The Thermal Structure of Jupiter from Infrared Spectral Measurements by Means of a Filtered Iterative Inversion Method

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ABSTRACT

The present study describes a filtering procedure to invert the radiative transfer equation which combines the advantages of the fast convergence of Chahine's method and the stability properties inherent in filtering. This procedure is used to invert available spectral infrared measurements of Jupiter. A set of Jovian thermal profiles is retrieved as a function of the temperature at the 0.1 mb level where temperature values from β Scorpio occultation measurements are available. The tropopause is distinctly sharper than the one existing in models computed with the assumption of convective radiative equilibrium. In order to fit the data of the 7.7 μm spectrum of Jupiter, the temperature at the 0.1 mb level cannot exceed 200 K. The relative abundance of Jovian molecular hydrogen was inferred and found to be q = 0.88 ± 0.06. The Jovian effective temperature corresponding to the inferred thermal profiles is T_e = 129.45 ± 1.2 K.

1. Introduction

Thermal profiles of planetary atmospheres can be derived from infrared spectral measurements by inversion of the radiative transfer equation (RTE). Such inversion techniques have been successfully applied to the earth's atmosphere and have also been used for the inference of temperature in the atmospheres of Mars and Jupiter. However, for the study of poorly known planets such as Jupiter and the other giant planets, one should prefer, among the existing methods, one which does not require a priori information about the thermal structure. For instance, one must be cautious with iterative methods, efficient in terrestrial meteorology, that yield solutions whose finest structure depends on the initial guess. It is therefore important to apply an iterative method with well-known conditions of convergence.

So far, infrared measurements of the giant planets are scarce and the computer time required to invert these data is not too excessive. However, in spatial experiments the data flow can be very large. For instance, an interferometry experiment such as the one planned aboard the coming 1977 mission to Jupiter, Saturn and possibly Uranus, will produce about 1200 spectra per day during 60 days. Thus a fast iterative method is essential.

With respect to these considerations, the iterative process of Chahine (1968, 1970) has very attractive properties. However, it can be unstable in the presence of experimental noise, for reasons which are analyzed in Section 2 by means of concepts developed by Gautier and Revah (1975, hereafter referred to as GR). This analysis leads us to elaborate a procedure for filtering Chahine's process. The procedure conserves the properties of fast convergence of this method but avoids instabilities. This filtered method is then applied to available infrared data of Jupiter, which are examined in Section 3. The computations are described in Section 4, and Section 5 is devoted to the results and discussion.

2. Method of inversion

Soundings of the atmospheres of the giant planets generally use infrared data extending over a very wide spectral range. Under these conditions the RTE cannot be accurately linearized and for this reason it is preferable to use methods applicable to nonlinear cases.

The nonlinear iterative method of Smith (1970) has been extensively used in terrestrial meteorology and often for planetary studies. However, it must be noticed that its rate of convergence is slow and that the finest structure of the retrieved thermal profile depends strongly on the shape of the initial guess of the iterative process (Chow, 1975). Such behavior, which is without serious consequences for the earth's atmosphere, where an initial guess close enough to the fore-
seen solution can be chosen, could be detrimental when exploring the atmosphere of a poorly known planet such as Jupiter and the other giant planets. Similarly, we did not use the procedure of Wallace and Smith (1976) since it imposes an a priori representation of the solution which can lead to a loss of information. Another possible method is that of Chahine which converges rapidly toward a solution for any reasonable initial guess (Barcilon 1975). However, it might lead, in the presence of experimental errors, to instabilities in the retrieved profile. This fact has been studied empirically, but up to now the reasons for this instability have not been clearly analyzed. The concepts presented in GR allow us to interpret it.

The integral equation to be inverted is

$$I(\nu) = \int_0^\infty B(\nu, T(P))K(\nu, P)dP,$$

where the weighting function $K(\nu, P)$ is defined at different pressure levels $P$ and wavenumbers $\nu$, and $T(P)$ is the unknown thermal profile to be recovered from a set of $N$ spectral data $I(\nu_i)$ with $i = 1, 2, \ldots, N$.

The method proposed by Chahine consists of establishing a relationship between the discrete number $N$ of wavenumbers at which the radiance is measured and an equivalent number of pressure levels such as

$$P_i = f(\nu_i), \quad i = 1, 2, \ldots, N,$$

and to determine iteratively a number $N$ of temperature values at levels $P_i$ by means of an algorithm. We symbolically write

$$T(P_i) = \mathcal{G}[I(\nu_i)].$$

An approximation $\hat{T}(P)$ of the unknown profile $T(P)$ is then obtained by interpolation between the $N$ points $T(P_i)$. Eq. (2) establishes a relationship between a sampling interval $\Delta \nu_i = |\nu_{i+1} - \nu_i|$ for $I(\nu)$ and the spacing $\Delta P_i = |P_{i+1} - P_i|$ between the pressure levels at which $T(P)$ is discretized. Following our analysis in GR we consider the Fourier spectrum $I^*(k)$ of $I(\nu)$ which consists of the spectrum $I^*_0(k)$ of the signal free of noise to which is superimposed a noise spectrum $N^*(k)$. $I^*_0(k)$ decreases as a function of $k$ and for a particular value $k_m$ one has

$$I^*(k_m) = N^*(k_m).$$

The spatial components $I^*(k)$ corresponding to values $k > k_m$ are completely masked by the noise background and a sampling interval $\Delta \nu_i$ chosen smaller than $\Delta \nu_m$, where

$$\Delta \nu_m = 1/2k_m$$

introduces components heavily contaminated by noise. As a consequence, if in the inversion scheme [Eq. (3)] one attempts to define the thermal profile at levels spaced by $\Delta P_i$ corresponding to the wavenumber interval $\Delta \nu_i < \Delta \nu_m$, erroneous Fourier components of $T(P)$ are taken into account: physically meaningless, they are responsible for the oscillations in the inferred approximation $T(P)$. Clearly, Chahine's method does not contain any filtering procedure and it is therefore necessary in some cases to increase the size of the step $\Delta \nu_i$ (and thus of $\Delta P_i$) to avoid oscillations and a divergence of the process. But if $T(P)$ has a small-scale structure such a choice for $\Delta P_i$ corresponds to an undersampling compared to the Shannon's minimum sampling (GR) and $T(P)$ is then a very crude approximation of $T(P)$. Therefore, filtering must be introduced in the course of the iterative process in order to try to obtain an optimal solution which will be the convolution of the actual profile by a band-limited function. The cutoff frequency $f_0$ of its spectrum then defines the spatial resolution of the retrieved thermal profile (GR).

A simple method to filter the high-frequency noise superimposed on the solution is to take at each iteration the convolution of the inferred profile $T(P)$ and a function whose Fourier properties are known. Because of its fast decrease in Fourier space, the Hanning window function (Blackman and Tuckey, 1958; Brault and White, 1971) was chosen. This function, characterized by its width $2T$, is defined as

$$W(X) = \begin{cases} \frac{1}{2}(1 - \cos 2\pi X/T), & 0 \leq X \leq T \\ 0, & |X| > T \end{cases}$$

(5)

whose Fourier transform is

$$D(f) = [Q(f)/2 + Q(f + 1/T)/4 + Q(f - 1/T)/4]^2/T,$$

(6)

where

$$Q(f) = T(\sin \pi f T/\pi f T).$$

(7)

The cutoff frequency $f_0$ of the spectrum, defined as the value of $f$ for the first zero of $D(f)$, is such that $f_0 T = 1$. The spatial resolution is then related to the width $T$ by

$$\rho = T/2 = \frac{1}{2}f_0.$$  (8)

The optimal $T$ can be deduced from the behavior of the Fourier spectrum of the nonfiltered retrieved thermal profile, or determined empirically by increasing the width of the "window" until the solution is exempt of oscillations. This filtering variation of Chahine's scheme was tested successfully for several cases of noise-contaminated spectra computed from planetary atmospheric models. A detailed analysis of the inversion procedure and of its behavior in Fourier space will be given in a forthcoming paper. Examples of thermal profiles of Jupiter determined from experimental data are shown in Fig. 1, with and without filtering. In addition to its simplicity this filtering process does not increase significantly the computer time required by Chahine's scheme. By a proper choice of the cutoff frequency the profile converges toward an approximation of the
actual solution which is close to the optimal one as far as the spatial resolution is concerned. As emphasized in GR any stable inversion method contains an explicit or hidden filtering. In the presently available iterative inversion methods, the filtering is \textit{a priori} and then cannot be optimized.

With soundings of the atmospheres of the giant planets, thermal profiles and hydrogen to helium ratios have to be simultaneously retrieved. In this case the filtered Chahine’s scheme is combined with the procedure of Encrenaz and Gautier (1973).

3. **Thermal sounding of the Jovian atmosphere: spectral data and opacities**

The absorbers to be considered in our approach are, on one hand, hydrogen and helium, and methane on the other. Ammonia data cannot be used since its unknown mixing ratio is expected to vary drastically with temperature wherever the latter is lower than the NH\textsubscript{3} triple-point temperature. Moreover, information from optical occultation measurements can be used to fix the thermal structure of the upper stratosphere above about the 0.1 mb level.

![Graph showing pressure level and frequency relationship](image-url)
a. Data from far-infrared H₂-H₂ and H₂-He spectra.

Data points corresponding to the hydrogen-helium absorption range were chosen from airborne spectral measurements of Houck et al. (1975) between 16 and 40 µm. Their results and these obtained from balloon measurements between 60 and 220 cm⁻¹ by Furniss et al. (1976) as well as theoretical computations (Encrenaz et al., 1971; Vapillon et al., 1977) show that NH₃ opacity is important or preponderant for wavenumbers <273 cm⁻¹.

Pressure-induced absorption coefficients for H₂-H₂ and H₂-He were interpolated from experimental data of Birnbaum and Cohen (1976). The correspondence defined by Eq. (2) between the wavenumbers νr at which observations are made, and the pressure levels P_r at the peak of the respective weighting functions is shown in Fig. 2.

Since the observed quantity is the total flux emitted by the planet, the weighting function is given by

\[ K(ν_r; P) = E_r[ν_r(P)] \partial ν_r/\partial \ln P, \]

where E_r is the second exponential integral function. The extent in pressure units of the height interval sounded is defined by the conditions that wavenumbers <273 cm⁻¹ cannot be used and that maximum absorption occurs at 600 cm⁻¹. This leads, for q = 0.85, to P_n = 192 mb and P_M = 568 mb so that the quantity

\[ Δξ = \ln(P_M/P_n), \]

takes the value 1.08. According to GR the minimum number of samples needed to restore the thermal profile without a loss of information is

\[ N_S = 1 + 2f_0Δξ, \]

where f_0 has been defined in Section 2. Numerical tests lead to limit the spatial resolution to 0.5 scale height, with a corresponding cutoff frequency f_0 [Eq. (8)] of

\[ f_0 = 1, \]

so that

\[ N_S = 3. \]

Thus, although five H₂-He frequency data points are used (Table 1) for inversion, only three points are significant in terms of information content.

<table>
<thead>
<tr>
<th>Wavenumber (cm⁻¹)</th>
<th>Brightness temperature (K)</th>
</tr>
</thead>
<tbody>
<tr>
<td>Methane point 1260</td>
<td>146.6</td>
</tr>
<tr>
<td>H₂-He point 600</td>
<td>124.47</td>
</tr>
<tr>
<td>348</td>
<td>123.52</td>
</tr>
<tr>
<td>322</td>
<td>126.6</td>
</tr>
<tr>
<td>298</td>
<td>132</td>
</tr>
<tr>
<td>273</td>
<td>140</td>
</tr>
</tbody>
</table>

b. Methane

In a first attempt to recover the thermal structure, we used the rather inaccurate data of Gillet et al. (1969) and Encrenaz et al. (1976), but the inferred thermal profiles led to brightness temperature spectra which did not fit the recent and more accurate measurements of Russel and Soifer (1977). Therefore the Russel and Soifer data were finally considered for inversion.

From the Fourier analysis developed in GR, the spatial resolution of the part of the profile retrieved from present available methane measurements is of the same order of magnitude as the observable height range. This result is in agreement with that based on a computation of the weighting functions made by Wallace et al. (1974). Therefore, we consider that the only indication which could be obtained from the presently available CH₄ 7.7 µm band measurements, independent of other information, concerns a mean value of the Jovian stratosphere, and that for inversion it is sufficient to use data at only one frequency [in disagreement with Orton (1977) who concludes from a Backus-Gilbert analysis that three methane data points are required to avoid a loss of information]. Thus, the retrieved thermal structure of the stratosphere depends strongly on the limiting conditions which are, on one hand, the value at the top of the troposphere resulting from the H₂-He spectrum, and, on the other, the temperatures deduced from measurements of the occultation of β Scorpio by Jupiter. Moreover, as also pointed out by Orton (1977), data must be chosen in the spectral range 1240–1290 cm⁻¹, since outside this interval additional opacities due to other components should be introduced (Russel and Soifer, 1977). In this study, the data point was chosen at 1260 cm⁻¹ where the experimental brightness temperature is 146.6 K. The methane opacity was calculated by means of a random band model as in Wallace et al. (1974). According to Orton (1977), the agreement between the random-band model and a line-by-line calculation is acceptable up to 1290 cm⁻¹. The mixing ratio of CH₄ was usually taken to be 6.2×10⁻⁴ according to commonly accepted values (Newburn and Gulkis, 1973), but tests were made with 3.2×10⁻⁴ and 12.4×10⁻⁴.

c. Data from occultation measurements

The structure of the higher Jovian stratosphere below the homopause has been inferred from measurements by Hubbard et al. (1972), Vapillon et al. (1973) and Veverka et al. (1974). Reviews of this subject were given recently by Hunten and Veverka (1976) and Combes et al. (1975). The most precise temperature determination concerns the level n = 3×10⁵ cm⁻³ (corresponding to about 7×10⁻⁸ mb at T = 170 ± 30 K. Below this level the temperature would increase, reaching a value of 200 K or more at 0.1 mb. As emphasized by Combes et al. (1975), these results are valid only
Table 2. Retrieved temperature profiles from infrared data points in Table 1 and various 0.1 mb temperatures.

<table>
<thead>
<tr>
<th>Profile number</th>
<th>Temperature imposed at the 0.1 mb level (K)</th>
<th>Retrieved profiles</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td></td>
<td>Results</td>
</tr>
<tr>
<td></td>
<td></td>
<td>rms value (%) of the residuals</td>
</tr>
<tr>
<td>1</td>
<td>170</td>
<td>2.03</td>
</tr>
<tr>
<td>2</td>
<td>180</td>
<td>1.03</td>
</tr>
<tr>
<td>3</td>
<td>190</td>
<td>0.61</td>
</tr>
<tr>
<td>4</td>
<td>200</td>
<td>2.12</td>
</tr>
</tbody>
</table>

*D$_{max}$ is the maximum variation of temperature between two successive iterations.

$^\dagger q = P(H_2)/[P(H_2)+P(He)].$

with the assumption of a thermal symmetry of the atmosphere of Jupiter at these levels, a hypothesis which has been questioned by Young (1976).

4. Computations and results

A set of temperature profiles has been obtained from the infrared data points given in Table 1, and with the limiting conditions given in Table 2 concerning the value of the temperature imposed at the 0.1 mb level. The temperature profile at pressures higher than 500 mb was obtained by adiabatic extrapolation. The iterative process was stopped when, between two successive iterations, the rms value $\sigma$ of the flux residuals changed by less than $4\times10^{-6}$ W m$^{-2}$ (cm$^{-1}$)$^{-1}$ and that, simultaneously, the maximal variation of the temperature was less than 0.4 K.

The abundance of hydrogen, defined by the parameter

$$q = \frac{P(H_2)}{P(H_2)+P(He)},$$

(10)

where $P(H_2)$ and $P(He)$ are partial pressures of molecular hydrogen and helium, is obtained at the minimum of the rms residuals as a function of $q$. The accuracy of the determination of $q$ was evaluated as in Orton (1975a) following Bevington (1969), by

$$\Delta q = \left(\frac{m-n}{\sigma} \frac{d\sigma}{dq}\right)^{-1}.$$  

(11)

Since the number of independent data is limited to 3 (Section 2), the number of degrees of freedom $m-n$ is equal to 2.

The following method was applied in the evaluation of the effective temperature corresponding to the obtained profiles: the monochromatic outgoing flux was computed from the thermal profile in the spectral range 275–800 cm$^{-1}$; for wavenumbers $<275$ cm$^{-1}$, where absorption is due to NH$_3$, we used the values of Houck et al. (1975) from 250 to 275 cm$^{-1}$. For 250 cm$^{-1}$,

5. Results and discussion

The retrieved temperature profiles are shown in Fig. 3. All the profiles exhibit a similar troposphere but differ in the upper stratosphere because of the temperature constraint at the 0.1 mb pressure level. As a consequence of the important width of the CH$_4$ weighting function and of the high sensitivity of the Planck function to temperature at 7.7 $\mu$m, the CH$_4$ brightness temperature at 1260 cm$^{-1}$ depends strongly on the temperature at the 0.1 mb level. Therefore for a given brightness temperature at 1260 cm$^{-1}$, the extent of the possible values of the 0.1 mb temperatures is limited since they might lead to contradictory constraints preventing the iterative process to converge. As shown in Table 2, the rms values of the residuals at the final

![Figure 3](image_url)
iteration increase when the value of the 0.1 mb temperature is too low or too high. The rms results indicate 190 K as an optimal value at this level and in any case exclude 0.1 mb temperatures significantly larger than 200 K. This is also confirmed by a lack of agreement between the brightness temperature spectrum corresponding to model 4 and the data of Russel and Soifer (1977) between 1240 and 1290 cm$^{-1}$ (Fig. 6). This result could be modified if value of the CH$_4$ mixing ratio was significantly changed. Increasing the CH$_4$ abundance would permit a lower value for the higher limit of the 0.1 mb temperature, while decreasing the CH$_4$ abundance would increase the value or the upper limit.

Changing the mixing ratio modifies the stratospheric part of the retrieved profile but is not critical for inferring the tropospheric range and the H$_2$/He mixing ratio. The upper stratospheric parts of the retrieved profiles 2 and 3 are in a good agreement with model C of Wallace et al. (1974), but differ from the profiles inferred by Wallace and Smith (1976) (Fig. 4). These were the first to propose the introduction of occultation data as thermal constraints; however, the value of 200 K they adopted at the $5 \times 10^{-3}$ mb level leads to a stratospheric gradient in disagreement with occultation results (Combes et al., 1975).

Our results are in strong disagreement with the upper atmospheric profile inferred by Houck et al. (1976) which imposed a too cold isothermal upper stratosphere above the 1 mb level (155 K).

On the other hand, while the tropospheric part of our retrieved profiles is very close to the ones obtained by most other authors, the position and the temperature value of the tropopause agree only with the results of Houck et al. (1975). Fig. 5 shows the brightness temperature spectrum corresponding to profile 2 in the spectral range where the H$_2$-He collision-induced absorption is predominant. The temperature spectra corresponding to the other retrieved profiles are very close to the one shown since they depend mainly on the tropospheric thermal structure which is practically the same for all the profiles.

These temperature spectra fit well the experimental data of Houck et al. (1975) in the spectral range 273–350 cm$^{-1}$, where the measurements are expected to be the most accurate. Below 273 cm$^{-1}$, ammonia absorption should be included to fit the data, while the dip centered at 23 μm (435 cm$^{-1}$) has been recognized as spurious by Houck et al. (1975). The feature between 550 and 600 cm$^{-1}$ does not appear in the experimental results of Aumann and Orton (1976). For temperature recovery, we have chosen a data point at 600 cm$^{-1}$, where H$_2$-He maximum absorption occurs. Choosing instead a data point at 550 cm$^{-1}$ would lower the tropopause temperature and as a consequence the resulting brightness temperatures in the 16–24 μm range. However, even if we consider the increase of 3 K in Aumann and Orton's data when scaled to the 20 μm Pioneer 10 results (Aumann and Orton, 1976), a discrepancy of several degrees Kelvin would hold at 500 cm$^{-1}$ between our temperature brightness spectra at the center of the disk and Orton's results. This discrepancy definitely reflects a difference in calibration between these authors and Houck et al. (1975). The low 13 μm brightness temperatures of Aumann and Orton (1976) and the low Pioneer 10 limb darkening measurements clearly imply a colder tropopause temperature than the one deduced from the Houck et al. results (Orton, 1975a,b, 1977).

The hydrogen abundance values $q$ [Eq. (10)] corresponding to the various retrieved profiles are given in Table 2. The most accurate result, corresponding to the lowest rms value obtained, is $q = 0.89 \pm 0.04$. However, taking into account the uncertainty in the boundary conditions, it is more reasonable to admit $q = 0.88 \pm 0.06$ in good agreement with the estimation of Orton (1975 a,b); in addition the upper limit of 0.94 is in agreement with the results obtained from UV Pioneer 10 measurements (Carlson and Judge, 1974).
Fig. 5. Brightness temperature spectra for the full disk of Jupiter and at the center of the disk calculated in the H$_2$–He absorption range from profile 2. Also shown are full disk experimental results of Houck et al. (1975) and of Aumann and Orton (1976) obtained with an aperture smaller than the disk size.

Obviously, these error bars do not take into account possible uncertainties due to bad calibration of the measurements, inaccuracy of the absorption coefficients or the possible influence of particle and cloud opacities. Ultimately, the thermal profiles deduced from Pioneer 10/11 radio occultations (Kliore et al., 1976) indicated error bars so large that they are compatible with any of the profiles retrieved from infrared data.

The effective temperature is similar for all the retrieved profiles since it is only slightly sensitive to the thermal structure of the upper atmosphere. The computed value, taking into account the uncertainty of $q$ and the high wavelength part of the spectrum, is $T_e = 129.45 \pm 1.2$ K.

6. Conclusion

The application of a filtered Chahine method to presently available infrared data of the Jovian spectrum leads to the inference of a set of thermal profiles, and to the evaluation of the H$_2$/He mixing ratio of Jupiter. Our results were compared to those obtained by various authors who used different iterative schemes.

With respect to the inferred tropospheric region of the profile, our results are similar to those obtained by most of the other authors. At the tropopause level
there is a discrepancy in temperature between the different determinations. This discrepancy does not result from a difference between the inversion techniques used but from the different data used for inversion and reflects the difficulty of precisely calibrating planetary infrared spectra.

On the other hand, there is a large spread in the stratospheric thermal profiles inferred by various authors due to the fact that the information which can be extracted from presently available methane spectra at 7.7 μm is poor and that the upper stratospheric thermal structure has to be imposed a priori. However, the combination of data from β Scorpii occultation measurements and from methane spectra allowed us to determine a mean stratospheric thermal structure in good agreement with warm models of Wallace et al. (1974) who take into account ultraviolet absorption by dust.

The Jovian H₂/He mixing ratio inferred from infrared measurements is clearly close to the solar value, but uncertainties in absorption coefficients and the possible contribution of aerosol absorption to the total infrared opacity will probably prevent improvement of the present accuracy of this mixing ratio determination as long as the radiative properties of the clouds of Jupiter are not better known.

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