Spectral Properties of the Baroclinic Waves in an Annulus with a Rigid Upper Surface

S. Trivikrama Rao[1] and Carl B. Ketchum[2]

Department of Atmospheric Science, State University of New York at Albany, Albany, N. Y. 12222

(Manuscript received 9 October 1975, in revised form 26 February 1976)

ABSTRACT

The spectral characteristics of the temperature field associated with the baroclinic waves in a rotating annulus of water with a rigid upper surface are presented as an extension of Rao and Ketchum (1975). The results indicate a clear nonlinear maintenance of the harmonics of the fundamental wave in the equilibrated state similar to the feature observed with a free upper surface. The period of the temperature oscillation in the case of rigid upper surface is approximately four times longer than that with a free upper surface.

1. Introduction

This paper is an extension of the results of Rao and Ketchum (1975, denoted hereafter as RK), from a free upper surface to a rigid upper surface on the spectral characteristics of the temperature wave in the wave regime in a rotating annulus of fluid with an impressed horizontal temperature gradient. The extension is undertaken to examine the effect of the rigid lid on the generation of higher harmonics in the basic temperature wave profile. The spectral and bispectral results indicate that the higher harmonics are present and maintained through a nonlinear interaction with the basic temperature wave mode similar to the results obtained in RK with the upper surface free. The main distinction between a free upper surface and a rigid upper surface is that the wave period in the latter case is roughly four times longer. The results are obtained using four thermocouples mounted at the center of the annular region at different azimuthal positions, and two additional thermocouples at different depths at one azimuthal position. A discussion of previous work and a detailed description of the apparatus and experimental technique used in this work is given in RK.

2. Apparatus and range of experiment

The annulus has a 9.99 ± 0.01 cm gap width and a depth of 20.00 ± 0.02 cm with a rigid plexiglass upper surface. The data are obtained from six thermocouples (see Fig. 1), four located at mid-depth and mid-radius with azimuthal positions of 0°, 90°, 162° and 280°. The two other thermocouples are mounted 5 cm above and below the thermocouple at 280°. The series of experimental runs (R series) for these data were set up to duplicate as nearly as possible run series A in RK. The mean horizontally impressed temperature difference is 7.92°C and the rotation rate varied from 0.354 to 4.485 s^{-1}. Specific details of each run are presented in Table 1.

3. Results

The temperature wave traces from thermocouple 6 for two runs in the wave regime and one in the vacillating regime (runs R-10, R-6 and R-9, respectively) are shown in Fig. 2. These traces are to be compared to

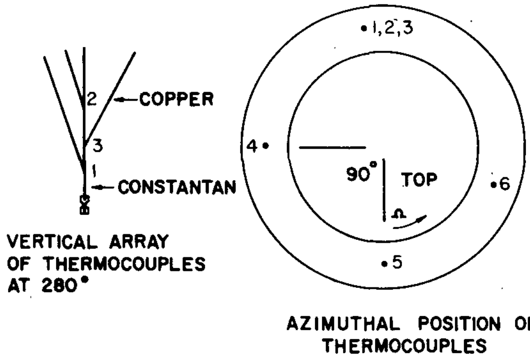


Fig. 1. Thermocouple setup for the R series data. The positions of the six thermocouples are the same as for the A series data presented in RK. The thermocouples are constructed from 0.0127 cm diameter copper-constantan wire. Thermocouples 3, 4, 5 and 6 are at the mid-depth (10 cm from the bottom) and mid-radius of the convection chamber; thermocouples 1 and 2 are 5 cm above and below thermocouple 3 at the 280° azimuthal position.

[1] Present affiliation: Air Resources Research, New York State Department of Environmental Conservation, Albany, N. Y. 12233.

[2] Present affiliation: College of the Atlantic, Bar Harbor, Maine 04609.

Unauthenticated | Downloaded 12/05/21 06:13 AM UTC

TABLE 1. Experimental conditions for the R series runs. The wavenumbers observed in the regular wave regime are shown by 2, 3 or 4 with V representing vacillating flow, and I irregular flow. The Rossby number is given by

$$\Theta=\frac{g}{\bar{\rho}}\frac{a[\rho(T_a)-\rho(T_b)]}{\Omega^2(b-a)^2}$$

and the Taylor number by

$$\mathcal{T}=\frac{4\Omega^2(b-a)^4}{\bar{\nu}^2}\frac{(b-a)}{d},$$

where d is the depth of the fluid, a and b are the radii of the inner and outer cylinders, respectively, $\bar{\rho}$ is the mean density, $\rho(T)$ the density of the fluid corresponding to the temperature T, T_a and T_b the temperatures of the inner and outer cylinders, $\bar{\nu}$ the average value of kinematic viscosity, g the acceleration of gravity, and Ω the rotation rate. The vertical temperature gradient is determined from thermocouples 1 and 2 which are separated by 10 cm in the vertical. The wave period is determined from the power spectral density estimates and the standard deviation is the standard deviation of the dimensional temperature record for the thermocouple 6 for that run.

Run no.	State	Temperature difference (K)	Rotation rate (s^{-1})	Rossby number Θ	Taylor number $\mathcal{T}$ ($\times 10^9$)	Vertical temperature gradient (K cm^{-1})	Wave period (s)	Standard deviation of temperature record (K)
				Rigid Upper Surface				
R-12	2	7.96	0.3544	2.884	0.02526	0.314	1280	0.320
R-10	2	7.93	0.3871	2.860	0.0323	0.299	768	0.303
R-7	2	7.97	0.4312	1.950	0.0374	0.296	960	0.325
R-6	3	7.87	0.6551	0.819	0.0889	0.290	768	0.285
R-8	4	7.83	1.3932	0.197	0.4198	0.247	768	0.228
R-9	V	8.05	2.2171	0.079	1.080	0.340	1280	0.470
R-1	I	7.85	4.4848	0.020	3.892	0.268	—	0.193

Figs. 4a, 4b and 4e of RK. Just as in Fig. 4a of RK, the temperature trace for run R-10 (see Fig. 2a) exhibits a fluctuating amplitude with a stable frequency. This run is near the transition from the upper symmetric to the wave regime. With an increase in rotation rate, the wave amplitude becomes more regular,

TABLE 2. Summary of the results of cross spectral analysis in the regular wave regime. A coherence near unity at a frequency estimate indicates that the relative phase shift can be used to calculate an azimuthal wavenumber corresponding to that frequency.

Run no.	Harmonic	Frequency (s^{-1})	Coherence	Observed phase difference for 90° separation	Corresponding wavenumber	Theoretical phase for 90° separation and given wavenumber
R-10	0	0.0065	0.992	3.07	2	−3.14
		0.0082	0.997	3.09		
	1	0.0147	0.808	0.51	4	0.00
	2	0.0213	0.785	3.11	6	3.14
		0.0229	0.836	−3.06		
R-6	0	0.0082	0.999	−1.81	3	−1.57
	1	0.0164	0.990	−0.30	6	3.14
	2	0.0245	0.997	1.47	9	1.57
	3	0.0327	0.985	1.71	?	0.00
R-9		0.0033	0.793 (0.890*)	−1.56 (−1.86*)	3	−1.57 (−2.51*)
		0.0049	0.782 (0.869*)	−1.71 (−1.85*)		
		0.0164	0.429 (0.688*)	0.07 (−1.30*)	4	0.00 (−1.26*)
		0.0180	0.523 (0.664*)	0.24 (−1.55*)		

* The numbers in parentheses represent the observed coherence and phase for a separation of 72°. Although the phase difference for both separations agree well with the theoretical phase, it is to be noted that the coherence drops off with distance.

Unauthenticated | Downloaded 12/05/21 06:13 AM UTC

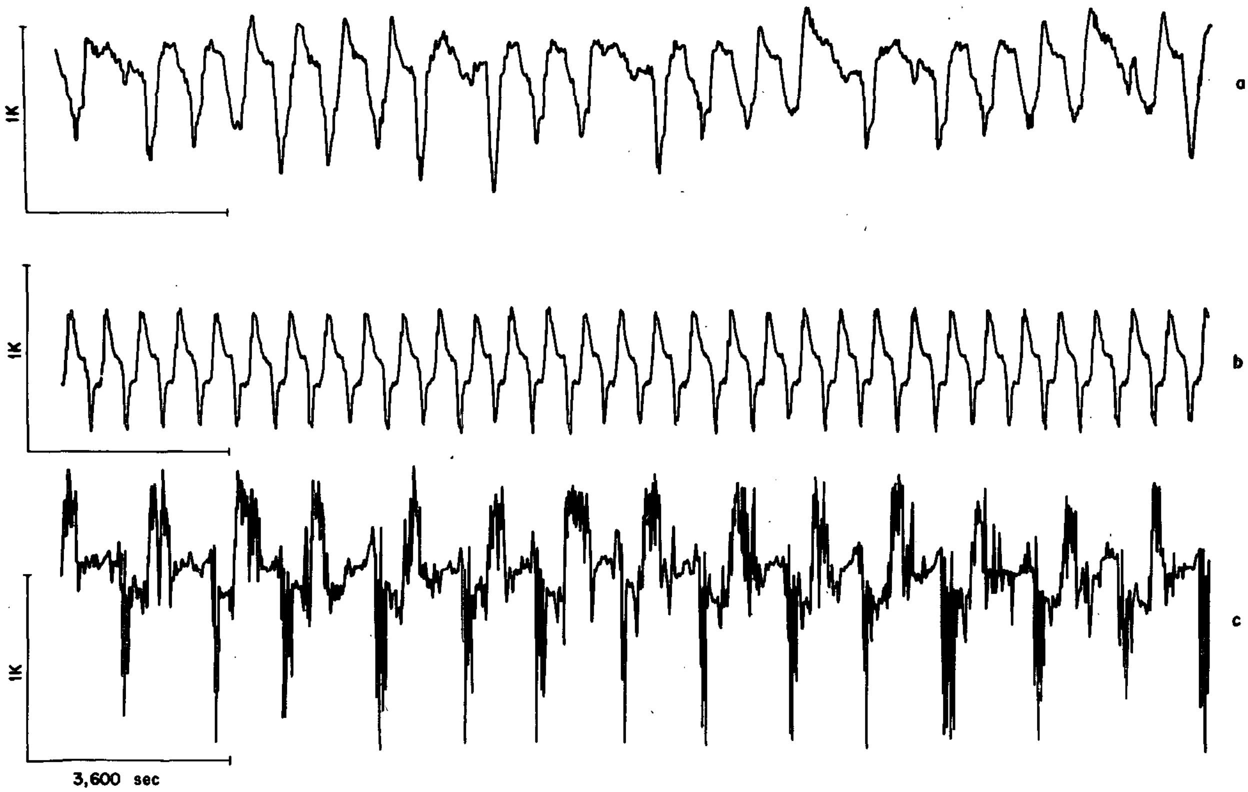


FIG. 2. Typical temperature traces from thermocouple 6. The length of the line along the time axis (abscissa) is equivalent to 3600s and the length of the line along the temperature axis (ordinate) is equivalent to 1 K. Experimental conditions are as follows:

Fig.	2a	2b	2c
Run	R-10	R-6	R-9
ΔT (K)	7.93	7.87	8.05
Ω (s^{-1})	0.387	0.655	2.217
State	2 waves	3 waves	vacillation

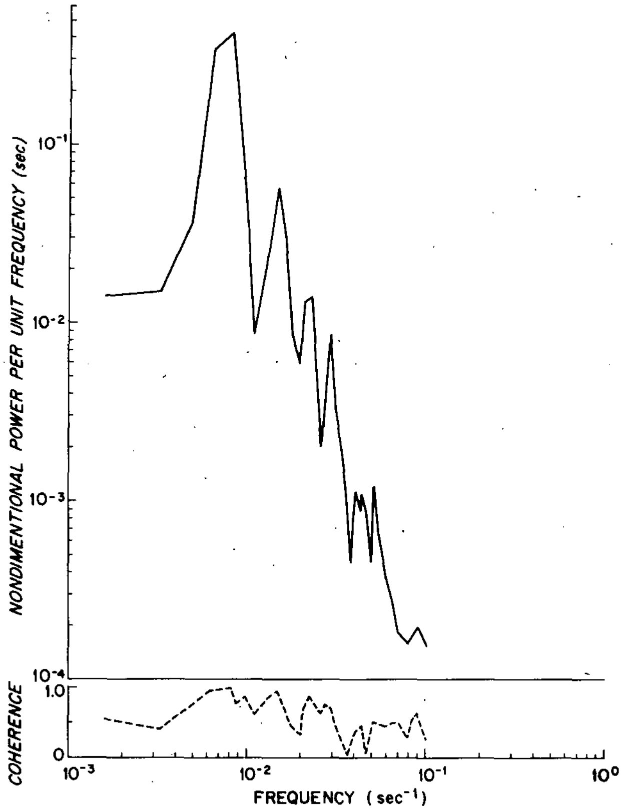
as in Fig. 2b. Fig. 2c represents the temperature trace within the vacillation regime. The spectral results for R-9 show a high azimutual coherence for both wavenumbers 3 and 4 indicating that this is an example of wavenumber vacillation. Although runs A-9 (in RK) and R-9 are nearly at the same external conditions, the former with free upper surface indicates wave form vacillation while the latter with rigid upper surface shows wavenumber vacillation. It is possible that the rigid upper surface might suppress wave form vacillation. However, it should be borne in mind that small changes in the impressed temperature difference and rotation rate can also affect the wavenumber present in addition to the presence of rigid lid. The data in this series are not sufficiently dense to exclude the possibility of wave form vacillation with an upper rigid lid.

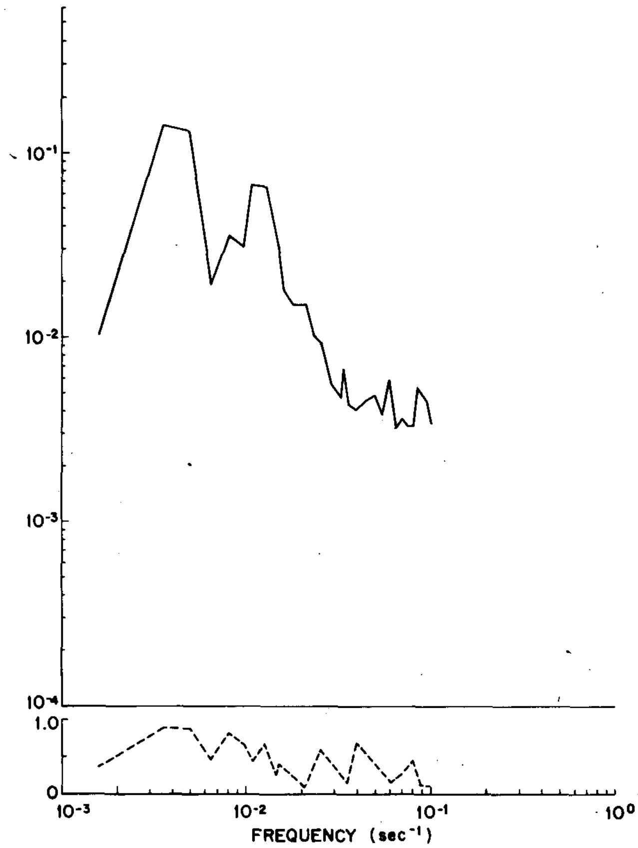
Although the shape of the temperature oscillation is similar to that in the case with a free upper surface, the period of oscillation is roughly four times longer than the periods observed with a free upper surface under nearly the same physical conditions. Thus, the drift velocity of the temperature wave is much smaller in the case of a rigid upper surface.

The spectral calculations are based on the thermocouple data sampled every 10 s, and then block-averaged to give an effective sampling interval of 30 s. The power spectra, cospectra, coherence, phase and bispectra of the temperature field within both the regular wave regime and the vacillation regime are computed following the method described in RK based on that of Hinich and Clay (1968). The block-averaged data are broken into 10 non-overlapping segments, and the Fourier transform of each segment is computed. The final estimate is the average of the 10 different estimates at each frequency. The resulting calculations have 20 degrees of freedom.

Fig. 3 illustrates the resulting power spectra for runs R-10, R-6 and R-9. The power spectra within the regular wave regime for runs R-10 and R-6 indicate that the temperature field has a fundamental wave period corresponding to the first peak in the spectra and several harmonics. The azimuthual coherence plotted underneath the spectra shows that these harmonics are highly coherent around the annulus. The two distinct nonharmonic peaks in the spectrum for run R-9 at frequencies of 0.0033 and 0.0164 s^{-1} corre-

Unauthenticated | Downloaded 12/05/21 06:13 AM UTC





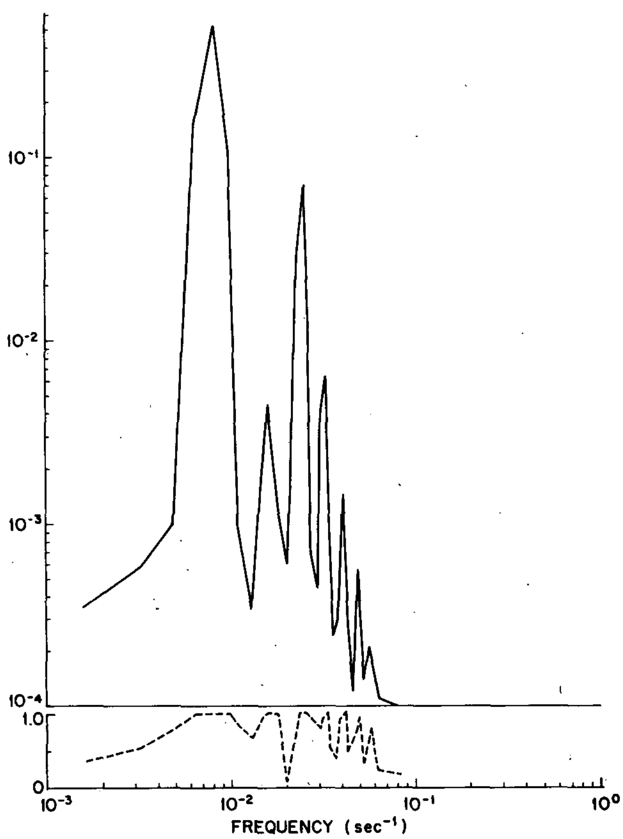


FIG. 3. Power spectral density estimates for the temperature data from thermocouple 6: (a) Run R-10, (b) R-6, (c) R-9. The coherence plotted underneath the spectrum corresponds to a separation of 118°, 90° and 72° in the azimuthal direction, respectively. The effective sampling interval for these calculations is 30 s. Although 10 runs within the wave regime are taken, the spectral properties of only two runs in the regular wave regime and one in the vacillation are presented since they are representative of the other results. If $S(f_k)$ is the true spectral estimate and S_k the estimated value, then there is a 90% confidence (set by a chi-squared distribution for 20 degrees of freedom) that $0.65S_k<S(f_k)<1.8S_k$. A dimensional spectral density can be obtained from these results by multiplying the given density by the square of the impressed temperature difference for that run.

spond to wavenumbers 3 and 4, respectively, indicating the possibility of a wavenumber vacillation. Table 2 summarizes the results of the spectral peaks for each of these runs, indicating the azimuthal coherence, the observed phase shift for azimuthal separations of 72° and 90°, the closest corresponding wavenumber for this observed phase, and the theoretical phase shift for that wavenumber. This indicates that for runs R-10 and R-6, the frequency harmonics are also wavenumber harmonics.

The results of the bispectral computations for the significant peaks in each spectrum for runs R-10, R-6 and R-9 are presented in Table 3. A bispectral skewness near unity indicates a strong possibility that non-

Unauthenticated | Downloaded 12/05/21 06:13 AM UTC

TABLE 3. Summary of the results of the bispectrum analysis for the temperature data recorded by thermocouple 6. A skewness near unity indicates a strong possibility that a nonlinear interaction between frequencies $\omega(j)$ and $\omega(k)$ produces the spectral component at a frequency of $\omega(j+k)$.

Run no.	Harmonic interaction	Interacting frequencies $\omega(j)$ (s^{-1})	Interacting frequencies $\omega(k)$ (s^{-1})	Resulting frequencies $\omega(j+k)$ (s^{-1})	Skewness
R-10	$0+0\rightarrow 1$	0.0065	0.0065	0.0130	1.0
		0.0065	0.0083	0.0147	1.0
		0.0082	0.0082	0.0164	0.96
	$0+1\rightarrow 2$	0.0065	0.0147	0.0212	0.95
	$1+1\rightarrow 3$	0.0147	0.0147	0.0294	0.94
R-6	$0+0\rightarrow 1$	0.0082	0.0082	0.0164	0.96
	$0+1\rightarrow 2$	0.0082	0.0164	0.0246	0.95
	$0+2\rightarrow 3$	0.0082	0.0245	0.0327	0.99
	$0+3\rightarrow 4$	0.0082	0.0327	0.0409	0.94
	$0+4\rightarrow 5$	0.0082	0.0409	0.0491	0.95
	$1+1\rightarrow 3$	0.0164	0.0164	0.0328	0.99
	$2+2\rightarrow 5$	0.0245	0.0245	0.0490	0.95
R-9	No interaction has skewness greater than 0.9				

linear interaction between the two interacting frequency components maintains the component at the sum or difference of the interacting frequencies. The high value of the skewness at the harmonic peaks for runs R-10 and R-6 is consistent with the assumption that the higher harmonics in the spectrum are the result of nonlinear interactions between temperature disturbances at lower frequencies. This process is not evident for run R-9 in the vacillation regime.

The average shape of the temperature wave over one wave cycle is obtained by superimposing 88 wave cycles in the temperature record. Fig. 4 shows the shape of the temperature wave at the 5, 10 and 15 cm levels at the azimuthal point of 280° for run R-6. The shape of these waves is similar to the shape of the waves with a free upper surface. The nonsinusoidal shape of the temperature wave is evident from this diagram.

The vertical variation of phase shown in Fig. 5 indicates that the phase of the wave in the upper level of the fluid leads the phase of the wave in the lower level consistent with the theory of Eady (1949). The phase lead decreases in the vacillation regime, similar to the feature observed with a free upper surface.

4. Summary and conclusions

The results indicate that the temperature wave features with a rigid upper lid are quite similar to those observed with a free upper surface. The major differ-

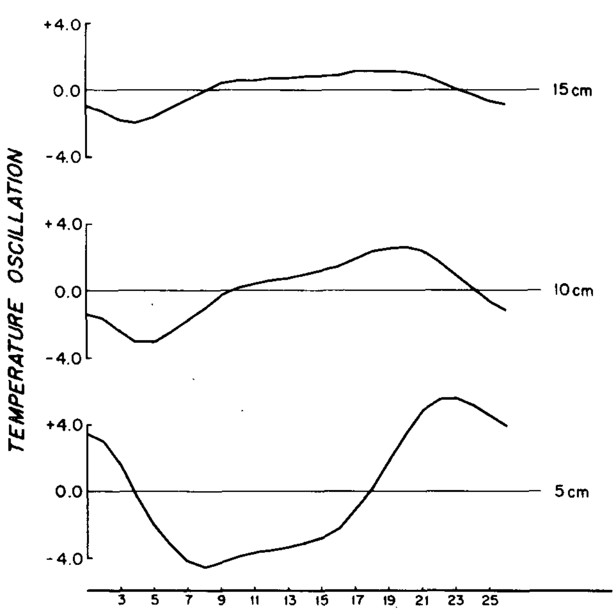


FIG. 4. The average shape of the temperature wave through a superposition of many wave cycles for Run R-6. Note the close resemblance to Fig. 7 of RK. If $T(J)$, $J=1,\ldots,n$, is a long record of total length n having a wave period τ (768 s in the present case) and the data are sampled at intervals of Δt (30 s), then $k=(\tau/\Delta_t)$[an integer]=26 (in the present case). The averaging over a wave period is done as

$$\bar{T}(l)=\sum_{m=1}^{n/k} T[l+(m-1)k],\quad \text{for } l=1,\ldots,k.$$

Unauthenticated | Downloaded 12/05/21 06:13 AM UTC

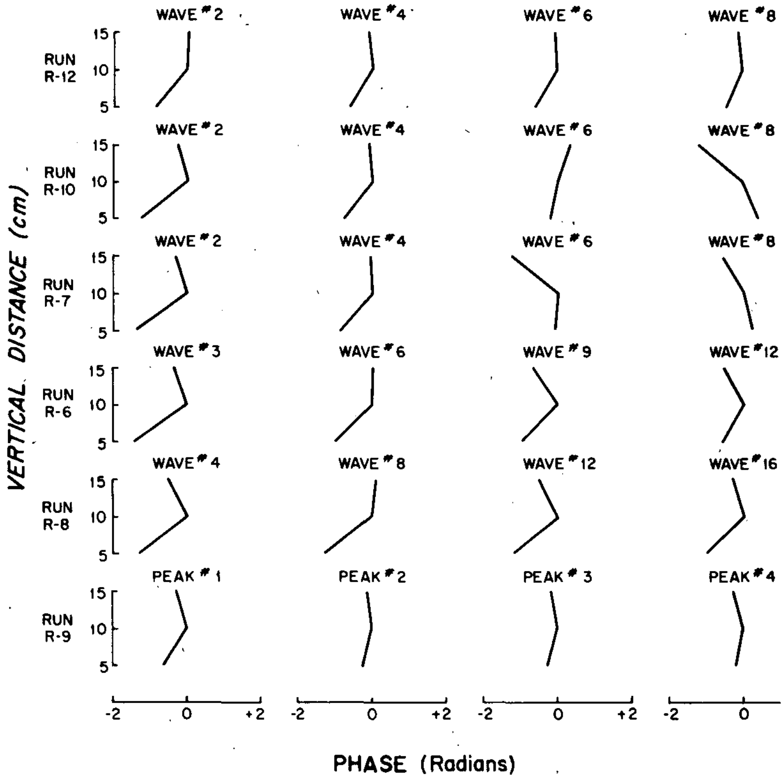


FIG. 5. Vertical variation of the phase from thermocouples 1–3 in the annulus. The curves on the extreme left correspond to the observed wavenumber (first peak in the spectral density estimate) in a run and those to its right are the harmonics (other distinct peaks in the spectral density estimate) of the observed wavenumber. In run R-9, since the peaks in the spectra do not represent a definite wavenumber, the curves correspond to the significant peaks in the spectrum. It can be seen that the phase difference decreases in the vacillation regime similar to that observed with a free upper surface.

ence between the rigid upper surface and free upper surface results is that the periods of the wave oscillation in the former case are much longer than those in the latter. The presence of the upper lid reduces the fluid velocity at the steering level of the mean flow to near zero, consequently decreasing the phase speed of the wave with respect to the free surface case. Of primary interest is the fact that in both cases, observations revealed that in the regular wave regime the fundamental wave produces several harmonics as a result of nonlinear interactions. Just as in free upper surface, the skewness of the interaction at higher frequencies affecting the low frequencies is much smaller, indicating little transfer of energy to the low-frequency side of the spectrum.

The finite-amplitude baroclinic theory of Pedlosky (1970) does not show production of higher harmonics. It is suspected that the circular geometry and/or the radial variation of the zonal velocity of the basic state, not considered in the above theory, may be responsible for the production of these harmonics.

Acknowledgments. The authors thank Diane Huff for typing the manuscript and Carol Clas for drafting the diagrams. The study was supported by the National Science Foundation under Grant GA-12734.

REFERENCES

Eady, E. T., 1949: Long waves and cyclone waves. *Tellus*, **1**, 33–52.

Hinich, M. J., and C. S. Clay, 1968: The application of the discrete Fourier transform in the estimation of power spectra, coherence and bispectra of geophysical data. *Rev. Geophys.*, **6**, 347–363.

Pedlosky, J., 1970: Finite-amplitude baroclinic waves. *J. Atmos. Sci.*, **27**, 15–30.

Rao, S. T., and C. B. Ketchum, 1975: Spectral characteristics of the baroclinic annulus waves. *J. Atmos. Sci.*, **32**, 698–711.

Unauthenticated | Downloaded 12/05/21 06:13 AM UTC