Estimating Momentum, Heat and Moisture Fluxes from Structure Parameters

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ABSTRACT

A theory is developed for obtaining the vertical fluxes of momentum, heat and moisture in a quasi-steady, locally-homogeneous surface layer from measurements of the structure parameters of velocity, temperature and humidity. The momentum flux is shown to be generally more sensitive to measurement errors in the structure parameters and to uncertainties in certain turbulence parameters than are the heat and moisture fluxes. The structure parameters can, in principle, be obtained from path-averaged wave propagation measurements; for example, the temperature structure parameter C_T^2 is readily obtained from optical scintillations. It is estimated that in the case of C_T^2 a path-averaged optical measurement requires about 1% of the averaging time of a conventional measurement of C_T^2 under typical conditions.

1. Introduction

The vertical fluxes of momentum, heat and moisture near the earth's surface are important both for boundary layer structure and dynamics and for larger scale atmospheric flow as well. Unfortunately, they are very difficult and expensive to measure directly, as discussed by Kaimal (1975). In addition, direct flux measurements require long averaging times (Wyngaard, 1973) and are apt to be unrepresentative over even mildly inhomogeneous terrain.

Indirect techniques, by which one infers fluxes from measurements of other flow properties, can be simpler and less expensive than direct measurements. Champagne *et al.* (1977) recently surveyed several of these indirect techniques, and showed that direct and indirect momentum, heat and moisture flux measurements compare well in the unstable surface layer over land.

Most of these indirect flux measurement techniques, when used with tower-based instrumentation, involve only point measurements. Such indirect techniques give little if any relief from the problems of long averaging times and spatial nonrepresentativeness. An optimum indirect approach would involve some spatial averaging, and would require only short time-averaging periods to give statistically reliable measurements.

With this in mind we recast the indirect flux measurement approach of Champagne *et al.* (1977) in terms of structure parameters, rather than dissipation rates, and extend it to the stably stratified case. This gives a theory, recently anticipated by Wesely (1976), for inferring surface-layer fluxes from structure parameters. For the case of the temperature structure parameter, we find that a significant reduction in averaging time is gained by dropping the conventional, point-measurement approach in favor of the measurement of path-averaged optical scintillations.

2. The indirect method

a. Theory

The vertical fluxes of horizontal momentum, temperature and water vapor are nearly independent of height in the lowest few tens of meters for typical conditions over uniform surfaces. We denote the surface values of these fluxes by τ_0, Q_0 and M_0, respectively. They define a scaling velocity u_*, a scaling temperature T_* and a scaling humidity q_* (Busch, 1973):

$$u_* = (\tau_0/\rho)^{\frac{1}{2}} \quad \text{[friction velocity (m s}^{-1}\text{)]}, \tag{1}$$

$$T_* = -Q_0/u_* \quad \text{[scaling temperature (K)]}, \tag{2}$$

$$q_* = -M_0/u_* \quad \text{[scaling humidity (g m}^{-3}\text{)]}, \tag{3}$$

where ρ is air density.

We denote the dissipation rates of turbulent kinetic energy, temperature variance and humidity variance by ϵ, χ_θ and χ_q. We define dimensionless dissipation rates by

$$\epsilon=\frac{u_*^3}{kz}\phi_\epsilon, \tag{4}$$

$$\chi_\theta=\frac{u_*T_*^2}{kz}X_\theta, \tag{5}$$

$$\chi_q=\frac{u_*q_*^2}{kz}X_q. \tag{6}$$

Here k is the von Kármán constant and z is height above the surface. Champagne *et al.* (1977) based their indirect flux measurement scheme on these dissipation rates, but we will now show that their scheme can be reworked to use structure parameters as inputs.

We consider structure functions for separation distances $r=|\mathbf{r}|$ in the inertial range of scales and we further assume these scales to be locally isotropic (Tennekes and Lumley, 1972). For temperature and humidity, these are related to the structure parameters C_T^2 and C_q^2 such that

$$D_T(\mathbf{r})=\overline{[T(\mathbf{x})-T(\mathbf{x}+\mathbf{r})]^2}=C_T^2r^{\frac{2}{3}}, \tag{7}$$

$$D_q(\mathbf{r})=\overline{[q(\mathbf{x})-q(\mathbf{x}+\mathbf{r})]^2}=C_q^2r^{\frac{2}{3}}. \tag{8}$$

The velocity structure function is a second-rank tensor, and as such the direction of the separation vector is important (Tatarskii, 1971). However, one can use the geometry of isotropic tensor functions to relate the various components (Batchelor, 1960). For simplicity we will use only one component, that for streamwise velocity and streamwise separation. In the inertial range this is

$$\overline{[u_1(\mathbf{x})-u_1(\mathbf{x}+\mathbf{r})]^2}=C_V^2r^{\frac{2}{3}}, \quad \mathbf{r}=(r,0,0), \tag{9}$$

where C_V^2 is the velocity structure parameter.

The structure functions are related to one-dimensional spectra through Fourier transforms (Tatarskii, 1971). In the inertial range these spectra are

$$F_u=0.25C_V^2\kappa_1^{-5/3}=\alpha_1\epsilon^{\frac{2}{3}}\kappa_1^{-5/3}, \tag{10}$$

$$F_\theta=0.25C_T^2\kappa_1^{-5/3}=\beta_{1\theta}\chi_\theta\epsilon^{-\frac{1}{3}}\kappa_1^{-5/3}, \tag{11}$$

$$F_q=0.25C_q^2\kappa_1^{-5/3}=\beta_{1q}\chi_q\epsilon^{-\frac{1}{3}}\kappa_1^{-5/3}, \tag{12}$$

where the spectra are scaled such that the integral over the half-line is the variance, e.g.,

$$\int_0^\infty F_u(\kappa_1)d\kappa_1=\overline{u_1^2}. \tag{13}$$

We can rewrite (10)–(12) as

$$\epsilon=0.125(C_V^2)^{\frac{3}{2}}\alpha_1^{-\frac{3}{2}}, \tag{14}$$

$$\chi_\theta=0.125C_T^2(C_V^2)^{\frac{1}{2}}\alpha_1^{-\frac{1}{2}}\beta_{1\theta}^{-1}, \tag{15}$$

$$\chi_q=0.125C_q^2(C_V^2)^{\frac{1}{2}}\alpha_1^{-\frac{1}{2}}\beta_{1q}^{-1}. \tag{16}$$

We now solve (4)–(6) for u_*, T_* and q_*, use (14)–(16) to express the dissipation rates, and arrive at the following equations for fluxes:

$$\tau_0/\rho=0.25k^{\frac{2}{3}}\alpha_1^{-1}z^{\frac{2}{3}}C_V^2\phi_\epsilon^{-\frac{2}{3}}, \tag{17}$$

$$Q_0=0.25k^{\frac{2}{3}}\alpha_1^{-\frac{1}{2}}\beta_{1\theta}^{-\frac{1}{2}}z^{\frac{2}{3}}(C_V^2)^{\frac{1}{2}}(C_T^2)^{\frac{1}{2}}X_\theta^{-\frac{1}{2}}\phi_\epsilon^{-\frac{1}{6}}, \tag{18}$$

$$M_0=0.25k^{\frac{2}{3}}\alpha_1^{-\frac{1}{2}}\beta_{1q}^{-\frac{1}{2}}z^{\frac{2}{3}}(C_V^2)^{\frac{1}{2}}(C_q^2)^{\frac{1}{2}}X_q^{-\frac{1}{2}}\phi_\epsilon^{-\frac{1}{6}}. \tag{19}$$

Note if Q_0 (or M_0) is negative, we must take the negative square root of C_T^2 (or C_q^2) in (18) [or (19)].

The expressions (17)–(19) for fluxes contain the dimensionless dissipation rates ϕ_ϵ, X_q and X_θ. In order to deal with these, we assume we have a quasi-steady, locally homogeneous surface layer whose structure follows Monin-Obukhov similarity. Thus ϕ_ϵ, X_θ and X_q depend only on z/L, where L is the Monin-Obukhov length

$$L=\frac{-u_*^3T}{kgQ_{0V}} \tag{20}$$

and $Q_{0V}=Q_0+0.61TM_0/\rho$. This use of the virtual temperature flux Q_{0V} accounts for the effects of humidity fluctuations on buoyancy (Lumley and Panofsky, 1964).

We now rewrite the right-side of the L definition (20) by using (17)–(19) to express u_*^3 and Q_{0V}. We also make a second assumption: that the scalar spectral constants, and the dimensionless scalar dissipation rates, are equal ($\beta_{1\theta}=\beta_{1q}=\beta_1$; $X_\theta=X_q=X$). Then (20) becomes

$$[(C_T^2)^{\frac{1}{2}}+(0.61T/\rho)(C_q^2)^{\frac{1}{2}}](g/T)z^{\frac{2}{3}}(C_V^2)^{-1} = -0.5k^{-\frac{2}{3}}\beta_1^{\frac{1}{2}}\alpha_1^{-1}zX^{\frac{1}{2}}\phi_\epsilon^{-5/6}L^{-1}. \tag{21}$$

The right side of (21) varies only with z/L, since k, β_1 and α_1 are constants. We define this to be $S(z/L)$:

$$S(z/L)\equiv-0.5k^{-\frac{2}{3}}\beta_1^{\frac{1}{2}}\alpha_1^{-1}zX^{\frac{1}{2}}\phi_\epsilon^{-5/6}L^{-1}. \tag{22}$$

We note from (21) that S can be calculated from measurements of structure parameters:

$$S(z/L)=[(C_T^2)^{\frac{1}{2}}+(0.61T/\rho)(C_q^2)^{\frac{1}{2}}](g/T)z^{\frac{2}{3}}(C_V^2)^{-1}. \tag{23}$$

Eqs. (22) and (23), in conjunction with (17)–(19), allow us to estimate fluxes from structure parameters. To do this, we calculate S from (23), using measured structure parameters. As explained in the next section, the right side of (22) has been determined experimentally; thus one knows $S(z/L)$, and having S one can infer z/L. Then having z/L, one knows X and ϕ_ϵ in (17)–(19), which with the structure parameters and the constants give the fluxes.

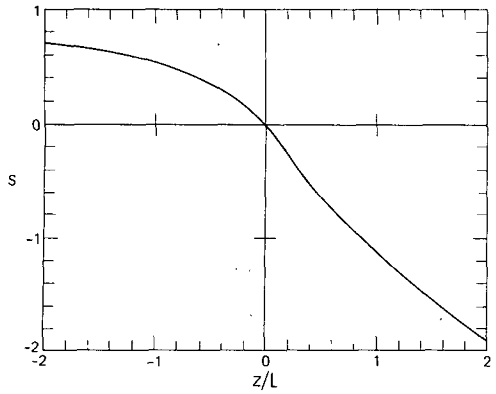

FIG. 1. The stability function S [defined in (22)] as a function of z/L, calculated from (24)–(27) with $k=0.35$, $\alpha_1=0.5$, $\beta_1=0.4$.

b. Examples

We will explore further this flux estimation technique by introducing some surface layer data. We take $k=0.35$ (Businger *et al.*, 1971) and $\alpha_1=0.5$, $\beta_1=0.4$ (Champagne *et al.*, 1977). For unstable conditions we will use (as did Champagne *et al.*, 1977)

$$\phi_\epsilon=[1+0.5(-z/L)^{\frac{2}{3}}]^{\frac{3}{2}}, \tag{24}$$

$$X=\frac{1.48}{[1-9(z/L)]^{\frac{1}{2}}}, \tag{25}$$

which are based on the 1968 Kansas experiments, and should be good approximations in the range $0\leqslant -z/L<2$. For stable conditions, the Kansas data indicate that the turbulent transport terms in the budgets of turbulent kinetic energy and temperature variance are negligible (Wyngaard and Coté, 1971). This implies that molecular destruction balances production caused by the mean wind and temperature gradients. Using the Kansas forms for these gradients (Businger *et al.*, 1971) gives for stable conditions in the range $0\leqslant z/L\leqslant 1$

$$\phi_\epsilon=1+3.7z/L, \tag{26}$$

$$X=2[0.74+4.7(z/L)]. \tag{27}$$

Fig. 1 shows a plot of $S(z/L)$ calculated from (22) using $k=0.35$, $\alpha_1=0.5$, $\beta_1=0.4$ and the expressions (24)–(27) for ϕ_ϵ and X. Note that S is well-behaved in the sense that one can infer a z/L value from the value for S. Note also that one must know *a priori* the signs of Q_0 and M_0.

In this example we used ϕ and X expressions approximating surface-layer behavior over a limited range, roughly $-2<z/L<1$. More stable conditions at $z=5$ m, say, would typically have very low turbulence levels and negligible fluxes, and we will not consider these. More unstable conditions can be found under very light winds and strong insolation, when the heat and moisture fluxes could be large. Thus it is useful to consider briefly the extension of the technique to very unstable conditions.

From the definition of z/L,

$$-\frac{z}{L}=\frac{(g/T)Q_{0V}}{u_*^3/kz}. \tag{28}$$

We can interpret large negative z/L values as meaning $(g/T)Q_{0V}\gg u_*^3/kz$, or that the rate of turbulent energy production by bouyancy effects is large compared with that by shear. Thus at large $-z/L$ we expect large velocity variances relative to u_*^2; in the convective limit $(L\rightarrow 0)$, u_*^2 vanishes but the velocity variances and C_V^2 are finite. Thus indirect stress measurements of the type discussed here are inherently limited to stabilities not far from neutral.

We can extend our indirect technique for Q_0 and M_0 to very unstable conditions, however. Rather than use asymptotic forms in the expressions (18) and (19) for Q_0 and M_0, it is simpler to start from the Kansas results for C_T^2 under unstable conditions (Wyngaard *et al.*, 1971):

$$C_T^2=\frac{4.9Q_0^2u_*^{-2}z^{-\frac{2}{3}}}{(1-7z/L)^{\frac{2}{3}}}. \tag{29}$$

This can be written, using (20), as

$$C_T^2=2.7Q_0^2z^{-\frac{4}{3}}(g/T)^{-\frac{2}{3}}Q_{0V}^{-\frac{2}{3}}\left(\frac{-7z/L}{1-7z/L}\right)^{\frac{2}{3}}. \tag{30}$$

Although the extension to C_q^2 has not been experimentally verified, according to the Monin-Obukhov similarity hypothesis (Busch, 1973) the analogous expression should hold:

$$C_q^2=2.7M_0^2z^{-\frac{4}{3}}(g/T)^{-\frac{2}{3}}Q_{0V}^{-\frac{2}{3}}\left(\frac{-7z/L}{1-7z/L}\right)^{\frac{2}{3}}. \tag{31}$$

At $-z/L=1$ the stability term in (30) and (31) is within 10% of its asymptotic value of 1.0. Hence measurements of C_T^2 and C_q^2 in the very unstable surface layer $(-z/L>1)$ would in principle give both Q_0 and M_0 through solution of

$$C_T^2=2.7Q_0^2z^{-\frac{4}{3}}(g/T)^{-\frac{2}{3}}[Q_0+0.61T(M_0/\rho)]^{-\frac{2}{3}}, \tag{32}$$

$$C_q^2=2.7M_0^2z^{-\frac{4}{3}}(g/T)^{-\frac{2}{3}}[Q_0+0.61T(M_0/\rho)]^{-\frac{2}{3}}. \tag{33}$$

Under dry $(0.61TM_0\ll\rho Q_0)$, very unstable conditions we find, from (32)

$$Q_0\approx 0.5(C_T^2)^{\frac{3}{4}}z(g/T)^{\frac{1}{2}}. \tag{34}$$

Measurements at Table Mountain, near Boulder, using C_T^2 inferred from optical scintillations over a 300 m path at $z=4$ m, support (34) (Wyngaard *et al.*, 1978a).

3. Error analysis

Fluxes estimated through our indirect technique are subject to errors from a number of sources. We will consider here in detail the flux errors from two such sources: the specification of universal constants and stability functions, and the measurement of the structure parameters.

We examine first the effect of errors in constants and stability functions. Assume for the moment that the structure parameters are known exactly, so one calculates S from (23) without error. However, (22) shows that a z/L value inferred from S will be in error if k, β_1, α_1, X or ϕ_ϵ is in error. We see from the flux expressions (17)–(19) that the fluxes will also be in error, both directly through the errors in k, β_1, α_1, X and ϕ_ϵ and indirectly through the changes in X and ϕ_ϵ caused by errors in their argument z/L.

In spite of intensive micrometeorological research in the past few decades, significant uncertainties remain in both the values of the universal constants k, α_1 and β_1, and the functions X and ϕ_ϵ. Currently used values of the von Kármán constant k range from the 0.35 found in Kansas (Businger *et al.*, 1971) to the traditional value of 0.40 or even slightly larger (Pruitt *et al.*, 1973; Frenzen, 1973, 1977). The spread in currently used values for the one-dimensional Kolmogorov velocity spectral constant α_1 is about as large, in percentage terms, at about 0.45–0.55. The temperature spectral constant β_1 has undergone larger excursions, partly because of recently discovered measurement problems. Hill's (1978) recent survey gives a range of 0.40–0.50 for β_1. The functions X and ϕ_ϵ are more difficult to measure accurately than are k, α_1 and β_1, so that their uncertainties are even larger. Yaglom (1977), in his review of flux-profile relationships, has discussed these and related uncertainties in detail. We stress, therefore, that our choices for k, α_1, β_1, X and ϕ_ϵ in the previous section are undoubtedly somewhat in error, and we turn now to an analysis of the effects of such errors on the flux estimates.

We consider separately the effects of errors in k, α_1, β_1, X and ϕ_ϵ, and allow no other errors. We assume the error level is sufficiently small that we can use a linearized analysis. It is convenient to do this by differential calculus, as we can illustrate by an example involving an error in β_1. We write the stability index S as, from (22),

$$S=S(k,\beta_1,\alpha_1,z/L,X,\phi_\epsilon). \tag{35}$$

We assume S is calculated exactly from structure parameters, using (23); thus $dS=0$, so that

$$0=\frac{\partial S}{\partial\beta_1}d\beta_1+\frac{\partial S}{\partial(z/L)}d(z/L). \tag{36}$$

Thus the error in z/L, $d(z/L)$, caused by the error in β_1, $d\beta_1$ is

$$d(z/L)=-\frac{\partial S}{\partial\beta_1}\left[\frac{\partial S}{\partial(z/L)}\right]^{-1}d\beta_1. \tag{37}$$

Now let F be any one of the fluxes. From (17)–(19), we have in general

$$F=F(k,\alpha_1,\beta_1,X,\phi_\epsilon). \tag{38}$$

The flux error is

$$dF=\frac{\partial F}{\partial\beta_1}d\beta_1+\frac{\partial F}{\partial X}dX+\frac{\partial F}{\partial\phi_\epsilon}d\phi_\epsilon, \tag{39}$$

which, on using

$$dX=\frac{dX}{d(z/L)}d(z/L), \tag{40}$$

$$d\phi_\epsilon=\frac{d\phi_\epsilon}{d(z/L)}d(z/L), \tag{41}$$

and (37), finally yields

$$dF=\left\{\frac{\partial F}{\partial\beta_1}-\left[\frac{\partial F}{\partial X}\frac{dX}{d(z/L)}+\frac{\partial F}{\partial\phi_\epsilon}\frac{d\phi_\epsilon}{d(z/L)}\right]\frac{\partial S}{\partial\beta_1}\left[\frac{\partial S}{\partial(z/L)}\right]^{-1}\right\}d\beta_1. \tag{42}$$

The calculation of the flux errors induced by errors in k, α_1, X and ϕ_ϵ proceeds similarly. The results of this error analysis appear in Fig. 2. The upper panel shows that momentum flux is relatively sensitive to errors in α_1, for example; the sign of the flux error is opposite that of the error in α_1, has the same percentage magnitude at $z/L=0$, and increases for stable or unstable stratification. The momentum flux is less sensitive to errors in ϕ_ϵ, and relatively insensitive to errors in k, β_1 and X. The lower panel indicates that the errors induced in the scalar fluxes (Q_0 and M_0) are, on the average, smaller than stress errors.

From our earlier discussion, the expected errors in k, α_1 and β_1 are of the order of 15% or less. Thus Fig. 2 suggests these should not cause serious flux uncertainties. The situation for X and ϕ_ϵ is less clear; the uncertainty in these is probably greater, perhaps of the order of 20–30%. In view of this, and the results in Fig. 2, it seems that under unstable conditions one can expect some difficulties in estimating momentum flux accurately (say to within 20%); accurate scalar flux estimates should be easier. While the individual curves are different on the stable side, the overall situation there should be about the same.

The second source of flux error that we will analyze lies in the measurement of the structure parameters. We denote the measured structure parameters with

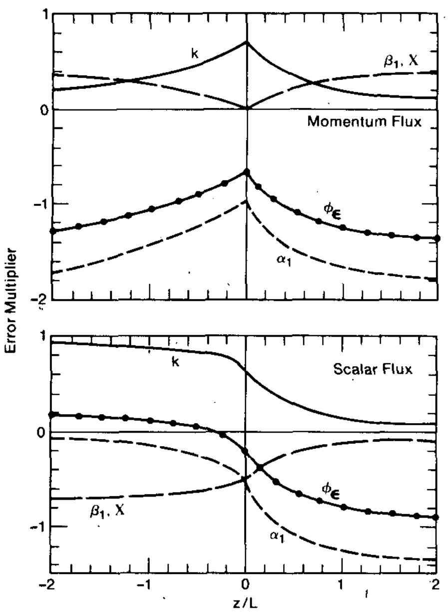

FIG. 2. Flux error multipliers (fractional flux error/fractional input parameter errors), with input parameters varied separately: (——) k varied; (- - - -) α_1 varied; (— —) β_1 or X varied; (—) ϕ_ϵ varied. Top panel, effect on momentum flux; bottom panel, effect on scalar fluxes.

a subscript m:

$$(C_T^2)_m = C_T^2(1+e_T), \quad (43)$$

$$(C_q^2)_m = C_q^2(1+e_q), \quad (44)$$

$$(C_V^2)_m = C_V^2(1+e_V). \quad (45)$$

Here e_T, e_q and e_V are the fractional errors incurred in the measurement of the structure parameters. We assume now that k, α_1, β_1, X and ϕ_ϵ are known exactly, and that there are no other errors. We consider a large ensemble of experiments and take e_T, e_q and e_V to be statistically independent and have zero mean; the root-mean-square (rms) fractional errors are assumed equal and denoted by e.

We calculated the rms, fractional errors in the fluxes by again assuming small input errors ($e<0.3$, say) and performing a linearized analysis. The results are presented in Fig. 3. The results shown are for the dry-land (large Bowen ratio) case, where

$$\frac{0.61TM_0}{\rho Q_0} \ll 1. \quad (46)$$

Results for typical over-sea conditions ($0.61TM_0 \approx \rho Q_0$) differ only slightly from those of Fig. 3. The results of Fig. 3 indicate that the momentum flux is most sensitive to structure parameter errors. This sensitivity is largest under unstable conditions, where the rms, fractional momentum flux error is about twice that in the structure parameters. The scalar fluxes are less sensitive, particularly under unstable conditions.

Finally, we made several key assumptions in our development of our indirect technique, and these are also a potential source of errors. We assumed $\beta_{1\theta} = \beta_{1q}$, which is plausible but not firmly verified. We also assumed $X_\theta = X_q$; Warhaft's (1976) arguments suggest they might differ slightly, but here again there appear to be no data suitable for a critical examination of this question. There is also the question of the extent to which the locally homogeneous, stationary surface layer follows M-O similarity. Businger (1973) has examined some of the effects on surface-layer structure caused by the large, convective-driven eddies which fill the unstable planetary boundary layer. Panofsky *et al.* (1977) used recent data to show that horizontal wind variances in the unstable surface layer are not M-O similar but are tied to these large eddies. It seems possible that they could also cause deviations from M-O similarity in the turbulent kinetic energy budget, and hence also in X and ϕ_ϵ, in the unstable surface layer. Further research on deviations from M-O similarity would be particularly useful.

4. Averaging times

While the statistical properties we are discussing, such as fluxes and structure parameters, are in principle determined from ensemble averages, in practice we must use time averages. It is known (Haugen *et al.*, 1971; Wyngaard, 1973) that very long averaging times are required in order that direct, single-point heat and momentum flux estimates in the unstable surface layer approach their true ensemble means. Indications are that several hours are required for accurate flux estimates in the mid-regions of a convective boundary layer. Higher moments require even longer averaging times, and this is a serious limitation of fixed-point boundary layer turbulence measurements.

Our indirect flux measurement method requires time-averaged structure parameters. These can be found in two ways—from paired, point measurements, using their definitions in (7)–(9), or, in prin-

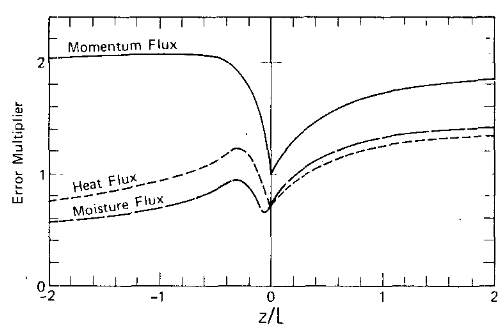

FIG. 3. Flux error multipliers (rms fractional flux error divided by rms fractional structure parameter errors). For simultaneous, statistically independent, equal-amplitude structure parameter errors, and $0.61TM_0 \ll \rho Q_0$ (dry land).

ciple, from path-averaged optical, acoustic or radio wave measurements (Tatarskii, 1971).

As an example of the second, C_T^2 can be determined from optical scintillations. Under normal conditions over dry land $(0.61TM_0 \ll \rho Q_0)$ optical refractive index fluctuations are caused almost entirely by temperature fluctuations (Friehe *et al.*, 1975), and the refractive index structure parameter C_n^2 is proportional to C_T^2. The log-intensity variance of a point detector receiving a spherical wave of wavenumber κ over a horizontal, uniform path of length R is (Lawrence and Strohbehn, 1970)

$$\sigma_{\ln I}^2 = 0.50 C_n^2 \kappa^{7/6} R^{11/6} \quad (47)$$

in the linear theory. Thus C_T^2 can be found from log-intensity variance measurements or from spaced temperature sensors.

The structure parameters C_V^2 and C_q^2 can also (in principle) be determined from other wave propagation measurements. Humidity contributes to both the optical and radio refractive index, so intensity and phase fluctuations at the detector also contain information on C_q^2. The time changes in intensity and phase are affected not only by the mean wind across the optical and radio path, but also by the velocity fluctuations along the path; thus the received beam also contains information on C_V^2. In a more direct way, acoustic intensity fluctuations are caused by C_V^2. Techniques for inferring C_q^2 and C_V^2 from wave propagation measurements are yet to be demonstrated, however, and we will confine ourselves here to C_T^2.

Each of the two ways of determining C_T^2—from spaced, point temperature measurements or from optical scintillations—requires time averaging of a random signal. Our question is: how do the required averaging times compare?

We base our analysis on the discussion by Lumley and Panofsky (1964). They show that the averaging time T required to determine within a fractional error e the average f_T, defined by

$$f_T = \frac{1}{T}\int_t^{t+T} f(t')dt', \quad (48)$$

is

$$T = \frac{2\overline{f'^2}\tau}{\bar{f}^2 e^2}, \quad (49)$$

where τ is the integral scale of $f(t)$ and $\overline{f'^2}$ is the ensemble average variance of f about its ensemble mean $\bar{f}$. In each case here f is a squared signal:

$$f_{\text{direct}} = [T(\mathbf{x},t) - T(\mathbf{x}+\mathbf{r},t)]^2 = \Delta T^2, \quad (50)$$

$$f_{\text{optical}} = (\ln I)^2. \quad (51)$$

Thus

$$\left.\frac{\overline{f'^2}}{\bar{f}^2}\right|_{\text{direct}} = \frac{\overline{(\Delta T^2 - \overline{\Delta T^2})^2}}{(\overline{\Delta T^2})^2} = K_{\Delta T} - 1, \quad (52)$$

$$\left.\frac{\overline{f'^2}}{\bar{f}^2}\right|_{\text{optical}} = \frac{\overline{[(\ln I)^2 - \overline{(\ln I)^2}]^2}}{[\overline{(\ln I)^2}]^2} = K_{\text{optical}} - 1, \quad (53)$$

where K is the kurtosis of the signal, i.e.,

$$K_x = \frac{\overline{x^4}}{(\overline{x^2})^2}. \quad (54)$$

If we fix the accuracy requirements, the ratio of required averaging times becomes

$$\frac{T_{\text{direct}}}{T_{\text{optical}}} = \frac{[\tau(K-1)]_{\text{direct}}}{[\tau(K-1)]_{\text{optical}}}. \quad (55)$$

We will now estimate K and τ for the two processes. Taking the direct measurement first, we assume that the separation vector $\mathbf{r}$ of the temperature sensors is perpendicular to the mean horizontal wind $\bar{\mathbf{V}}$. The integral scale of the time signal $\Delta T^2(t) \equiv [T(\mathbf{x},t) - T(\mathbf{x}+\mathbf{r},t)]^2$ is defined in terms of its correlation function $R_{\Delta T^2}(\tau)$, i.e.,

$$\tau_{\Delta T^2} = \int_0^\infty R_{\Delta T^2}(\tau)d\tau \Big/ R_{\Delta T^2}(0), \quad (56)$$

where $R_{\Delta T^2}(\tau)$ is

$$R_{\Delta T^2}(\tau) = \overline{[\Delta T^2(0) - \overline{\Delta T^2}(0)][\Delta T^2(\tau) - \overline{\Delta T^2}(\tau)]}. \quad (57)$$

In order to make this tractable we will assume, for estimation purposes, that $\Delta T^2(t)$ is a Gaussian process. When (57) is expanded the fourth moments can be expressed as products of second moments, and we find

$$R_{\Delta T^2}(\tau) = 2R_{\Delta T}^2(\tau), \quad (58)$$

where $R_{\Delta T}(\tau)$ is the correlation function of ΔT, i.e.,

$$R_{\Delta T}(\tau) = \overline{[T(\mathbf{x},0) - T(\mathbf{x}+\mathbf{r},0)][T(\mathbf{x},\tau) - T(\mathbf{x}+\mathbf{r},\tau)]}. \quad (59)$$

Expanding (59), assuming Taylor's hypothesis in order to relate temporal and streamwise spatial correlations, and assuming local isotropy gives

$$R_{\Delta T}(\tau) = D_T(\mathbf{r} - \bar{\mathbf{V}}\tau) - D_T(\bar{\mathbf{V}}\tau). \quad (60)$$

In this range of separation we have

$$D_T(\mathbf{r}) = D_T(r) = C_T^2 r^{\frac{2}{3}}, \quad (61)$$

$$R_{\Delta T}(\tau) = C_T^2 |\mathbf{r} - \bar{\mathbf{V}}\tau|^{\frac{2}{3}} - C_T^2 |\bar{\mathbf{V}}\tau|^{\frac{2}{3}}. \quad (62)$$

Combining (56), (58) and (62) and integrating gives finally

$$\tau_{\Delta T^2} \approx r/3U, \tag{63}$$

where $U = |\vec{\mathbf{V}}|$.

We also need the kurtosis of ΔT. Park (1976) has measured structure functions of velocity and temperature at 3.8 and 4.8 m above the ocean, and finds that for streamwise separations in the range 1 cm$<r$ $<$100 cm,

$$K_{\Delta T} \approx 100\ (r/\eta)^{-0.4}, \tag{64}$$

where $\eta = (\nu^3/\epsilon)^{\frac{1}{4}}$ is the Kolmogorov microscale and ν the kinematic viscosity. Measurements at 1.4 m over a wheat canopy by Antonia and Van Atta (1978) suggest a constant in (64) closer to 80 and an exponent $-\frac{1}{3}$, but the difference for our purposes is negligible and we will use (64).

The expression for the averaging time for direct measurement of C_T^2 becomes, from (49), (51), (63) and (64),

$$T_{\text{direct}} \propto (30/U) r^{0.6} \eta^{0.4}. \tag{65}$$

This estimate should be useful throughout the boundary layer, although we will use it only in the surface layer. Note that the $\eta^{0.4}$ term is essentially constant there ($\eta^{0.4} \approx \epsilon^{-0.1}$) so the dominant influences on T_{direct} are r and U. We should not expect (65) to hold as we approach free convection ($U \rightarrow 0$) under unstable conditions. There the large eddies give short term mean horizontal wind speeds of the order of $w_* = [(g/T_0)Q_0 z_i]^{\frac{1}{3}}$ in the surface layer, and thus T_{direct} does not grow without bound in this case as $U \rightarrow 0$, as implied by (65).

We turn now to the optical scintillations. Log-intensity fluctuations are closely Gaussian (Lawrence and Strohbehn, 1970) so $K_{\text{optical}} = 3$, and

$$R_{(\ln I)^2}(\tau) = 2R_{\ln I}^2(\tau). \tag{66}$$

The correlation function $R_{\ln I}$ has been calculated (Clifford *et al.*, 1974). For a point detector and spherical waves, we find by numerical integration

$$\tau_{\text{optical}} \approx \frac{0.27(\lambda R)^{\frac{1}{2}}}{U_n}, \tag{67}$$

where λ is the optical wavelength, R the optical path length, and U_n the mean wind transverse to the path. Combining these results for K and τ for the optical process gives

$$T_{\text{optical}} \propto \frac{0.5(\lambda R)^{\frac{1}{2}}}{U_n}. \tag{68}$$

Note that U_n here plays the role of U in the expression (65) for T_{direct}, and we should not expect that $T_{\text{optical}} \rightarrow \infty$ as $U_n \rightarrow 0$ under very unstable conditions. Note also that T_{optical} is predicted to increase as the path length R increases. This is because optical technique essentially measures the intensity of eddies of size near $(\lambda R)^{\frac{1}{2}}$, and the averaging time to determine this intensity increases linearly with the eddy size. The increased averaging time indicated for longer paths should be offset by the increased representativeness of the longer path estimate. The path weighting function for the optical scintillation technique is closely parabolic, varying as $[(x/R)(1-x/R)]^{5/6}$, where x is the position along the optical path.

Combining (65) and (68) gives

$$\frac{T_{\text{direct}}}{T_{\text{optical}}} \approx \frac{60 r^{0.6} \eta^{0.4} U_n}{(\lambda R)^{\frac{1}{2}} U}. \tag{69}$$

For a typical case $r = 20$ cm, $\eta = 0.1$ cm, $U_n = U$, $\lambda = 0.6 \times 10^{-6}$ m, $R = 300$ m, giving

$$\frac{T_{\text{direct}}}{T_{\text{optical}}} \approx 100. \tag{70}$$

This indicates that the optical technique for determining C_T^2 under these conditions will require only 1% the averaging time of the direct measurement, for the same accuracy. Our experience indicates that at $z = 5$ m under typical conditions this means 10–20 s as opposed to 15–30 min. This estimate should hold throughout the surface layer.

5. Discussion

Other parameters which can in principle be determined from propagation measurements, such as the mean vertical gradients of wind, temperature and humidity, can also be used to infer fluxes indirectly. In fact this is the traditional approach to indirect flux measurements, as discussed by Champagne *et al.* (1977). We cast our theory in terms of structure parameters because they enter directly into the theory of wave propagation through turbulence; thus one can (in principle) infer structure parameters by observing turbulence effects on propagated waves.

Up to this point we have made no mention of the joint temperature-humidity structure parameter C_{Tq} which arises through the correlation between temperature and humidity fluctuations. C_{Tq} also affects electromagnetic and acoustic wave propagation (Gossard, 1960; Friehe *et al.*, 1975), since the refractive index fluctuation n can be expressed in general as

$$n = -C(\theta + aq), \tag{71}$$

where C is a conversion constant and the factor a depends on the wavelength of the radiation. Eq. (71) implies that the structure parameters are related by (Wyngaard *et al.*, 1978b)

$$C_n^2 \propto C_T^2 + 2aC_{Tq} + a^2 C_q^2. \tag{72}$$

Wyngaard *et al.* (1978b) discuss the surface layer behavior of C_{Tq}, showing that it is maintained by processes analogous to those which maintain C_T^2 and C_q^2. Our theory here could have been posed in terms of C_{Tq} rather than C_q^2, but we chose not to do this since at present there is no demonstrated technique for determining either one from propagation measurements. As propagation effects become better explored, however, the structure parameter C_{Tq} might well be a useful key to surface layer fluxes.

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