Radiative Transfer Within the Earth's Troposphere and Stratosphere: A Simplified Radiative-Convective Model

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ABSTRACT

A simplified radiative transfer model for the earth's atmosphere is presented. The simplification is achieved by a combination of band absorptance and emissivity formulation for treating radiative transfer due to H2O, CO2, and O3. The model incorporates the major and minor radiative transfer processes due to H2O, CO2, and O3. The radiative model is used to develop an efficient and accurate radiative-convective model. Results for the global surface temperature, stratospheric thermal structure, and the net outgoing longwave flux are presented.

The computed thermal structure of the stratosphere and the stratospheric cooling rates are in excellent agreement with previous studies. The amplitude of the diurnal temperature difference in the upper stratosphere obtained from the present model is larger by about 50% than Leovy's (1964) results. This difference is due to the inclusion of Doppler broadening effects and CO2 hot and minor isotopic bands in the present model.

The flux calculations indicate that the relatively minor bands like the CO2 hot and minor isotopic bands and the e-type absorption by the H2O continuum band have to be included in order to compute the outgoing flux F to within 1% accuracy. Results are also presented for the sensitivity of F to surface temperature. It is shown that the H2O e-type absorption has a substantial influence on the sensitivity parameter dF/dT.*

1. Introduction

It would greatly facilitate the development of global atmospheric models that treat the three interactive processes, radiative transfer, dynamics and chemical kinetics, if simplified formulations can be developed for these three processes. The simplification should be achieved without compromising the accuracy of the results. Toward this goal, a simplified radiative transfer model has been developed at Langley Research Center that incorporates the major and minor radiative transfer processes due to H2O, CO2, and O3. The present paper describes the details of this radiative transfer model.

Detailed radiative transfer analyses for the earth's atmosphere have been presented by Ellingson (1972) and Dickinson (1973). These analyses clearly illustrate the complex nature of atmospheric radiative transfer problem. Cess and Ramanathan (1972), Ramanathan and Cess (1974) and Cess (1974) have developed simplified techniques to treat this complex problem. Based on these techniques, Ramanathan (1974) developed an efficient stratospheric radiative transfer model. The simplification is achieved in these analyses by employing the emissivity formulation for H2O bands and by employing the band absorptance formulation for CO2 and O3 bands. The present analysis extends these simplified techniques to develop a radiative transfer model for the earth's lower atmosphere. The lower atmosphere refers to the region that includes the troposphere and the stratosphere.

The present model includes both longwave cooling and solar heating due to H2O, CO2 and O3. The simplicity of formulation facilitates inclusion of radiative transfer processes, such as Doppler broadening, H2O continuum bands, hot and minor isotopic bands of CO2, and overlap of H2O bands with the CO2 and O3 bands. Results for the longwave cooling rates and distribution of longwave flux within the atmosphere obtained from the present model are in excellent agreement with more detailed calculations of Dickinson (1973) and Ellingson (1972). The model is employed to perform global energy balance studies and to investigate the sensitivity of climate models.

2. Theoretical model

For the purposes of the present work, the lower atmosphere is defined as the region extending upward from the ground to 50 km in altitude. The main contribution to radiative energy transfer within the lower atmosphere arises from H2O, CO2 and O3. Contribution from
these gases arise both from transmission of longwave radiation and solar absorption. A schematic outline of features included in the present work is shown in Fig. 1.

a. Longwave radiation

The vibration-rotation bands of CO₂ and O₃ and the vibration-rotation band, pure rotation bands and the continuum bands of H₂O contribute to the transfer, i.e., absorption and emission, of longwave energy within the atmosphere. An exact treatment of these bands in radiative transfer calculations would involve line-by-line integrations over several hundred spectral lines in each of these bands. This procedure is, of course, very time-consuming and tedious, and the conventional, as well as well-established, procedure adopted in the literature to greatly simplify the calculation is to treat the bands mentioned above in terms of spectrally integrated emissivity or spectrally integrated band absorption formulation. In the present work, a combination of the emissivity and band absorption formulation is adopted.

1. Water vapor

Water vapor has a vibration-rotation band centered at 6.3 μm, pure rotation bands at wavelengths greater than 12 μm, and two continuum bands in the 8.3–12.5 μm and 16.7–20.83 μm spectral regions. Radiative transfer due to atmospheric H₂O has been formulated in terms of the emissivity formulation by Rodgers (1967) and Cess (1974). An alternate formulation, in terms of the modified emissivity, has been employed by Manabe and Strickler (1964). In these analyses, the emissivity or the modified emissivity is expressed in terms of a single variable \( \bar{U}_1 \), defined as

\[
\bar{U}_1 = U_1 F(T) (\bar{P})^n,
\]

where \( U_1 \) is the H₂O amount (g cm⁻²), \( F(T) \) is some function of temperature which accounts for the temperature dependence of the absorption coefficient and the rotational line half-width, and \( \bar{P} \) is the effective broadening pressure. The value for the exponent \( n \) varies anywhere from 1 (Cess, 1974; Rodgers and Walsh, 1966) to 0.7 (Manabe and Strickler, 1964).

The experimental results of Bignell (1970) show that the absorption coefficient for the two continuum bands of H₂O depends on the total pressure as well as the partial pressure of H₂O. Hence, it follows that the emissivity formulation expressed in terms of a single variable \( \bar{U}_1 \) cannot adequately describe the continuum bands. In the present work the continuum bands are treated separately by dividing the H₂O absorption spectrum into three parts. The first part consists of the vibration-rotation and pure rotation bands and the second and third parts consist of the two continuum bands.

\[
E_1(\bar{U}_1, T) = 0.59\left(\frac{T_0}{T}\right)^{n/2} \frac{1}{\bar{P}_0} \left(\frac{T_0}{T}\right)^{1/2} \left[ 1 - \frac{1}{2 \bar{P}_0 T_0} \frac{1}{1 + A_1(\bar{U}_1)} \right]
\]

where \( A_1 = 19, A_2 = 3.5 \) and \( \bar{U}_1 \) is defined as

\[
\bar{U}_1 = \int \left( \frac{P}{\bar{P}_0} \right) \left( \frac{T_0}{T} \right)^{1/2} dU_1,
\]

where \( P_0 = 1 \) atm, \( T_0 = 293 \) K and \( P \) and \( T \) are the atmospheric pressure and temperature respectively. The factor \( (T_0/T)^{1/2} \) in Eq. (3) can be reconciled as accounting for the variation of line half-width with temperature. \( U_1 \) is the water vapor amount (g cm⁻²) such that \( \bar{U}_1 \) is the effective water vapor amount. Theoretical justification for expressing the emissivity in terms of \( \bar{U}_1 \) has been given by Cess (1974). The emissivity given by Eq. (2) is compared with Staley and Jurica’s emissivity table in Fig. 2 for ranges of \( U_1 \) and \( T \) relevant to the atmosphere. As seen from Fig. 2, Eq. (2) agrees well with Staley and Jurica’s table.

For the two continuum bands, Bignell’s (1970) experimental results for the absorption coefficient has been adopted. The equation for the absorption coefficient \( K_\lambda \) (g⁻¹ cm² atm⁻¹), given by Bignell, is

\[
K_\lambda(P, e) = K_{1\lambda} P + K_{2\lambda} e,
\]

where \( K_{1\lambda} \) and \( K_{2\lambda} \) are wavelength-dependent and \( e \) is the partial pressure of water vapor. This dependency of \( K_{2\lambda} \) on \( e \) is usually referred to as \( e \)-type absorption (Bignell, 1970). The emissivity values for the two
continuum bands have been computed by adopting the wavelength-dependent values of $K_{2A}$ given by Bignell (1970) and by treating the ratio $K_{1A}/K_{2A}$ as a constant. This procedure of treating $K_{1A}/K_{2A}$ as a constant has been suggested by McClatchey et al. (1972) for the continuum band in the 8.3-12.5 $\mu$m region. The approximations involved in this procedure and the justifications for this procedure are discussed below.

Bignell compares the results for $K_{1A}$ published in the literature and shows that there are significant differences between the various values [see Fig. 8, Bignell (1970)]. Bignell presents about six curves for values of $K_{1A}$ plotted as a function of wavelength, as obtained from previous publications. Depending on which of these curves is chosen, the ratio $K_{1A}/K_{2A}$ becomes a very weak or a very strong function of wavelength. Further, it is not clear which of these curves are more reliable. Hence, in the present model, the easier approach of treating the ratio as a constant is followed. This approximation can be justified for the longwave flux and cooling rate calculations. The contribution to the atmospheric opacity due to the continuum bands is significant mainly in the lowest 1 or 2 km above the ground. As discussed by Kunde et al. (1974), in this altitude region the $\epsilon$-type absorption is more important than the absorption due to the $K_{1A}$ term such that any error introduced in the $K_{1A}$ term would be negligible for the flux or cooling rate calculations.

The constant value for the ratio $K_{1A}/K_{2A}$ is chosen by the following procedure. For the 16.7-20.83 $\mu$m band, the emissivities were computed by adopting Bignell’s results for $K_{1A}$ and $K_{2A}$. The emissivities were recomputed by adopting the same $K_{2A}$ values and by assuming a constant value for $K_{1A}/K_{2A}$. The two emissivities were calculated for $P=1$ atm and for values of water vapor amount between 10 and 0.5 g cm$^{-2}$ and for values of $\epsilon$ between 0.04 and 0.005 atm. The value of the ratio that yielded the best overall agreement between the two emissivities was chosen. The range of water vapor amount and $\epsilon$ mentioned above covers the values for which the continuum bands are important for atmospheric flux and cooling rate calculations. For the continuum band in the 8.3-12.5 $\mu$m region, Bignell has measured only $K_{2A}$. Hence, for this band the value of $K_{1A}/K_{2A}$ is taken from McClatchey et al. (1972). Adopting McClatchey et al.’s value for $K_{1A}/K_{2A}$ and Bignell’s results for $K_{2A}$, the emissivity was computed for the range of water vapor amount and $\epsilon$ indicated above.

Since $K_{1A}/K_{2A}$ has been treated as a constant, the expression for the two emissivities can be represented in terms of a single variable $\bar{U}_N$ where $\bar{U}_N$ is given by

$$\bar{U}_N \text{[for } N=2 \text{ and } 3]=\int(e+\alpha N)\text{d}U,$$

where $\alpha_3$ denotes the constant ratio and $N=2$ and 3 denote the two continuum bands in the regions 8.3-12.5 $\mu$m and 16.7-20.83 $\mu$m, respectively, with $\alpha_2=0.005$ and $\alpha_3=0.02$. Bignell’s values for $K_{1A}$ and $K_{2A}$ are for a temperature of 303 K. Since the temperature dependencies of $K_{1A}$ and $K_{2A}$ are not precisely known and the continuum bands are important only in the first 2 km from the ground where the temperature is close to 300 K, the temperature dependencies of $K_{1A}$ and $K_{2A}$ were neglected in computing the emissivity. The dependency of the Planck function on temperature was included. The two computed emissivities have been fitted by analytical expressions and are given by

$$E_2=[0.272+8.7610^{-4}(T-T_0)][1-\exp(-20\bar{U}_2)]$$

for the band in the region 8.3-12.5 $\mu$m and

$$E_3=[0.124+4.9210^{-4}(T_0-T)][1-\exp(-90\bar{U}_3)]$$

for the band in the region 16.7-20.83 $\mu$m, where $T_0=300$ K. Eqs. (2), (6) and (7) agree within 1% with the computed emissivities. It should be noted that Eqs. (2), (6) and (7) include the diffusivity factor. Following Rodgers (1967), the diffusivity factor is taken as 1.66. The diffusivity factor is an approximate way of accounting for the integration of the optical pathlength over all solid angles. The continuum band in the region 16.7-20.83 $\mu$m overlaps with the pure rotational band of H$_2$O and this overlap is accounted for by multiplying the emissivity $E_3$ with the average transmissivity of rotational band of H$_2$O in the region 16.7-20.83 $\mu$m. The transmissivity formulation given by Rodgers and Walsh (1966) has been adopted.

As mentioned earlier, Eq. (2) gives the emissivity for the rotation band and the vibration-rotation band. However, Staley and Jurica’s emissivity for the rotation band, which was used in deriving Eq. (2), considers only wavelengths $>15.15$ $\mu$m. Since the con-
tinum band extends up to 12.5 \textmu m, additional relations for \text{H}_2\text{O} absorption are needed for the wavelength interval 12.5–15.15 \textmu m. For this region, the band formulation given by Rodgers and Walsh (1966) has been adopted.

It would be preferable to compare the emissivity formulations derived here with the emissivity formulations given in the literature but a meaningful comparison is precluded by the fact that the previous formulations (Rodgers, 1967; Manabe and Strickler, 1964; Cess, 1974) do not include the \epsilon-type absorption of water vapor. Hence, the applicability of the present formulation to the atmosphere will be discussed later by comparing the radiative flux distribution within the atmosphere computed from the present model with the more exact calculations of Ellingson (1972).

2) Carbon Dioxide

Carbon dioxide contributes to the transmission of longwave radiation within the atmosphere in the 15 \textmu m region. As shown in Fig. 1, \text{CO}_2 has several hot bands and isotopic bands in this region. Radiative transfer due to \text{CO}_2 bands is described by a formulation similar to the total band absorptance formulation given by Cess and Ramanathan (1972). The difference between the two formulations is explained in Appendix A. Considering first the case for which the rotational lines within the band are pressure broadened, the band absorptance is given by

\[ A(U_\epsilon) = 2A_0 \ln \left( 1 + \frac{U_\epsilon}{4+U_\epsilon(1+1/\beta)} \right), \]  

where

\[ U_\epsilon = \int \frac{S}{A_0} P dz, \]

\[ \beta = \frac{4\nu_0}{U_\epsilon D} \int P dU_\epsilon, \]

where \( A_0(\text{cm}^{-1}), U_\epsilon \) and \( \beta \) are the bandwidth parameter, dimensionless optical pathlength and the linewidth-parameter, respectively, and \( \nu_0 \) (\text{cm}^{-1} \text{ atm}^{-1}), \( D \) (\text{cm}) and \( S \) (\text{cm}^{-2} \text{ atm}^{-1}) are the mean line half-width at 1 atm pressure, mean line spacing and band intensity, respectively. The partial pressure of \text{CO}_2 is denoted by \( P_\text{e} \) and \( z \) is the physical coordinate in the vertical direction. Considering first the fundamental band of \text{^{13}CO}_2, it is seen that \( U/\beta = \Omega(10^3) \). As discussed by Cess and Ramanathan (1972), \( U/\beta \gg 1 \) corresponds to the strong line limit and in this limit Eq. (8) simplifies to

\[ A = 2A_0 \ln (1 + \xi^4), \]

where \( \xi = \beta U_\epsilon \) is the strong line parameter. Following Edwards (1965), Eq. (11) can be extended to account for hot and minor isotopic bands in the 15 \textmu m region by letting

\[ A = 2A_0 \ln \left( 1 + \sum_{i=1}^{10} \xi_i^4 \right), \]

\[ \xi_i = 1.66 \left( \frac{4\nu_0}{A_0 D} \right) \int S q_i P_{i} P dz, \]

where the subscript \( i \) refers to the individual band. The fundamental band of \text{^{13}CO}_2 is denoted by \( i=1 \) and \( i=2–7 \) refers to the 6 hot bands and \( i=8–10 \) refers to the minor isotopic bands (see Table 2). The parameter \( q_i \) is the ratio of the abundance of the individual isotopes to the total abundance of \text{CO}_2. The \( q_i \)'s are obtained from Goody (1964). The values of \( D_i \) and \( S_i \) are taken from Dickinson (1972), \( A_0 \) is taken from Cess and Ramanathan (1972), and \( \nu_0 = 0.064 \text{ cm}^{-1} \text{ atm}^{-1} \). Ramanathan and Cess (1974) have applied Eq. (12) to the atmosphere of Venus which consists primarily of \text{CO}_2, and have shown that the above formulation is in excellent agreement with the more detailed line-by-line calculations of Dickinson (1972).

3) Ozone

The 9.6 \textmu m band of \text{O}_3 is also described by the band absorptance formulation given by Eq. (8). The values for the band parameters are chosen as \( \nu_0 = 0.076 \text{ cm}^{-1} \) (Goody, 1964); \( D = 0.1 \text{ cm}^{-1} \) (Rodgers, 1968); \( S = 387 \text{ cm}^{-2} \text{ atm}^{-1} \) STP (Aida, 1975); and \( A_0 = 39 \text{ cm}^{-1} \). With these values, the band absorptance given by the present formulation agrees within 5% with Walshaw’s (1957) experimental results for ozone amounts less than about 0.3 atm-cm. For ozone amounts >0.3 atm-cm, Eq. (8) yields larger values for \( A \) than Walshaw’s values with the percentage difference between Eq. (8) and Walshaw’s values becoming larger for larger values of \( A \). Based on Aida’s (1975) recent findings, the above-mentioned feature of the present formulation seems desirable. Aida’s (1975) analysis shows that due to neglect of the weaker \( v_1 \) band located in the wings of the 9.6 \mu m band (i.e., the \( v_1 \) band), Walshaw’s results underpredict the absorption of the 9.6 \textmu m band for ozone amounts >0.1 atm-cm. Based on detailed theoretical calculations Aida has derived an empirical formulation which is in good agreement with the measurements of Walshaw for ozone amounts <0.1 atm-cm. The band absorptance given by Eq. (8) is compared with Aida’s formulation in Fig. 3 for two values of pressure. The present formulation is in good agreement with Aida’s formulation except for ozone amounts between 0.2 and 0.4 atm-cm at \( P = 1 \). The present formulation is smaller by about 10% in the above range of \text{O}_3 amount. This difference could be attributed to the inadequate treatment of the \( v_1 \) band of \text{O}_3 in the present formulation. Since the combination of large amounts of

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\(^2\) For the flux calculations the hot bands have been accounted for slightly differently as discussed in Appendix A.
O₃ (i.e., greater than 0.2 atm-cm) and large pressures (∼1 atm) does not occur within the atmosphere the present formulation should be sufficient for atmospheric applications.

Consideration must be given to account for the overlap of H₂O band in the 15 μm region with CO₂ and for the overlap of the continuum band of H₂O in the 9.6 μm region with O₃. To account for this overlap, the transmissivity of H₂O in the 15 and 9.6 μm regions is multiplied by the band absorptance of CO₂ and O₃, respectively. The transmissivity of H₂O for the 15 μm region is obtained from the formulation given by Rodgers (1967). For the 9.6 μm region, the transmissivity of H₂O is obtained from Bignell’s (1970) measurements as

$$T(\tilde{U}_2) = \exp(-15U_2), \quad (14)$$

where $\tilde{U}_2$ is given by Eq. (5).

4) Doppler Broadening

Within the upper stratosphere, i.e., the region above 35 km in altitude, the pressure is so low that Doppler broadening becomes the dominant line-broadening mechanism. The procedure suggested by Ramanathan and Cess (1974) is followed for including Doppler-broadening effects. Following Ramanathan and Cess (1974), the pressure-Doppler matching of the band absorptance is performed at the altitude level ($Z$) for which

$$\frac{dA}{dZ}_{\text{Pressure}} = \frac{dA}{dZ}_{\text{Doppler}}, \quad (15)$$

where $A$ for pressure-broadened lines is given by Eq. (8) and the band absorptance formulation for Doppler-broadened lines is similar to the formulation given by Cess (1973). The band absorptance formulation for Doppler-broadened lines is given by

$$A(U) = A_0U(1 - 0.18U/\delta), \quad U/\delta \leq 1.5, \quad (16)$$

$$A(U) = 0.753A_0\delta([\ln(U/\delta)]^4 + 1.21), \quad U/\delta \geq 1.5, \quad (17)$$

where $\delta = \alpha \sqrt{4\pi n_d}/D$ with $\alpha = 2$ for bands whose alternate rotational lines are missing (most of the CO₂ bands fall in this category) and $\alpha = 1$ for all other bands, and $n_d$ is the Doppler half-width. The present formulation differs slightly from that given by Cess (1973) in the definition of $\delta$. Further discussions on the difference between the two formulations and the details of derivations of Eqs. (16) and (17) are given in Appendix B. Doppler broadening effects are considered only for CO₂ and O₃. For O₃, the mean line spacing $D$ is taken as 0.07 cm⁻¹ as opposed to the value of 0.1 cm⁻¹ assumed previously for pressure-broadened lines. The smaller value of $D$ for Doppler-broadened lines gives a better agreement with previous theoretical calculation and, in addition, seems justifiable from theoretical considerations. At the low pressures and absorber amounts characteristic of the upper stratosphere the main contribution to the cooling rates comes from the strong lines close to the band center and from Goody (1964) and Aida (1975), it is seen that $D$ is smaller near the band center for the 9.6 μm band of O₃. Since Doppler broadening becomes important only within the upper stratosphere the smaller value of $D$ seems justified. The Doppler-broadening effect for H₂O is neglected since H₂O contributes less than 10% of the total longwave cooling rates within the upper stratosphere.

It should be noted that Eqs. (8), (16) and (17) do not include the diffusivity factor in the definition of $U$. The diffusivity factor is assumed as 1.66 (Rodgers, 1967) for Eqs. (8) and (17). Since Eq. (16) corresponds to the linear limit, the diffusivity factor is taken as 2. The optical path length is multiplied by the diffusivity factor to account for the slant path within the atmosphere.

![Fig. 4. Temperature distribution used for the radiative flux distribution shown in Fig. 7.](image-url)
b. Solar absorption

Solar absorption by H$_2$O, CO$_2$ and O$_3$ contributes to the heating within the atmosphere. The various solar bands included in the present work are listed in Fig. 1. The band absorbance formulation given by Lacis and Hansen (1974) has been adopted for the solar absorption by H$_2$O and O$_3$. For the CO$_2$ bands, the formulation given by Ramanathan and Cess (1974) is used. The effect on the solar heating due to reflection by clouds and the surface and the effects due to Rayleigh scattering have been included according to the procedure suggested by Lacis and Hansen (1974).

3. Validation of the longwave radiative transfer model

The most detailed calculations currently available for the radiative flux distribution within the lower atmosphere have been performed by Ellingson (1972). Ellingson has considered all the bands listed in Fig. 1 in detail by dividing the longwave spectrum into 100 spectral steps. Similar detailed radiative heating calculations for the upper stratosphere have been reported by Dickinson (1973). Hence, in order to verify the various simplifications and approximations introduced in the present model, the model has been compared with the results of Ellingson and Dickinson.

Equations for describing radiative flux and radiative heating within the atmosphere, incorporating the emissivity and band absorbance formulations, have been given by Rodgers (1967) and Cess and Ramanathan (1972) and, hence, will not be reproduced here. Distributions of temperature, water vapor and ozone that are used in the comparison study are taken from Ellingson and are shown in Figs. 4, 5 and 6, respectively. The CO$_2$ mixing ratio is assumed to be uniform throughout the atmosphere with a value of 320 ppm by volume. In Fig. 7 we compare the distribution of upward and downward flux computed from the present model with Ellingson’s results. As shown in Fig. 7, the present model is in good agreement with Ellingson’s

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**Fig. 6.** Vertical O$_3$ distribution used for the radiative flux distribution shown in Fig. 7.

**Fig. 7.** Comparison of the results for the upward and downward flux distribution between the present model and Ellingson’s (1972) model. The flux distribution is for clear sky conditions.
energy transfer within the stratosphere and the radiative equilibrium condition also implies that the net radiative flux (i.e., the difference between the upward and downward fluxes of longwave and solar radiation) is a constant within the stratosphere; in the globally averaged case it is easily shown that this constant should be zero.

2) At the top of the atmosphere, the outgoing longwave flux is in balance with the net solar flux. The net solar flux is the difference between the downward incoming solar flux and the upward solar flux which arises due to the reflection by the atmosphere, surface and clouds.

In addition to these assumptions, the following parameters and variables are specified in the model. The thermal rate within the troposphere is specified to be (6.5 K km⁻¹) in accordance with Manabe and Strickler (1964). For H₂O, we have used the relative humidity distribution given by Manabe and Wetherald (1967), i.e.,

$$R(Z) = R_0 \left( \frac{A - 0.02}{1 - 0.02} \right)$$

where $R(Z)$ is the relative humidity, $R_0$ the surface relative humidity, and $A = P/P_0$ where $P_0$ is the surface pressure. Following Manabe and Wetherald, $R_0$ is set equal to 0.77. Since Eq. (18) predicts a negative $R$ when $P < 0.02$, following Manabe and Wetherald it is assumed that the minimum water vapor mass mixing ratio in the atmosphere is 3 ppm (parts per million), i.e., when the mass mixing ratio computed from Eq. (18) is less than 3 ppm the mass mixing ratio is set equal to 3 ppm. The mixing ratio of CO₂ is assumed to be uniform with a value of 320 ppm (by volume). The ozone density profile is taken from Krueger and Minzner (1975) and is shown in Fig. 9. The effect of clouds is incorporated by adopting the equivalent single cloud model suggested by Cess (1974). This equivalent single cloud represents a global average over many cloud layers. Adopting the three-level cloud model of Manabe and Strickler (1964), and performing the averaging procedure suggested by Cess (1974), the three-level clouds are replaced by a single cloud with a cloud top altitude of 6.25 km and a fractional cloud cover of 0.45.

![Figure 9](image)

**Fig. 9.** O₃ concentration in the unperturbed atmosphere.
Further, the cloud is assumed to be black in the long-wave spectrum. For solar radiation, the reflectivities of the cloud and the surface are assumed to be 0.52 and 0.1, respectively. The solar constant is taken to be 1360.4 W m\(^{-2}\) and for the solar zenith angle a globally averaged value of cos \(\tau\) = 0.5 is used and the fractional length of daytime is assumed to be 0.5. The above-mentioned values for the various quantities have been chosen to correspond as closely as possible to globally averaged values.

The model computes the surface temperature, tropopause height or (equivalently) the tropopause temperature, and the vertical distribution or temperature within the stratosphere. The procedure adopted for calculating the tropopause height and the stratospheric temperature distribution will be described first. The model extends from the ground to 50 km and for the numerical computations reported in this paper, the model has nine equally spaced levels 1.25 km apart between the ground and 10 km and within the region from 10 to 50 km the levels are located at an interval of 1 km.

a. Stratospheric thermal structure

The procedure adopted for computing the temperature distribution is a modified version of the procedure described in Ramanathan (1974). The present technique will be described only for CO\(_2\) since the same procedure can be extended to H\(_2\)O and O\(_3\). Adopting the band absorbance formulation given by Eq. (11), the net divergence of the radiative flux due to the fundamental band of CO\(_2\) can be written as [see Cess and Ramanathan (1972) for the derivation]

\[
\frac{dq_R(\xi)}{d\xi} = -\varepsilon_\alpha(0) \frac{dA(\xi)}{d\xi} - \int_0^\xi F(\xi, \xi')d\xi' \int_\xi^{\xi'} F(\xi', 0)d\xi',
\]

(19)

where \(F(\xi, \xi')\) and \(\xi\) are defined by

\[
F(\xi, \xi') = \frac{d\varepsilon_\alpha(\xi')}{d\xi'} \frac{dA(\xi)}{d\xi} \left( |\xi - \xi'| \right).
\]

(20)

In Eq. (19), \(q_R\) is the radiative flux and \(\xi\) is the dimensionless optical pathlength, and as defined in Eq. (20), \(\xi\) is measured from the top of the atmosphere downward and the parameters appearing in Eq. (20) have been defined earlier in the discussions following Eqs. (9), (10) and (11). The quantities \(\xi_T\) and \(\xi_0\) denote the value of \(\xi\) at the tropopause and at the surface, respectively. The Planck function evaluated at the band center is given by \(\varepsilon_\alpha\), and \(\varepsilon_\alpha(0)\) is the Planck function corresponding to the temperature at the top of the atmosphere.

The band absorbance \(A(\xi)\) is given by Eq. (11). Since the temperature lapse rate within the troposphere is prescribed, \((d\varepsilon_\alpha/d\xi')\) is a known function within the region \(\xi_T \leq \xi \leq \xi_0\). However, it should be noted that \(\xi_T\) is as yet unknown since the tropopause height is yet to be calculated. The procedure for calculating the tropopause height will be explained later. Assuming for the time being that \(\xi_T\) is known, it is seen that the last integral on the right-hand side (rhs) of Eq. (19) is a known function of \(\xi\). To distinguish this integral from the first two rhs integrals in which the integrand is as yet an unknown function of \(\xi\), the three integrals are written explicitly.

Eq. (19) is an integral equation whose solution is computationally time consuming and tedious. Ramanathan (1974) simplified Eq. (19) by replacing this equation with an algebraic equation given by

\[
\frac{dq_R(\xi)}{d\xi} = -\varepsilon_\alpha(\xi) \frac{dA(\xi)}{d\xi} - \int_\xi^{\xi_T} F(\xi, \xi')d\xi'.
\]

(21)

The first three rhs terms in Eq. (19) are replaced by the first rhs term in Eq. (21). Since the integrand on the right-hand side of Eq. (21) is a known function of \(\xi\), Eq. (21) is an algebraic equation. The first rhs term of Eq. (21) is usually referred to as the cool-to-space term (Rogers and Walshaw, 1966). As discussed by Ramanathan (1974), a combination of two conditions within the stratosphere permits the simplification accomplished in Eq. (21). First, within the upper stratosphere, i.e., the region above 30 km in altitude, \(\xi < 1\) and the small value of \(\xi\) implies that the individual rotational lines within the band are non-overlapping. Cess and Ramanathan (1972) and Ramanathan and Cess (1974) have shown that in the limit of non-overlapping lines the integral equation describing the divergence of the radiative flux vector reduces to an algebraic equation similar to Eq. (21). Second, the temperature lapse rate within the lower stratosphere is less than 1 K km\(^{-1}\). Due to the small temperature lapse rate, the contribution of the surrounding regions within the lower stratosphere to the net radiative cooling at any particular level within the lower stratosphere will be negligible when compared to the local cooling-to-space given by the first term in Eq. (21). The second condition imposes a severe restriction on the applicability of Eq. (21) since it requires the temperature lapse rate within the lower stratosphere to be small. Hence, we consider here a more general form of Eq. (21) which at the same time retains the algebraic nature of Eq. (21). The equation for \(dq_R/d\xi\) considered in the present work is

\[
\frac{dq_R(\xi)}{d\xi} = -\varepsilon_\alpha(\xi) \frac{dA(\xi)}{d\xi} \int_\xi^{\xi_T} F(\xi, \xi')d\xi' - \int_\xi^{\xi_T} F(\xi, \xi')d\xi'.
\]

(22)
The first two rhs terms in Eq. (19) are replaced by the first rhs term in Eq. (22). The second rhs term in Eq. (22) represents the exchange of energy between the level \( \xi \) under consideration and the region of the stratosphere below the level \( \xi \). In Fig. 10 the cooling rates computed from Eqs. (21) and (22) are compared with the cooling rates computed from the exact formulation given by Eq. (19). It is seen that Eq. (22) is in better agreement with Eq. (19). The only term neglected in Eq. (22) is the exchange of energy between the level \( \xi \) and the region of the stratosphere above the level \( \xi \). Since the absorber amount and the atmospheric pressure decrease exponentially with altitude the exchange of energy between the level \( \xi \) and the upper region is relatively negligible when compared with the other terms retained in Eq. (22).

The net radiative heating (\( Q \)) within the stratosphere is given by

\[
Q = -\frac{1}{\rho c_p} \frac{dq_R}{dz} = -\frac{1}{\rho c_p} \left[ -\left( \frac{dU}{dz} \right)_{CO_2} - \left( \frac{dU}{dz} \right)_{O_3} + \left( \frac{dU}{dz} \right)_{H_2O} + S \right],
\]

where \( \rho \) is the density, \( c_p \) the specific heat at constant pressure, and \( Z \) altitude. The first three terms within the large parentheses denote longwave cooling due to \( CO_2 \), \( O_3 \), and \( H_2O \). The quantities \( U \) and \( U_1 \) are defined respectively in Eqs. (3) and (9). The equations for \( dq_R/dU \) and \( dq_R/dU_1 \) are similar to Eq. (22). The contribution to the stratospheric longwave cooling rates from the \( H_2O \) continuum bands is negligible and, hence, the \( H_2O \) continuum bands are neglected in Eq. (23).

The solar heating due to \( CO_2 \), \( H_2O \) and \( O_3 \) is denoted by \( S \).

The temperature distribution within the stratosphere is obtained by letting \( Q = 0 \) in Eq. (23) since \( Q = 0 \) for radiative equilibrium. Eq. (23) can be treated as an algebraic equation [i.e., Eq. (23) can be solved separately for each altitude level within the stratosphere] provided it is solved first for the lowest altitude level within the stratosphere and then solved for the next higher altitude level and so on. This conclusion follows from an inspection of Eq. (22). The integrand within the two integrals in Eq. (22) is only a function of the temperature distribution within the region of the atmosphere whose altitude levels are lower than the altitude level for which \( dq_R/d\xi \) is evaluated. Consequently, Eq. (23) reduces to an algebraic equation if the procedure suggested above for solving the temperature distribution is followed. The algebraic nature of Eq. (23) permits a significant reduction in the computational time required to solve for the temperature distribution.

The stratospheric temperatures are computed by a Newton-Raphson iteration technique which is given by

\[
T_{n+1}(Z) = T_n(Z) - \frac{Q(Z)}{[dQ(Z)/dT]},
\]

In Eq. (24), \( T_0(Z) \) is the value of the temperature obtained from the previous iteration (or the initial guess value for the first iteration), \( Q(Z) \) is the value of radiative heating computed from Eq. (23) corresponding to the temperature \( T_0(Z) \), and \( dQ(Z)/dT \) is evaluated numerically by computing \( Q \) at a temperature \( T_0(Z) + \Delta T \), and letting \( dQ(Z)/dT = \Delta Q/\Delta T \) where

\[
\Delta Q = Q[Z, T = T_0(Z) + \Delta T] - Q[Z, T = T_0(Z)].
\]

For the calculations, \( \Delta T \) is chosen as 0.5 K. This value of \( \Delta T \) is not very critical since calculations performed with \( \Delta T \) varying from 0.1 to 1 K yielded about the same value for \( dQ/dT \). The temperature computed from the iteration is given by \( T_n(Z) \) and the above procedure is repeated until the temperature difference between two successive iterations is less than 0.1 K. Convergence is usually obtained in about three iterations are less.

As mentioned earlier, the tropopause height is as yet unknown. The following procedure is adopted for calculating the tropopause height. Initially, a value is assumed for the tropopause height and the temperature at an altitude which is one level above the level of the assumed tropopause is computed by solving Eq. (23). The temperature lapse rate in this layer, i.e., the layer between the assumed tropopause level and the level above this tropopause level, is computed next. If this lapse rate is supercritical or critical, i.e., if \( [dT/dz] + 6.5 \leq 0 \), where 6.5 K km\(^{-1}\), is the assumed lapse rate within the troposphere, the tropopause level is increased by one level and the procedure repeated.
until the lapse rate in the layer just above the tropopause satisfies the condition \( \left[ (dT/dZ) + 6.5 \right] \geq 0 \). If, on the other hand, the initially assumed value of the tropopause level is such that \( \left[ (dT/dZ) + 6.5 \right] > 0 \) above this level, then the tropopause level is reduced by one level and the procedure repeated until the lapse rate in a layer becomes critical or supercritical and the tropopause is taken as the level overlying this layer since above this level the atmosphere is subcritical and below this level the atmosphere is critical or supercritical. The latter possibility did not arise in the present calculations since the initial guess value of the tropopause is chosen to be 7.5 km which is well below the expected value of the tropopause height.

To summarize the computational procedure, the tropopause height is computed first and the stratospheric temperatures are computed next beginning from the level just above the computed tropopause.

The stratospheric thermal structure for globally averaged conditions computed from Eq. (23) is compared with Manabe and Wetherald's (1967) theoretical results and with the 1962 U. S. Standard Atmosphere in Fig. 11. The two theoretical models are in good agreement. The present model is in excellent agreement with the U. S. Standard Atmosphere above 30 km which indicates the validity of the radiative equilibrium assumption for the region above 35 km.

The present model requires approximately 1 s in a CDC 6600 computer to compute the thermal structure of the stratosphere. In view of the fact that the present model considers all the important physics of the radiative transfer problem within the atmosphere, it can be concluded that the present model is extremely efficient in terms of the computational time.

b. Diurnal calculations

Theoretical analyses of diurnal temperature distribution within the stratosphere are scarce. Leovy (1964) has presented results for diurnal temperature changes which assume radiative equilibrium, but Leovy has not included CO₂ hot and isotopic bands. Since these weak bands contribute significantly to the cooling within the upper stratosphere, the diurnal temperature distribution within the stratosphere was calculated including these bands. Results for the diurnal temperature difference obtained from the present model is compared with Leovy's (1964) results in Fig. 12. Leovy's results agree well with the present model below 40 km but above 40 km the present model predicts a larger diurnal temperature difference. The diurnal temperature difference at 50 km predicted by the present model is higher by about 50% than that of Leovy's analysis. This difference can be attributed to the inclusion of several CO₂ hot and isotopic bands and Doppler-broadening effects in the present model.

c. Global energy balance studies

Results for the longwave flux and the surface temperature computed from the present model are presented in this section to indicate the degree of similarity

![Fig. 11. Comparison of vertical thermal structures for the globally averaged radiative-convective equilibrium condition.](image)

![Fig. 12. Comparison of the maximum diurnal temperature differences. The solar zenith angle corresponds to 30°N, equinox.](image)
between the model and the observed globally averaged atmospheric conditions.

1) Outgoing longwave flux

As suggested by Schneider (1972), the similarities between the model and atmosphere are examined by comparing the outgoing longwave flux $F$ and the dependence of $F$ on the surface temperature $T_s$, with the observed values. In addition, results will be presented to indicate the individual contribution to the outgoing flux due to the minor infrared bands. Some of these minor bands, especially CO$_2$ hot bands and H$_2$O e-type absorption bands, are usually neglected in global energy balance models; hence, it would be useful to estimate the error involved in neglecting these bands.

Since the purpose here is to compare the outgoing flux for globally averaged conditions, the temperature distribution has also been specified in addition to the parameters previously mentioned. The vertical temperature profile is given by the 1962 U. S. Standard Atmosphere. As shown by Cess (1974), the outgoing longwave flux $F$ may be expressed as

$$F = C_1 - C_2 A_e,$$  \hspace{1cm} (25)

where $A_e$ is the fractional cloud cover and $C_1$ is the clear sky flux. The quantities $C_1$ and $C_2$ are functions of the vertical distribution of temperature and the vertical distribution of H$_2$O, CO$_2$ and O$_3$. In addition, $C_2$ is also a function of the cloud top altitude. Unless otherwise mentioned, the cloud top height has been assumed to be 6.25 km. The outgoing flux obtained from the present model is

$$F = 259.42 - A_e 71.39,$$  \hspace{1cm} (26)

where $F$ is in W m$^{-2}$. The numerical values given in Eq. (26) is compared with Budyko's (1969) empirical formulation given by

$$F = 222.5 + 2.23 T_s - A_e (47.419 + 1.604 T_s),$$  \hspace{1cm} (27)

where $T_s$ is the surface temperature ($^\circ$C). Budyko's formulation is based upon monthly mean data from 260 meteorological stations. Upon substituting $T_s=15^\circ$C in Eq. (27) it is seen that the value for $C_1$ and $C_2$ given by Eqs. (26) and (27) agree within 1.5%. It should be noted that the agreement between the two equations in the value for $C_2$ has been forced. As mentioned earlier, $C_2$ is a function of the cloud top altitude and the value of 6.25 km for the cloud top altitude has been chosen so that $C_2$ predicted by the model agrees with Budyko's $C_1$ value. However, since $C_1$ is unaffected by the choice of the cloud top altitude, the above exercise of comparing the two equations is indicative of the degree of agreement between the two equations. Detailed discussions on the dependency of $C_2$ on cloud top altitude are given in Cess (1974).

To assess the importance of the infrared bands of CO$_2$ and O$_3$ and the e-type absorption of H$_2$O, flux calculations were performed after neglecting each of these bands separately. Results are listed in Table 1. It is seen that CO$_2$ contributes about 11% to the clear sky flux while O$_3$ contributes 3% to the clear sky flux. The table indicates that even the relatively minor bands like the CO$_2$ hot bands and the e-type absorption by the H$_2$O continuum band have to be included in order to estimate $F$ within 1% accuracy.

As a second test of the present radiation model, we compare the dependence of $F$ on surface temperature given by the quantity $dF/dT_s$. As suggested by Schneider (1972), $dF/dT_s$ for the real atmospheric conditions is obtained from Budyko's formulation given by Eq. (27). On differentiating Eq. (27) with respect to $T_s$ we have

$$\frac{dF}{dT_s} = 2.23 - 1.604 A_e.$$  \hspace{1cm} (28)

For the present model, $dF/dT_s$ is obtained numerically after holding the stratospheric temperature fixed and uniformly changing the surface temperature and tropospheric temperature. In addition, as suggested by Cess (1974), the cloud top temperature is held fixed while changing the tropospheric temperature. The present model yields

$$\frac{dF}{dT_s} = 2.162 - 1.75 A_e.$$  \hspace{1cm} (29)

As indicated by Eqs. (28) and (29), the model and Budyko's formulation are again in good agreement. It should be noted that the cloud correction term in Eq. (28) is not very sensitive to the cloud top altitude and, hence, the agreement in this term is not due to the choice of the cloud top altitude.
The e-type absorption by the H₂O continuum band has an appreciable effect on \( dc_c/dT_s \), where \( dc_c/dT_s \) is the sensitivity of the clear sky flux to the surface temperature. The effect of H₂O e-type absorption on \( dc_c/dT_s \) is shown in Fig. 13. In Fig. 13, \( dc_c/dT_s \) is shown as a function of \( T_s \). The curve marked “Complete Model” includes all the infrared bands shown in Fig. 1. The curve marked “Without H₂O e-type Absorption” includes all the infrared bands except the e-type absorption by the H₂O continuum band. As seen from Fig. 13, there is a substantial difference between the two curves, particularly for values of \( T_s \geq 280 \) K.

As seen from Eqs. (5), (6) and (7), the e-type absorption increases as the square of the partial pressure of H₂O. Since the H₂O relative humidity is fixed, the H₂O partial pressure (e) increases with \( T_s \). Consequently, the e-type absorption increases rapidly with \( T_s \) and the two curves thus differ substantially at larger \( T_s \). Further, since the e-type absorption increases with \( T_s \), \( dc_c/dT_s \) is smaller for the curves marked “Complete Model”. Fig. 3 also indicates the nonlinear relationship between \( C_1 \) and \( T_s \) as opposed to Budyko’s (1969) and Cess’s (1974) formulations which imply a linear relationship between \( C_1 \) and \( T_s \).

The range of \( T_s \) covered in Fig. 13 is representative of the latitudinal variation of \( T_s \), i.e., \( T_s = 250, 288 \) and 310 K corresponds respectively to the polar, mid-latitude and equatorial surface temperatures. Hence, the results shown in Fig. 13 can be interpreted as indicating the sensitivity of longwave flux to \( T_s \) as a function of latitude. It then follows that the predictions by global climatic models that neglect the e-type absorption could be substantially different (at least at the lower latitudes) if these models were to include the e-type absorption.

2) Surface temperature

The procedure for calculating the surface temperature \( T_s \) is straightforward and involves the following steps: (i) assume a surface temperature; (ii) calculate the tropopause and stratospheric temperatures by the procedure explained previously; (iii) calculate the net radiative flux \( F^a \) at the top of the atmosphere defined by \( F^a = F - F^s \), where \( F \) is the outgoing longwave flux and \( F^s \) the net solar flux \( (F^s = F - F^s) \) at the top of the atmosphere; and (iv) calculate a new surface temperature by the Newton-Raphson interaction technique given by

\[
T_s(\text{New}) = T_s(\text{Old}) - \frac{F^a}{(dF/dT_s)}.
\]  

Since, for globally averaged conditions, \( F^a = 0 \), steps (ii)–(iv) are repeated until the condition \( F^a = 0 \) is satisfied. An accurate calculation of \( F^a \) should consider Rayleigh scattering effects, scattering by clouds and the lower surface, and multiple reflection between clouds and the surface. Since the main purpose of this work is to illustrate the applicability of the present longwave model for climatic studies, in the surface temperature calculations reported here the albedo of the earth-atmosphere system has been specified such that \( F^a \) can be written as

\[
F^a = \frac{1360.4}{4} (1 - a),
\]

where the factor 1360.4 on the right-hand side is the solar constant and the factor 4 in the denominator accounts for the product of the fractional day of sunshine and the cosine of mean zenith angle. The albedo \( a \) has been assumed to be 0.31 and this value is in good agreement with the measured value of the global albedo (Vonder Haar and Suomi, 1971). The computed surface temperature depends on the assumed values of cloud top height, surface relative humidity and the lapse rate. This dependency is illustrated in Table 2. It is not clear which of the values indicated in Table 2 are more appropriate for global climate models. For example, Schneider (1972) uses a value of 5.5 km for the cloud top altitude while Cess (1974) suggests a value of 6.25 km. Global measurements of these parameters have not been made to the desired accuracy.

Results such as the ones presented in Table 2 should serve as a useful guideline in the design of global climatic measurement programs. For example, if the

<table>
<thead>
<tr>
<th>Cloud top altitude (km)</th>
<th>Tropospheric lapse rate (K km⁻¹)</th>
<th>Surface relative humidity</th>
<th>Computed surface temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>5.5</td>
<td>6.5</td>
<td>0.77</td>
<td>289.6</td>
</tr>
<tr>
<td>6.0</td>
<td>6.5</td>
<td>0.77</td>
<td>290.9</td>
</tr>
<tr>
<td>6.25</td>
<td>6.5</td>
<td>0.77</td>
<td>291.5</td>
</tr>
<tr>
<td>5.5</td>
<td>6.5</td>
<td>0.71</td>
<td>289.1</td>
</tr>
<tr>
<td>5.5</td>
<td>6.0</td>
<td>0.83</td>
<td>290.1</td>
</tr>
<tr>
<td>5.5</td>
<td>7.0</td>
<td>0.77</td>
<td>288.0</td>
</tr>
</tbody>
</table>

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purpose of a measurement program is to monitor small variations in surface temperature of the order of 1 K or less and to identify the causal mechanisms for such variations, then it is clear from Table 2 that simultaneous measurements of the three parameters indicated in Table 2 are essential. Further, Table 2 gives an approximate estimate of the accuracy with which these measurements have to be made for a desired accuracy in the measurement of the surface temperature variations. It should be noted that the parameters considered in Table 2 are only a small fraction of the number of the possible atmospheric parameters that can affect the surface temperature. It is cautioned that, due to the approximations made in a radiative-convective model, surface temperature sensitivity indicated by the radiative-convective model should be considered as indicative of the actual sensitivity rather than as a prediction of the sensitivity of the actual earth-atmosphere system.

5. Conclusions

A simple formulation for treating radiative energy transfer within the lower atmosphere is presented. In addition to including the major bands of H$_2$O, CO$_2$ and O$_3$, the model includes the two continuum bands of H$_2$O (including the e-type absorption), the hot and isotopic bands of CO$_2$, and Doppler-broadening effects. When compared with more detailed theoretical models the simple model shows that the simplification has been achieved without a significant loss of accuracy, and the simplification is also significant in terms of computing time. The computer time required by the present model (\(\sim 1\) s in a CDC 6600 computer to compute the thermal structure within the stratosphere) should be smaller at least by an order of magnitude than that required by more detailed models.

The detailed models referred to above are those of Ellingson (1972) and Dickinson (1973). Ellingson divides the longwave spectrum into 100 spectral steps while the procedure adopted in the present model is equivalent to a model with six spectral steps (one step for each CO$_2$ and O$_3$ and four for H$_2$O); hence, based on this comparison alone the present model should require an order of magnitude less computer time. A direct comparison between the present model and Dickinson’s model is difficult since Dickinson’s model does not include H$_2$O and is only applicable within the stratosphere. Dickinson includes all of the bands of CO$_2$ and O$_3$ included in the present analysis but Dickinson’s procedure is much more detailed. Dickinson combines several CO$_2$ lines into a group and considers several such groups (probably more than ten) while the present model employs one band model for CO$_2$; yet, as illustrated previously, the stratospheric cooling rates computed from the present model are in excellent agreement with Dickinson’s calculations, and the agreement is particularly significant within the upper stratosphere where the cooling due to hot and isotopic bands and Doppler-broadening effects become appreciable.

It is of interest to compare the efficiency and accuracy of the present model with that of Manabe and Wetherald’s (1967) model. Manabe and Wetherald also employ simplified techniques (but different from the present techniques) to treat the radiative transfer problem. To obtain the temperature distribution within the atmosphere, Manabe and Wetherald’s formulation would require the solution of an integral equation which is tedious and computationally time-consuming when compared with the solution of an algebraic equation as employed in the present formulation. This advantage may not be significant when the radiative transfer model is used in a general circulation model to compute the longwave cooling rates. But, since Manabe and Wetherald’s model does not consider Doppler-broadening effects and does not include an explicit treatment of CO$_2$ hot bands, it is concluded that the present model should yield more accurate longwave cooling rates in the region of the stratosphere above 25 km thus enabling a better treatment of the interaction between radiation and dynamics within the stratosphere. The preceding statement is not intended as a criticism of the excellent work reported in Manabe and Wetherald but is mentioned mainly to indicate the difference between the two models in the treatment of the radiative transfer within the stratosphere.

To summarize the preceding discussions, the salient features of the present model are accuracy and negligible computer time for temperature and cooling rate calculations. In view of these two features the present model should greatly facilitate the inclusion of radiative transfer processes in global climatic models and in interactive atmospheric models that treat the dynamical, radiative transfer and photochemical processes simultaneously.

It is shown that minor absorption bands like the hot and minor isotopic bands of CO$_2$ and the e-type absorption by H$_2$O continuum band must be included in the model to estimate the outgoing flux $F$ within 1%. The global energy balance studies indicate that the sensitivity parameter $dF/dT_s$ should be compared for validating the longwave radiative transfer model used in global climatic studies. The e-type absorption by the continuum band is shown to have an appreciable effect on the sensitivity of the model at equatorial latitudes.

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several useful suggestions which have contributed to the clarity of the presentation.

APPENDIX A

Band Model for Partially Overlapping CO₂ Bands

The comparison of the atmospheric flux distribution computed from the present model with Ellingson’s (1972) results indicated that the present model underpredicts the downward flux by about 30% in the lower stratosphere (the region between 15 and 25 km). Since CO₂ contributes more than 70% to the downward flux in this region, the above-mentioned discrepancy is due to the difference in the CO₂ fluxes. After a detailed examination it is concluded that the procedure given by Eq. (12) to account for the hot and minor isotopic CO₂ bands underestimates the band absorptance in the lower stratosphere. It is implicitly assumed in Eq. (12) that all the CO₂ bands completely overlap each other. However, as indicated in Table 2, the band centers of some of the bands are separated by more than 50 cm⁻¹ and this reveals the varying degree of overlap between the bands. We will now illustrate that Eq. (12) is a good approximation to the total band absorptance within the upper stratosphere and troposphere while underpredicting the band absorptance within the lower stratosphere. Within the upper stratosphere $\xi_i \ll 1$, such that the total band absorptance $A$ can be written as

$$A = 2A_0 \sum_{i=1}^{10} \xi_i^4, \quad \xi_i \ll 1. \quad (A1)$$

It can be seen that Eq. (12) reduces to this limit. Within the troposphere $\xi_i > 1$ and it is reasonable to expect that all the bands completely overlap when $\xi_i > 1$; hence Eq. (12) which assumes completely overlap should be a good approximation. Within the lower stratosphere $\xi = 10$ and hence bands $B_2$ and $B_3$, whose band centers are separated by more than 50 cm⁻¹ from the rest of the bands in the 15 $\mu$m region, would not be completely overlapped by rest of the bands listed in Table 3.

We propose here an approximate formulation that accounts for the partially overlapping bands. It will be shown shortly that the present approximate formulation is in excellent agreement with experimental measurements. This will be the sole justification for the proposed formulation. To account for the overlap between the bands, we make use of the multiplicative property of the transmissivity of overlapping bands (Goody, 1964) and let

$$A = 2A_0 \left[\ln(1+\xi_4^4+\sum_{i=1}^{10} \xi_i^4) + T_2 \ln(1+\xi_4^4) + \frac{T_3 \ln(1+\xi_4^4)}{1+\xi_4^4}\right], \quad (A2)$$

where $T_2$ and $T_3$ are respectively the average transmissivity of the fundamental band ($B_4$) centered at 667 cm⁻¹ over the spectral regions of bands $B_2$ and $B_3$. Bands $B_4-B_9$ are assumed to completely overlap with band $B_1$ and this can be justified from Table 2 which shows that the band centers of these bands nearly coincide. Following Goody (1964) the average transmissivity of a band is related to the band absorptance by

$$\tilde{T} = \exp(-\tilde{A}), \quad (A3)$$

where $\tilde{T}$ is the transmissivity and $\tilde{A}$ the average band absorptance over the spectral region under consideration; in the present nomenclature $\tilde{A} = A/2A_0$. The average band absorptance $\tilde{A}$ can be obtained from Eq. (11) provided an appropriate value is chosen for the band intensity $S$. For $B_3$, the appropriate value of $S$ is the effective intensity of $B_1$ centered at $\omega_1 = 667$ cm⁻¹ in the spectral region of band $B_3$ centered at $\omega_2 = 720$ cm⁻¹ with an effective band width of $2A_0$. The effective intensity $S_{\text{eff}}$ is obtained from Edwards and Menard's (1964) line intensity distribution given by

$$S_\omega = \frac{S}{2A_0} \exp(-|\omega - \omega_1|/A_0), \quad (A4)$$

where $S_\omega$ is the line intensity distribution. To obtain $S_{\text{eff}}$ from Eq. (A4), we let $S_{\text{eff}} = S_\omega \Delta \omega$, where $\Delta \omega$ is the effective band width of $B_2$ and $S_\omega$ the mean line intensity of $B_2$ over the band $B_3$ centered at $\omega_2$. The mean line intensity $S_\omega$ is obtained by letting $\omega = \omega_2$ in Eq. (A4) and by letting $\Delta \omega = 2A_0$. Upon making use of these approximations we have

$$S_{\text{eff}} = S_1 \exp(-|\omega - \omega_2|/A_0), \quad (A5)$$

where $S_1$ is the band intensity of $B_1$. Combining Eqs. (A5), (A4) and (A3) yields

$$\tilde{T}_2 = \frac{1}{1+\xi_4^4} \quad (A6)$$

where $\xi_4$ is obtained from Eq. (13) after replacing $S$ by $S_{\text{eff}}$ given by (A5). Further, since $|\omega_2 - \omega_1| \approx |\omega_2 - \omega_1|$, we can let $\tilde{T}_2 = \tilde{T}_3$. The band absorptance computed
Table 4. Comparison of total band absorptance of CO\textsubscript{2} in the 15 \mu m region.*

<table>
<thead>
<tr>
<th>Amount of CO\textsubscript{2} (g m\textsuperscript{-2})</th>
<th>Total pressure (atm)</th>
<th>A[Exp. (cm\textsuperscript{-1})]</th>
<th>A[Eq. (A2)] (cm\textsuperscript{-1})</th>
<th>A[Eq. (12)] (cm\textsuperscript{-1})</th>
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<tr>
<td>24 500</td>
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<td>220</td>
<td>219</td>
</tr>
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<td>11 800</td>
<td>6.5</td>
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</tr>
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</tr>
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<td>183</td>
<td>190</td>
<td>184</td>
</tr>
<tr>
<td>2 440</td>
<td>10.3</td>
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<td>184</td>
<td>180</td>
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* The total pressure \( P \) is defined as \( P = P_{\text{mix}} + 1.3 P_e \), where \( P_{\text{mix}} \) is the partial pressure of the mixture of gases other than CO\textsubscript{2} and \( P_e \) is the partial pressure of CO\textsubscript{2}; the factor 1.3 is the self-broadening coefficient. Edward's (1965) results are indicated by \( A[\text{Exp.}] \).

The results from (12) and from (A2) are compared with experimental measurements in Table 4. The experimental results are indicated by \( A[\text{Exp.}] \) and are taken from Edwards (1965). It is seen that (A2) is in excellent agreement with the experimental results. It is further seen that for large absorber amounts (\( > 3000 \) g m\textsuperscript{-2}) and large pressures (\( > 2 \) atm) both (12) and (B2) are in good agreement with the experimental results, while for intermediate path lengths (i.e., between 2000 and 100 g m\textsuperscript{-2}) and intermediate pressures (i.e., between 0.5 and 2 atm), Eq. (A2) is in much better agreement with the experimental results. This verifies our earlier conclusion that Eq. (12) is a good approximation for \( A \) when \( \xi > 1 \) while underpredicting \( A \) for intermediate values of \( \xi \).

The results of the flux calculations shown in Fig. 4 were computed using (A2) which again indicate the excellent agreement between Eq. (A2) and the detailed theoretical calculations of Ellingson (1972). Since the lower stratosphere heating rates computed from (12) and (A2) differ only by about 10\%, the simpler formulation given by Eq. (12) is adopted for the calculations involving stratospheric structure.

APPENDIX B

Band Model for Doppler-Broadened Lines

As discussed by Cess (1974), Doppler broadening of rotational lines within a vibration-rotation band becomes important only at extremely low atmospheric pressures. For sufficiently low pressures the individual lines can be assumed to be nonoverlapping. For nonoverlapping lines, the total band absorptance can be expressed as (Cess, 1973)

\[
A = 2 \sum_{j=0}^{w} A_j, \tag{B1}
\]

where \( j \) denotes the rotational quantum number and \( A_j \) is the absorptance of a single line. First, we will reproduce the derivation given by Cess (1973) and then derive the modified formulation given in Eq. (16). To obtain a closed form expression for \( A \), Cess (1974) prescribes

\[
A_j = \pi \nu_B (W - \ln 2W^2), \quad W \leq 1.5, \tag{B2}
\]

\[
A_j = 2 \nu_B \ln W^4, \quad W > 1.5, \tag{B3}
\]

\[
S_j = \frac{PH}{D}, \tag{B4}
\]

\[
W = \frac{SD}{A_0}, \exp(-2jD/A_0), \tag{B5}
\]

where \( A_j \) is the expression for the Doppler line absorptance, \( S_j \) the line intensity distribution, \( S \) the total band intensity, \( D \) the line spacing, \( \gamma_B \) the Doppler line half-width, \( A_0 \) the bandwidth parameter, \( PH \) the pressure path length, and \( w \) the mean optical path length of the line. The summation in (B1) is replaced by

\[
A = 2 \sum_{j=0}^{w} A_j = 2 \int_{0}^{w} A \rho \rho \tag{B6}
\]

such that by combining (B2)--(B6) Cess obtains

\[
A = A_0 U (1 - 0.18 U/\delta), \quad U/\delta \leq 1.5, \tag{B7}
\]

\[
A = 0.753 A_0 U \{ \ln (U/\delta) \}^{1 + 1.21}, \quad U/\delta \geq 1.5, \tag{B8}
\]

where

\[
\delta = \pi \nu_B / D, \quad U = SPH/A_0 \tag{B9}
\]

The mean line spacing \( D \) is usually set equal to 2\( B \) (Cess and Tiwari, 1972) where \( B \) is the rotational line constant. However, for CO\textsubscript{2}, alternate lines are missing (Goody, 1964) such that Cess lets \( D = 4B \) in (B7)--(B9) to account for the missing lines. Upon comparing the CO\textsubscript{2} cooling rates computed from (B7) and (B8) with those of Dickinson (1973) it is seen that (B8) underpredicts the cooling rates within the upper stratosphere by as much as 50\% in some instances. In addition, the pressure level at which the transition occurs from pressure to Doppler broadening is considerably smaller for (B8). We have tried to remedy this discrepancy by adjusting the parameters in (B7)--(B9). The only parameter available for this purpose is \( \delta \) since \( S \) and \( A_0 \) are determined from the band absorptance formulation.
for pressure-broadened lines. It is seen that the best agreement with Dickinson’s (1973) results are obtained if δ is redefined as

$$\delta = 2 \sqrt{\nu v_p} / D. \quad (B10)$$

On comparing (B9) and (B10), it is seen that δ given by (B10) is twice that of (B9). There seems to be some theoretical justification for (B10). In Eq. (B6) Cess has implicitly assumed that all the rotational lines are present. For CO₂, since alternate lines are missing, Eq. (B6) must read

$$A = 2 \sum_{j=0}^{\infty} A_j, \quad A_1 = A_3 = A_5 = \ldots = 0, \quad (B11)$$

where j = 0, 2 implies summation over alternate lines. The factor 2 outside the summation sign accounts for the symmetric P and R branches.

Assuming the P and R branches to be symmetric about the band center, Eq. (B11) can be written as

$$A = 2 \sum_{j=0}^{\infty} A_j = 2 \int_0^{\infty} A \rho d \lambda. \quad (B12)$$

To be consistent with Eq. (B12), Eq. (B5) should be modified to read

$$S_j = S_0 \left( \frac{A \rho d \lambda}{A_0} \right), \quad (B13)$$

such that

$$\int_0^{\infty} S \rho d \lambda = S. \quad (B14)$$

It should be noted that in deriving (B12) we have implicitly replaced the P and R branches by one branch that has no missing lines, such that D = 2B in Eq. (B13). Combining (B12), (B13), (B2) and (B3), yields

$$A = A_0 \left( 1 - 0.18 U / \delta \right), \quad U / \delta \leq 1.5, \quad (B15)$$

$$A = 0.753 A_0 \left[ \ln \left( U / \delta \right) \right]^{1.21}, \quad U / \delta \geq 1.5, \quad (B16)$$

where δ = \sqrt{\pi} \nu D / 2B. It is seen that (B15)–(B17) are similar to (B7)–(B9) except for the fact that δ = 2B in Eq. (B17) whereas δ = 4B in Eq. (B9). To retain the formulation given by Cess (1973), Eq. (B17) can be written as

$$\delta = \frac{\sqrt{\pi} \nu D}{2B} = \frac{2 \sqrt{\pi} v_p}{4B} = \frac{2 \sqrt{\pi} v_p}{D}, \quad (B18)$$

where D = 4B.

In conclusion, it is suggested that the parameter δ, given by Cess, be multiplied by a factor 2 when applying Cess’s Doppler band absorbance formulation for bands whose alternate rotational lines are missing.

REFERENCES


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