R \lesssim 25\,\mu m$ (however, the concept of a cutoff size is not reinstated). de Almeida attributes this difference to an instability in the numerical scheme used by Klett–Davis to integrate the trajectory equations. He found that for small drop sizes they belong to the category of the so-called "stiff equations," for which even the standard Adams–Bashworth and Adams–Moulton predictor corrector methods may prove to be unstable (Lin and Lee use the Adams–Moulton method). de Almeida conjectures that for $R < 20\,\mu m$ the values of Davis–Sartor also may be affected by the same source of error.

Let us now consider briefly the expected qualitative behavior of the superposition model for r/R near unity. Since under superposition the individual flow fields do not interact, the strength of wake formation behind the leading sphere of two spheres of comparable size falling in close proximity will be overestimated. This effect will be enhanced also by the underestimate of the strength of viscous interaction between the spheres. This leads to spuriously low drag, hence higher velocities, and hence stronger wakes (this effect is not eliminated by Lin–Lee's drag coefficient adjustments). In short, all of the deficiencies of superposition lead to an expectation for overestimated wake capture. And accordingly, all of Lin–Lee's computed values of Y_c for r/R near unity are higher than the corresponding values of Klett–Davis. For example, for $R = 20$ and $30\,\mu m$, Klett–Davis find $Y_c \approx 0.70$ and 1.3, respectively, for $r/R = 1$. By extrapolation of Lin–Lee's curves shown in their Fig. 6, their corresponding values for Y_c are >2 and >4, respectively.

At this point an objection may be raised about the inherent limitations of the modified Oseen equations employed by Klett–Davis: the fluid inertia forces which lead to wake capture for nearly equal drops cannot be represented rigorously by the Oseen equations. However, the errors are relatively small, so long as $\mathrm{Re} = O(1)$. In this case the net inertial and viscous forces acting on the drops are comparable in magnitude. But of the force arising from the fluid inertia, at worst (for close separations) only a fraction roughly proportional to the fractional velocity difference of the drops is misrepresented in the Klett–Davis formulation. This fractional velocity difference is typically less than 0.1 (e.g., see Fig. 3 of Klett–Davis). Thus the inertial forces may be in error by as much as about 10% while, as we have seen, the viscous forces are about 3% too low. This compares with about a 50% error in the viscous forces and an undetermined but apparently large error in the inertial forces as provided by the method of superposition.

Some further support for the Klett–Davis model comes from the experimental work of Abbott (1974), who found surprisingly good agreement between his observed wake-capture collision efficiencies and the corresponding Klett–Davis values. [However, the interested reader may also want to note the somewhat inconclusive exchange of view on Abbott's experimental arrangement by Sartor (1975a,b) and Abbott (1975).]

So the argument that Lin and Lee strongly overestimate wake capture for $\mathrm{Re} \lesssim 1$ appears to stand. In fact, they probably overestimate it for all Re. Nevertheless, their wake capture results do seem to represent a definite improvement over the earlier work of Shafrir and Neiburger, whose application of superposition led to zero values of Y_c for r/R near unity.

Acknowledgments. I would like to thank Mrs. Pearl Sanders for typing the manuscript. This research has been supported by the National Science Foundation under Grant GA-42465.

REFERENCES

Abbott, C. E., 1974: Experimental cloud droplet collection efficiences. *J. Geophys. Res.*, **79**, 3098–3100.

——, 1975: Reply (to Sartor). *J. Geophys. Res.*, **80**, 1699.

de Almeida, F. C., 1975: On the effects of turbulent fluid motion in the collisional growth of aerosol particles. Ph.D. dissertation, University of Wisconsin, 185 pp.

Davis, M. H., and J. D. Sartor, 1967: Theoretical collision efficiencies for small cloud droplets in Stokes flow. *Nature*, **215**, 1371–1372.

Hocking, L. M., 1958: Three-dimensional viscous flow problems solved by the Stokes and Oseen approximations. Ph.D. dissertation, University of London, 100 pp.

——, and P. R. Jonas, 1970; The collision efficiency of small drops. *Quart. J. Roy. Meteor. Soc.*, **96**, 722–729.

Klett, J. D., and M. H. Davis, 1973: Theoretical collision efficiencies of cloud droplets at small Reynolds numbers. *J. Atmos. Sci.*, **30**, 107–117.

Lin, C. L., and S. C. Lee, 1975: Collision efficiency of water drops in the atmosphere. *J. Atmos. Sci.*, **32**, 1412–1418.

Sartor, J. D., 1975a: Comment on "Experimental cloud droplet collection efficiencies" by C. E. Abbott. *J. Geophys. Res.*, **80**, 1697–1968.

——, 1975b: Comment on "Reply" by C. E. Abbott. *J. Geophys. Res.*, **80**, 3508.

Shafrir, U., and M. Neiburger, 1963: Collision efficiencies of two spheres falling in a viscous medium. *J. Geophys. Res.*, **68**, 4141–4147.

Stimson, M., and G. B. Jeffery, 1926: The motion of two spheres in a viscous fluid. *Proc. Roy. Soc. London*, **A111**, 110–116.