

Comparative Effects of Stability, Baroclinity and the Scale-Height Ratio on Drag Laws for the Atmospheric Boundary Layer

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ABSTRACT

The effects of baroclinity and the scale-height ratio on the drag laws of the planetary boundary layer (PBL) are examined theoretically and compared to those of stability. The similarity drag relations using surface geostrophic winds are found to be more sensitive to these parameters than the drag relations based on the layer-averaged winds. Since baroclinity can be more safely ignored in the latter, these are considered more suitable for parameterizing the PBL in general circulation models. The geostrophic drag relations based on the generalized similarity theory are used to explain (simulate) the observed increasing trend of the surface cross-isobar angle in going toward the equator. It is shown that this trend is partly due to the change in the scale-height ratio and partly due to baroclinity. Clarke and Hess (1975), on the other hand, have suggested that baroclinity is wholly responsible for this trend. It is shown here that baroclinity effects are very much exaggerated in their formulation.

1. Introduction

In a related paper (Arya, 1977; hereafter called A), we proposed a scheme of parameterizing the boundary layer fluxes in large-scale models of the atmosphere. The effects of baroclinity on the parametric drag relations based on a generalized PBL similarity theory (Arya and Sundararajan, 1976) were ignored for the sake of simplicity and only the effects of thermal stability and the scale-height ratio were considered. Consequently, the drag coefficient and the orientation angle of the surface stress could be determined as functions of a bulk Richardson number, the ratio h/z_0 (h representing the boundary layer height and z_0 the surface roughness parameter) and, for the unstable boundary layer capped by inversion, also the scale-height ratio fh/u_* , where f denotes the magnitude of the Coriolis parameter and u_* the so-called friction velocity. Here we examine the comparative effects of baroclinity vis-à-vis other parameters considered in A and determine the criteria when the former can be ignored. Certain hypotheses on the variation of the surface cross-isobar angle (α_0) with latitude, particularly over the tropical oceans, are also examined.

2. Effects of baroclinity on drag laws

Following A, we consider the two alternative forms of the drag laws which can be obtained from the usual

similarity matching arguments:

$$\left. \begin{aligned} ku_{g0}/u_* &= \ln \frac{h}{z_0} - A_0 \\ kv_{g0}/u_* &= -B_0 \operatorname{sgn} f \end{aligned} \right\}, \quad (1)$$

$$\left. \begin{aligned} ku_m/u_* &= \ln \frac{h}{z_0} - A_m \\ kv_m/u_* &= -B_m \operatorname{sgn} f \end{aligned} \right\}, \quad (2)$$

where u_{g0} and v_{g0} are the components of the surface geostrophic wind in x and y directions, respectively (we use a right-handed coordinate system for the Northern Hemisphere with the x axis oriented in the direction of surface shear and the z axis in the vertical), u and v are the actual wind components and u_m and v_m their layer-averaged (over the depth h) values, and A_0 , B_0 , etc., are universal similarity functions of certain dimensionless similarity parameters (these are listed in Table 1 of A for the two competitive similarity theories).

The effect of baroclinity on drag laws has not been well understood. Experimental studies have generally suffered from large sampling and other observational errors. Simultaneous measurements of all the relevant parameters are also lacking. Consequently, the functional dependence of the similarity functions A_0 , B_0 ,

etc., on the dimensionless baroclinity parameters does not appear to be well determined from observations alone, although some rough estimates have recently been made (Clarke and Hess, 1974). Here we shall evaluate these functions on the basis of certain theoretical and numerical modelling studies. But first, some useful relations are derived from the equations of mean motion without reference to any particular model.

Ignoring the entrainment effects and assuming that the momentum flux vanishes at the top of the boundary layer, which occurs somewhat above the inversion base, the integration of the equations of mean motion from $z=0$ to h gives

$$\left. \begin{aligned} \int_0^h v_g dz &= -(u_*^2/f) + hv_m \\ \int_0^h u_g dz &= hu_m \end{aligned} \right\} \quad (3)$$

The left-hand sides of Eqs. (3) can be expressed in terms of the surface components of geostrophic wind and shear for any given distribution of geostrophic shear in the form

$$\left. \begin{aligned} \frac{\partial u_g}{\partial z} &= \left(\frac{\partial u_g}{\partial z} \right)_0 F_1 \left(\frac{z}{h} \right) \\ \frac{\partial v_g}{\partial z} &= \left(\frac{\partial v_g}{\partial z} \right)_0 F_2 \left(\frac{z}{h} \right) \end{aligned} \right\} \quad (4)$$

Substituting from Eqs. (4) into (3), we obtain after some rearrangement of terms,

$$\left. \begin{aligned} \frac{v_{g0}}{u_*} = \frac{v_m}{u_*} - \left(\frac{fh}{u_*} \right)^{-1} - m_2 \left(\frac{\partial v_g}{\partial z} \right)_0 \\ \frac{u_{g0}}{u_*} = \frac{u_m}{u_*} - m_1 \left(\frac{\partial u_g}{\partial z} \right)_0 \end{aligned} \right\} \quad (5)$$

where m_1 and m_2 are related to the geostrophic shear distribution functions F_1 and F_2 as

$$m_1 = \int_0^1 \int_0^\xi F_1(\xi') d\xi' d\xi, \quad m_2 = \int_0^1 \int_0^\xi F_2(\xi') d\xi' d\xi. \quad (6)$$

Note that $m_1 = m_2 = \frac{1}{2}$ for the case of constant geostrophic shear, and $\frac{1}{3}$ for a linearly decreasing shear ($F_1 = F_2 = 1 - \xi$). Substituting from (5) into (1) and (2) yields

$$\left. \begin{aligned} A_0 &= \left(\ln \frac{h}{z_0} - k \frac{u_m}{u_*} \right) + m_1 k M_0 \cos \beta_0 \\ &= A_m + m_1 k M_0 \cos \beta_0 \\ B_0 &= -k \frac{v_m}{u_*} \operatorname{sgn} f + k \left(\frac{fh}{u_*} \right)^{-1} + m_2 k M_0 \sin \beta_0 \\ &= B_m + k \left(\frac{fh}{u_*} \right)^{-1} + m_2 k M_0 \sin \beta_0 \end{aligned} \right\} \quad (7)$$

in which the two baroclinity parameters are defined as

$$\left. \begin{aligned} M_0 &\equiv - \frac{h}{u_*} \left[\left(\frac{\partial u_g}{\partial z} \right)_0^2 + \left(\frac{\partial v_g}{\partial z} \right)_0^2 \right]^{1/2} \\ \beta_0 &\equiv \tan^{-1} \frac{(\partial v_g / \partial z)_0}{(\partial u_g / \partial z)_0} \end{aligned} \right\} \quad (8)$$

a. Unstable PBL

Note that Eqs. (3)–(7) are generally valid for any stability condition. To specialize them for the convective case, we take $v_m = 0$. A simple expression for the velocity in the mixed layer (u_m) can be obtained by assuming it to be equal to the velocity at the top of the surface layer and using the Monin-Obukhov similarity profile relations to this height h_s , i.e.,

$$\frac{u_m}{u_*} = \frac{1}{k} \left[\ln \frac{h_s}{z_0} - \psi_M \left(\frac{h_s}{L} \right) \right], \quad (9)$$

in which the function ψ_M is a known similarity function (Businger *et al.*, 1971). Following Wyngaard *et al.* (1974a), we take $h_s = -a_1 L$ with the proportionality coefficient a_1 being of the order 10. Then Eq. (9) yields

$$\frac{u_m}{u_*} = \frac{1}{k} \left[\ln \left(-\frac{L}{z_0} \right) - a_2 \right], \quad (10)$$

in which a_2 is a constant which is weakly dependent on the value of a_1 .

After substituting from Eq. (10), Eqs. (7) give

$$\left. \begin{aligned} A_0 &= \ln \left(-\frac{h}{L} \right) - a_2 + m_1 k M_0 \cos \beta_0 \\ B_0 &= k \left(\frac{fh}{u_*} \right)^{-1} + m_2 k M_0 \sin \beta_0 \end{aligned} \right\} \quad (11)$$

Without the last terms due to baroclinity, Eqs. (11) are consistent with our adopted parameterizations for the barotropic convective PBL (Arya, 1977). The effect of baroclinity is simply expressed by the additional terms which are proportional to the dimensionless magnitude of the surface geostrophic shear and cosine or sine of the angle between the same and the surface wind. From the similarity considerations alone, one could have expected A_0 and B_0 to be rather complicated functions of h/L , fh/u_* , M_0 and β_0 .

The above derivation is valid only in the free convection limit, which is reached in the actual atmosphere during periods of strong surface heating and light winds, i.e., when $-h/L \gtrsim 100$. The results of a second-order closure model used by Arya and Wyngaard (1975) have confirmed Eqs. (11). They also show that for moderately convective or unstable conditions, the numerical model results are better described by slightly modified rela-

tions of the form

$$\left. \begin{aligned} A_0 &= \bar{A}_0 \left(\frac{h}{L}, \frac{fh}{u_*} \right) + m_1 k M_0 \cos(\beta_0 - \delta) \\ B_0 &= \bar{B}_0 \left(\frac{h}{L}, \frac{fh}{u_*} \right) + m_2 k M_0 \sin(\beta_0 - \delta) \end{aligned} \right\} \quad (12)$$

in which A_0 and B_0 have been expressed as sums of barotropic parts \bar{A}_0 and \bar{B}_0 , and the deviations $A'_0 \equiv m_1 k M_0 \cos(\beta_0 - \delta)$, and $B'_0 \equiv m_2 k M_0 \sin(\beta_0 - \delta)$ due to baroclinity, and δ is a phase angle which essentially depends on h/L . Our numerical model results indicate δ values of 0° , 15° and 30° for $h/L = -1000$, -50 and 0 , respectively (Arya and Wyngaard, 1975).

b. Stable PBL

The stably stratified baroclinic PBL has not received as much attention from theoretical modelers as its convective counterpart has. Using a stability-dependent mixing-length model, Fiedler (1972) has shown that even under extreme conditions of cold or warm air advection the effect of baroclinity on A_0 and B_0 decreases rapidly with increasing stability (this does not, however, imply that the effect of baroclinity on wind and stress profiles in the PBL becomes any weaker—quite the contrary). Wippermann (1972), using a different type of mixing length model, has confirmed this result. His computed A'_0 and B'_0 for the case of constant geostrophic shear can be closely approximated by

$$\left. \begin{aligned} A'_0 &= n S_0 \cos(\beta_0 - \delta) \\ B'_0 &= n S_0 \sin(\beta_0 - \delta) \end{aligned} \right\} \quad (13)$$

where S_0 is the magnitude of geostrophic shear normalized by f , n is a function of stability parameter $\mu_* \equiv u_* / fL$ (for $\mu_* = 0, 17.5, 37.5$ and 75 , the calculated $n = 0.028, 0.021, 0.014$ and 0.009 , respectively), and $\delta \approx 35^\circ$ independent of stability. Alternatively, A'_0 and B'_0 can also be expressed in the form we have adopted for the unstable PBL with the same constant coefficients m_1 and m_2 ($m_1 = m_2 = 0.5$ for the case of constant geostrophic shear), i.e.,

$$\left. \begin{aligned} A'_0 &= m_1 k M_0 \cos(\beta_0 - \delta) \\ B'_0 &= m_2 k M_0 \sin(\beta_0 - \delta) \end{aligned} \right\} \quad (14)$$

Thus, (12) may be considered generally valid, in which stability determines the value of δ .

The similarity functions A_0 and B_0 , which relate the surface stress vector to the surface geostrophic wind, are related to the alternative pair A_m and B_m through Eqs. (7). The latter forms are more suitable for parameterizing surface fluxes in global models, since geostrophic winds become irrelevant and meaningless in low lati-

TABLE 1. Computed drag parameters under different stability and baroclinity conditions.

| Specified parameters | | | | Computed parameters | | | | |
|----------------------|----------|-------|-----------------|---------------------|----------------------|------------------|----------------------|-------|
| h/L | fh/u_* | M_0 | β_0 (deg) | α_0 (deg) | $C_{D0} \times 10^3$ | α_m (deg) | $C_{Dm} \times 10^3$ | |
| -50 | 1.0 | | 0 | — | 3.3 | 3.01 | 0.0 | 3.29 |
| | | | 10 | 0 | -1.4 | 5.78 | -4.2 | 3.22 |
| | | | 10 | 90 | 19.9 | 3.12 | -0.6 | 3.83 |
| | | | 10 | 180 | 5.9 | 1.79 | 4.3 | 3.35 |
| | | 10 | 270 | -11.6 | 2.51 | 0.5 | 2.86 | |
| 10 | 0.10 | | 0 | — | 34.1 | 0.242 | 26.1 | 0.285 |
| | | | 10 | 0 | 34.0 | 0.284 | 23.1 | 0.289 |
| | | | 10 | 90 | 38.6 | 0.241 | 26.5 | 0.315 |
| | | | 10 | 180 | 34.2 | 0.208 | 28.9 | 0.279 |
| | | | 10 | 270 | 29.7 | 0.239 | 25.7 | 0.259 |

tudes. From Eqs. (7) and (12), we have (writing $A_m = \bar{A}_m + A'_m$ and $B_m = \bar{B}_m + B'_m$)

$$\left. \begin{aligned} A'_m &= m_1 k M_0 [\cos(\beta_0 - \delta) - \cos\beta_0] \\ B'_m &= m_2 k M_0 [\sin(\beta_0 - \delta) - \sin\beta_0] \end{aligned} \right\} \quad (15)$$

where A'_m and B'_m represent the deviations due to baroclinity; we expect these to vanish under convective conditions when $\delta \approx 0$. Even for the near-neutral and stably stratified conditions, the amplitudes of A'_m and B'_m are likely to be only about half those of A'_0 and B'_0 . Thus, when baroclinity has to be ignored, for reasons of simplicity for example, the drag relations (2) should be preferable to (1). The errors introduced by the neglect of baroclinic effects would of course depend on the magnitude of the surface geostrophic shear (more appropriately, on M_0) as well as on stability. Table 1 below brings out the effect of baroclinity on various quantities for two different stability conditions ($h/L = -50$ and 10). Here we have assumed $m_1 = m_2 = 0.50$ and $k = 0.35$. Also we have taken a fixed value of $h/z_0 = 10^5$, although for a given land site this ratio could differ by an order of magnitude under widely different stability conditions. For the barotropic parts of similarity functions we have used the following relations taken from our paper A:

$$\left. \begin{aligned} \bar{A}_0 &= \bar{A}_m = \ln(-h/L) + \ln(fh/u_*) + 1.5 \\ \bar{B}_0 &= k(fh/u_*)^{-1} + 1.8(fh/u_*) \exp(0.2h/L) \\ \bar{B}_m &= 1.8(fh/u_*) \exp(0.2h/L) \end{aligned} \right\} \text{for } \frac{h}{L} \leq -2, \quad (16)$$

$$\left. \begin{aligned} \bar{A}_0 &= \bar{A}_m = -0.96(h/L) + 2.5 \\ \bar{B}_0 &= 1.15(h/L) + 1.1 \\ \bar{B}_m &= 0.80(h/L) + 1.1 \end{aligned} \right\} \text{for } \frac{h}{L} \geq 2. \quad (17)$$

The angle α_m between the layer-averaged wind vector $V_m \equiv (u_m, v_m)$ and the surface shear vector τ_0 (α_m measured clockwise from the direction of the latter is con-

sidered positive), and the drag coefficient $C_{Dm} \equiv u_*^2/V_m^2$ are given by Eqs. (2) as

$$\left. \begin{aligned} \tan \alpha_m &= B_m / [\ln(h/z_0) - A_m] \\ C_{Dm} &= k^2 / \{ [\ln(h/z_0) - A_m]^2 + B_m^2 \} \end{aligned} \right\} \quad (18)$$

The surface cross-isobar angle (α_0) and the geostrophic drag coefficient (C_{D0}) are similarly expressed in terms of A_0 and B_0 using Eqs. (1).

Table 1 illustrates that α_m and C_{Dm} are much less sensitive to baroclinity as compared to α_0 and C_{D0} . For $M_0 = 10$, which represents a substantial geostrophic shear ($\partial V_g / \partial z = 1$ and $10 \text{ m s}^{-1} \text{ km}^{-1}$ for the unstable and stable cases, respectively), α_m differs from its barotropic value by less than 5° and C_{Dm} by less than 15%. Therefore, one may perhaps be justified in ignoring the effects of baroclinity while parameterizing the surface momentum fluxes in a GCM in terms of the layer-averaged GCM computed variables (wind, temperature and humidity). Details of such a parameterization scheme are given in A (see also Deardorff, 1972b).

3. The variation of α_0 in low latitudes

Observations over the oceans clearly indicate that on the average the surface cross-isobar angle (α_0) increases toward the equator (Gordon, 1952; Riehl, 1954; Brummer *et al.*, 1974). The simplest physical explanation for this in qualitative terms was given by Gordon (1952), *viz.*, the observed trend is due to the decrease in the Coriolis parameter f toward the equator (see also Arya and Wyngaard, 1975). On the other hand, Clarke and Hess (1975) have suggested that the observed trend in α_0 is entirely due to the effects of baroclinity. The comparative effects of baroclinity and rotation (more appropriately, the scale-height ratio) on α_0 are examined further to resolve the above controversy.

a. Theoretical considerations

It is clear that the earth's rotation is primarily responsible for making the free atmospheric flow isobaric or geostrophic. It is mainly the friction which deflects (veers) this flow and produces a cross-isobaric component in the lowest layer (PBL). A good measure of the relative effects of rotation and friction in the PBL is provided by the parameter fh/u_* , which may be interpreted as the ratio of the two height scales h and u_*/f . In the absence of rotation ($fh/u_* = 0$) the flow is expected to be essentially down the pressure gradient ($\alpha_0 = 90^\circ$) as in the case of laboratory pipe and channel flows.

In the atmosphere rotational effects do vanish in going toward the equator. However, the rapid change of f near the equator, which gives rise to advective accelerations, makes the channel flow analogy (Csanady, 1974) somewhat weaker. In the PBL, thermal stratification and baroclinity are additional

factors which influence the cross-isobaric flow and hence α_0 . Their relative influence in determining the variation of α_0 with latitude, excluding the narrow equatorial region (so that acceleration effects may be ignored), can be assessed from the parametric relations (1) which give

$$\tan \alpha_0 = \frac{B_0(fh/u_*, h/L, M_0, \beta_0)}{\ln(h/z_0) - A_0(fh/u_*, h/L, M_0, \beta_0)}$$

or using Eqs. (12) with $m_1 = m_2 = m$,

$$\tan \alpha_0 = \frac{\bar{B}_0(fh/u_*, h/L) + mkM_0 \sin(\beta_0 - \delta)}{\ln(h/z_0) - \bar{A}_0(fh/u_*, h/L) - mkM_0 \cos(\beta_0 - \delta)} \quad (19)$$

Here we are interested only in the zonally averaged flow conditions. In low latitudes, where thermal winds oppose the easterly geostrophic flow, we have according to the conventional definition of α_0 (it is considered positive when the surface cross-isobaric flow is toward the low pressure)

$$\beta_0 = 180 - \alpha_0. \quad (20)$$

After substituting from (20) and some algebra, Eq. (19) reduces to

$$[\ln(h/z_0) - \bar{A}_0] \sin \alpha_0 - \bar{B}_0 \cos \alpha_0 = mkM_0 \sin \delta. \quad (21)$$

Expressing (21) as a quadratic equation for $\sin \alpha_0$, the two roots are given as

$$\left. \begin{aligned} \sin \alpha_0 &= \frac{acM_0}{a^2 + \bar{B}_0^2} \pm \left[\frac{a^2c^2M_0^2}{(a^2 + \bar{B}_0^2)^2} - \frac{c^2M_0^2 - \bar{B}_0^2}{a^2 + \bar{B}_0^2} \right]^{1/2} \\ \text{or} \\ \sin \alpha_0 &= \frac{acM_0}{a^2 + \bar{B}_0^2} \pm \frac{\bar{B}_0}{(a^2 + \bar{B}_0^2)^{1/2}} \left[1 - \frac{c^2M_0^2}{a^2 + \bar{B}_0^2} \right]^{1/2} \end{aligned} \right\} \quad (22)$$

in which

$$a \equiv \ln(h/z_0) - \bar{A}_0, \quad c \equiv mk \sin \delta. \quad (23)$$

Noting that the coefficient $c \lesssim 0.1$, and over the ocean surface $a^2 + \bar{B}_0^2 \gtrsim 150$, the following condition is likely to be satisfied for $M_0 \lesssim 50$ (this value will be reached only under conditions of extreme geostrophic shears):

$$c^2M_0^2 \ll a^2 + \bar{B}_0^2. \quad (24)$$

Then (22) can be approximated to

$$\sin \alpha_0 \approx \frac{\bar{B}_0}{(a^2 + \bar{B}_0^2)^{1/2}} + \frac{acM_0}{a^2 + \bar{B}_0^2}. \quad (25)$$

Here we have retained only one root of (22) which is physically consistent with the requirement that $\sin \alpha_0 \geq 0$ under barotropic conditions (this requirement follows from our definition of α_0).

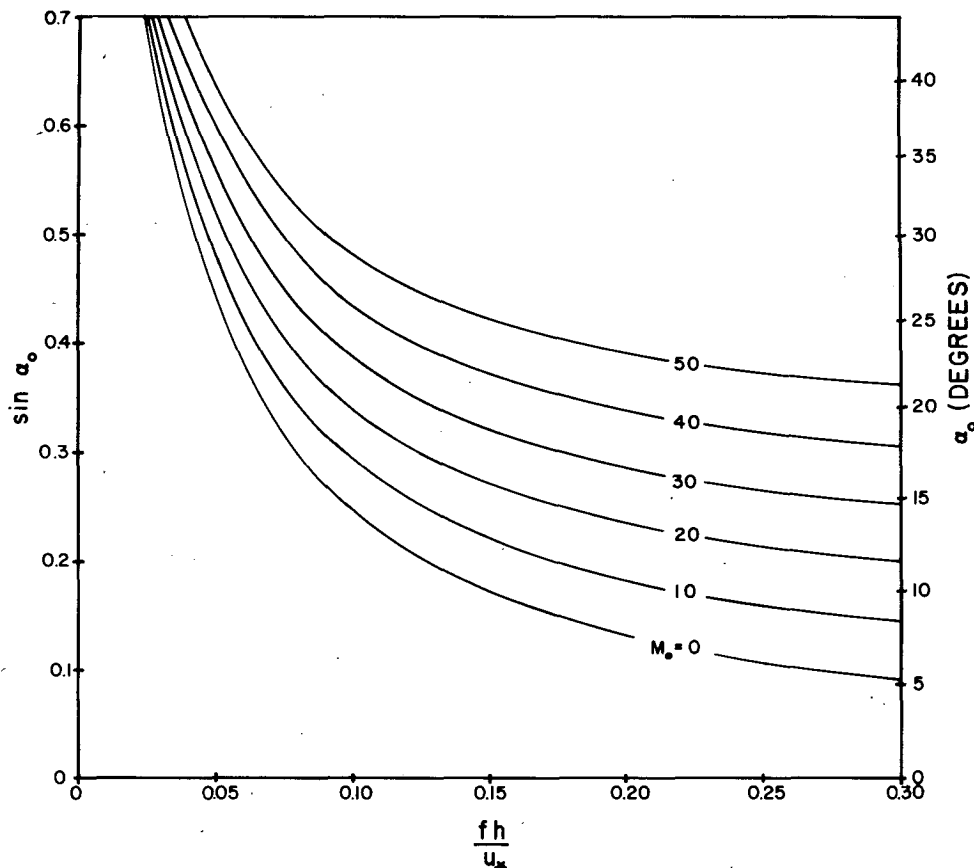


FIG. 1. The variation of α_0 with fh/u_* and M_0 ; $h/z_0=10^7$ and $h/L=-20$.

Eq. (25) is valid only when the surface geostrophic and thermal winds are parallel and opposite, as often occurs in the trades (Brummer *et al.*, 1974). The first term in the above expression of $\sin\alpha_0$ is obviously the barotropic part $\sin\alpha_0$, while the second term represents the deviation due to baroclinity $(\sin\alpha_0)'$. The relative contribution of the latter is directly proportional to the baroclinity parameter M_0 and also depends on h/z_0 , h/L and fh/u_* .

b. Some results of simulations

Boundary layer parameters being highly variable for land areas, any systematic trend of α_0 with latitude can be expected only over the oceans. This is quite evident from climatological data (Gordon, 1952) and also from the averaged data from several field experiments (Brummer *et al.*, 1974). We do not intend to simulate such climatic trends as Clarke and Hess (1975) have done, because there appears to be no way of determining or even defining the appropriate averages of the relevant PBL similarity parameters. Instead, we shall use Eq. (25) to simulate the trends of α_0 with latitude and the relative contributions of baroclinity and fh/u_* to the same under a variety of assumed conditions. The comparatively minor effects of the changes in h/z_0 and

h/L on this trend can be ignored by assuming some fixed values for these parameters (α_0 is rather insensitive to even an order of magnitude variations in h/z_0 and, under moderately convective conditions, also to similar changes in h/L). For the tropical oceans in the winter hemisphere, typically we assume $h/z_0 \approx 10^7$ and $h/L \approx -20$. We also assume a linear variation of geostrophic wind with height ($m=0.5$). For these conditions, the result of (25) is graphically represented in Fig. 1, which clearly brings out the relative effects of variations in fh/u_* and M_0 on the surface cross-isobar angle. The effect of latitude is implicit in both these parameters—more directly so in fh/u_* .

The boundary layer height h is perhaps the most important quantity which appears in all the similarity parameters. There is much controversy on the variation of h with latitude. For an idealized steady, homogeneous and neutral PBL, which is not constrained by any inversion from above, the simplest dimensional arguments yield

$$h \sim u_*/f. \quad (26)$$

This indeed is the basic premise of the similarity theory proposed by Kazanski and Monin (1960), which has also been extended to stratified PBL's (Zilitinkevich *et al.*, 1967). It has been lately recognized, however,

that (26) is not valid for the unstable PBL (Deardorff, 1972a; Wyngaard *et al.*, 1974b) and also for the near-neutral PBL capped by a low-level inversion (Pollard *et al.*, 1973). In these cases, h is essentially determined by the average height of the inversion base z_i , which in low latitudes is much smaller than the scale height u_*/f . The limited expedition data that are available (Riehl, 1954; Augstein *et al.*, 1974) indicate that h varies considerably both in space and time; in certain regions it does have an increasing trend toward the equator, but certainly not in proportion to u_*/f . Even on the equator, h rarely exceeds 2000 m during fair weather conditions (see Augstein *et al.*, 1974).

For the purpose of our simulations, we consider three simple models of the variation of h/u_* with latitude as shown in Fig. 2. Of these, the curve III representing Eq. (26) and the curve I representing $h/u_* = \text{constant}$ are the two extremes, while the middle curve II is probably more close to reality. The corresponding variations of α_0 with latitude for the same three models, according to Eq. (25), are shown in Fig. 3. Here the full curves pertain to barotropic conditions, which clearly show that, unless the boundary layer height increases as dramatically or sharply as implied by Eq. (26), α_0 must increase with decreasing latitude essentially due to the change in the rotational parameter.

In order to investigate the effect of baroclinity, we need to know about the variation of the surface geostrophic shear $(\partial V_g/\partial z)_0$ with latitude. The geostrophic shear, being related to the spatial distribution of the surface air temperature, is highly seasonal and also spatially variable. The estimated values of $(\partial V_g/\partial z)_0$

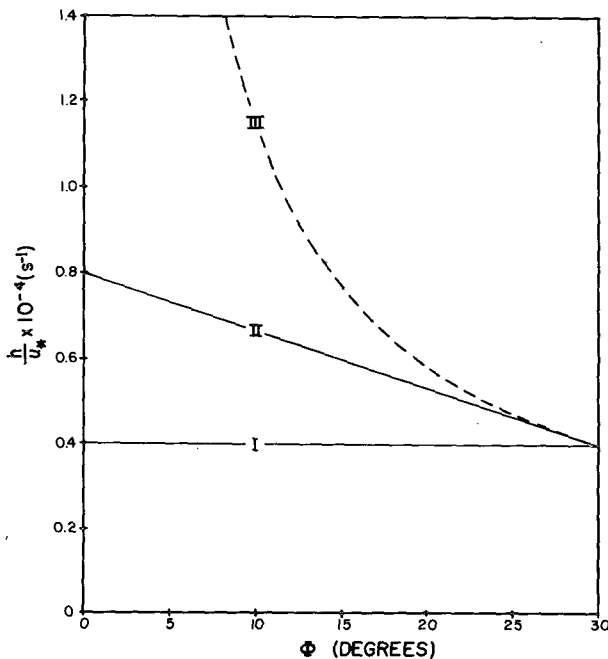


FIG. 2. Assumed distributions of h/u_* with latitude.

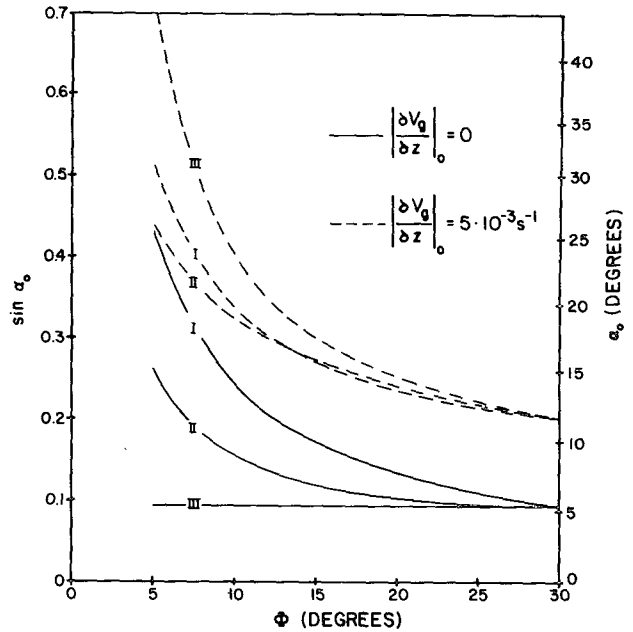


FIG. 3. Simulated variations of α_0 with latitude for the assumed h/u_* distributions and $(\partial V_g/\partial z)_0 = 0$ and $5 \times 10^{-3} \text{ s}^{-1}$.

from the zonally averaged surface air temperatures for the winter hemisphere range from 1.7×10^{-3} to $3.0 \times 10^{-3} \text{ s}^{-1}$ with the maximum occurring around 10° latitude (Clarke and Hess, 1975). The daily averaged values during the Atlantic Trade Wind Experiment (carried out in February near 10°N) ranged from 3.6×10^{-3} to $6.3 \times 10^{-3} \text{ s}^{-1}$ with an average value of about $5 \times 10^{-3} \text{ s}^{-1}$ (Brunner, 1976).

The dotted curves in Fig. 3 correspond to a constant geostrophic shear of $5 \times 10^{-3} \text{ s}^{-1}$, independent of latitude; their upward shift from the corresponding full curves would be proportionately reduced wherever smaller geostrophic shears were to prevail. Note that the comparative effects of baroclinity and fh/u_* in the simulated trends of α_0 with latitude depend strongly on the assumed variation of h/u_* as well as on $(\partial V_g/\partial z)_0$. Curiously, for a particular choice of geostrophic shear (say, about $4 \times 10^{-3} \text{ s}^{-1}$), the two extreme distributions (I and III) of h/u_* vs Φ would yield very similar results of α_0 vs Φ , but for completely different reasons. In I, the increase in α_0 toward the equator is entirely due to changes in the Coriolis parameter, while in III this trend is entirely due to baroclinity (see e.g., Clarke and Hess, 1975). Of course, the truth lies somewhere in between these two extremes; for distribution II the contributions of baroclinity and the ratio fh/u_* to the trend of α_0 with latitude are of the same order of magnitude.

4. Conclusions

The effects of baroclinity on the similarity forms of drag laws of the atmospheric boundary layer are investigated theoretically. The results for all stability

conditions indicate that the baroclinity effect on any similarity function can be simply considered by adding a baroclinic perturbation to its barotropic counterpart. This is a generalization of the result obtained earlier for the unstable case (Arya and Wyngaard, 1975). Furthermore, it is shown that the geostrophic drag laws are more sensitive to baroclinity than the drag laws based on the layer-averaged mean variables. The use of the latter has been particularly recommended for the parameterization of boundary layer in atmospheric circulation models (Deardorff, 1972b; Arya, 1977). Here it is shown that the baroclinity effects may be safely ignored so that the simple parameterization schemes proposed in A are probably adequate.

The geostrophic drag law based on the generalized similarity theory is used to simulate the observed trend with latitude of the surface cross-isobar angle (α_0) over the oceans (Gordon, 1952; Brummer *et al.*, 1974). Gordon's intuitive explanation that α_0 increases toward the equator mainly in response to the decrease in the Coriolis force is found to be partly correct (see Arya and Wyngaard, 1975). This has been questioned recently by Clarke and Hess (1975) who attributed the observed trend in α_0 wholly to the effects of baroclinity. It is shown here that the baroclinity effects are much exaggerated in the similarity formulation used by Clarke and Hess, because the boundary layer height is wrongly assumed to be equal to $0.3 u_* / f$ even in low latitudes while there is an overwhelming evidence against such an assumption (Augstein *et al.*, 1974). It appears that the trends of both M_0 and fh/u_* with latitude are important in causing the surface cross-isobar angle to increase toward the equator.

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