A Radiative-Convective Model Study of the CO_2 Climate Problem

T. Augustsson

George Washington University, NASA-Langley Research Center, Hampton, Va. 23665

V. Ramanathan

National Center for Atmospheric Research,[1] Boulder, Colo. 80307

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ABSTRACT

A radiative-convective model study of the increase in global surface temperature ΔT_s due to an increase in the CO_2 concentration is presented. The model considers several weak bands of CO_2 which contribute about 30% to ΔT_s. A comparison study of the various published results with the present analysis indicates that, for the CO_2 bands in the 12–18 μm region, the best estimate of ΔT_s for the constant cloud top radiative-convective model is about 1.9 K for a doubling of the CO_2 concentration. The inclusion of the CO_2 bands in the 10 and 7.6 μm regions increases the value of ΔT_s to about 2 K. The computed value of ΔT_s is very sensitive to the radiative-convective model assumptions regarding cloud top and relative humidity. Because of this sensitivity the estimated value of ΔT_s for a doubling of the CO_2 concentration ranges from 1.98 to 3.2 K.

1. Introduction

Radiative-convective models usually employ approximate techniques to treat the radiative transfer processes due to H_2O, CO_2 and O_3. The models also adopt numerous simplifying assumptions to account for the various climatic feedback mechanisms. The purpose of the present paper is to quantify the errors and uncertainties introduced by some of these approximate techniques and simplifying assumptions into the results predicted by the model for the global surface temperature increase (ΔT_s) that may result due to an increase in the atmospheric CO_2 concentration. Toward this goal, results are presented on the following aspects of the CO_2-climate problem: (1) the sensitivity of ΔT_s to the temperature dependency of the CO_2 15 μm band absorptance, (2) the contribution to ΔT_s by the weak bands of CO_2, and (3) the strong sensitivity of ΔT_s to some of the assumptions inherent in radiative-convective models. The radiative-convective model adopted for the present study is the same as the model described in Ramanathan (1976, hereafter identified as R).

2. Results and discussions

a. The temperature dependency of the 15 μm band absorptance.

In the 15 μm region the model includes the fundamental bands of four isotopes of CO_2 and six hot bands of $C^{12}O_2^{16}$ (the band centers of these bands are given in Table 3 of R). The temperature dependency of the band absorptance A of these bands arises from the temperature dependency of the intensity and the half-width of the individual rotational lines within a band. In the present model, the temperature dependent parameters, namely the bandwidth parameter A_0, the mean line half-width ν_0 and the band intensity S [see Eqs. (8)–(13) in R], account for the temperature dependency of A. The temperature dependence of A_0 and ν_0 are given respectively in Edwards and Menard (1964) and Cess and Ramanathan (1972). The temperature dependence of S is given by

$$S=\begin{cases} S_0\left(\dfrac{T_0}{T}\right). & \text{for the fundamental band}\\[2ex] S_0\left(\dfrac{T_0}{T}\right)\exp\left[K\left(\dfrac{1}{T_0}-\dfrac{1}{T}\right)\right] & \text{for the hot bands}\end{cases} \tag{1}$$

where $K\approx 960$ for the first hot bands and $K\approx 1920$ for the second hot bands. In Eq. (1) S_0 [$\mathrm{cm^{-2}\,atm^{-1}}$] is the band intensity defined at the reference temperature T_0 and T is the actual temperature.

The importance of the temperature dependence of A to the model estimated values for ΔT_s is indicated in Table 1. In the calculations that neglect the temperature dependence of the individual bands, a constant value of $T=288$ K was assumed for the temperature-dependent parameters A_0, ν_0 and S. By comparing the first and last row of Table 1, it is seen that neglecting

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the temperature dependence of A introduces an error of about 25% in the value of ΔT_s. Further, comparing rows 2 and 3 with row 1, it is seen that the hot bands T dependence is as important as the T dependence of the fundamental bands for the ΔT_s calculations.

The atmospheric temperature is much less than 288 K and hence setting $T=288$ K for the band parameters overestimates the band absorptance and consequently, as shown in Table 1, overestimates ΔT_s. The results also indicate that the models that employ the laboratory data for A without accounting for the temperature dependence of A would tend to overestimate the value of ΔT_s because the laboratory data for A [such as the data in Burch *et al.* (1961)] are usually measured at $T>288$ K.

b. The weak bands of CO_2

In addition to the bands considered thus far, CO_2 has several tens of weak bands centered between the regions 12–18 μm, 9–10 μm and 7–8 μm [see McClatchey *et al.* (1973) for a complete listing of these bands]. The effect of these bands on the ΔT_s is described below.

The following bands were included. The vibrational transitions associated with each band are given within parentheses: (i) four hot bands in the 12–18 μm region centered at 544.3 cm^{-1} (100^1–11^10^2), 597.3 cm^{-1} (02^20^1–11^10^2), 741.8 cm^{-1} (02^20^1–11^10^1), 791.5 cm^{-1} (100^2–11^10^1); (ii) two hot bands in the 10 μm region centered at 961. cm^{-1} (100^1–001^1), 1064.7 cm^{-1} (100^2–001^1); and (iii) the pressure-induced fundamental band in the 7.6 μm region centered at 1388.1 cm^{-1} (00^00–10^00). All of the bands described above belong to the isotope $C^{12}O_2^{16}$. The rest of the weak bands were neglected for one or both of the following two reasons: the band intensity is too small or the band is completely overlapped by the stronger $C^{12}O_2^{16}$ bands. The band intensities of the 12–18 μm and the 10 μm bands are taken from McClatchey *et al.* (1973) and these bands have been treated by the band absorptance formulation given in *R* [Eq. (8)]. For the 7.6 μm band the formulation given by Edwards (1960) is adopted.

TABLE 1. Sensitivity of the increase in surface temperature to the temperature dependency of the 15 μm band absorptance. The results are for a doubling of the CO_2 concentration from the present day value of 320 ppm (by volume). The weak bands have not been included. T_s is the surface temperature.

Description	Increase in T_s (°K)
1. Includes the temperature dependency of A for both hot and fundamental bands	1.53
2. Neglects the temperature dependency of A for the fundamental bands	1.68
3. Neglects the temperature dependency of A for the hot bands	1.72
4. Neglects the temperature dependency of A for both the fundamental and hot bands	1.91

The ΔT_s computed after including these weak bands in the present analysis is 1.98 K for a doubling of the CO_2 mixing ratio from the present day value of 320 ppmv. Upon comparing this value with the previously mentioned value of 1.53 K (as given in the first row in Table 1) it is seen that the weak bands contribute about 30% to the magnitude of the increase in T_s. The four hot bands in the 12–18 μm region contribute about 0.33 K to ΔT_s while the 10 μm hot bands and the 7.6 μm band contribute respectively 0.095 K and 0.025 K to ΔT_s.

It is instructive at this point to compare the present results with the previously published results in the literature. The previous results are summarized in Schneider (1975), in which Schneider indicates that the radiative convective models of Manabe (1971), Schneider (1975) and Ramanathan (1975) yield the most reliable estimates for ΔT_s. For a doubling of CO_2 concentration, Manabe (1971) obtains a value of 1.9 K while Schneider (1975) and Ramanathan (1975) each obtain a value of 1.5 K. Schneider's result is a revision of the earlier estimate of his model reported in Rasool and Schneider (1971) and the reasons for this revision are stated clearly in Schneider (1975, p. 2064). Ramanathan's result is computed from the model described in R. Both of these models do not include the four weak bands in the 12–18 μm region and the weak bands in the 10 and 7.6 μm regions. The CO_2 radiation model of Manabe adopts the laboratory measurements of Burch *et al.* (1961) in the 12–18 μm region and hence Manabe's model implicitly accounts for all of the bands included in the present analysis excepting the 10 and 7.6 μm bands. Upon neglecting the 10 and 7.6 μm bands, the present analysis computes $\Delta T_s=1.86$ K while adding the contribution of 0.33 K by the four minor bands to the results of Schneider (1975) and Ramanathan (1975), these two models yield $\Delta T_s=1.83$ K. Thus, considering only the bands in the 12–18 μm region, it is seen that the models of Manabe, Schneider and Ramanathan are in almost exact agreement with the present model. Although this is an encouraging conclusion, such a close agreement is rather surprising and is possibly fortuitious because there are substantial differences between the above models in the techniques adopted for treating the radiative transfer effects of CO_2 bands.

The weak bands of CO_2 have an interesting consequence for the CO_2 climate problem. This can be illustrated with the aid of Fig. 1 where the contribution to ΔT_s by the 15 μm and the weak bands are drawn separately as a function of the fractional increase in the CO_2 concentration (δCO_2). It is seen that the ΔT_s due to the weak bands increases almost linearly with respect to δCO_2 while the rate of increase of ΔT_s with respect to δCO_2 due to the 15 μm bands is much less than linear and, in fact, for values of $\delta CO_2 > 8$ the ΔT_s due to the 15 μm bands almost levels off. The 15 μm bands are

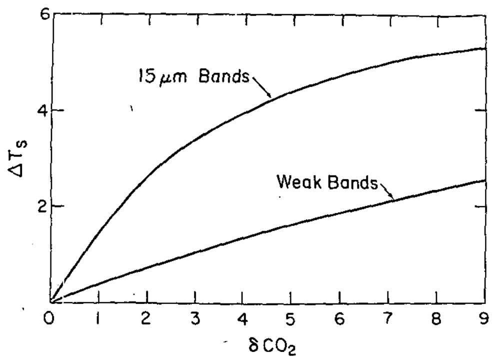

FIG. 1. Individual contribution to the increase in surface temperature by the 15 μm bands and the weak bands. The results are for the constant cloud top altitude model. $\delta CO_2 = [CO_2\text{(perturbed)} - CO_2\text{(ambient)}]/CO_2\text{(ambient)}$. CO_2(ambient) = 320 ppm (by volume); T_s = surface temperature. The number of bands and their band centers included in the category of weak bands is given in the text.

optically thick while the weak bands are optically thin. Consequently, the greenhouse effect due to the 15 μm bands increases logarithmically with respect to CO_2 and the weak bands greenhouse effect increases almost linearly with CO_2 concentration. Based on Fig. 1, it can be concluded that the warming effect of CO_2 on the global surface temperature may never saturate out even for large increases in CO_2 concentrations.

c. *Radiative-convective model assumptions*

Based on the initial suggestion by Manabe and Wetherald (1967), it has become conventional in radiative-convective models to employ a constant cloud top altitude (CTA) model and a relative humidity distribution which is independent of surface temperature. According to Manabe and Wetherald the relative humidity distribution RH is given by

$$\mathrm{RH} = 0.77\left(\frac{P-0.02}{1-0.02}\right)^{\Omega}, \tag{2}$$

where P is the atmospheric pressure and $\Omega = 1$. However, the recent studies of Cess (1974, 1976) indicate the existence of alternate assumptions for the cloud top and RH. For the cloud top, Cess (1974) proposes the constant cloud top temperature (CTT) model. For RH, Cess indicates that RH should be made a function of T_s by letting

$$\Omega = 1.0 - 0.03\,\Delta T_s. \tag{3}$$

The effect of the two alternate assumptions suggested by Cess on the model estimated value of ΔT_s is shown in Table 2. The substantial difference between the results of various experiments shown in Table 2 is clearly evident. The basic reason for this difference can be explained by considering the difference in the sensitivity parameter dF/dT_s shown in the last column of Table 2, where F denotes the outgoing longwave flux at the top of the atmosphere. The computed value of ΔT_s should be inversely proportional to the parameter dF/dT_s. Upon comparing rows 3 and 4 in Table 2, it is seen that ΔT_s is also sensitive to the value of Ω assumed for the ambient atmosphere. The extreme case of a CTT model with the R as a function of T_s has not been considered because the dF/dT_s obtained from such a model is too small when compared with the observed value of 1.57 W m^{-2} K^{-1}, proposed by Cess (1976).

There are no clear-cut theoretical justifications for choosing any one of the models shown in Table 2 as a more representative model for the real world. The results only indicate the large uncertainty in the model results introduced by the assumptions made in radiative-convective models. There is another potentially serious source of uncertainty in the ΔT_s results due to the neglect of the coupling between the surface temperature and fractional cloud cover in radiative-convective models. Finally, the radiative-convective model is a one-dimensional globally averaged model and hence does not give any information concerning the surface temperature response of the individual latitude zones. The recent three-dimensional model study of Manabe and Wetherald (1975) and the discussions given in Schneider (1975) indicate that the polar regions, due to their relative stable thermal stratification and to the positive feedback mechanism between ice albedo and surface temperature, may be subject to a significantly larger increase in surface temperature than that indicated by the globally averaged models.

3. Concluding remarks

The results presented in this study when considered in conjunction with Schneider's study provide satisfactory explanations for almost all of the differences in ΔT_s estimated by various radiative-convective models. The present analysis indicates that, for the bands in the 12–18 μm region, the best CTA radiative-convective model estimate is $\Delta T_s \approx 1.9$ K for a doubling of the CO_2 concentration. Inclusion of the CO_2 bands in the 10 and 7.6 μm regions increases the value of ΔT_s to 1.98 K.

TABLE 2. Increase in T_s due to a doubling of CO_2 concentration. The results include the weak bands of CO_2 mentioned in Section 2.2 of the text. (CTA = cloud top altitude; CTT = cloud top temperature.) See text for the definition of the symbol Ω.

No.	Experiment description	Increase in T_s (K)	dF/dT_s (W m^{-2} K^{-1})
1.	Constant CTA model $\Omega = 1$.	1.98	2.24
2.	Constant CTT model $\Omega = 1$.	3.2	1.38
3.	Constant CTA model $\Omega = 1 - 0.03\Delta T_s$	2.6	1.87
4.	Constant CTA model $\Omega = 0.8 - 0.03\Delta T_s$	3.0	1.63

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