A Severe Downslope Windstorm and Aircraft Turbulence Event Induced by a Mountain Wave

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ABSTRACT

A detailed analysis is presented of the large-scale, mesoscale and turbulent-scale features of a major downslope windstorm event in central Colorado on 11 January 1972. The storm is found to be associated with a moderate amplitude baroclinic disturbance moving across the northwestern United States within an intense zonal current. Optimal conditions for strong mountain wave generation are detectable from sounding data 12–24 h in advance and about 1000 km upstream. The mesoscale structure is dominated by a single quasi-hydrostatic wave of extreme amplitude and variable location, with corresponding variations in the windstorm structure.

Severe to extreme aircraft turbulence is observed in a deep boundary layer over the region of strong surface winds and also in a separate mid-tropospheric turbulence zone. Analysis of the latter shows that it originates in a region of intense wave-generated shear and is then carried downstream by the mean flow and upward by the wave motion. Energy generation and dissipation rates of order 1 m² s⁻³ are observed. Comparisons of the turbulence features with the theoretical solutions for shearing instability by Tanaka and by Lee and Merkine show fair agreement.

Effects of the wave-windstorm-turbulence event on the larger scales are complex, involving both a substantial removal of westerly momentum and a three-dimensional redistribution of mass.

Hazards to aircraft from this kind of event are illustrated and discussed. Avoidance by vertical path deviation is found to be impractical.

1. Introduction

On 10 and 11 January 1972, a moderately intense middle-tropospheric weather system embedded in a persistently strong northwesterly flow regime passed across the northwestern United States, accompanied by a number of reported windstorm events and significant aircraft turbulence. In Colorado the system produced a major downslope windstorm along the eastern slope of the Rocky Mountains on 11 January, with wind gusts of over 50 m s⁻¹ in Boulder and substantial attendant damage. A meteorological narrative of the windstorm and a partial analysis of data collected by two NCAR aircraft was given by Lilly and Zipser (1972). During and near the time of the surface windstorm a large number of moderate and severe clear air turbulence incidents were reported by other aircraft flying over central Colorado.

A theoretical analysis of the downslope windstorm phenomenon as observed in Colorado has been presented by Klemp and Lilly (1975). These storms, including the 11 January event, were shown to be reasonably well explained by inviscid hydrostatic wave dynamics, in spite of the large values of energy dissipation involved in the turbulent breakdown regions. Linear theory was shown to give useful predictions of the form and intensity of the waves, and appeared to be more suitable for this purpose than two-layer hydraulic jump theory. The effect of terrain shape on wave amplitude has been shown (Smith, 1977; Lilly and Klemp, 1978) to be largely controlled by nonlinear dynamics. In this paper we present a more detailed observational analysis of the large-scale, mesoscale and especially the turbulent-scale features of the 11 January event in the middle and upper troposphere.

This analysis has three principal objectives. The first is to exhibit the scale interactive aspects of the downslope windstorm phenomenon. We will show that the source of energy and the vertical structure which allows that energy to be tapped arises out of the global, synoptic and meso-α scales [250–2500 km, as defined by I. Orlanski (personal communication)], while the principal storm energetics reside on the meso-β (25–250 km) scale, with important elements on the meso-γ (2.5–25 km) and turbulent microscales, on which we concentrate special attention. The release of energy in the mesoscales and microscales is accompanied by
significant feedback effects to the larger scales in the form of downward momentum transport and alterations of the thermodynamic and kinematic structure. The downslope windstorm in these respects is similar to severe convective storm systems, though dynamically very different. As with the convective storm problem, accurate 6–24 h prediction of downslope windstorms requires accuracy in predicting the meso-α scale evolution which establishes the critical vertical profiles of wind and temperature, while short-term local warning effectiveness probably requires high-resolution observing networks and rapid communication links.

The second objective is to describe as completely and accurately as the data allow the dynamical structure and evolution of the unique observational feature found in this event, a region of intense turbulence generated by instability of the primary standing gravity wave. Although the existence of this feature was not anticipated, our observational procedures were in some respects almost ideally suited to its definition. Nevertheless, there are fairly severe limitations to the quantity and quality of the data, and the gaps have been filled in some cases with plausible assumptions. The data fit into a coherent picture, and there are enough cross-checks available to provide reasonable evidence that it is the correct picture. The attempt to squeeze as much as possible out of this data set is justified by its uniqueness, since it was obtained during the course of an unusual and important event with a return period of several years.

The third objective is to document the hazard to aircraft of this uncommonly dangerous meteorological phenomenon. Each year many small “general aviation” aircraft are lost over mountainous terrain in the western United States due to weather-related accidents, many of which involve wave downdrafts and turbulence. Even large commercial carriers are not immune to a risk of accident in these conditions, as will be shown.

The synoptic and mesoscale features of the 11 January wave and windstorm event are examined in detail in Section 2, which extends the early results reported by Lilly and Zipser (1972). Section 3 contains analyses of the structure, dynamics and energy budget of the mid and upper tropospheric turbulence zone. Section 4 discusses the feedback problem, the effects of this and other strong wave events on the larger scales. Section 5 discusses the aspects of air safety in severe wave events.

2. Synoptic and mesoscale structure

The large-scale meteorology of January 1972 was dominated in the western United States by strong westerly and northwesterly flow. This is illustrated by Figs. 1 and 2, which show the monthly mean contours at 700 mb and their anomalies from a long-term mean.

![Figure 1](image_url)

**Fig. 1.** Mean height contours (dam) at 700 mb for January 1972 (from Wagner, 1972).
The mean tropospheric profile of the westerly wind component for the month at Grand Junction, Colo., about 200 km west of the Continental Divide, is shown in Fig. 3, together with the 1948–70 January mean profile for Grand Junction and the profile observed on 11 January 1972, upstream of the storm observations to be presented. A profile like that of 11 January is somewhat unusual, but one would expect it to occur fairly frequently during a month with anomalously strong westerlies. About one-fifth of the January 1972 readings showed similar wind profiles. Several strong downslope storms occurred during that month. That of the 11th was by far the most damaging and widespread, however, with gale force or stronger winds observed at most stations between Casper, Wyo., and Pueblo, Colo., i.e., within 250 km north and south of Denver and Boulder.

An additional climatological aspect of some interest is the wind speed on Mines Peak, a mountain-top station along the Continental Divide 70 km west of Denver at 3809 m MSL (pressure ~640 mb). This station recorded a mean wind speed during January 1972 of 20 m s⁻¹, the highest monthly mean of the eight years of observation before that time and the five years since (A. Judson, U. S. Forest Service, Fort Collins, Colo., personal communication). This speed is substantially greater than the mean westerly component observed at the same altitude above Grand Junction, presumably because of acceleration over the mountain and also because the wind direction was not always westerly.

The large-scale meteorological sequence just before and during the 11 January storm is shown by the 12 h surface and 500 mb maps from 0000 on 11 January to 0000 (all times GMT) 12 January (Fig. 4). Although the upper air flow pattern appeared to change little during this time, a surge of warmer air moved across the northwestern United States at 500 mb, accompanied by rapid pressure falls and strong surface cyclogenesis in the lee of the Rocky Mountains.

The changes in tropospheric structure are shown by spatial cross sections from 0000 on 11 January to 0000 on 12 January (Fig. 5) along an approximate mean streamline from Quillayute, Wash. (48°N, 124°W, symbol UIL), to Dodge City, Kans. (38°N, 100°W, DDC), running through Boise, Idaho (BOI) through a point midway between Salt Lake City, Utah, and Lander, Wyo. (LND/SLC), then through Denver (DEN) and Dodge City. These cross sections show the eastward motion of the mid-tropospheric warm air, as seen from the contours of 12 h temperature change. The strong temperature increase at Denver between 1200 of 11 January and 0000 of 12 January was caused in part, however, by the local wave downslope motions. Time sections at Quillayute, Boise and Denver (Fig. 6) further illustrate the sequence, with the motion of the 310 K isentrope indicating the advection of warm air in the mid-troposphere, accompanied by a rising tropopause and somewhat increased stability in the lower troposphere. This feature moved from the Washington coast to Colorado in about 18 h with a propagation speed of about 30 m s⁻¹, quite rapid by most standards but considerably less than the speed of the mid and upper tropospheric flow. Computations of various isentropic trajectories arriving in Denver on
11 and 12 January (performed by D. Deaven of NCAR) typically show the sources to be over the eastern Pacific only 12 h earlier.

The changes in tropospheric thermal structure, along with the more subtle wind profile developments, evidently controlled the outbreak of the downslope windstorm. Brinkmann (1974) showed that the atmospheric vertical profiles associated with strong downslope winds usually contain a layer of high thermal stability just above mountaintop level. Klemp and Lilly (1978) showed that this feature, plus a tropospheric temperature and wind speed profile such that the distance from the mountaintops to the tropopause is approximately half the vertical wavelength of a stationary hydrostatic wave, produces maximum wave and surface wind amplitude. At the bottom of Fig. 5 we have shown at various points values of the predictions of potential downslope windstorm intensity made
from the Klemp-Lilly linear numerical model using the atmospheric structure at the corresponding point and an assumed mountain height of 2 km above a lee plain. From this figure we see that the potential for strong windstorm conditions could have been detected from the Boise sounding at 0000 on 11 January, about 18 h before the occurrence in eastern Colorado. A straightforward advective extrapolation of the vertical structure would be of doubtful accuracy in most situations, but the data give encouragement to the application of high-resolution numerical prediction models to downslope windstorm forecasting.

The mesoscale structure of the flow over and near the Front Range west of Denver and Boulder has been deduced principally from flight data from NCAR aircraft, with the aid of surface pressure and wind observations from a number of special recording stations operated by or for NCAR. Fig. 7 shows a composite analysis of potential temperature from the aircraft flight data, presented earlier by Lilly and Zipser. The dashed lines show the aircraft tracks. The lower right-hand portion of the analysis was produced from data obtained from a Beech Queen Air twin engine propeller aircraft which took off from the Jefferson County (Jeffco) airport at about 2000 GMT (1300 local time) and returned at about 2200 GMT. The remainder of the analysis utilized data obtained by a North American Sabreliner, a twin engine jet, which left Jeffco about 0000 and returned to Denver Stapleton Airport at about 0300 (all times GMT), Jeffco being unusable due to high winds and hangar damage at that time. The noncoincidence of the flights was caused by equipment problems and airport operational hazards associated with the windstorm. The heavy dashed line between the two analysis sections indicates that they should probably not be combined, as was initially suspected from the very large upstream tilt between the waves at the 15,000 and 20,000 ft (4.5 and 6 km) levels.

Surface anemometer and pressure observations strongly support this conclusion. Winds in the Boulder area and at Jeffco showed strong maxima during the period of the Queen Air flight, but exhibited a general lull, especially at Jeffco, during the Sabreliner flight. On the other hand, stations in the foothills west of Boulder experienced maximum wind speeds during the Sabreliner flight. These data are consistent with an oscillation of the position of the principal wave trough and surface windspeed maximum, which apparently almost coincide, from near or east of Boulder between 1800 and 2200 to west of Boulder in the foothills from 2200 to 0300, and then back again.

Additional evidence supporting this oscillation is obtained from the records of barometric pressure at stations in and near the mountains. Fig. 8 (solid line) shows the measured pressure difference between two mountain stations, one located just upwind of the crest of the Front Range (the Sitzmark Ski Lodge at Hideaway Park, altitude 2660 m) and one at just 30 m higher in the eastern foothills, named Fritz Peak, but actually located on fairly level terrain upwind of an
isolated hill. The dashed line in Fig. 8 shows the pressure difference between Sitzmark and the NCAR building in Boulder, at an altitude difference of 830 m.

These records show that the pressure at Boulder fell about 10 mb relative to that at Sitzmark during the morning of the 11th, coinciding with the beginning of the strong wave-windstorm situation, remained low through the afternoon and evening, and returned rapidly on the morning of the 12th. The Arctic front passed Boulder about 1000Z on the 12th, accounting for the further pressure rise at that time. The Sitzmark-Fritz Peak difference plot is rather more complex, with the dominant features during the windstorm period a maximum in the morning of the 11th, a sharp minimum near noon and a very strong maximum in the afternoon followed again by a sharp drop. The altitude difference of the stations (30 m) corresponds to about a 3 mb positive difference, which is close to that observed during most of the 10th and 12th. The strong windstorm periods in Boulder occurred during the pressure difference minima, near midday and in the evening of the 11th, while the difference maximum coincided with the lull just before and during the Sabreliner flight.

The quantitative pressure differences must be treated with some caution since the barographs are located inside buildings and do not necessarily measure ambient static pressure. It is believed that the net effect of strong winds on a building is to reduce the average pressure inside, by some fraction of the Bernoulli pressure head, \( \rho v^2/2 \). Since strong winds at the downwind station coincided with the pressure difference minima between Sitzmark and NCAR and probably with the maxima between Hideaway Park and Fritz Peak, the observed pressure difference may be somewhat exaggerated.

To the extent that it is quantitatively valid, the pressure difference data between Sitzmark and Fritz Peak can be interpreted in terms of the momentum exchange between the earth and atmosphere. The vertical integral of the pressure differences \( \delta p \) across a mountain range of height \( H \)

\[
\int_0^H (\delta p)dz
\]
is the force exerted on a unit length of mountain range by the air and vice versa. If we assume that $\delta p$ decreases linearly with height to the top of the Front Range, about 1200 m above the two barograph stations, then the net force per unit length is $\delta p \times 600$ m. At times on 11 January the pressure difference (after removal of the effect of the 30 m altitude difference) attained 12 mb, and if two-thirds of this is real (excluding a rather large assumed building effect), the net force is about $5 \times 10^6$ N m$^{-1}$.

On the other hand, if we assume that the observed $\delta p$ remains constant to the top of the ridge and neglect the building effect, the net force is about $14 \times 10^6$ N m$^{-1}$. As will be shown, the momentum exchange observed by the aircraft lies within this range. During the strong downslope wind periods in Boulder, the pressure difference between Sitzmark and Fritz Peak becomes much smaller, but the total cross-mountain pressure force remains large, indicating that the momentum exchange remains large, but acts on a lower part of the mountain slope.

The surface drag of the downslope wind on roughness elements, foothills, etc. should be added to the cross-mountain pressure force, or wave drag, as these quantities are essentially independent. Even a frictionless mountain can produce wave drag in stratified air flowing over it. On the other hand, a neutral barotropic fluid crossing a rough mountain range will show no wave response and no cross-mountain pressure gradient if the geostrophic wind is perpendicular to the mountain range. We estimate the direct wind drag to be about $3 \times 10^6$ N m$^{-1}$, based on an rms surface velocity of 30 m s$^{-1}$ over 35 km of lee slope and a drag coefficient of 0.01. Thus it appears that this term is significantly smaller than the wave drag, though the uncertainties in both are rather large.

In Fig. 9 we show an analysis of the westerly wind component, slightly revised from a similar figure presented earlier by Klemp and Lilly (1975). This analysis used the Sabreliner data where available, and at lower levels is made to be consistent with the presumed coincidence of mean surface streamlines and isentropes with the contours on the lee slope. It is assumed that the flow is two-dimensional and steady. The total mass flow through the troposphere between the 200 and 700 mb levels upstream of the mountains must evidently pass through a channel of
weight 260 mb between the wave trough at the 6 km level and the ground surface, leading to a computed mean flow in that layer of greater than 60 m s\(^{-1}\). The procedure leading to the low-level analysis of Fig. 9 has previously been applied to the analysis of long mountain waves over the Colorado Front Range (Lilly and Kennedy, 1973) and is believed to be reasonably valid for use over this rather two-dimensional topography.

The coexistence of the observed weak winds in the wave trough above the 6 km level and the deduced very strong winds below that level requires a strong negative wind shear just downstream of the mountain crest. Although the details of the velocity distribution shown in our analysis cannot be directly verified, it appears that the maximum shear amplitude is at least 0.05 s\(^{-1}\), which occurs in conjunction with a Brunt-Väisälä frequency \(\frac{\partial^2 \theta}{\partial z^2}\), of order 10\(^{-2}\) s\(^{-1}\). Thus the Richardson number is apparently well under 0.25 for a horizontal distance of order 10 km, creating an environment favorable for generation of turbulent energy. Further quantitative discussion of these matters will be presented in the next section. In addition Klemp and Lilly (1978) have successfully simulated the principal elements of the observed wave pattern, using real upstream data and a nonlinear two-dimensional model.

3. The mid-tropospheric turbulence zone

Turbulence was observed in two distinct zones during the NCAR aircraft flights, as is indicated by the +'s on the aircraft trajectories in Figs. 7 and 9. The boundary layer over the strong surface winds was turbulent to a depth of about 1 km. This turbulence was probably generated principally by surface roughness, augmented by shears developed in the separation region, and then carried aloft to a depth of 2-3 km. Some of the data obtained from the lower turbulence zone of the 11 January flights were analyzed by Lester and Fingerhut (1974).

A second separate region of intense turbulence generation occurred about 2 km above and a few kilometers downstream of the crest of the mountains in
mountaintops, i.e., nearly to the tropopause. This mid-tropospheric turbulence zone is the primary subject of this section. The experience acquired from numerous research and commercial flights indicates that intense mid-tropospheric turbulence zones are not normal features of mountain waves in Colorado but occur only in very strong waves. Apparently more common is the development of turbulent breakdown near or above the tropopause, where wave energy combines with pre-existing strong wind shear or approaches a critical layer.

Fig. 10 summarizes the turbulence reports received on 11 January from flights over central Colorado. Most of these reports were collected and made available by Paul Kadlec of Continental Airlines. The locations of the reports cannot always be assumed to accurately reflect where the strong turbulence occurred, and the distribution is highly biased by traffic frequency. Nonetheless, it is clear that turbulence reports are heavily concentrated east of the Continental Divide at levels up to 39,000 ft (12,000 m). Most of these can probably be accounted for by either the boundary layer roughness zone or the mid-tropospheric turbulent zone. For one especially notable incident, identified with a star on Fig. 10, we were able to acquire the flight recorder traces of vertical acceleration and altitude. We present and discuss this material in Section 5. The analysis to be discussed in the remainder of this section

![Graph](image URL)

Fig. 8. Surface pressure at Sitzmark Ski Lodge, Hideaway Park, Colo., altitude 2660 m MSL, 10 km west of the Front Range, minus that at Fritz Peak, altitude 2690 m MSL, 10 km east of the Front Range (solid line); and surface pressure at Sitzmark minus that at NCAR, altitude 1860 m MSL, 40 km east of Sitzmark (dashed). Data taken from 9-12 January 1972. Stippled area indicates time of Sabreliner flight.

![Graph](image URL)

Fig. 9. Analysis of westerly wind component (m s^{-1}) on 11 January 1972, made from Sabreliner and sonde data only. The analysis below 470 mb over the eastern slope was deduced from assumptions indicated in the text.
is based on data recorded on the NCAR Sabreliner, principally from flight legs at approximately 6 and 9 km MSL. The lower of these two legs was apparently near the center of the generation region of the midtropospheric turbulent zone at the bottom of the primary wave. From the recorded data and their spectra we attempt to deduce the scale and growth characteristics of the turbulent eddies, and compare them with predictions of linear and nonlinear theory for shearing instability. From further analysis of the 6 km level plus that at 9 km, which contained a large area of decaying turbulence carried upward from the generation region, we estimate important terms in the momentum, energy and thermal variance budgets.

The principal Sabreliner instrumentation included a differential pitot gust probe system (North American Rockwell) for measuring airspeed, angle of attack and sideslip, and a fast response thermometer (Rosemount No. 101E2AL), both installed at the end of a 5 m boom. A Litton LTN-51 inertial navigation system measured ground speed, all relevant orientation angles, vertical and horizontal acceleration and position with sufficient accuracy to allow horizontal ground speed determination of order 1 m s⁻¹ in absolute precision and substantially better resolution at high frequencies. The data were recorded digitally and later processed numerically.

Because this flight program was one of the first using the Sabreliner gust probe system, several hardware and recording problems were discovered, most of which were remedied by digital processing techniques. Though
the gust probe velocity, temperature and pressure sampling rates were either 4 or 8 s⁻¹, the effects of boom vibration and some almost irremedial data handling problems restricted our analysis to frequencies <0.5 Hz. No serious aliasing effects are believed to be present in the final data, but some excessive smoothing may have occurred. Another problem involved drift of the gust probe data due to sensitivity to temperature changes. For angle of attack it was found possible to combine the direct gust probe measurements at high frequencies (>0.01 Hz) with calculations made from aircraft lift curves at lower frequencies, as described by Lilly and Kennedy (1973). No long-term information on sideslip angle was available, however, so that reliable data are limited to frequencies near and greater than 0.01 Hz. Since pilots have difficulty in controlling sideslip angle to less than about half a degree, there remains an unavoidable low-frequency uncertainty in the cross-wind of about 3 m s⁻¹. The other principal problem involved the removal of data recording “spikes,” caused by various kinds of electrical noise. These were especially severe during the very turbulent periods. The spikes were removed by objective and subjective methods, but there was probably some consequent distortion of the highest frequency parts of the spectra.

Some of the conclusions to be drawn are based on power and correlation spectra calculated for all or parts of the aircraft flight legs. Most of these spectra were calculated in the following manner: The means and least-squares trends were removed from each run, and the front and back ends were “tapered,” respectively, by multiplication by

\[ \frac{1 - \cos[2\pi(t - t_0)/\tau]}{2} \text{ and } \frac{1 - \cos[2\pi(t_f - t)/\tau]}{2}, \]

where \( t_0 \) and \( t_f \) are the front and back ends of the record, \( \tau = 0.2(t_f - t_0) \), and the tapering was done over half a period, i.e., 10% of the total record at each end. The processed record was then subjected to a complete fast Fourier transform. The resulting squared amplitudes and coproducts and quadrature products were averaged over the adjacent 11 temporal wavenumbers before plotting, except for the first and last five wavenumbers. Here we define the temporal wavenumber \( n \) as the product of frequency (Hz) and the total time interval \( t_f - t_0 \) (s). For wavenumbers \( 1 \leq n \leq 5 \) or \( N - 4 \leq n \leq N \), the averaging was done from wavenumber 1 to \( 2n - 1 \) or from \( 2n - N \) to \( N \), respectively, where \( N = 0.5(t_f - t_0) \) [s] is the maximum wavenumber. All of the time spectral results were also converted to spatial spectra. The platform velocity appropriate for making this conversion depends on the propagation velocity of the phenomena observed. We used mean aircraft ground speed for the full flight leg analyses on the assumption that most of the variance is associated with standing waves. For the turbulent region analyses we used airspeed since we are most interested in phenomenon which are being carried approximately with the mean.
the westbound 6 km flight level. Due to the 1 s averaging, the magnitudes of acceleration are considerably reduced from their peak values, but the duration of the strong turbulence is well defined and is almost identical with the duration of potential temperatures > 318 K. Figs. 12–14 show the vertical, westerly and northerly air velocity components, again with potential temperature superimposed. In the vertical velocity record there appear to be four well-defined maxima and minima, with amplitudes of 10–20 m s\(^{-1}\) and a period of about 15 s (3 km wavelength). Most of the major and minor peaks are almost exactly out of phase with similar peaks in the potential temperature indicating significant downward heat flux. Periodicities in the \(u\) and \(v\) fields are less clear, but \(u\) and \(\theta\) show a strong negative correlation with each other.

Along with the high-amplitude turbulence, larger scale trends are evident in the \(v\), \(\theta\) and \(u\) fields. The bottom of the primary wave trough is apparent as a large maximum in \(\theta\) and a minimum in \(u\), with a strong downdraft at the upstream end and an almost equally strong updraft at the downstream end. Thus it is evident that the flow is curved. The aircraft track is also curved, mainly in the opposite direction, due to control difficulties in the strong turbulence field. From the vertical and horizontal velocities near the ends of the turbulent region we estimate the mean streamline radius of curvature to be between 6 and 12 km. The contribution to \(\gamma\) vorticity from this curvature is therefore less than 0.01 s\(^{-1}\), considerably smaller than our estimates of the vertical shear. Thus the deviation from horizontal flow is believed not to greatly affect the applicability of Kelvin-Helmholtz wave theory.

Fig. 15 shows power spectra calculated for each of the velocity variables and potential temperature records shown on Figs. 12–14. Each of these spectra has a noticeable hump near the 15 s period and none shows much evidence of an inertial (−5/3 power law) range.
These results were initially considered questionable because of the rather arbitrary trend removal and tapering procedure applied to the data before the Fourier transformations were carried out. We therefore calculated spectra in two other ways. For the first, the original record was doubled in length by reflection, with the double record then transformed without trend removal or tapering. For the second variation, the record length was doubled after subtracting the mean, by adding 35 s of zeros at each end. Except for the expected changes in the number of modes and amplitude levels of the spectra, the results from these alternative calculations are almost identical to those arising from the standard method. Thus we conclude that a quasi-periodic structure of 2.5-3.5 km exists in all the parameters. We believe, however, that the lack of a visible inertial range is due to data recording and handling problems, in particular the effects of data "spikes" and their removal.

Calculations of growth rates for linear and nonlinear Kelvin-Helmholtz instability modes have been carried out by Tanaka (1975) and Liu and Merkine (1976). In both cases hyperbolic tangent wind profiles were used. Tanaka assumed a constant buoyant stability, while Liu and Merkine used a hyperbolic tangent density profile, symmetric with the velocity profile but with not necessarily the same thickness. In both cases the wavelength of maximum growth rate at all unstable Richardson numbers is nearly the same as that at the stability boundary $\text{Ri} = 0.25$, about $6 \Delta U/(\partial U/\partial z)_{\text{max}}$, where $\Delta U$ is the amplitude of the mean velocity difference across the shear layer. Applying this result to the present data, where $\Delta U$ is at least 50 m s$^{-1}$ and the observed wavelength is about 3 km, we are led to postulate a shear amplitude of $\sim 0.1$ s$^{-1}$, an extremely large value but perhaps not beyond the realm of possibility. There could be other explanations however. Certainly the shear and stability conditions are not likely to satisfy the symmetry conditions assumed in the theory, since the deformation of the fairly uniform velocity and stability structure air upstream into a strongly sheared layer must be accompanied by development of a strong stability gradient. In addition the upstream perturbations may have most of their energy in the smaller scales.

The actual growth rate calculated from the linear or nonlinear theory is a function of Richardson number. In our case the stability in the turbulent region at 6 km is probably fairly close to that in the mean flow upstream, since the density increase by compression from 250 to 450 mb is about the same as the mean velocity decrease from 45 to 30 m s$^{-1}$. Thus $N^2 \approx 10^{-4}$ s$^{-2}$, so that the Richardson number is less than 0.04, perhaps as low as 0.01. Linear theory then predicts that the exponential growth rate at the most unstable wavelength is about 0.15 times the maximum shear amplitude, and if we accept the shear amplitude of 0.1 s$^{-1}$, the e-folding time is about 65 s. The mean transit time of air through the observed turbulent region is of order 400 s, so that there would be time for the turbulence amplitude to amplify by more than $e^{0.15 \sim 400}$. The unstable region probably also extends further upstream into the descending air above our flight level, but the growth rate decreases as the disturbance becomes nonlinear. From his nonlinear calculations, Tanaka found that for $\text{Ri} = 0.08$, a period of about $90/(\partial U/\partial z)_{\text{max}}$ is required to increase the velocity amplitude to its maximum value after starting three orders of magnitude $(\varepsilon')$ smaller, while Liu and Merkine predicted a period of about $35/(\partial U/\partial z)_{\text{max}}$ to increase amplitude by a factor of 50, i.e., $(\varepsilon')$. There is apparently some difference between these results, and also the time scale would be about 25% smaller for $\text{Ri} = 0$, but both seem to agree reasonably well with our data. There is a larger discrepancy on the calculated maximum amplitudes, with Tanaka predicting a maximum kinetic energy of about 0.02$(\Delta U)^2$, while Liu and Merkine obtain 0.23$(\Delta U)^2$. From our data, the mean turbulent energy in the turbulent zone at 6 km is about 150 m$^2$s$^{-2}$, which was obtained after removing the means and least-squares trends from the records but without tapering. Thus if we assume $\Delta U = 50$ m s$^{-1}$, this value falls in the range between the two theoretical predictions. From our data the $\theta$ variance is 3.1 K$^2$ at 6 km, while the total mean $\theta$ difference across the shear layer is about 10 K. Probably the variance should be reduced somewhat, because subtraction of a least-squares trend does not fully account for the effects of the large-scale gradients which the airplane encounters while flying across the chord of the curved turbulent zone. Thus we estimate $\sigma^2(\Delta \theta)^2 < 0.03$. It is not clear from either the Tanaka or Liu and Merkine papers, however, what the maximum ratio of density variance to total density difference is predicted to be.

Some of the important terms of the momentum, energy and thermal variance budgets are accessible from the observational data. We calculated the relevant covariances $\langle u'w' \rangle$ and $\langle w'\theta' \rangle$ by two different methods. After removal of the least-squares trend, the running products

\[
\int_{t_0}^{t} u\' w' C dt \quad \text{and} \quad \int_{t_0}^{t} w'\theta' C dt
\]

were calculated directly, and the results for the 6 km level are presented on Figs. 16 and 17, for the turbulence zone and for the entire leg, respectively. The velocity C was chosen to be the true airspeed for the turbulence zone calculations and the true ground speed for those of the full leg. Similar computations were made for the 9 km leg. The second method was by use of covariate spectra. The real and imaginary parts of each wave-number component of one variable were multiplied by
Fig. 16. Running spatial integral of momentum flux $\int \rho \Delta \cdot \Delta \, ds$ (solid) and potential temperature flux $\int \Delta \cdot \Delta \, ds$ (dashed) over the turbulent zone of the 6 km flight leg, with $\Delta$ and $\Delta'$ the deviations from the means and least squares trends. The spatial increments $ds$ are taken to be equal to $Cd_t$, where $C$ is the true airspeed of the aircraft.

Fig. 17. As in Fig. 16 except over the entire 6 km flight leg and with $C$ equal to the aircraft ground speed. The turbulent zone is indicated by stippling.
Fig. 13. The momentum flux is strongly positive (i.e., downgradient) and the heat flux negative (i.e., upgradient) as would be expected from shearing instability theory. The appearance of the curves suggests that virtually all of the flux contributions are due to the last $1/4$ min of turbulence, i.e., the upstream end. Such a conclusion would not necessarily be justified, however, since the detailed shape of the running integral curves is a strong function of the trend removal procedure. If a somewhat more complex representation of the larger scale flow were removed from the velocity and temperature fields, say, a sine curve for the $w$ field and a negative cosine curve plus a constant trend for $u$ and $\theta$, the contributions from the middle regions would be apparently enhanced and those from both ends diminished. Even so, it is obvious that a major part of all the correlations must arise from the region near time 1800, where the major deviations in potential temperature and all velocity components are found.

The corresponding running covariance integrals for the entire 6 km leg (Fig. 17) show that the total momentum flux is of opposite sign to the contributions from the turbulence zone (shaded region). This is to be expected, since the momentum flux from standing mountain waves should usually be negative. It is not at all obvious, however, that the wave momentum flux can be accurately estimated by simply removing the contributions from the turbulent region, though that is perhaps our best estimate. The spectral analysis of $u w$ covariance (Fig. 18) which is plotted on a variance area conserving graph, may help the interpretation a bit. It shows that most of the negative contributions are associated with wavelengths $>20$ km, while the positive contributions from wavelengths between 5 and 20 km cancel out about 80% of the larger scale parts. Although Fourier modes are calculated from the entire field, it is apparent that most of the amplitude in both the large- and small-scale components arises from the principal wave and turbulence zone. Thus both the spectra and the running covariance integral strongly suggest, but do not prove, that without the high-frequency turbulence components the momentum flux would be substantially larger than its observed net value.

From the above results it is possible to estimate some terms of the kinetic energy and potential temperature variance budgets, and these are summarized on Table 1. From Fig. 16 we find the integrated momentum flux to be about 360 N m$^{-1}$. After dividing by $f a C_d l$, the average momentum flux in kinematic units is $u' w' = 30$ m s$^{-2}$. Thus if the mean shear in the turbulent zone is 0.05 s$^{-1}$, the energy generation rate is 1.5 m$^2$ s$^{-2}$, while it is twice that large if the shear is 0.1 s$^{-1}$, as suggested by linear stability theory. Similarly, the average heat flux from Fig. 16 is $-6$ K m s$^{-1}$, leading to the buoyant energy transformation term of 0.2 m$^2$ s$^{-2}$. The estimate of energy dissipation is more tenuous. The value listed, 0.8 m$^2$ s$^{-2}$, is based on the assumption that an inertial range, or at least a longitudinal velocity component spectrum satisfying the $-5/3$ power law, exists from frequencies of 0.1 Hz and greater (wavelengths $<1500$ m). The dissipation magnitude is then estimated from values of the westerly (approximately longitudinal) spectrum for wavelengths between 1000 and 2000 m, using the Kolmogoroff relationship $F(k) = \alpha k^{5/3}$, where $F(k)$ is the spectrum function shown on the first curve of Fig. 15 and $\alpha$ is taken to be 0.5. This value is taken to be an upper limit since use of other velocity components in the same way would yield values about a factor of 2 smaller, while use of (dubious) spectral estimates at higher frequencies would lead to much smaller estimates. The advection term is based on the assumption that the mean value of turbulence energy, 150 m$^2$ s$^{-2}$, was attained in the time it takes the air to traverse one-half the total extent of the turbulent region (7200 m) at a mean velocity of 30 m s$^{-1}$, i.e., 240 s. This value is also taken to be an upper limit because the complete turbulent region probably extends further upstream above the aircraft flight and the

<table>
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<th>Table 1. Terms of kinetic energy and potential temperature variance equations for turbulent region at 6 km MSL.</th>
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<tr>
<td>Turbulence kinetic energy $[\sqrt{\frac{1}{2}}]$</td>
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<tr>
<td>Kinetic energy generation $[-u w (\partial \theta / \partial z)]$</td>
</tr>
<tr>
<td>Transformation to potential energy $[-g u w / \theta]$</td>
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<tr>
<td>Kinetic energy dissipation $[\nu \nabla^2 \theta]$</td>
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<tr>
<td>Kinetic energy advection $[\nabla \cdot \mathbf{K} \mathbf{E}]$</td>
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<td>$\theta$-variance $[\theta^2]$</td>
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<tr>
<td>$\theta$-variance generation $[-2 u w (\partial \theta / \partial z)]$</td>
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<tr>
<td>$\theta$-variance dissipation $[\kappa \theta^2]$</td>
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<tr>
<td>$\theta$-variance advection $[\nabla \cdot \mathbf{K} \theta^2]$</td>
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Fig. 18. The cospectrum of westerly and vertical velocity components, multiplied by wavenumber, for the entire 6 km flight leg.
Fig. 19. As in Fig. 15 except for the 9 km turbulent zone. An airspeed of 165 m s⁻¹ was used to obtain wavelength.

The estimated mean turbulent energy probably includes part of what should be regarded as mean flow.

Thus it appears that the principal energy source term is larger than the sum of the transformation, dissipation and advection terms. If so, the difference is probably associated with vertical transport terms, including pressure velocity correlations and turbulent diffusion, with the latter believed to be most important. Vertical transport of energy by pressure forces does not occur significantly for short wavelength Kelvin-Helmholtz instability, but rapid thickening of the turbulent layer by diffusion seems very likely, though not directly observable by a single aircraft.

Essentially the same uncertainties arise in the estimation of terms of the θ variance budget. The vertical mean gradient is assumed to be uncertain within a range of 2–4 K km⁻¹, and again the dissipation rate is estimated from the wavelength octave between 1 and 2 km. Again it is suggested that the generation term is too large to be balanced by the others, with vertical diffusion likely to be significant.

The possibility of checking these figures further arises from the existence of additional flight legs in the downstream turbulent decay region, although the long separation in time (> 1 h) between the 6 and 9 km legs may lead to some uncertainty. The time required for air to flow from the downstream part of the turbulent region at 6 km to the turbulent region at 9 km is 400±100 s, and the mean kinetic energy and potential temperature variance in the latter are 85 m² s⁻² and 0.45 K², respectively. If we estimate the kinetic energy and potential temperature variance at the downstream end of the 6 km leg to be 200 m² s⁻² and 4 K², respectively, then the average rate of removal of kinetic energy between the two layers is 0.3 m² s⁻³, and the average θ variance removal is 0.01 K² s⁻¹. Fig. 19 shows the power spectra for the three velocity components and potential temperature in the 9 km turbulent zone, treated in the same way as those at 6 km. From these curves it is apparent that the spectra fit inertial range criteria more accurately than those at 6 km, though the contributions from frequencies >0.2 s⁻¹ still show some doubtful features. Using the fit to a −5/3 power law between 0.03 and 0.1 Hz, we estimate the energy dissipation to be 0.1±0.02 m² s⁻³ and the θ variance dissipation to be 0.0013±0.0003 K² s⁻¹. The heat and momentum fluxes at the 9 km level turbulent zone, as estimated from either direct correlation or cospectra, are insignificantly different from zero. Thus the energy and θ variance dissipation rates between the 6 and 9 km levels, as estimated from end points of the trajectory, evidently lie between those estimated from spectra at the two levels.

4. Interactions with the larger scales

Bretherton (1969) and Lilly (1972) previously hypothesized that mountain waves and their associated surface and upper air turbulence could produce significant effects on their larger scale meteorological environment, principally through an exchange of momentum between the earth and the atmosphere. In the previous section the integrated wave momentum flux was estimated to be 0.7×10⁶ N m⁻¹ at the 6 km level and about twice that large at the 9 km level 1 h later. The torque calculated from cross-mountain pressure differences and surface drag was of the same magnitude. These values are ½ to ¾ the product of the nominal mean surface stress in middle latitudes (0.1 Pa) and the length of a latitude circle at 40°N (3×10⁴ km). Therefore, this event had a substantial effect on the earth’s momentum budget, comparable to that of a much larger scale weather event, such as a major mid-latitude cyclone. It is clear that wave momentum flux can be extremely variable and intermittent. This fact does not provide much guidance as to its long-term influence on the momentum budget, however, nor does it aid much in tracing the influence of terrain fluctuations on flow evolution downstream.

During the winter of 1973 an observation program, the Wave Momentum Flux Experiment (WAMFLEX), was carried out in an attempt to gather a larger and
more complete sampling of mountain waves and their associated momentum transports over the Colorado Rocky Mountains. The results of that experiment, including comparison with model calculations, will be reported elsewhere. Some of the problems of measurement and interpretation found there are, however, well illustrated by the results of this case study.

The measurement problems include those associated with instrumental errors and sampling inadequacies. For the present case and most strong wave situations observed by well-equipped aircraft, the actual instrumental errors are probably not of major significance, i.e., they do not contribute more than 20% error to estimates of wave momentum flux and most other important quantities. The surface pressure measurements are probably less reliable, because of the uncertain effects of dynamic pressure inside a building or shelter under the influence of strong winds. These errors are largely removable by improved instrumentation techniques, such as use of a well-designed static pressure port and proper external exposure. Future measurements of cross-mountain torques should utilize such techniques.

Sampling errors are much more pervasive, arising from inadequate coverage and resolution in all three spatial dimensions and time. It is likely that about half of the momentum transported to the earth during the 11 January storm was abstracted from either the stratosphere or the boundary layer. The available research aircraft were, however, not capable of safely carrying out observational flights closer than a few thousand feet above the mountain peaks or in the stratosphere, and that will generally be the case unless unusually high performance (usually military) aircraft are available.

The problems of interpretation of wave momentum flux and the feedback to the larger scale flow seem at least equally knotty with those of its measurement. In the case of continuous spatially periodic forcing one assumes that the mean flow velocity will decrease with time at the levels of wave breakdown where momentum is removed by wave drag, as in the computations carried out by Breeding (1971) and Jones and Houghton (1971). For an array of quasi-periodic mountain ranges one might postulate in the steady-state limit a similar downstream decrease in horizontal momentum transport at the levels of wave breakdown. The adjustment cannot be that simple, however, because continuity requires that the vertical integral of mass transport remain constant downstream across the mountain range. Thus net downstream differences in the pressure and temperature profiles are to be expected. Examination of the present data indicates that these downstream changes are quite large and are probably strongly influenced by blocking, together with three-dimensional and probably rotational effects.

From Fig. 7 it is evident that the potential tempera-

Fig. 20. The trace of vertical acceleration taken from the flight recorder on a 707 jet cargo aircraft flying over the Front Range at 0226 GMT. The ordinate scale is in units of standard gravity force.

tures observed at the downwind end of the flight legs are all higher than those at the upwind end. Very little, if any, of this heating is believed to have been caused by latent heat release, so that a net downward displacement across the mountain range is apparent throughout most of the troposphere. Fig. 5c shows that the same is true on a somewhat larger scale also, except that the sign of the displacement is reversed above 400 mb. From looking at the other cross sections and time sections it is evident that the air reaching the surface at Denver was never before below the 700 mb level over North America. Thus the low-level air was effectively blocked from crossing over the Colorado Rockies. The Continental Divide is not, however, a uniform barrier, and in southern Wyoming it drops to about the 760 mb level. During the windstorm period strong surface southwesterlies were observed at Laramie, Wyo., 200 km north-northwest of Denver, but with a surface potential temperature 8°C lower than that at Denver and about the same as that at Grand Junction, Colo., west of Denver. Thus one effect of the Colorado Rockies is to laterally separate layers of air which are originally flowing in vertical columns with little directional shear and to divert the lower layers northward through the Wyoming gap, where persistently strong surface westerlies are observed in winter. From the viewpoint of the air which flows over them, however, the Colorado Rockies look like a small rising ramp followed by a large and steep descent.

5. Hazards to aircraft

The turbulent region encountered by the Sabreliner at 6 km MSL was one of, if not the, most severe ever
Even so the maximum aircraft vertical velocity exceeded that of the air, suggesting some degree of human or mechanical control instability. The commercial jet was evidently in real peril for a minute or so. Since events of this magnitude are extremely rare for the average commercial, military or general aviation pilot, it is essential that the proper responses must be learned through others' experience. At least one airline has utilized some of our data from the 11 January event in its pilot training program.

Regarding avoidance procedures, major airlines in the United States are generally well aware of the areas where turbulence generated by mountain waves is frequently intense. The Colorado Front Range is possibly the most important of these. It is largely avoidable for through traffic by lateral deviations of 100–200 km. Our results suggest, however, that vertical deviations may not be practical in some cases. In the flow pattern of Fig. 7, the only tropospheric flight pattern that could avoid turbulence would involve following the isentropes between 310 and 315 K, thus making large altitude changes in the presence of strong up and downdrafts only a few thousand feet above the terrain.

6. Conclusions

The 11 January downslope windstorm was the strongest such event over at least a seven-year period in Boulder, Colo., a site notorious for such storms. The precursors of the storm included a favorable planetary wave and jet stream pattern together with a moderate amplitude and probably predictable synoptic-scale wave which produced an upstream environment highly responsive to topographic forcing. The mesoscale structure of the mountain wave and windstorm system are shown to have undergone an important oscillation in wavelength which is closely related to the question of where and how wave momentum flux is ultimately transported to the ground. The integrated wave momentum flux varied from 0.7 to 1.6×10⁶ N m⁻¹ from the aircraft data.

The 11 January wave was found to be exceptional not only by its overall intensity but by the existence within it of a region of dynamically unstable shear in the middle troposphere. The mid-tropospheric turbulence zone associated with this shear was found to contain levels of turbulent energy, generation and dissipation among the largest reported by well-instrumented aircraft, equalled or exceeded regularly only in severe convective storms. All evidence indicates that the classical mechanism of shearing instability in the presence of stable thermal stratification was responsible for generation of the turbulence, with the shear produced by the large-amplitude and quasi-steady mountain wave. The spatial scale of the turbulence differed substantially from that of the driving wave and the statistical
properties of the two forms of motion could be partially separated, though uncertainties of up to a factor of 2 remain in some of the terms in the momentum, energy and thermal variance equations.

The observed wave drag was found to be of order \( \frac{1}{2} \) the average force per latitudinal meter extent exerted by the earth on the atmosphere. Thus this event contributed substantially to the global momentum balance during its brief lifetime, although the principal altitude and mode of interaction remains somewhat uncertain. The effects of blocking apparently caused lateral separation of different layers and net descent of air crossing the Colorado Rockies.

The observed large vertical motions and severe turbulence were found to be hazardous to aircraft, especially when unaware of the location of the principal wave and turbulence regions. Since severe turbulence occurred at nearly all levels in the troposphere, avoidance by vertical deviation was impractical.

Acknowledgments. The list of significant contributors to the work reported here is long and varied. Ed Zipser was the sparkplug who initiated the plan for the flight on the morning of 11 January and participated as scientific observer on the Queen Air. The NCAR Research Aviation Facility, under its manager Jack Hinkelman, was able to effectively respond to the two-aircraft flight plan request within a few hours after its transmittal to the facility. The flight crew on the Sabreliner consisted of Bill Zinser as pilot and Pete Orum as copilot, with Al Rodi on board as instrumentation observer. Bob Burris and Jim Covington served as the flight crew on the Queen Air. The data processing had to be carried out in several stages since several instrument systems required recalibration or special data handling to provide useful information. Most of this reduction was carried out by Neil Kelley, Ron Ruth and Bonnie Gacnik, while most of the scientific data analysis, calculation of turbulent fluxes, spectra, etc., was accomplished by Patrick Kennedy. I am also indebted to the following colleagues who read and commented on earlier versions of the paper: Joe Klemp, Jim Deardorff, Bill Blumen, Bob Chervin, Ed Zipser, Don Lenschow and two anonymous reviewers whose criticisms were very helpful.

REFERENCES


