The Effect of the Meridional Circulation on the Baroclinic Instability of the Winter Zonal Flow

RICHARD BLAKESLEE AND ROBERT GALL

Institute of Atmospheric Physics, The University of Arizona, Tucson, AZ 85721
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ABSTRACT

The linear instability of the observed winter zonal mean flow, with and without the inclusion of the meridional circulation, is presented. The purpose of this note is to determine whether the meridional circulation has an important effect on the growth-rate spectrum and structure of the linear baroclinic waves. It is found that the inclusion of the meridional circulation produced only slight changes in the growth-rate spectrum, as compared to the case where the meridional flow is neglected. The presence of the meridional flow destabilized all wavenumbers greater than 5, although the change was less than 7% for all wavenumbers. The maximum growth rate, with or without considering meridional circulation, occurred between wavenumbers 12 and 15. The wave structures were also very similar in the two cases. The maximum amplitude of the geopotential perturbation for wavenumbers 5-7 remains at the earth's surface when the meridional circulation is included; however, the position of the maximum geopotential perturbation at the earth's surface is displaced northward about 250 km.

1. Introduction

Since its introduction with the studies of Charney (1947) and Eady (1949), linear baroclinic instability theory has often been assumed to explain not only the existence of the eddies that appear in the general circulation of the middle latitudes, but a number of their important features as well. For example, certain of the basic structural features, such as the correlation between the temperature and vertical velocity fields and the vertical distribution of the temperature perturbation, appear to be explained by this theory. Recently, however, Gall (1976a), Green (1970) and Simmons and Hoskins (1978) described a number of significant discrepancies between the zonal scale and details of the structure of the waves predicted by linear theory and the eddies that are observed in the middle latitudes. For instance, the atmospheric eddies display a primary maximum amplitude of kinetic energy and geopotential perturbation near the tropopause (Oort and Rasmussen, 1971), while linear theory predicts these maxima to be at the earth's surface. Furthermore, there is evidence to suggest that the wavelength of the wave with the maximum growth rate in the linear theory may be shorter than the scale of the eddies which dominate the middle-latitude circulation (Simmons, 1970; Simmons and Hoskins, 1977; Gall and Blakeslee, 1977; Gall, 1976a).

Gall (1976b) and Simmons and Hoskins (1978) discussed some reasons why there may be differences between the scale and structure of the waves predicted by linear theory and the eddies of the general circulation. In particular, it was shown that weak nonlinear processes involving the interaction between the growing waves and the zonal flow account for many of these differences. It is tempting, however, to explore the possibility that factors left out of the linear analyses of Gall and Simmons and Hoskins could account for at least some of the differences.

In particular, a logical question is whether the presence of a mean meridional circulation could have a significant effect on the stability and structure of the linear waves. This flow is, of course, always present in realistic situations.

In this study, two experiments are conducted to determine the effects of the mean meridional circulation on the linear baroclinic instability of winter zonal flow. The first experiment determines the linear instability of wave perturbations to the mean winter zonal flow as a function of zonal wavenumber for the case when the meridional circulation is absent. In the second experiment, the linear instability is again determined, but with the mean meridional flow present.

The inclusion of the meridional circulation in the base state may be regarded as the addition of a purely divergent field of zonal wavenumber zero. Inclusion of divergent components in the base state is no problem, even though the base state is no longer exactly "balanced." Mathematically, a linear growth rate for perturbations can still be computed. Physically, it accepts the fact that small perturbations in the real

1 We will refer to the first experiment as the zonal experiment and to the second experiment as the meridional experiment.
atmosphere will “feel” these divergent components as they grow. By noting the differences between the two experiments, the changes included by the inclusion of the meridional circulation are easily deduced.

The primary question to be considered is whether the presence of the meridional flow causes significant differences. These differences will be outlined; however, the reader should keep in mind that the details of these differences could change somewhat if other slightly different zonal flows and meridional circulations had been considered. The flows considered here are those observed in the Northern Hemisphere winter.

2. The linear model

The linearized primitive equations in pressure coordinates, which retain the effects of spherical geometry, are utilized. The equations are written in “semi-spectral” form, in which the perturbations to the mean zonal flow are expressed as a Fourier series in zonal wavenumber. Transformation of the primitive equations into the semi-spectral form is presented in detail in Saltzman (1957).

For the numerical integrations, the meridional and vertical derivatives are evaluated on a grid by an energy-conservative centered difference scheme patterned after the “box method” (Kurihara and Holloway, 1967), while the zonal derivatives are evaluated analytically. The time derivatives are replaced by centered differences with time-smoothing (Blek, 1974) applied each time step to reduce high-frequency oscillations due to gravity waves, as well as to suppress the computational mode in time.

The grid consists of 38 points in the meridional direction, spaced 2.4° apart, beginning at the equator and extending to the North Pole. Thirteen levels are used in the vertical. The velocity components are staggered on the grid to reduce the spatial computational mode.

A horizontal boundary condition in the vicinity of the equator is required since the model covers only the region of the Northern Hemisphere. Therefore, a closed boundary is placed one-half grid interval south of the equator, where \( \nu_n = 0 \), where \( \nu_n \) is the velocity component normal to the wall. At the upper boundary, \( p = 0 \), \( \omega_n = 0 \), is assumed. No topography is included on the lower boundary and frictional terms are excluded.

The stability analysis uses the initial value technique adopted in a number of recent studies (Brown, 1969; Gall, 1976a; Simmons and Hoskins, 1976) to determine the most unstable mode for a given zonal wavenumber in a specified mean zonal flow. A temperature perturbation of amplitude 0.2 K and constant phase in the north–south vertical cross section is introduced initially onto the geostrophically balanced mean zonal flow, and the model is integrated numerically in time until the most unstable mode dominates all the other modes that are present. In the present study, this was found to be the case by the time the surface pressure perturbation had reached 20 mb.

3. The mean atmosphere

The winter mean zonal flow used in this study was derived in the following manner. Taking the observed winter mean zonal winds given by Newell et al. (1969),
a geostrophically balanced temperature field was calculated by integrating the geostrophic relation

$$\frac{\partial \bar{a}}{\partial \phi} = f\bar{u} + \frac{\tan \phi}{a}$$

(1)

and relating geopotential to temperature with the hydrostatic equation

$$\frac{\partial \bar{a}}{\partial \rho} = -\frac{RT}{\rho}.$$  

(2)

The vertical profile of mean temperature at 36°N (Newell et al., 1969) provided the necessary boundary condition for the integration of (1). Derivatives in (1) and (2) were replaced by finite differences identical to those used in the model.

The zonally averaged meridional wind \( \bar{u} \) was obtained by direct interpolation to the model grid of the mean winter meridional winds published by Oort and Rasmusson (1971, Table F3). The mean omega field \( \bar{\omega} \) was calculated from the continuity equation

$$\frac{\partial \bar{u}}{\partial \phi} + \frac{\partial \bar{\omega}}{\partial \rho} = 0.$$  

(3)

The streamlines of the meridional circulation are shown in Fig. 1.

4. Results

a. Growth rates and phase speeds

The growth-rate spectrum of the linear perturbations to the observed winter mean zonal flow, with and without the meridional wind, are presented in Fig. 2. In both cases, the maximum instability occurs in the short waves between wavenumbers 12 and 15, corresponding to waves with wavelengths on the order of 2000 km.

The inclusion of the meridional circulation produces only slight changes in the growth-rate spectrum, as compared to the case where the meridional flow is neglected. The two growth-rate spectra are everywhere within 7% of each other, with a maximum difference occurring at wavenumber 19. In general, the difference increases with increasing wavenumber. Except for the ultralong waves, the growth rates are higher when the meridional flow is present.

The phase speeds of the linear waves in the two experiments are also shown in Fig. 2. With the addition of the mean meridional circulation to the base state, the phase speeds increase slightly at all wavenumbers except 9–11, but they never exceed the phase speeds of the zonal experiment by more than 1° longitude per day.

b. Energetics

The baroclinic and barotropic contributions to the kinetic-energy budget of the linear perturbations in the two experiments are shown in Fig. 3. In both experiments, the baroclinic conversion \( (\bar{\omega} \bar{a})^2 \) is the dominant process that supplied energy to the growing waves, though it is slightly smaller at all wavelengths

\(^{*}\) The overbar denotes an average over the mass of the atmosphere of the Northern Hemisphere.
when the meridional circulation is present. The baroclinic conversion spectra display bimodal distributions, with relative maxima occurring at wavenumbers 7 and 13 and a local minimum falling near wavenumber 11. This distribution of the baroclinic energy conversion is consistent with the slight indentation in the growth-rate curve apparent in Fig. 1.

The barotropic conversion ($K_x K_z$) is illustrated in the lower portion of Fig. 3, where $(K_x K_z)$ represents the transfer of eddy kinetic energy to zonal kinetic energy. In the zonal experiment, barotropic damping (negative barotropic conversion) occurs for waves longer than wavenumber 11, while for the shorter waves (except for wavenumber 19) the barotropic conversion is positive. The addition of the meridional flow decreases the barotropic damping of the longer waves and increases the barotropic instability of the shorter waves. Note that the increased growth rates when the meridional flow is present are the result of increased barotropic instability or decreased barotropic stability. The baroclinic instability of all wavelengths is decreased in the presence of the meridional flow.

The barotropic terms involving the mean meridional wind $\overline{v}$ are an order of magnitude smaller and, therefore, negligible when compared to the other terms present in $(K_x K_z)$. Hence, it may be concluded that the changes in the barotropic kinetic energy conversion noted above are not associated with any direct barotropic contribution from the mean meridional wind. Rather the changes observed in all the kinetic energy conversion terms must instead be associated with a modification in the horizontal structure of the waves induced by the meridional circulation. Evidence that changes in the structure of the linear waves occur is discussed in the next section.

Previous work by the authors and others (Gall, 1976a; Simmons and Hoskins, 1977; Simons, 1970; Gall and Blakeslee, 1977; Staley and Gall, 1977) suggests that the growth-rate spectrum may be rather sensitive to small changes in the low-level structure of the zonal flow. The growth rate of the short waves appears to be particularly sensitive to these changes. Thus, some of the detail in the growth-rate spectrum (Fig. 2) and the kinetic-energy conversion (Fig. 3) may be particular to the zonal flow considered here. An example is the change in slope of the growth-rates curve at wave-number 10.

c. Structure

The maximum kinetic energy occurs at the surface, whether or not the meridional circulation is present. This is in contrast to the waves in the general circulation where the maximum kinetic energy is usually found at the tropopause. Fig. 4 shows the differences between the normalized vertical kinetic-energy distributions between the meridional and the zonal experiments as a function of zonal wavenumber. Although Fig. 4 indicates that the kinetic-energy distribution in the
longer waves is enhanced at the higher levels with respect to the surface maximum, the general structural features of the kinetic-energy distribution of the longer waves remains unchanged in the meridional experiment. The kinetic-energy structure of the shorter waves is affected to an even smaller degree by the inclusion of the meridional circulation.

The addition of the meridional flow slightly modifies the north–south orientation of the axes of maximum geopotential amplitude (trough or ridge axes) in the north–south cross sections. In the absence of the meridional circulation, the trough axes are very nearly vertical near the surface, with a slight equatorward tilt at higher levels. With the inclusion of the meridional flow, the surface geopotential-amplitude maxima shift northward, and the equatorward tilt of the trough axes increases. However, the magnitudes of the trough shifts are only on the order of 250 km at the surface.

5. Summary and conclusion

In this note we have shown that the addition of the meridional circulation to the winter zonal mean flow results in only minor changes in the growth rate and structure of the linear waves, as compared to the case when the meridional flow is not included. Examination of the energetics reveals that the barotropic conversion terms associated with \( \nu \) are negligible; therefore, the meridional circulation is unimportant as a direct source (or sink) of kinetic energy for the linear waves. The major effect of the meridional flow is a weak modification of the structure of the perturbation by the meridional wind field; however, even this effect must be considered small.

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