A Numerical Investigation of the Effect of Electric Charges and Vertical External Electric Fields on the Collision Efficiency of Cloud Drops

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ABSTRACT

The aerodynamic interaction between electrically charged cloud drops in the presence of vertical external electric fields was numerically investigated for 800 mb and +10°C. The collector drops had radii between 11.4 and 74.3 μm while the collected drops had radii between 1 and 66 μm. The external electric fields considered ranged between 0 and 3429 V cm^{-1} ($=3.429\times10^5$ V m^{-1}), and the electric charge on the cloud drops ranged between 0 and 1.1×10^{-4} esu ($=3.7\times10^{-14}$ C). The results demonstrate that the presence of electric charges and fields of magnitudes observed during thunderstorm and pre-thunderstorm conditions drastically enhance the collision efficiency of cloud drops. The enhancement was found to be most pronounced for the smallest collector drops studied.

1. Introduction

It is now well established that some convective clouds become weakly electrified in their early stages of development, the electrification increasing progressively as the clouds reach their mature stage. An electrified cloud implies that the cloud particles carry an electric charge and are exposed to an external electric field. Electric field strength in mature convective thunderstorm clouds may be as high as several kilovolts per centimeter (Mason, 1971). Takahashi (1972, 1973) summarized observations of his own and others on the electric charge typically carried by cloud drops, drizzle drops and raindrops of varying size. One notices from Fig. 7 of his summary that the mean drop charge for thunderstorms can be expressed as $Q_{A,\mathrm{Mean}}\approx 2A^2$, where $Q_{A,\mathrm{Mean}}$ is expressed in esu and A in cm (Grover and Beard, 1975).[1] It is certainly physically reasonable to find that $Q_{A,\mathrm{Mean}}\propto A^2$, since a cloud drop can be considered to behave electrically as a conducting sphere which in the presence of a sufficiently large electric field acquires most of its charge through conduction charging. The constant of proportionality will depend on the maturity of the convective cloud system. Thus observations suggest that in the early stages of cloud development processes which control the growth of cloud particles may be affected by the electric state of the cloud system in which the growth takes place.

At temperatures at which clouds are still not yet glaciated, cloud drops grow predominantly by the collision-coalescence process. Intuitive physical reasoning tells us that electric charges and fields will enhance the collision-coalescence growth. Unfortunately, only a few studies have been reported which elaborate on this intuition in a quantitative manner. Sartor and Miller (1965) and Davis (1965) studied the interaction between electrically charged cloud drops in an external electric field in Stokes flow. The force between two charged water drops was approximated by the force between two charged, spherical electric conductors as determined by Davis (1962, 1964a, b). Lindblad and Semonin (1963), Plumlee and Semonin (1965) and Semonin and Plumlee (1966) also used the electrostatic force model of Davis (1962, 1964a, b) in their study of the interaction between electrically charged cloud drops in an external electric field. They assumed that the drops were embedded in flow which could be described by the streamfunction around an isolated sphere determined analytically by Proudman and Pearson (1957). Since the Stokes streamfunction as well as the streamfunction of Proudman and Pearson are only accurate at Reynolds numbers much less than unity (LeClair *et al.*, 1970; Pruppacher *et al.*, 1970), the above-mentioned studies only apply to very small cloud

[1] Since the theoretical results of Davis (1964a), given in the form of complicated analytic expressions and large number of tables, have been expressed in electrostatic units, and since the results are an integral part of our computations we have used the electrostatic units rather than the AMS recommended MKS units in several places of our paper.

drops. In order to improve the presently available models for computing the effect of electric charges and fields on the collision efficiency of cloud drops, and to extend the applicability of the results of such models to larger cloud drops, we have combined the electrostatic force model of Davis (1962, 1964a, b) with a numerical model for describing viscous flow past a sphere.

2. The aerodynamic and electrical model

The aerodynamic model used in the present computations to determine the trajectory of two interacting cloud drops is basically the same as that used previously by Shafrir and Neiburger (1963), Shafrir and Gal-Chen (1971) and Lin and Lee (1975). The model is based on Langmuir's (1948) superposition model which assumes that each body moves in the stream caused by the fluid motion around the other body in isolation. This model had been considerably criticized since the collision efficiencies derived from it by Shafrir and Neiburger decrease to zero for all drops investigated as $p=a/A$ approaches unity. This result is in contrast to the observations of Woods and Mason (1965), Cataneo *et al.* (1971) and List and Hand (1971) who experimentally demonstrated that due to a wake effect the collision efficiency of nearly equal size water drops is finite even for small drops and sharply increases as $p=a/A$ approaches unity. Essentially the same result was obtained by Steinberger *et al.* (1968) who showed by means of a model experiment that equal sized rigid spheres of Reynolds number as small as 0.06 falling in oil along their line of centers accelerate as they fall. Additional computations by Neiburger (1967), designed to improve the collision efficiencies of Shafrir and Neiburger (1963), did not lead to a better agreement of theory with experiment in that for all collector drops $<63\,\mu$m, the revised collision efficiencies still decreased to zero as $p=a/A$ approached unity. Recently, the computations of Lin and Lee (1975) suggested that the collision efficiencies of Shafrir and Neiburger (1963) and Neiburger (1967) for the range $0.8\leqslant(p=a/A)\leqslant 1.0$ probably were biased by some computational errors and therefore do not necessarily reflect any serious physical deficiencies inherent in the superposition model. The results of Lin and Lee, supported by the present results for uncharged cloud drops in the absence of external electric fields (Fig. 1), show that for all collector drops investigated ($A\gtrsim 10\,\mu$m) the collision efficiency sharply increases as $p=a/A$ approaches unity, in agreement with experiment. This result is also in agreement with the computations of Klett and Davis (1973) who used an analytic model based on modified Oseen equations. By these findings we felt justified to use the superposition scheme in determining the effect of electric charges and fields on the collision efficiency of cloud drops.[2]

The equations of motion for the collector drop of radius A and for the collected drop of radius a can then be written as

$$m_A\frac{d\mathbf{v}_A}{dt}=m_A\mathbf{g}^*-\frac{\pi}{4}c_{D,A}N_{\mathrm{Re},A}A\mu(\mathbf{v}_A-\mathbf{u}_a)+\mathbf{F}_{e,A},\quad (1)$$

$$m_a\frac{d\mathbf{v}_a}{dt}=m_a\mathbf{g}^*-\frac{\pi}{4C_{SC}}c_{D,a}N_{\mathrm{Re},a}a\mu(\mathbf{v}_a-\mathbf{u}_A)+\mathbf{F}_{e,a}.\quad (2)$$

We may non-dimensionalize Eqs. (1) and (2) by setting

$$\left.\begin{array}{c}\mathbf{v}'=\mathbf{v}/V_{\infty,A},\quad \mathbf{u}'=\mathbf{u}/V_{\infty,A},\\ t'=tV_{\infty,A}/A,\quad \mu'=\mu A^2/m_AV_{\infty,A},\\ (\mathbf{g}^*)'=\mathbf{g}^*A/V^2_{\infty,A},\quad p=a/A,\\ \mathbf{F}_e'=A\mathbf{F}_e/m_AV^2_{\infty,A}\end{array}\right\}.$$

One then finds

$$\frac{d\mathbf{v}_A'}{dt'}=(\mathbf{g}^*)'-\frac{\pi}{4}c_{D,A}N_{\mathrm{Re},A}\mu'(\mathbf{v}_A'-\mathbf{u}_a')+\mathbf{F}_{e,A},\quad (3)$$

$$\frac{d\mathbf{v}_a'}{dt'}=(\mathbf{g}^*)'-\frac{\pi}{4p^2C_{SC}}c_{D,a}N_{\mathrm{Re},a}\mu'(\mathbf{v}_a'-\mathbf{u}_A')+\frac{\mathbf{F}'_{e,a}}{p^3}.\quad (4)$$

Setting further $d\mathbf{R}'/dt'=\mathbf{v}'$, Eqs. (3) and (4) may be written in component form in a fixed reference frame as

$$\frac{dR'_{A,x}}{dt'}=v'_{A,x};\quad \frac{dv'_{A,x}}{dt'}=(g^*)'-B_2(v'_{A,x}-u'_{a,x})+(F'_{e,A})_x,\quad (5)$$

$$\frac{dR'_{A,y}}{dt'}=v'_{A,y};\quad \frac{dv'_{A,y}}{dt'}=-B_2(v'_{A,y}-u'_{a,y})+(F'_{e,A})_y\quad (6)$$

and

$$\frac{dR'_{a,x}}{dt'}=v'_{a,x};\quad \frac{dv'_{a,x}}{dt'}=(g^*)'-B_1(v'_{a,x}-u'_{A,x})+\frac{(F'_{e,a})_x}{p^3},\quad (7)$$

$$\frac{dR'_{a,y}}{dt'}=v'_{a,y};\quad \frac{dv'_{a,y}}{dt'}=-B_1(v'_{a,y}-u'_{A,y})+\frac{(F'_{e,a})_y}{p^3},\quad (8)$$

where

$$B_1=(\pi/4p^2C_{SC})c_{D,a}N_{\mathrm{Re},a}\mu',$$
$$B_2=(\pi/4)c_{D,A}N_{\mathrm{Re},A}\mu'.$$

Eqs. (5)–(8) determine the trajectories of two interacting drops.

Since the two drops accelerate as they approach each other, it was necessary to continually update the quantities B_1 and B_2. In order to do this, $N_{\mathrm{Re},A}$ and $N_{\mathrm{Re},a}$ were recalculated at each time step along the tra-

[2] A list of symbols is given in an appendix.

jectory from

$$N_{\mathrm{Re},A}=\frac{2A\rho|\mathbf{v}_A-\mathbf{u}_a|}{\mu}, \tag{9}$$

$$N_{\mathrm{Re},a}=\frac{2a\rho|\mathbf{v}_a-\mathbf{u}_A|}{\mu}, \tag{10}$$

and the drag force coefficients $c_{D,A}$ and $c_{D,a}$, a function only of the respective Reynolds numbers, were then computed from the relations for water drops in air given by Beard (1976).

Values for the velocity field $\mathbf{u}_A$, $\mathbf{u}_a$ around the drops were derived from the streamfunction fields given by LeClair *et al.* (1970) for selected Reynolds numbers from a numerical solution of the Navier-Stokes equation of motion for air flowing past a rigid sphere. Numerically computed flow fields were used for the following Reynolds numbers: 0.02, 0.03, 0.04, 0.07, 0.1, 0.2, 0.3, 0.4, 0.5, 0.6, 0.8, 1.0, 1.5, 1.75, 2.0, 2.5, 3.0, 4.0, 4.5 and 6.0. For drops with $N_{\mathrm{Re},a}<0.02$ we assumed that the flow field could be approximated by Proudman and Pearson's (1957) analytic solution to the Navier-Stokes equation of motion, which by considering only the first two terms of the Stokes expansion reads

$$\psi_{PP}=\frac{1}{4}a^2V_{\infty,a}\left(\frac{r}{a}-1\right)^2\sin^2\theta\left[\left(1+\frac{3}{16}N_{\mathrm{Re},a}\right)\left(2+\frac{a}{r}\right)+\frac{3}{16}N_{\mathrm{Re},a}\left(2+\frac{a}{r}+\frac{a^2}{r^2}\right)\cos\theta\right]. \tag{11}$$

The surprising accuracy with which this streamfunction describes the actual length of the standing eddy at the downstream end of a sphere has been pointed out by Van Dyke (1964) and Pruppacher *et al.* (1970).

For the purpose of describing the electrostatic forces $\mathbf{F}_{e,A}$ and $\mathbf{F}_{e,a}$, we represented both interacting drops by conducting spheres, an assumption which has been justified by Davis (1969). Davis (1964b) solved for the force between two conducting spheres. Expressing the model of Davis (1964b) in slightly different notation, we find for the electrostatic force on the spherical a drop

$$\begin{aligned}\mathbf{F}_{e,a}=-\Big[&\epsilon a^2E_0{}^2(F_1\cos^2\gamma+F_2\sin^2\gamma)\\&+E_0\cos\gamma(F_3Q_A+F_4Q_a)\\&+\frac{1}{\epsilon a^2}(F_5Q_A{}^2+F_6Q_AQ_a+F_7Q_a{}^2)+E_0Q_a\cos\gamma\Big]\hat{\mathbf{e}}_r\\&+[\epsilon a^2E_0{}^2F_8\sin2\gamma+E_0\sin\gamma(F_9Q_A+F_{10}Q_a)\\&+E_0Q_A\sin\gamma]\hat{\mathbf{e}}_\gamma,\end{aligned} \tag{12}$$

TABLE 1. Combinations of collector drop radius, Reynolds number and electric force used in the present work.

A (μm)	$N_{\mathrm{Re},A}$	Q_AE_0 (C V m^{-1})
11.4	0.02	0.0
11.4	0.04	-6.1×10^{-11}
11.4	0.1	-2.5×10^{-10}
19.5	0.1	0.0
19.5	0.2	-3.1×10^{-10}
19.5	0.3	-6.3×10^{-10}
24.7	0.2	0.0
31.4	0.4	0.0
31.4	0.6	-6.8×10^{-10}
31.4	1.0	-2.1×10^{-9}
40.2	0.8	0.0
40.2	1.0	-7.4×10^{-10}
40.2	1.75	-3.7×10^{-9}
50.7	1.5	0.0
50.7	2.0	-2.1×10^{-9}
50.7	2.5	-4.3×10^{-9}
61.7	2.5	0.0
61.7	3.0	-2.3×10^{-9}
61.7	4.0	-7.2×10^{-9}
74.3	4.0	0.0
74.3	4.5	-2.6×10^{-9}
74.3	6.0	-1.1×10^{-8}

where γ is the angle between the local vertical, given by the direction of $\mathbf{g}$, and the line connecting the centers of the two interacting spheres. We are considering only a vertical electric field, where E_0 is taken to be negative when the field points from a positively charged region in the top of the cloud to a negatively charged region in the base of the cloud. The force coefficients F_1–F_{10} in Eq. (12) are the same as in Davis (1964b), and the analytic expressions for them, which are complicated functions of $p=a/A$ and the distance between the two spheres, are given in Davis (1964a). The term in the first brackets of Eq. (12) is the force component $(F_{e,a})_r$ along the line connecting the sphere centers, while the term in the second brackets of Eq. (12) is the force component $(F_{e,a})_\gamma$ perpendicular to the line of centers. The corresponding x and y components of $\mathbf{F}_{e,a}$ in Cartesian coordinates are given by

$$(F_{e,a})_x=(F_{e,a})_r\cos\gamma-(F_{e,a})_\gamma\sin\gamma, \tag{13}$$

$$(F_{e,a})_y=(F_{e,a})_r\sin\gamma+(F_{e,a})_\gamma\cos\gamma. \tag{14}$$

Once $\mathbf{F}_{e,a}$ is found the force on the A sphere, according to Davis (1964b), is given by

$$\mathbf{F}_{e,A}=\mathbf{E}_0(Q_A+Q_a)-\mathbf{F}_{e,a}. \tag{15}$$

The choices of collector drop radii and Reynolds numbers were constrained by the facts that numerical flow fields of only certain Reynolds numbers were

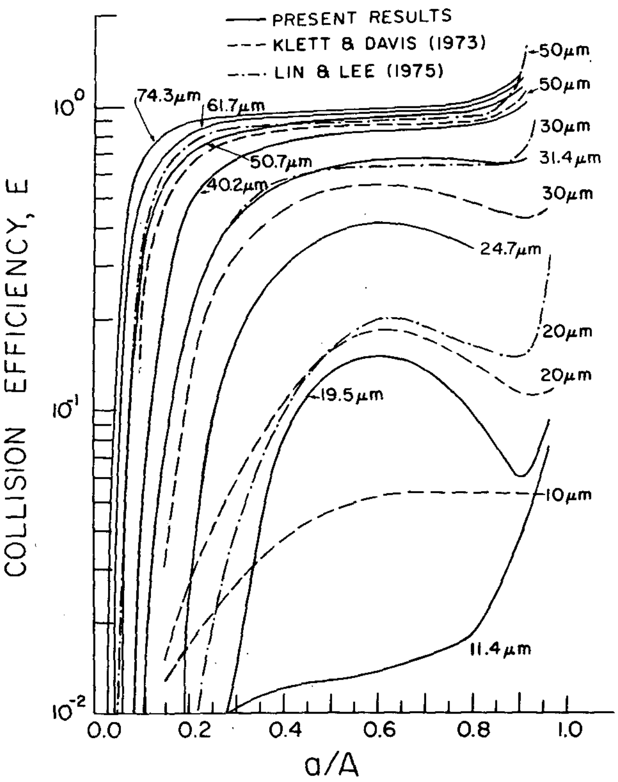

FIG. 1. Comparison between the present numerically computed collision efficiencies with recent published results for electrically uncharged water drops in air with no external electric field present.

available and that the collector drop radius and Reynolds number must satisfy the relations

$$\frac{c_{D,A}N_{\mathrm{Re},A}}{24}6\pi\mu A V_{\infty,A}=m_A g^*-Q_A E_0, \tag{16}$$

$$V_{\infty,A}=\mu N_{\mathrm{Re},A}/(2A\rho), \tag{17}$$

where the minus sign in Eq. (16) is the result of our convention which takes E_0 as positive for an upward-pointing electric field. Note that according to Eq. (16) a change in the product Q_AE_0 requires a change in the Reynolds number if we wish to keep the collector drop radius constant. In order to study atmospherically realistic cases which at the same time satisfied the above-mentioned constraints, we therefore constructed and used the combinations of collector drop radius and Reynolds number and the product Q_AE_0 listed in Table 1.

Similarly, the collected drop radius and Reynolds number were constrained by the relations

$$\frac{c_{D,a}N_{\mathrm{Re},a}}{24C_{SC}}6\pi\mu a V_{\infty,a}=m_a g^*-Q_a E_0, \tag{18}$$

$$V_{\infty,a}=\mu N_{\mathrm{Re},a}/(2a\rho). \tag{19}$$

Again, the choice of the collected drop radius was dictated by the available Reynolds number flow fields.

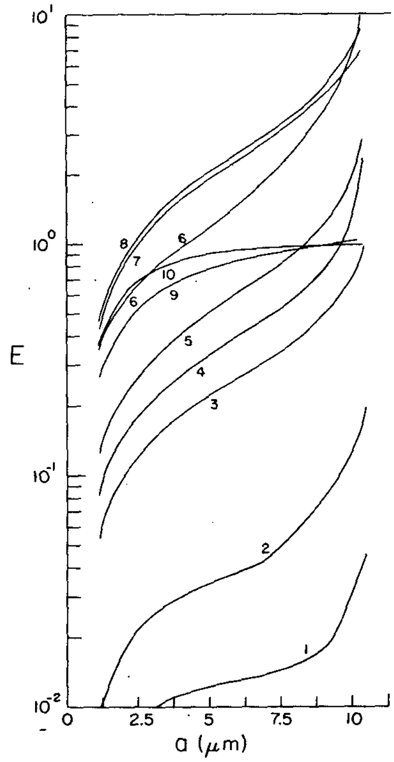

FIG. 2. Present results for the efficiency with which a cloud drop of radius $A=11.4$ μm collides with smaller cloud drops of radius a.

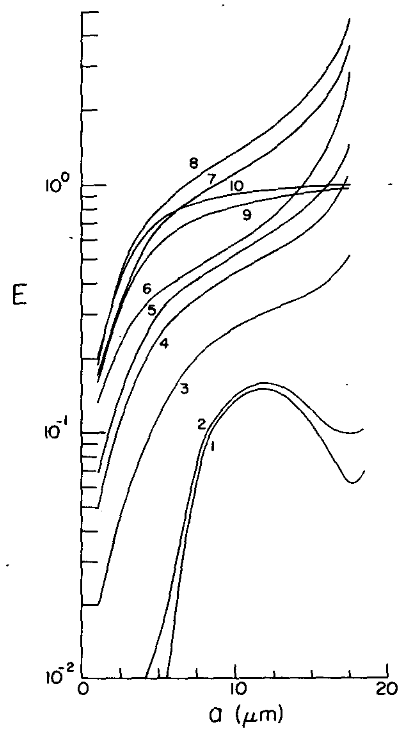

FIG. 3. As in Fig. 2 except for $A=19.5$ μm.

3. Results

Eqs. (5)–(8) were numerically integrated using a stable Hamming predictor-corrector-modifier scheme which was accurate to fourth order. We chose to carry out the present computations for 800 mb and +10°C, for which $\mu=1.768\times10^{-4}$ poise $(=1.768\times10^{-7}\ \mathrm{kg\ m^{-1}\ s^{-1}})$, $\rho=9.84\times10^{-4}\ \mathrm{g\ cm^{-3}}$ $(=9.84\times10^{-1}\ \mathrm{kg\ m^{-3}})$ and $\rho_w=0.9997\ \mathrm{g\ cm^{-3}}$ $(=0.9997\times10^{3}\ \mathrm{kg\ m^{-3}})$. We also set $g=9.8\ \mathrm{m\ s^{-2}}$. The initial vertical separation between the two interacting drops was chosen to be 70 collector drop radii. Larger initial vertical separations did not alter the trajectories.

In each case the trajectory computation was repeated until the "critical trajectory" for grazing contact of the two interacting spheres was found. This allowed us to determine the corresponding critical horizontal offset R_c of the center of the a sphere from the center of the A sphere when the a sphere is 70 collector drop radii upstream. From a knowledge of R_c, $y_c=R_c/A$ could then be computed. From this the collision efficiency, defined by

$$E\equiv\frac{\pi R_c^2}{\pi(A+a)^2}=\frac{y_c^2}{(1+p)^2}, \tag{20}$$

was determined.

The results of our computations are summarized in Figs. 1–8. In Fig. 1 the collision efficiency is plotted in the traditional manner as a function of $p=a/A$. It is seen, for all collector drops studied, that E for p-ratios close to unity increases sharply as p approaches 1.0, in agreement with experiment and in agreement with the analytical results of Klett and Davis (1973). One further notes that with decreasing size of the collector drop, E decreases without exhibiting a cutoff. This is in agreement with the analytical Stokes flow results of Hocking and Jonas (1970), Jonas (1972) and Davis and Sartor (1967) and with the low Reynolds number results of Klett and Davis (1973). It is also seen that the present values for E are in fair agreement with those determined by Lin and Lee (1975), who used a similar method of computation. The disagreement at the smallest collector drop radii is possibly due to the fact that, in the present computation, numerically determined flow fields were available for *all collector drops studied*, while Lin and Lee apparently only had available numerical flow fields for $N_{\mathrm{Re}}>1.6$. The present values for E are also in fair agreement with those analytically determined by Klett and Davis (1973), except for the values of E determined for an 11.4 μm collector drop. The reason for this disagreement is unclear. Unfortunately, Lin and Lee did not study collector drops of $A<20\ \mu$m to help assess the discrepancy with the results of Klett and Davis.

The present results for E are shown in Figs. 2–8 for various electric charge and field combinations with the numbered curves identified in Tables 2a–2g. It should be noted in Table 2 that the results for several cases are coincidental and are therefore displayed as a

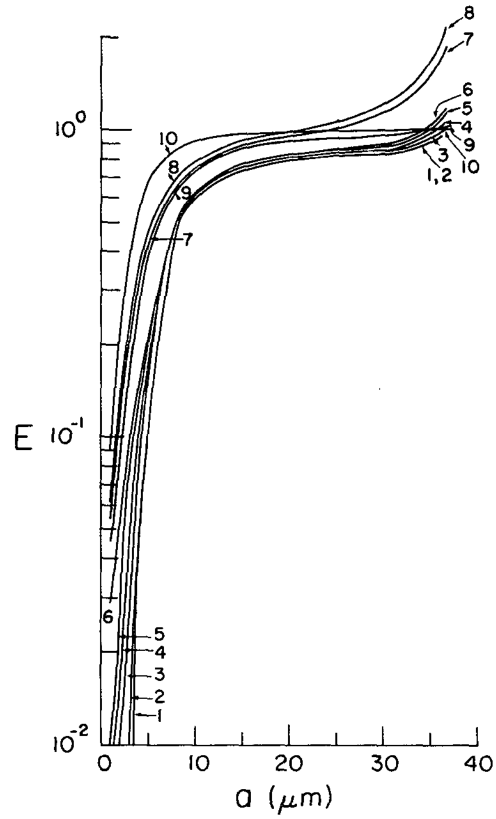

FIG. 5. As in Fig. 2 except for $A=40.2\ \mu$m.

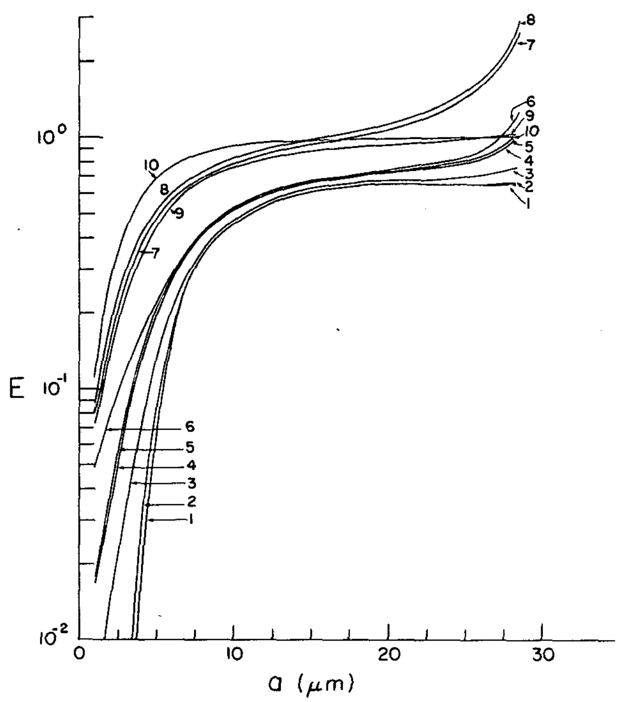

FIG. 4. As in Fig. 2 except for $A=31.4\ \mu$m.

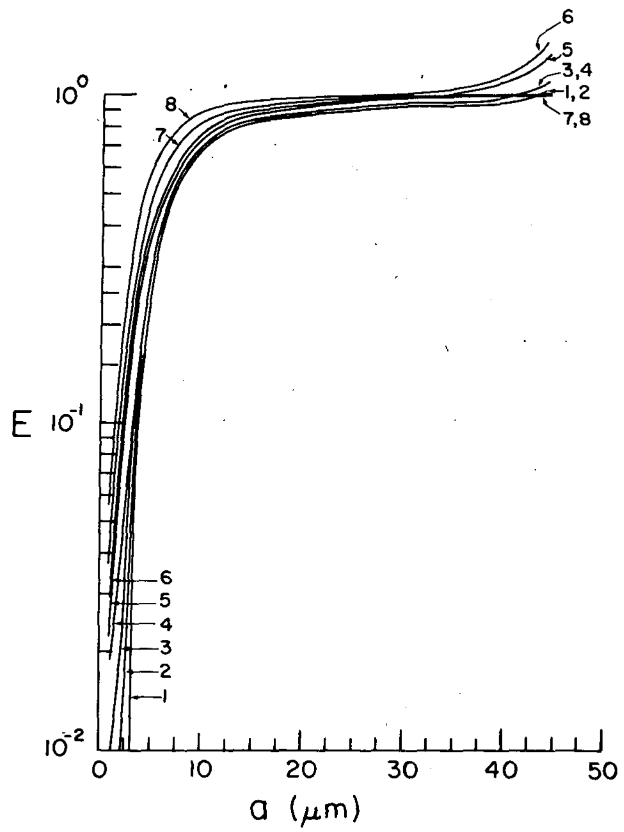

FIG. 6. As in Fig. 2 except for $A = 50.7$ μm.

single curve. The results can be summarized as follows:

1) External electric fields and electric charges residing on interacting cloud drops may have a profound

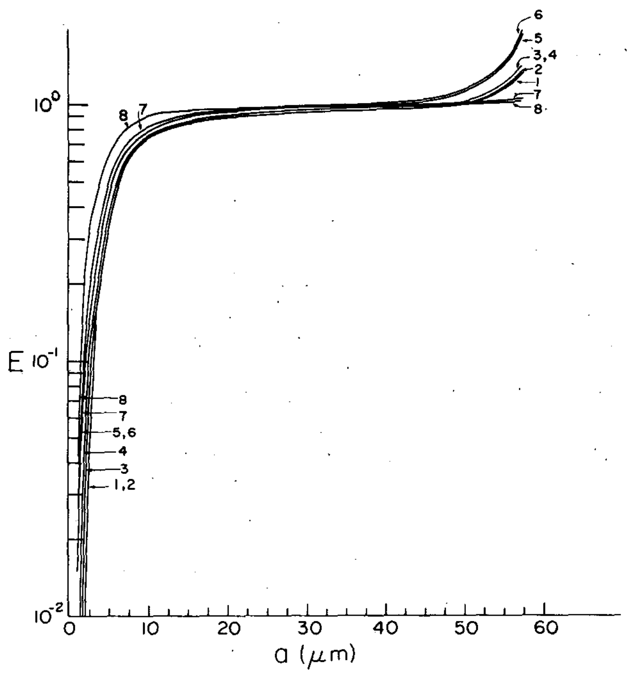

FIG. 7. As in Fig. 2 except for $A = 61.7$ μm.

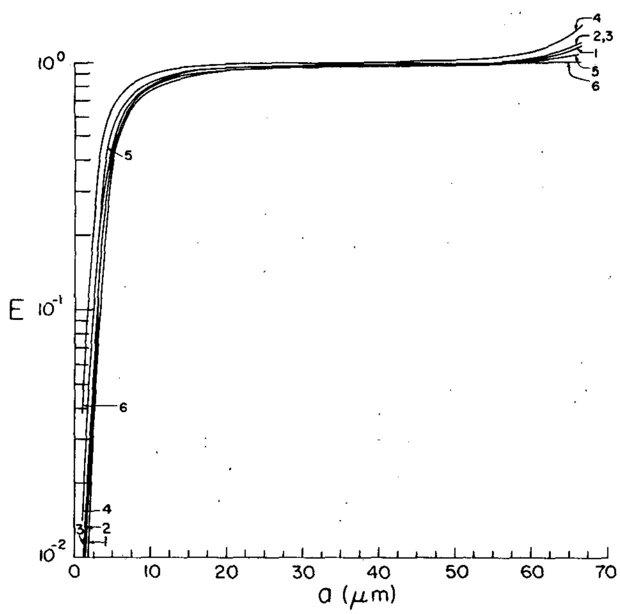

FIG. 8. As in Fig. 2 except for $A = 74.3$ μm.

effect on the collision efficiency of the interacting drops.

2) External electric fields in the presence of electrically neutral drops invariably enhance the collision efficiency of cloud drops, with the effect being most pronounced for the smallest collector drops. This implies that the critical electric field strength necessary to affect the collision efficiency increases with increasing collector drop size. Thus an external field of 500 V cm^{-1} significantly raises the collision efficiency of an 11.4 μm collector drop while it has a negligible effect if $A \gtrsim 30$ μm. On the other hand, an external electric field of 3000 V cm^{-1} significantly increases E for all collector drops of $A < 50$ μm, while for $A \gtrsim 50$ μm the effect of these high intensity fields is negligible.

3) Electric charges residing on drops in the absence of an external electric field invariably raise the collision efficiency of cloud drops if the two interacting drops are oppositely charged, the effect being most pronounced

TABLE 2a. Legend for curves in Fig. 2. Multiply electric field by 100 to convert to volts per meter.

$A = 11.4$ μm Curve	q_A (esu cm^{-2})	q_a (esu cm^{-2})	E_0 (V cm^{-1})
1	0.0	0.0	0.0
2	±0.2	∓0.2	0.0
3	0.0	0.0	∓500
4	0.0	0.0	∓706
5	0.0	0.0	∓1000
6	±2.0	∓2.0	0.0
7	0.0	0.0	∓2847
8	0.0	0.0	∓3000
9	±2.0	∓2.0	∓706
10	±2.0	∓2.0	∓2847

TABLE 2b. As in Table 2a except for Fig. 3.

$A=19.5$ μm Curve	q_A (esu cm^{-2})	q_a (esu cm^{-2})	E_0 (V cm^{-1})
1	0.0	0.0	0.0
2	±0.2	∓0.2	0.0
3	0.0	0.0	∓500
4	0.0	0.0	∓1000
5	0.0	0.0	∓1236
6	±2.0	∓2.0	0.0
7	0.0	0.0	∓2504
8	0.0	0.0	∓3000
9	±2.0	∓2.0	∓1236
10	±2.0	∓2.0	∓2504

TABLE 2d. As in Table 2a except for Fig. 5.

$A=40.2$ μm Curve	q_A (esu cm^{-2})	q_a (esu cm^{-2})	E_0 (V cm^{-1})
1	0.0	0.0	0.0
2	±0.2	∓0.2	0.0
3	0.0	0.0	∓500
4	0.0	0.0	∓683
5	0.0	0.0	∓1000
6	±2.0	∓2.0	0.0
7	0.0	0.0	∓3000
8	0.0	0.0	∓3529
9	±2.0	∓2.0	∓683
10	±2.0	∓2.0	∓3529

for the smallest collector drops. This implies that the critical electric charge necessary to affect the collision efficiency increases with increasing collector drop size. Thus, oppositely charged drops which carry a charge of magnitude equal to one-tenth the mean thunderstorm charge, as defined above, significantly raises the collision efficiency for collector drops of $A<40$ μm while having negligible effect on drops of $A\gtrsim 40$ μm.

4) In contrast to the results when either electric fields or charges are present, Figs. 2–8 indicate a reduction in the collision efficiency to near-geometric values when both fields and charges are present. This reduction is due to the substantially decreased interaction time during which mutually attractive charges on the two drops can act to force a collision. This decreased interaction time is the result of the vertical component of the relative velocity of the two drops, which is caused by the external vertical field acting on the charges carried by the drops.

5) For $A\gtrsim 70$ μm the effect of electric fields and charges on collision efficiency is negligible even though the field strengths and charges may be as large as those found in thunderclouds close to electric breakdown.

All of the above results would remain unchanged if E_0, Q_A and Q_a were all multiplied by (-1), for as shown by Eqs. (12), (16) and (18), the electrostatic force depends only on the products Q_A^2, Q_a^2, Q_AQ_a, Q_AE_0 and Q_aE_0. Nevertheless, it must be stressed that we have not treated all possible cases of interest to the collision efficiency problem. In particular the effects of horizontal fields, charges of like sign on the drops, and positive (negative) values of the product Q_AE_0 (Q_aE_0) were not studied. However, the presently outlined computational method can easily be adapted to any desired configuration. The present study merely attempted to demonstrate the usefulness of the method and to give a few results which allow some conclusions as to the electrical effects on collision efficiency which can be expected. Whether or not the effects for the present electric configuration represent in fact maximum effects cannot be stated at the present time. It certainly would be desirable to study another interesting configuration, namely the case of large drops carrying negative charge down to a negative charge layer.

It would also be desirable to compare the presently computed collision efficiencies with experiment. Unfortunately, no data are available which would allow a quantitative comparison for controlled conditions of external electric fields and electric charges on the interacting drops. The reason for the lack of experimental data lies in the fact that thus far a collision can only

TABLE 2c. As in Table 2a except for Fig. 4.

$A=31.4$ μm Curve	q_A (esu cm^{-2})	q_a (esu cm^{-2})	E_0 (V cm^{-1})
1	0.0	0.0	0.0
2	±0.2	∓0.2	0.0
3	0.0	0.0	∓500
4	0.0	0.0	∓1000
5	0.0	0.0	∓1038
6	±2.0	∓2.0	0.0
7	0.0	0.0	∓3000
8	0.0	0.0	∓3235
9	±2.0	∓2.0	∓1038
10	±2.0	∓2.0	∓3235

TABLE 2e. As in Table 2a except for Fig. 6.

$A=50.7$ μm Curve	q_A (esu cm^{-2})	q_a (esu cm^{-2})	E_0 (V cm^{-1})
1	{ 0.0	0.0	0.0
	{ ±0.2	∓0.2	0.0
2	0.0	0.0	∓500
3	{ 0.0	0.0	∓1000
	{ 0.0	0.0	∓1208
4	±2.0	∓2.0	0.0
5	0.0	0.0	∓2485
6	0.0	0.0	∓3000
7	±2.0	∓2.0	∓1208
8	±2.0	∓2.0	∓2485

TABLE 2f. As in Table 2a except for Fig. 7.

$A=61.7\ \mu$m Curve	q_A (esu cm^{-2})	q_a (esu cm^{-2})	E_0 (V cm^{-1})
1	0.0	0.0	0.0
	±0.2	∓0.2	0.0
2	0.0	0.0	∓500
3	0.0	0.0	∓907
	0.0	0.0	∓1000
4	±2.0	∓2.0	0.0
5	0.0	0.0	∓2842
6	0.0	0.0	∓3000
7	±2.0	∓2.0	∓907
8	±2.0	∓2.0	∓2842

be detected experimentally if the collision event between the two interacting water drops is followed by coalescence. It is well known, however, that not only the efficiency with which drops collide but also the efficiency with which they coalesce is strongly dependent on the electric charges residing on the drops and on any external electric fields present. It is sincerely hoped that despite the difficulties involved, such experimental data become available soon.

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TABLE 2g. As in Table 2a except for Fig. 8.

$A=74.3\ \mu$m Curve	q_A (esu cm^{-2})	q_a (esu cm^{-2})	E_0 (V cm^{-1})
1	0.0	0.0	0.0
	±0.2	∓0.2	0.0
	0.0	0.0	∓500
2	0.0	0.0	∓705
	0.0	0.0	∓1000
3	±2.0	∓2.0	0.0
4	0.0	0.0	∓2960
	0.0	0.0	∓3000
5	±2.0	∓2.0	∓705
6	±2.0	∓2.0	∓2960

APPENDIX

List of Symbols

- A, a — radius of collector drop, of collected drop
- $c_{D,A}$, $c_{D,a}$ — drag force coefficient of collector drop, of collected drop
- C_{SC} — Stokes-Cunningham slip correction factor $[=1+\alpha(\lambda/a)]$
- E — collision efficiency
- E_0 — external electric field
- $\hat{\mathbf{e}}_r$ — unit vector pointing from center of large drop to center of small drop
- $\hat{\mathbf{e}}_\gamma$ — unit vector perpendicular to $\hat{\mathbf{e}}_r$, in direction of increasing γ
- $\mathbf{g}$ — gravity
- g — $[=|\mathbf{g}|]$
- $\mathbf{g}^*$ — $\{=\mathbf{g}[(\rho_w-\rho)/\rho_w]\}$
- g^* — $[=|\mathbf{g}^*|]$
- m_A, m_a — mass of collector drop, of collected drop
- $N_{\mathrm{Re},A}$ — Reynolds number of collector drop $[=2AV_{\infty,A}\rho/\mu]$
- $N_{\mathrm{Re},a}$ — Reynolds number of collected drop $[=2aV_{\infty,a}\rho/\mu]$
- Q_A, Q_a — electric charge on collector drop, on collected drop
- $Q_{A,\mathrm{Mean}}$ — magnitude of mean thunderstorm charge on drop of radius A
- q_A — charge parameter for collector drop $[=Q_A/A^2]$
- q_a — charge parameter for collected drop $[=Q_a/a^2]$
- $R'_{A,x}$, $R'_{A,y}$ — position coordinates of collector drop in Cartesian coordinates
- $R'_{a,x}$, $R'_{a,y}$ — position coordinates of collected drop in Cartesian coordinates
- r — radial coordinate from center of sphere
- t — time
- $\mathbf{u}_A$,$\mathbf{u}_a$ — velocity of flow past collector drop, past collected drop
- $V_{\infty,A}$, $V_{\infty,a}$ — terminal velocity of collector drop, of collected drop
- $\mathbf{v}_A$,$\mathbf{v}_a$ — instantaneous velocity of collector drop, of collected drop
- α — $[=0.257+0.400\exp(-1.10a/\lambda)]$
- γ — angle between the directions of $\mathbf{g}$ and $\hat{\mathbf{e}}_r$
- ϵ — dielectric constant of air
- θ — polar angle measured from the upstream stagnation streamline on a sphere
- λ — mean free path of air molecules
- μ — dynamic viscosity of air
- ρ — density of air
- ρ_w — density of water
- ψ — streamfunction of flow past a rigid sphere

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