Mechanism for the Growth and Decay of Long- and Synoptic-Scale Waves in the Mid-Troposphere

S. K. KAO AND C. N. CHI

Department of Meteorology, University of Utah, Salt Lake City 84112

(Manuscript received 12 October 1977, in final form 4 April 1978)

ABSTRACT

An analysis of the forces and motion at 500 mb, between 30 and 60°N, in wavenumber-frequency domain, indicates that there exist definite cycles in the generation, transport and dissipation of the kinetic and available potential energies associated with long- and synoptic-scale waves. The growth and decay of the kinetic energy of long- and synoptic-scale waves are primarily controlled by the transport of kinetic energy to and from the waves through the nonlinear wave interactions, while the contribution to the kinetic energy through energy conversion tends to balance the effects of the Reynolds and frictional stresses. The evolution of the available potential energy associated with the long and synoptic waves is essentially the consequence of the transfer of thermal energy to and from the wave through the interaction between the velocity and temperature waves, while the transfer of thermal energy through the interactions between the velocity waves and the gradient of the zonal mean temperature tends to balance the effects of diabatic heating or cooling and energy conversion. The growth and decay of the kinetic energy of the zonal flow are primarily the result of the interaction between the velocity waves and the gradient of the mean zonal velocity, while the energy conversion from available potential to kinetic energy tends to balance the effects of the Reynolds and frictional stresses. The evolution of available potential energy associated with the zonal flow is essentially controlled by the interaction between the velocity waves and the gradient of the zonal mean temperature, while the effect of diabatic heating tends to balance the effect of energy conversion between the kinetic and available potential energies.

1. Introduction

Analyses of the transfer, conversion and dissipation of kinetic and available potential energies in a fluid are basic to the understanding of the mechanism for the wave motion in the fluid. For unstable waves of small amplitude in a barotropic fluid, it is known that the transfer of kinetic energy is generally from the mean flow to the waves. In a baroclinic fluid, an amplifying wave of small amplitude receives kinetic energy through the conversion of the available potential energy of the mean flow. The linear theory of instability describes well the growth of waves while their amplitudes remain small.

However, for waves of finite amplitude the transfers of energies due to nonlinear interactions become complex. In a barotropic fluid, interactions of two-dimensional waves of finite amplitude transfer a large portion of the kinetic energy to waves of larger scale and a small fraction of the energy to waves of smaller scale (Fjortoft, 1953), and the exchanges of kinetic energy between a zonal flow and two waves lead to a process similar to the index cycle in the atmosphere (Lorenz, 1960). Indeed, nonlinear interactions play an important role in the transfer of kinetic energy in wave motion.

Because of the effects of baroclinicity and topography of the earth's surface, the nonlinear interactions of waves in the atmosphere are even more complex. And because of the mathematical complexity, the characteristics of atmospheric waves have not been thoroughly analyzed. It is felt that to understand properly the mechanism for the large-scale wave motion in the atmosphere, it is necessary to analyze the processes of conversion transfer and dissipation of kinetic and available potential energies.

In recent years, a great deal of research has been carried out on the characteristics and energetics of large-scale atmospheric waves, and many valuable results have been obtained (Wendell, 1969; Wiin-Nielsen, 1967; Saltzman, 1970; Yang, 1967; Kao, 1968, 1970; Kao and Wendell, 1970; Steinberg *et al.*, 1971; Gruber, 1975; Pratt, 1976; Tenenbaum, 1976; Burrows, 1976; Kao and Lee, 1977). However, these studies have been confined mostly to the maintenance of the seasonal or yearly mean state of the waves. Questions arise as to what the mechanism is for the growth and decay of synoptic and long waves in the atmosphere, and what the contribution is of energy conversion, wave interactions and energy dissipation to the evolution of these waves. A preliminary analysis of such a mechanism for the evolution of large-scale atmospheric waves with the use of composite average of each term in the kinetic energy equation indicates that there exists a definite cycle in the transfer, con-

0022-4928/78/1375-1387$06.50

version and dissipation of kinetic energy at various stages of the evolution (Tsay and Kao, 1978). However, the composite average approach can only provide an estimate of the linear and nonlinear contributions to the evolution of the kinetic and available potential energies of the waves. To gain a deeper insight into the mechanism for the growth and decay of large-scale atmospheric waves, it is necessary to analyze the kinetic and available potential energies in the wavenumber-frequency domain. In this paper, we report some of the results obtained from our analysis of linear and nonlinear contributions to the growth and decay of wave energies at 500 mb, 30–60°N, winter 1975. Analyses of wave mechanism at various levels in the tropics and higher latitudes are being made and will be reported in a later paper.

2. Theoretical consideration

To analyze the mechanism for the evolution of the kinetic and available potential energies of large-scale waves in the atmosphere, we shall first determine the spectral kinetic and available potential energies of waves of various wavenumbers and frequencies, which may respectively be calculated from

$$E_K(k,\omega) = [2\pi(\sin\phi_2-\sin\phi_1)]^{-1} \times \int_{\phi_1}^{\phi_2} \cos\phi \int_0^{2\pi} \{|U(k,t)|^2+|V(k,t)|^2\} \times e^{-i\omega t}dtd\phi, \quad (1)$$

$$E_A(k,\omega) = [2\pi(\sin\phi_2-\sin\phi_1)]^{-1} \times \int_{\phi_1}^{\phi_2} c_p\gamma \cos\phi \int_0^{2\pi} |\theta(k,t)|^2 e^{-i\omega t}dtd\phi, \quad (2)$$

where

$$U(k,t)=\frac{1}{2\pi}\int_0^{2\pi} u(\lambda,t)e^{-ik\lambda}d\lambda, \quad (3)$$

$$V(k,t)=\frac{1}{2\pi}\int_0^{2\pi} v(\lambda,t)e^{-ik\lambda}d\lambda, \quad (4)$$

are respectively the Fourier coefficients of the zonal and meridional components of the wind velocities at the pressure level in question, and

$$\theta(k,t)=(1/2\pi)\int_0^{2\pi} \{T(\lambda,t)-\langle\bar{T}\rangle\}e^{-ik\lambda}d\lambda \quad (5)$$

is the Fourier coefficient of the temperature deviation from the meridional-zonal mean temperature $\langle\bar{T}\rangle$ at the pressure level in question. In (1) and (2) $E_K(k,\omega)$ and $E_A(k,\omega)$ are respectively the kinetic and available potential energy spectra in wavenumber (k) and frequency (ω) domain averaged over the latitudinal belt between latitude ϕ_1, and ϕ_2, c_p is the specific heat at constant pressure, $\gamma=(\bar{T}-c_pR^{-1}p\partial\bar{T}/\partial p)^{-1}$, $\mu=Rc_p^{-1}$.

To gain insights into the mechanism for the evolution of the kinetic and available potential energies of large-scale waves in the atmosphere, it is necessary to analyze the effects of nonlinear interactions, energy conversion, eddy stresses and frictional forces on the growth and decay of the atmospheric waves. To do so, we need to transform the equations governing the kinetic and available potential energies to the wavenumber-frequency domain.

Applying the Fourier transform, first expressing the equations for kinetic and available potential energies in wavenumber domain and then in wavenumber-frequency space, the evolution and rate of change in the kinetic and available potential energies associated with waves of wavenumber k and oscillating amplitude at frequency ω can be shown to take the form (Kao, 1977)

$$\begin{aligned}
\frac{\partial}{\partial t}[|E_K(k,\omega)|\sin(\omega t-\pi/2)] &= \omega|E_K(k,\omega)|\sin\omega t \\
&= |N_K(k,\omega)|\sin[\omega t+\alpha_{NK}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2] \\
&+|M_K(k,\omega)|\sin[\omega t+\alpha_{MK}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2] \\
&+|A_K(k,\omega)|\sin[\omega t+\alpha_{AK}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2] \\
&+|G_K(k,\omega)|\sin[\omega t+\alpha_{GK}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2] \\
&+|N_{KB}(k,\omega)|\sin[\omega t+\alpha_{NKB}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2] \\
&+|A_{KB}(k,\omega)|\sin[\omega t+\alpha_{AKB}(k,\omega) \\
&\qquad -\alpha_{EK}(k,\omega)-\pi/2], \quad k,\omega\neq 0, \quad (6)
\end{aligned}$$

$$\begin{aligned}
\frac{\partial}{\partial t}[|E_A(k,\omega)|\sin(\omega t-\pi/2)] &= \omega|E_A(k,\omega)|\sin\omega t \\
&= |N_{A1}(k,\omega)|\sin[\omega t+\alpha_{NA1}(k,\omega)-\alpha_{EA}(k,\omega)-\pi/2] \\
&+|N_{A2}(k,\omega)|\sin[\omega t+\alpha_{NA2}(k,\omega)-\alpha_{EA}(k,\omega)-\pi/2] \\
&+|M_A(k,\omega)|\sin[\omega t+\alpha_{MA}(k,\omega)-\alpha_{EA}(k,\omega)-\pi/2] \\
&+|K_A(k,\omega)|\sin[\omega t+\alpha_{KA}(k,\omega)-\alpha_{EA}(k,\omega)-\pi/2] \\
&+|H_A(k,\omega)|\sin[\omega t+\alpha_{HA}(k,\omega)-\alpha_{EA}(k,\omega)-\pi/2] \\
&+|N_{AB}(k,\omega)|\sin[\omega t+\alpha_{NAB}(k,\omega) \\
&\qquad -\alpha_{EA}(k,\omega)-\pi/2], \quad k,\omega\neq 0, \quad (7)
\end{aligned}$$

where

$$N_K(k,\omega)=[2\pi(\sin\phi_2-\sin\phi_1)]^{-1}\int_{\phi_1}^{\phi_2}\cos\phi\sum_{\substack{j=-\infty\\ \neq 0}}^{+\infty}\int_0^{2\pi}\Big(\frac{-ik}{\cos\phi}\{U(j,t)[U(-k,t)U(k-j,t)$$
$$-U(k,t)U(-k-j,t)]+V(j,t)[V(-k,t)U(k-j,t)-V(k,t)U(-k-j,t)]\}$$
$$+\{U(j,t)[U_\phi(-k,t)V(k-j,t)+U_\phi(k,t)V(-k-j,t)]+V(j,t)[V_\phi(-k,t)V(k-j,t)$$
$$+V_\phi(k,t)V(-k-j,t)]\}+\{V(j,t)[U(-k,t)U(k-j,t)+U(k,t)U(-k-j,t)]$$
$$-U(j,t)[V(-k,t)U(k-j,t)+V(k,t)U(-k-j,t)]\}\tan\phi-a\{U(j,t)[U_p(-k,t)W(k-j,t)$$
$$+U_p(k,t)W(-k-j,t)]+V(j,t)[V_p(-k,t)W(k-j,t)+V_p(k,t)W(-k-j,t)]\}\Big)e^{-i\omega t}dtd\phi \quad (8)$$

is the contribution of the nonlinear interactions of wave motion associated with the advections; subscripts ϕ and p denote partial differentiation with respect to latitude and pressure, respectively;

$$M_K(k,\omega)=[-2\pi(\sin\phi_2-\sin\phi_1)]^{-1}\int_{\phi_1}^{\phi_2}\int_0^{2\pi}\cos\phi\Big\{\frac{\cos\phi}{a}[V(k,t)U(-k,t)+V(-k,t)U(k,t)]\frac{\partial}{\partial\phi}\Big(\frac{\bar{u}}{\cos\phi}\Big)$$
$$+2|V(k,t)|^2\frac{1}{a}\frac{\partial\bar{v}}{\partial\phi}+2|U(k,t)|^2\bar{v}\frac{\tan\phi}{a}+[U(k,t)W(-k,t)+U(-k,t)W(k,t)]\frac{\partial\bar{u}}{\partial p}$$
$$+[V(k,t)W(k,t)+V(-k,t)W(k,t)]\frac{\partial\bar{v}}{\partial p}\Big\}e^{-i\omega t}dtd\phi \quad (9)$$

is the contribution of the nonlinear interactions associated with the velocity shears of the zonal mean flow, $\bar{u}$ and $\bar{v}$;

$$A_K(k,\omega)=[-R/2\pi p(\sin\phi_2-\sin\phi_1)]\times\int_{\phi_1}^{\phi_2}\int_0^{2\pi}[\theta(k,t)W(-k,t)+\theta(-k,t)W(k,t)]\times\cos\phi e^{-i\omega t}dtd\phi \quad (10)$$

is the contribution through the conversion of available potential energy to the kinetic energy;

$$N_{KB}(k,\omega)=[-1/2\pi a(\sin\phi_2-\sin\phi_1)]\int_0^{2\pi}\sum_{\substack{j=-\infty\\ \neq 0}}^{+\infty}\{U(j,t)\times[U(k,t)V(-k-j,t)+U(-k,t)V(k-j,t)]+V(j,t)[V(k,t)V(-k-j,t)+V(-k,t)V(k-j,t)]\}\cos\phi|_{\phi=\phi_1}^{\phi=\phi_2}e^{-i\omega t}dt \quad (11)$$

is the contribution of the horizontal boundary flux arising from the nonlinear interactions of the wave motion; $A_{KB}(k,\omega)$ is the contribution of the horizontal boundary flux arising from the conversion of the available potential energy into the kinetic energy; and $G_K(k,\omega)$ is the contribution of the Reynolds-dissipation stresses.

The Fourier coefficient $J_s(k,\omega)$ may be expressed in terms of its real and imaginary parts as

$$J_s(k,\omega)=J_s(k,\omega)_r+iJ_s(k,\omega)_i. \quad (12)$$

Thus,

$$|J_s(k,\omega)|=[J_s^2(k,\omega)_r+J_s^2(k,\omega)_i]^{\frac{1}{2}}, \quad (13)$$

and

$$\alpha_{J_s}(k,\omega)=\arctan\frac{J_s(k,\omega)_i}{J_s(k,\omega)_r} \quad (14)$$

is the phase angle of J_s wave at $t=0$. We express

$$N_A(k,\omega)=N_{A1}(k,\omega)+N_{A2}(k,\omega), \quad (15)$$

where

$$N_{A1}(k,\omega)=[2\pi a(\sin\phi_2-\sin\phi_1)]^{-1}\times\int_{\phi_1}^{\phi_2}\sum_{\substack{j=-\infty\\ \neq 0}}^{+\infty}\int_0^{2\pi}c_p\gamma\{-ik\theta(j,t)[\theta(-k,t)U(k-j,t)-\theta(k,t)U(-k-j,t)]+\theta(j,t)[\theta_\phi(k,t)V(-k-j,t)+\theta_\phi(-k,t)V(k-j,t)]\cos\phi\}e^{-i\omega t}dtd\phi \quad (16)$$

is the contribution of the nonlinear interactions of thermal waves associated with the horizontal advection;

and

$$N_{A2}(k,\omega)=[2\pi a(\sin\phi_2-\sin\phi_1)]^{-1}\times\int_{\phi_1}^{\phi_2}\sum_{\substack{j=-\infty\\ \neq 0}}^{+\infty}\int_0^{2\pi}\{c_p\gamma\theta(j,t)[\theta(k,t)W(-k-j,t)+\theta(-k,t)W(k-j,t)]_p+(R\gamma/p)\theta(j,t)\times[\theta(k,t)W(-k-j,t)+\theta(-k,t)W(k-j,t)]\}\times\cos\phi e^{-i\omega t}dtd\phi \quad (17)$$

is the contribution of the nonlinear interactions of thermal waves associated with the vertical advection. We may express

$$M_A(k,\omega)=M_{A1}(k,\omega)+M_{A2}(k,\omega), \quad (18)$$

where

$$M_{A1}(k,\omega)=[-2\pi a(\sin\phi_2-\sin\phi_1)]^{-1}\times\int_{\phi_1}^{\phi_2}\int_0^{2\pi}c_p\gamma[\theta(k,t)V(-k,t)+\theta(-k,t)V(k,t)]\frac{\partial\bar{T}}{\partial\phi}\times\cos\phi e^{-i\omega t}dtd\phi \quad (19)$$

is the contribution associated with the latitudinal gradient of the zonal mean temperature; and

$$M_{A2}(k,\omega)=[-2\pi a(\sin\phi_2-\sin\phi_1)]^{-1}\times\int_{\phi_1}^{\phi_2}\int_0^{2\pi}c_p\gamma[\theta(k,t)W(-k,t)+\theta(-k,t)W(k,t)]\frac{\partial\bar{T}}{\partial p}\times\cos\phi e^{-i\omega t}dtd\phi \quad (20)$$

is the contribution associated with the vertical gradient of the zonal mean temperature. Notice that

$$K_A(k,\omega)=-A_K(k,\omega); \quad (21)$$

$$N_{AB}(k,\omega)=[-2\pi a(\sin\phi_2-\sin\phi_1)]^{-1}\times\int_0^{2\pi}c_p\gamma\sum_{\substack{j=-\infty\\ \neq 0}}^{+\infty}\theta(j,t)[\theta(k,t)V(-k-j,t)+\theta(-k,t)V(k-j,t)]\cos\phi\Big|_{\phi=\phi_1}^{\phi=\phi_2}e^{-i\omega t}dt \quad (22)$$

is the contribution due to the horizontal flux of thermal energy at the boundaries; and $H_A(k,\omega)$ is the contribution of the diabatic heating or cooling to the rate of change in the available potential energy associated with waves of wavenumber k oscillating at frequency ω. Eqs. (8)–(22) provide the expressions and interpretation of terms in Eqs. (6) and (7).

Eq. (6) indicates that the growth and decay of the kinetic energy associated with waves of wavenumber k, oscillating at frequency ω, depends on the eddy transport of kinetic energy resulting from the nonlinear wave interaction (N_K), the interaction between waves and the gradient of the mean zonal flow (M_K), the rate of conversion of the available potential to kinetic energy (A_K), the effects of Reynolds and frictional stresses (G_K), and the effects of flux of kinetic energy at the boundaries (N_{KB}). Eq. (7) shows that the evolution of the available potential energy associated with waves of wavenumber k, oscillating at ω, depends on the eddy transport of available potential energy resulting from the interaction between the velocity and temperature waves (N_A), the interaction between the velocity wave and the gradient of the zonal mean temperature (M_A), the rate of conversion from available potential to kinetic energy (K_A), the effect of diabatic heating or cooling (H_A), and the flux of available potential energy at the boundaries (N_{AB},K_{AB}).

By following the same procedure in obtaining (6) and (7), the equations for the evolution and rate of change of the kinetic and available potential energies associated with the zonal mean flow can be shown to be

$$\frac{\partial}{\partial t}\{|E_K(0,\omega)|\sin(\omega t-\pi/2)\}=\omega|E_K(0,\omega)|\sin\omega t$$
$$=|M_K(0,\omega)|\sin[\omega t+\alpha_{MK}(0,\omega)-\alpha_{EK}(0,\omega)-\pi/2]+|A_K(0,\omega)|\sin[\omega t+\alpha_{AK}(0,\omega)-\alpha_{EK}(0,\omega)-\pi/2]+|G_K(0,\omega)|\sin[\omega t+\alpha_{GK}(0,\omega)-\alpha_{EK}(0,\omega)-\pi/2]+|M_{KB}(0,\omega)|\sin[\omega t+\alpha_{MKB}(0,\omega)-\alpha_{EK}(0,\omega)-\pi/2]+|A_{KB}(0,\omega)|\sin[\omega t+\alpha_{AKB}(0,\omega)-\alpha_{EK}(0,\omega)-\pi/2],\quad \omega\neq 0, \quad (23)$$

$$\frac{\partial}{\partial t}\{|E_A(0,\omega)|\sin(\omega t-\pi/2)\}=\omega|E_A(0,\omega)|\sin\omega t$$
$$=|M_A(0,\omega)|\sin[\omega t+\alpha_{MA}(0,\omega)-\alpha_{EA}(0,\omega)-\pi/2]+|K_A(0,\omega)|\sin[\omega t+\alpha_{KA}(0,\omega)-\alpha_{EA}(0,\omega)-\pi/2]+|H_A(0,\omega)|\sin[\omega t+\alpha_{HA}(0,\omega)-\alpha_{EA}(0,\omega)-\pi/2]+|M_{AB}(0,\omega)|\sin[\omega t+\alpha_{MAB}(0,\omega)-\alpha_{EA}(0,\omega)-\pi/2],\quad \omega\neq 0, \quad (24)$$

where

$$M_K(0,\omega)=-\sum_{k=1}^{+\infty}M_K(k,\omega), \quad (25)$$

$$M_A(0,\omega)=-\sum_{k=1}^{\infty}M_A(k,\omega). \quad (26)$$

It may be noted from (23)–(26) that the rate of change of the kinetic energy of the zonal mean flow depends on the interactions of the gradient of the zonal mean velocity field and the velocity waves of all wavenumbers, and the rate of change of the available potential energy of the mean zonal flow depends on the interactions of the gradient of the zonal mean temperature field and the thermal and velocity waves of all wavenumbers.

It can be shown that for stationary amplitude waves ($\omega=0$), the mechanism for the maintenance of the kinetic and available potential energies is respectively governed by

$$N_K(k,o)+M_K(k,o)+A_K(k,o)+G_K(k,o)+A_{KB}(k,o)+M_{KB}(k,o)=0,\quad k\neq 0, \tag{27}$$

$$N_A(k,o)+M_A(k,o)+K_A(k,o)+H_A(k,o)+N_{AB}(k,o)=0,\quad k\neq 0. \tag{28}$$

For stationary mean zonal flow, the mechanism for the maintenance of the kinetic and available potential energies is respectively governed by

$$M_K(o,o)+A_K(o,o)+G_K(o,o)+A_{KB}(o,o)+M_{KB}(o,o)=0, \tag{29}$$

$$M_A(o,o)+K_A(o,o)+H_A(o,o)+M_{AB}(o,o)+K_{AB}(o,o)=0. \tag{30}$$

The above four equations show that the mechanism for the maintenance of kinetic and available potential energies of stationary amplitude waves and stationary mean zonal flow depends on the balance of the contributions of interactions of waves (N_K,N_A), interactions between waves and mean velocity and temperature fields (M_K,M_A), conversion of energies (A_K,K_A), Reynolds and frictional stresses (G_K), diabatic heating or cooling (H_A) and fluxes of kinetic and available potential energies at boundaries (N_{KB},N_{AB},M_{KB}).

Eqs. (6), (7), (23), (24), (27), (28), (29) and (30) will be used to analyze the growth and decay of the kinetic and available potential energies of large-scale atmospheric waves and the mean zonal flow.

Before we start to analyze the mechanism for the evolutions of the kinetic and available potential energies of transient waves and the zonal mean flow, we shall develop a relation between the evolution of the kinetic and available potential energies of transient waves. Such a relation may be obtained by eliminating the energy conversion term from the equations of kinetic and available potential energies [(6) and (7)] and can be shown to take the form

$$\begin{aligned}
&\frac{\partial}{\partial t}\{|E_K(k,\omega)|\sin\omega t+|E_A(k,\omega)|\sin[\omega t+\alpha_{EA}(k,\omega)-\alpha_{EK}(k,\omega)]\}\\
&=|N_K(k,\omega)|\sin[\omega t+\alpha_{NK}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2]+|M_K(k,\omega)|\sin[\omega t+\alpha_{MK}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2]\\
&\quad+|G_K(k,\omega)|\sin[\omega t+\alpha_{GK}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2]+|N_{KB}(k,\omega)|\sin[\omega t+\alpha_{NKB}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2]\\
&\quad+|A_{KB}(k,\omega)|\sin[\omega t+\alpha_{AKB}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2]+|N_{A1}(k,\omega)|\sin[\omega t+\alpha_{NA1}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2]\\
&\quad+|N_{K2}(k,\omega)|\sin[\omega t+\alpha_{NK2}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2]+|M_A(k,\omega)|\sin[\omega t+\alpha_{MA}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2]\\
&\quad+|H_A(k,\omega)|\sin[\omega t+\alpha_{HA}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2]\\
&\quad+|N_{AB}(k,\omega)|\sin[\omega t+\alpha_{NAB}(k,\omega)-\alpha_{EK}(k,\omega)-\pi/2],\quad k,\omega\neq 0.
\end{aligned} \tag{31}$$

It may be noted that there is a phase shift of $[\alpha_{EA}(k,\omega)-\alpha_{EK}(k,\omega)]$ between the evolutions of the kinetic and available potential energies of the transient waves.

3. Data and computational method

The data used in this analysis are extracted from the objective analysis of the National Meteorological Center (NMC) for the period 0000 GMT 1 December 1975 to 1200 GMT 29 February 1976, at 200, 500 and 850 mb pressure levels. The horizontal wind, temperature and height fields are objectively analyzed from observed data by means of Flattery's method (1967). The ω fields are obtained by computing ω values at the mid-point of each σ layer in the forecast model (Shuman and Hovermale, 1968).

The procedure of computation involves first the calculation of Fourier coefficients in the wavenumber domain with the use of (3), (4) and (5) by applying the fast Fourier transform, next the computation of the energy spectra in the wavenumber-frequency domain with the use of (1) and (2), and finally the application of the so-called "tapering" spectral window (Tukey, 1967) to the spectra. The dissipation-Reynolds-stresses term $G_K(k,\omega)$ and the diabatic heating term $H_A(k,\omega)$ are computed as the residual terms from the kinetic and available potential energy equations [(6) and (7)], respectively.

4. Maintenance of the kinetic and available potential energies of the stationary-amplitude waves and stationary mean zonal flow

To analyze the mechanism for the maintenance of kinetic and available potential energies associated with

TABLE 1. Values of the terms in Eqs. (27) and (29) for stationary-amplitude waves and mean zonal flow [units: $(m\ s^{-1})^2\ day^{-1}$].

k	$N_K\ (k,0)$	$M_K\ (k,0)$	$A_K\ (k,0)$	$A_{KB}\ (k,0)$	$N_{KB}\ (k,0)$	$M_{KB}\ (k,0)$	$G_K\ (k,0)$
0	0.00	1.89	18.07	−1.65	0.00	11.48	−29.92
1	5.69	−0.31	4.09	−0.25	−2.89	0.00	− 6.33
2	−2.93	−0.48	13.79	−0.08	1.30	0.00	−11.57
3	3.51	−0.50	8.31	−0.99	0.04	0.00	−10.36
4	−0.26	0.28	5.27	−1.13	0.20	0.00	− 4.36
5	−2.25	−0.01	5.91	−0.67	0.70	0.00	− 3.58
6	−1.95	−0.29	6.32	−0.79	0.46	0.00	− 3.74
7	−0.60	−0.22	8.06	−1.42	−0.07	0.00	− 5.76
8	−0.62	−0.08	7.74	−0.41	−0.69	0.00	− 5.94
9	0.03	−0.08	4.06	−0.50	−0.41	0.00	− 3.16
10	−0.40	−0.04	2.22	0.13	0.07	0.00	− 1.97

the stationary-amplitude waves and stationary mean zonal flow, we have computed the seasonal mean values of the terms in Eqs. (27)–(30) for wavenumbers 0–20. Since the magnitude of the energy spectra decreases about two orders of magnitude from wavenumber 0 to 10, only those values for waves in this wavenumber range are listed in Tables 1 and 2.

It is seen from Table 1 that the maintenance of the kinetic energy of the stationary amplitude waves is essentially through the balance of 1) the energy gain by converting available potential to kinetic energy (A_K,A_{KB}), 2) the energy loss through the effects of the Reynolds and molecular frictional stresses (G_K), and through the interaction between velocity waves and the gradient of the mean zonal velocity (M_K), and (3) energy transport through nonlinear wave interactions (N_K). Since there is no net gain or loss of kinetic energy through nonlinear wave interactions, the kinetic energy must be transferred from waves with negative values of N_K to those with positive values. Table 1 indicates that nonlinear wave interaction transfers kinetic energy from waves of wavenumbers 2, 4, 5, 6, 7, 8 and 10 to the ultralong waves of wavenumbers 1 and 3.

Table 1 indicates that the kinetic energy of the stationary zonal flow is primarily maintained by the balance of 1) the energy gain through the conversion of available potential to kinetic energy (A_K) and the interaction of the velocity waves and the gradient of the mean zonal velocity (M_K,M_{KB}), and 2) the energy loss through the effects of the Reynolds and molecular frictional stresses (G_K). It may be noted that the energy gain of the stationary zonal flow through M_K is equal to the total energy loss of all stationary amplitude waves through M_K.

It is seen from Table 2 that the maintenance of the available potential energy of the stationary-amplitude waves is essentially through the balance of 1) the gain of available potential energy through the interaction between the velocity waves and the gradient of the zonal mean temperature (M_A) and the diabatic effect (H_A), 2) the loss of available potential energy through the energy conversion (K_A), and 3) the transfer of the available potential energy through the interaction of the velocity and temperature waves and the boundary effect (N_A,N_{AB}), which transfers the available potential energy from waves of wavenumbers 1, 2 and 5 to those of wavenumbers 4, 6, 7, 8, 9 and 10.

Table 2 also indicates that the available potential energy of the stationary mean zonal flow is primarily maintained by the gain of the available potential energy through diabatic heating (H_A) and the loss of available potential energy through the interaction between the velocity waves and the gradient of the zonal mean temperature (M_A), and through energy conversion (K_A). It may be noted that the loss of

TABLE 2. As in Table 1 except for Eqs. (28) and (30).

k	$N_{A1}\ (k,0)$	$N_{A2}\ (k,0)$	$M_A\ (k,0)$	$K_A\ (k,0)$	$N_{AB}\ (k,0)$	$M_{AB}\ (k,0)$	$H_A\ (k,0)$
0	0.00	0.00	−68.48	−18.07	0.00	−0.05	86.60
1	−3.14	−0.58	7.51	− 4.09	−1.89	0.00	2.19
2	−2.35	−1.35	9.61	−13.79	1.22	0.00	6.66
3	0.03	−0.17	15.39	− 8.31	−6.84	0.00	−0.09
4	0.77	−0.94	9.67	− 5.27	−2.62	0.00	−1.60
5	−0.39	−0.14	5.52	− 5.91	0.83	0.00	0.10
6	0.62	−0.01	5.14	− 6.32	−0.04	0.00	0.61
7	0.96	−0.14	7.08	− 8.06	−0.36	0.00	0.52
8	1.33	−0.38	5.73	− 7.74	0.14	0.00	0.92
9	1.62	−0.31	2.27	− 4.06	−0.30	0.00	0.79
10	1.02	−0.35	.78	− 2.22	−0.13	0.00	0.90

available potential energy of the stationary mean zonal flow through M_A is equal to the total gain of available potential energy of the stationary amplitude waves through M_A.

The values listed in Tables 1 and 2 are in general agreement with those reported by Saltzman (1970) and Steinberg *et al.* (1971), except that earlier analyses considered the kinetic and available potential energies of the entire atmosphere; therefore, the terms involving the fluxes at the boundaries disappear.

For reference, the spectra of the kinetic and available potential energies of the stationary-amplitude waves are computed and shown in Fig. 1. It may be noted that both the spectra generally decrease with increasing wavenumber and are approximately proportional to the −3 power of the wavenumber in the range of wavenumbers 7–20.

5. The growth and decay of the large-scale atmospheric waves

To analyze the growth and decay of large-scale waves, the daily values of the kinetic and available potential energies associated with waves of wavenumbers 1–8 are computed and plotted in Fig. 2. It may be noted that the growth and decay of the kinetic energy are almost in phase with that of the available potential energy, and that they show certain periodicities.

Eqs. (6) and (7) were used for the computation of the rate of change of the kinetic and available potential energy spectra on 500 mb surface for the domain 30–60°N, which represents middle latitudes in the mid-troposphere. Since waves having a small rate of change in kinetic energy are not of interest here, we analyzed those waves having large rates of change in kinetic energy. To do so, we selected waves of the following wavenumber-periods (k,τ): (1,9.1), (2,7.1), (3,9.1), (4,5.6), (5,5.1), (6,4.9), (7,4.4) and (8,4.9). The amplitudes and phase angles of various terms in (6) and (7) are shown in Table 3.

Several characteristic features of the values of the amplitudes and phase angles in Table 3 may be noted. First of all, the nonlinear interaction term N_K has generally the largest amplitude as compared with other terms in the table, and the phase angle between N_K and $(\partial E_K/\partial t)$ is generally small. These indicate that the growth and decay in the kinetic energy of the large-scale atmospheric waves are primarily the consequence of the nonlinear interactions. With respect to the available potential energy, the interaction between the temperature and horizontal velocity waves (N_{A1}) and those between these waves and the gradient of the zonal mean temperature (M_A) have the comparatively large amplitudes.

To examine the effects of various terms in (6), (7), (23) and (24) on the maximum rate of change in the kinetic and available potential energies, letting $\omega t=\pi/2$

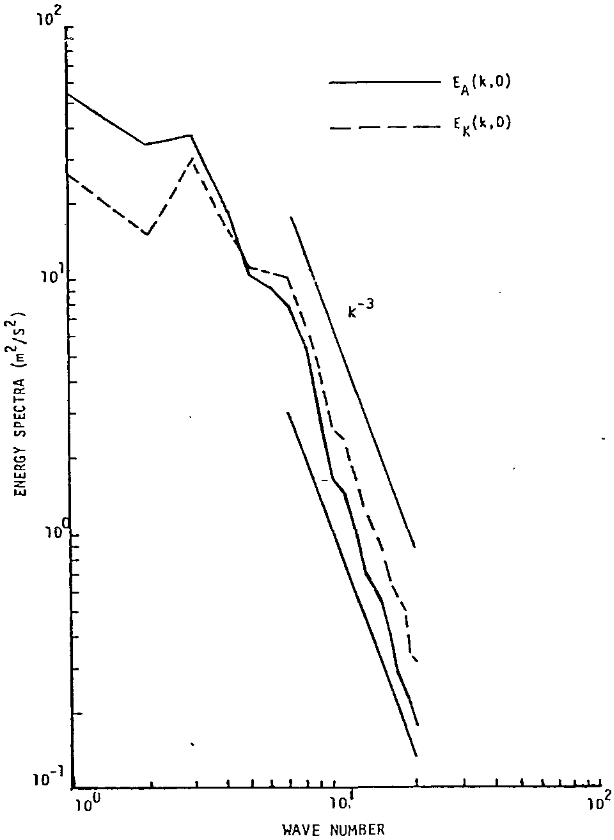

FIG. 1. Stationary parts of kinetic energy (dashed) and available potential energy (solid) spectra for the belt 30–60°N, 500 mb, winter 1975/76. The two solid straight lines have k^{-3} dependence.

in these two equations, we have

$$\left(\frac{\partial E_K(k,\omega)}{\partial t}\right)_{\max} =\omega|E_K(k,\omega)|=a_1|N_K(k,\omega)|+a_2|M_K(k,\omega)| +a_3|A_K(k,\omega)|+a_4|A_{KB}(k,\omega)|+a_5|N_{KB}(k,\omega)| +a_6|G_K(k,\omega)| \quad k\neq 0, \quad (32)$$

$$\left(\frac{\partial E_A(k,\omega)}{\partial t}\right)_{\max} =\omega|E_A(k,\omega)|=b_1|N_{A1}(k,\omega)|+b_2|N_{A2}(k,\omega)| +b_3|M_A(k,\omega)|+b_4|K_A(k,\omega)|+b_5|N_{AB}(k,\omega)| +b_6|H_A(k,\omega)| \quad k\neq 0, \quad (33)$$

$$\left(\frac{\partial E_K(0,\omega)}{\partial t}\right)_{\max} =\omega|E_K(0,\omega)|=a_2|M_K(0,\omega)|+a_3|A_K(0,\omega)| +a_4|A_{KB}(0,\omega)|+a_6|G_K(0,\omega)| +a_7|M_{KB}(0,\omega)|, \quad (34)$$

$$\left(\frac{\partial E_A(0,\omega)}{\partial t}\right)_{\max} =\omega|E_A(0,\omega)|=b_3|M_A(0,\omega)|+b_4|K_A(0,\omega)| +b_7|M_{AB}(0,\omega)|, \quad (35)$$

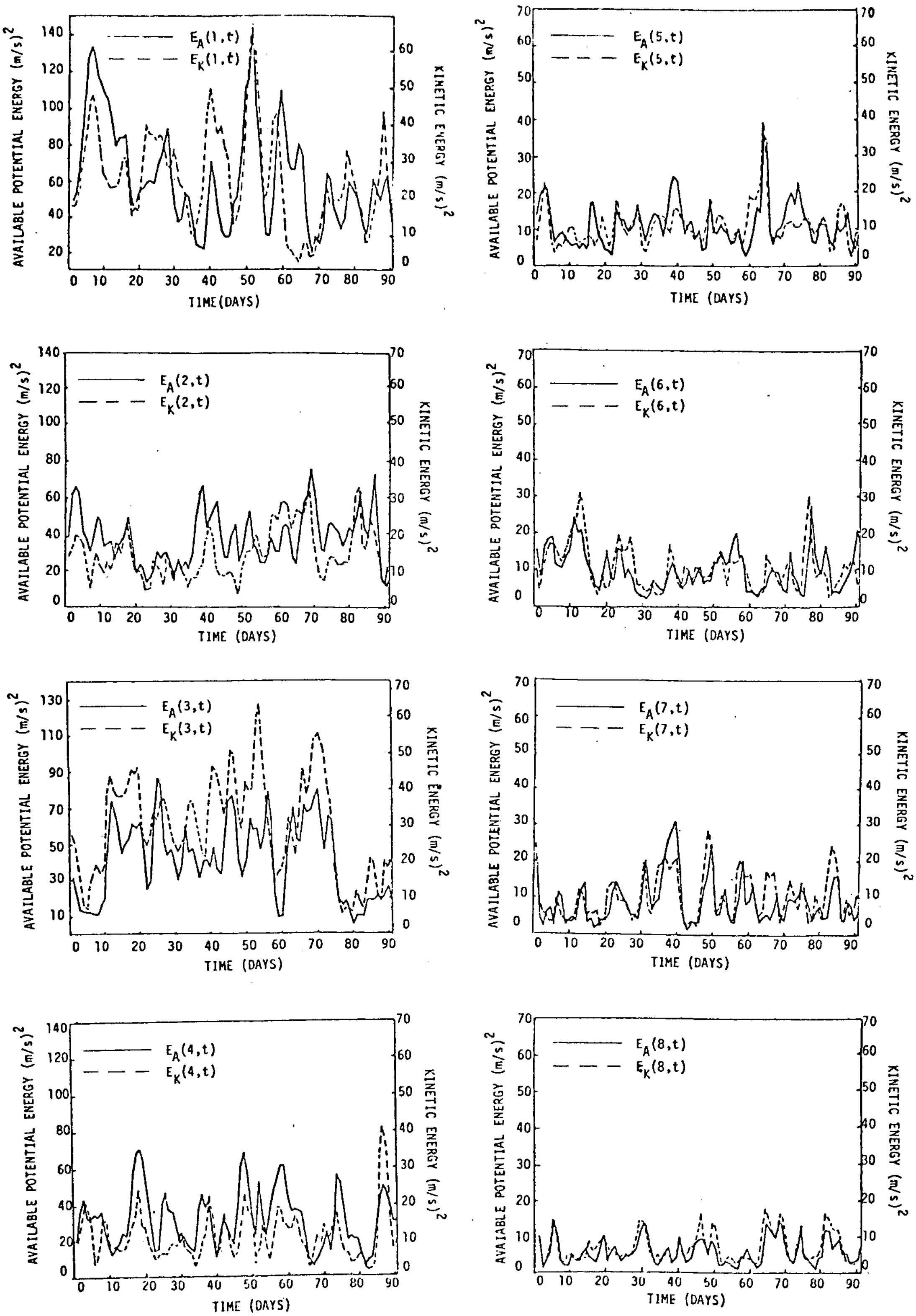

FIG. 2. Variations of the available potential energy (solid curves) and kinetic energy (dashed curves) associated with waves of wavenumbers 1–8.

TABLE 3. The amplitude and phase angle relative to the rate of change in the kinetic and available energies in Eqs. (6), (7), (23) and (24). Units: (m s^{-1})^2 day^{-1}.

k	Period (days)		$\left(\frac{\partial E_K}{\partial t}\right)_{max}$	N_K	M_K	A_K	A_{KB}	N_{KB}	G_K	$\left(\frac{\partial E_A}{\partial t}\right)_{max}$	N_{A1}	N_{A2}	M_A	K_A	N_{AB}	H_A
0	9.1	Ap	1.55	0	0.74	4.86	.93	1.14	5.3−	6.42	0	0	6.01	4.86	0	5.05
		α	(177)	0	−88	48	77	−172	−137	(−111)	0	0	−125	−132	0	31
1	9.1	Ap	2.15	2.75	0.11	1.56	0.31	1.02	1.86	2.77	1.06	0.27	0.85	1.56	0.06	1.14
		α	(66)	5	−71	101	141	−168	−61	(67)	−11	29	58	−80	102	47
2	7.1	Ap	1.14	1.47	0.18	0.81	0.28	0.70	0.93	1.64	1.21	0.06	1.83	0.81	0.12	1.78
		α	(50)	46	−79	112	−102	−38	−96	(34)	−14	124	71	−52	−118	−94
3	9.1	Ap	1.24	1.61	0.74	0.32	0.60	0.52	1.33	1.76	2.39	0.17	0.75	0.32	0.01	1.13
		α	(177)	17	−42	47	115	105	−116	(−154)	2	56	24	−162	28	−173
4	5.6	Ap	1.84	1.77	0.19	0.33	0.37	0.67	0.63	2.28	1.16	0.15	1.91	0.33	0.05	0.48
		α	(11)	−15	1	18	−140	38	162	(31)	−22	−41	30	−182	−195	−112
5	5.1	Ap	1.14	0.90	0.13	0.38	0.42	0.46	0.99	1.28	0.59	0.08	0.63	0.38	0.06	0.74
		α	(61)	−38	11	−123	127	−52	61	(41)	−30	144	57	77	10	−62
6	4.9	Ap	1.01	0.87	0.08	0.82	0.41	0.23	0.72	0.77	0.83	0.07	0.81	0.82	0.02	0.41
		α	(34)	−7	−33	61	−168	97	−82	(25)	−13	145	91	−110	145	16
7	4.4	Ap	2.13	1.97	0.03	1.12	0.24	0.27	1.28	1.75	0.97	0.17	1.23	1.12	0.02	0.59
		α	(173)	−19	−6	116	159	171	−23	(−170)	−14	2	45	−83	81	129
8	4.9	Ap	0.93	0.46	0.09	0.86	0.24	0.12	0.82	0.73	0.19	0.12	1.32	0.86	0.02	0.62
		α	(−165)	−5	−48	83	38	35	−88	−(161)	−20	−157	78	−100	−18	−32

Ap = Amplitude; α = phase angle shift from the phase of rate of change; (α) = phase angle of rate of change.

TABLE 4. Linear and nonlinear contributions to the maximum rate of change in the kinetic and available potential energies. Units: $(\mathrm{m\ s^{-1}})^2\ \mathrm{day^{-1}}$.

Wave-number k	Period (days) τ	$\left(\frac{\partial E_K}{\partial t}\right)_{max}$	$=a_1N_K$	$+a_2M_K$	$+a_3A_K$	$+a_4A_{KB}$	$+a_5N_{KB}$	$+a_6G_K$	$+a_7M_{KB}$	$\left(\frac{\partial E_A}{\partial t}\right)_{max}$	$=b_1N_{A1}$	$+b_2N_{A2}$	$+b_3M_A$	$+b_4K_A$	$+b_5N_{AB}$	$+b_6H_A$	$+b_7M_{AB}$
0	9.1	1.55	=0	−0.06	−3.06	+0	+0	+3.71	+1.12	6.42	=0	+0	+5.84	+4.55	+0	−3.97	+0
1	9.1	2.15	=2.74	+0.04	−0.30	−0.24	−1.00	+0.90	+0	2.77	=1.04	+0.24	+0.45	+0.27	+0.01	+0.78	+0
2	7.1	1.14	=1.02	+0.03	−0.30	−0.06	+0.55	−0.10	+0	1.64	=1.00	−0.04	+0.35	+0.50	−0.06	−0.13	+0
3	9.1	1.24	=1.53	+0.55	+0.21	−0.34	−0.13	−0.58	+0	1.76	=2.38	+0.10	+0.69	−0.30	+0.01	−1.16	+0
4	5.6	1.84	=1.70	+0.19	+0.31	−0.29	+0.53	−0.60	+0	2.28	=1.08	+0.11	+1.65	−0.33	−0.05	−0.18	+0
5	5.1	1.14	=0.72	+0.12	−0.21	−0.26	+0.28	+0.48	+0	1.28	=0.51	−0.06	+0.34	+0.09	+0.06	+0.35	+0
6	4.9	1.01	=0.86	+0.07	+0.40	−0.40	−0.03	+0.10	+0	0.77	=0.81	−0.06	−0.08	−0.28	−0.02	+0.39	+0
7	4.4	2.13	=1.86	+0.03	−0.45	−0.22	−0.27	+1.17	+0	1.75	=0.94	+0.17	+0.87	+0.14	+0	−0.37	+0
8	4.9	0.93	=0.46	+0.06	+0.10	+0.19	+0.10	+0.03	+0	0.73	=0.18	−0.11	+0.27	−0.15	+0.02	+0.53	+0

where

$$a_1=\sin[\alpha_{NK}(k,\omega)-\alpha_{EK}(k,\omega)]$$
$$a_2=\sin[\alpha_{MK}(k,\omega)-\alpha_{EK}(k,\omega)]$$
$$a_3=\sin[\alpha_{AK}(k,\omega)-\alpha_{EK}(k,\omega)]$$
$$a_4=\sin[\alpha_{AKB}(k,\omega)-\alpha_{EK}(k,\omega)]$$
$$a_5=\sin[\alpha_{NKB}(k,\omega)-\alpha_{EK}(k,\omega)]$$
$$a_6=\sin[\alpha_{GK}(k,\omega)-\alpha_{EK}(k,\omega)]$$
$$a_7=\sin[\alpha_{MKB}(k,\omega)-\alpha_{EK}(k,\omega)]$$
$$b_1=\sin[\alpha_{NA1}(k,\omega)-\alpha_{EA}(k,\omega)]$$
$$b_2=\sin[\alpha_{NA2}(k,\omega)-\alpha_{EA}(k,\omega)]$$
$$b_3=\sin[\alpha_{MA}(k,\omega)-\alpha_{EA}(k,\omega)]$$
$$b_4=\sin[\alpha_{KA}(k,\omega)-\alpha_{EA}(k,\omega)]$$
$$b_5=\sin[\alpha_{NAB}(k,\omega)-\alpha_{EA}(k,\omega)]$$
$$b_6=\sin[\alpha_{HA}(k,\omega)-\alpha_{EA}(k,\omega)]$$
$$b_7=\sin[\alpha_{MAB}(k,\omega)-\alpha_{EA}(k,\omega)].$$

The values of various terms in (32)–(35) for the positive maximum rate of change in the kinetic and available potential energies are shown in Table 4. Changing the sign of each term in Table 6 gives the negative maximum rate of change in the kinetic and available potential energies.

It is seen from Table 4 that the nonlinear interaction term (a_1N_K), which has the largest value and has the same sign as the maximum rate of change in the kinetic energy, apparently provides the main mechanism for the growth and decay of the kinetic energy of the long and synoptic waves. The term involving interactions between velocity waves and the gradient of the mean zonal velocity (a_2M_K) has also the same sign as the maximum rate of change in the kinetic energy but has much smaller value than the nonlinear interaction term (a_1N_K). Unlike the stationary case, the energy conversion ($a_3A_K+a_4A_{KB}$) has little effect on the evolution of the kinetic energy of long and synoptic waves. The term representing the Reynolds and frictional stresses (a_6G_K) has the same sign as the maximum rate of change in the kinetic energy for ultralong waves (1,9.1) and synoptic waves (5,5.1), (6,4.9), (7,4.4) and (8,4.9), but has the opposite sign for waves (2,7.1), (3,9.1) and (4,5.6).

For the growth and decay of the available potential energy of the long and synoptic waves, the interactions between the velocity and temperature waves ($b_1N_{A1}+b_2N_{A2}$) apparently provide the major mechanism for

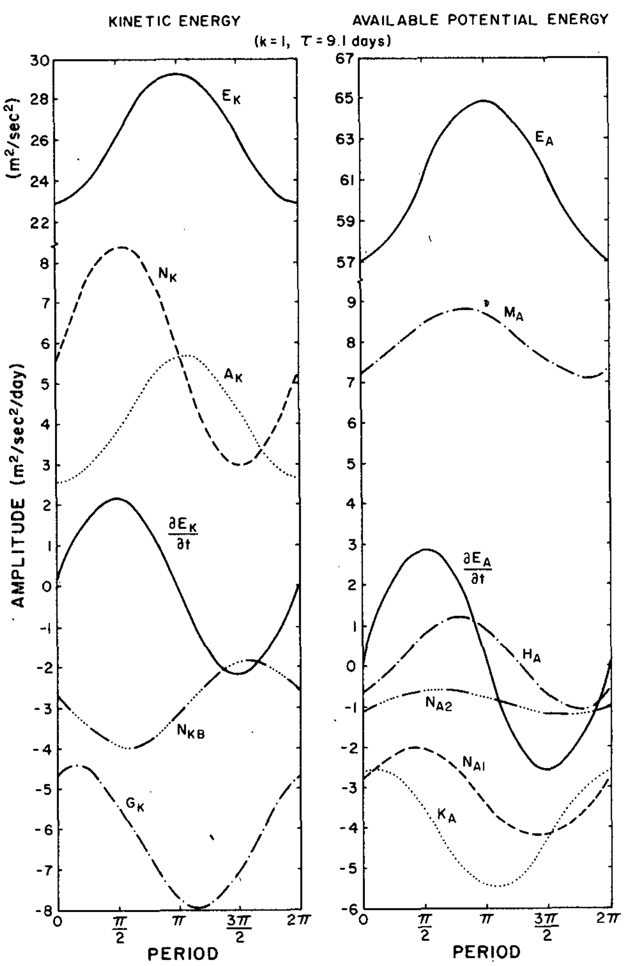

FIG. 3. Variations of the kinetic and available potential energies in relation to the linear and nonlinear contributions to waves of wavenumber 1, period 9.1 days.

transfer of the energy to and from the waves, respectively. The interactions between the velocity waves and the gradient of the zonal mean temperature (b_3M_A) generally have the same sign as the maximum rate of change in the available potential energy. The sign of the terms involving the energy conversion (K_A) and diabatic effect (H_A) varies for waves of different wavenumber and frequency.

More discussion regarding the mechanisms for the growth and decay of kinetic and available potential energies associated with long and synoptic waves will be made with the use of Figs. 3–6, which show the evolution of the kinetic and available potential energies of large-scale atmospheric waves in relation to linear and nonlinear contributions. In these figures, the values for the stationary amplitude waves have been added to the time-change of these quantities. The following similar characteristics for these long and synoptic waves may be noted:

1) The growth and decay of the kinetic energy of the long waves are practically in phase with those of the available potential energy.

2) The fact that N_K is practically in phase with $\partial EK/\partial t$ indicates that the growth and decay of the kinetic energy of the long atmospheric waves are greatly affected by the nonlinear wave interactions.

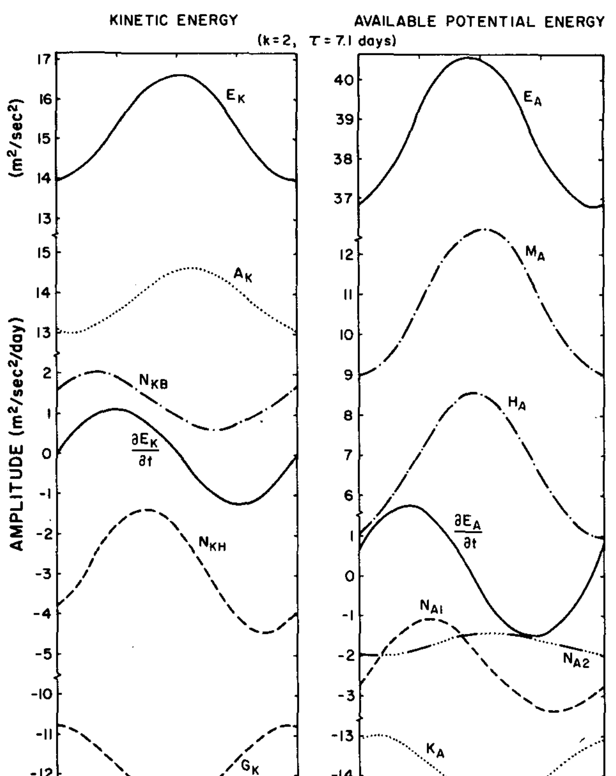

FIG. 4. As in Fig. 3 except for wavenumber 2, period 7.1 days.

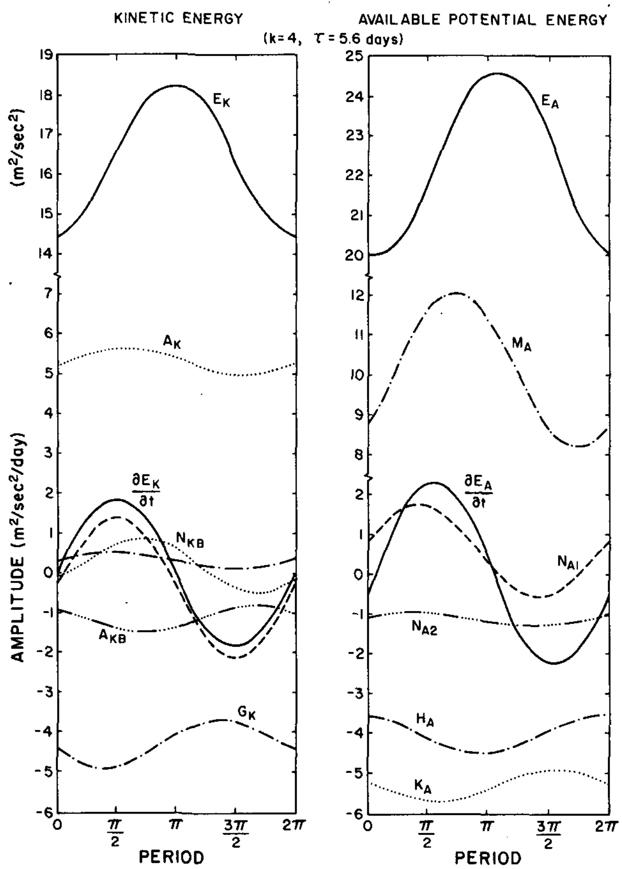

FIG. 5. As in Fig. 3 except for wavenumber 4, period 5.6 days.

3) Since A_K is of opposite sign to and 180° out of phase from G_K, it would indicate that the kinetic energy converted from the available potential energy tends to compensate the energy dissipation through the eddy and frictional stresses.

4) The fact that N_A is almost in phase with $\partial E_A/\partial t$ indicates that the evolution of the available potential energy of the long waves is greatly affected by the interaction between the temperature and velocity waves.

5) In most cases M_A tends to compensate the loss of available potential energy due to energy conversion K_A.

Therefore, the growth and decay of the kinetic energy associated with waves of a certain wavenumber are primarily the consequence of the nonlinear interaction which transports eddy kinetic energy to and from the waves respectively. The growth and decay of the available potential energy associated with waves of a certain wavenumber are essentially the result of the interaction between the velocity and temperature waves, which transfers the available potential energy to and from the waves respectively, while the transport of available potential energy through the interaction

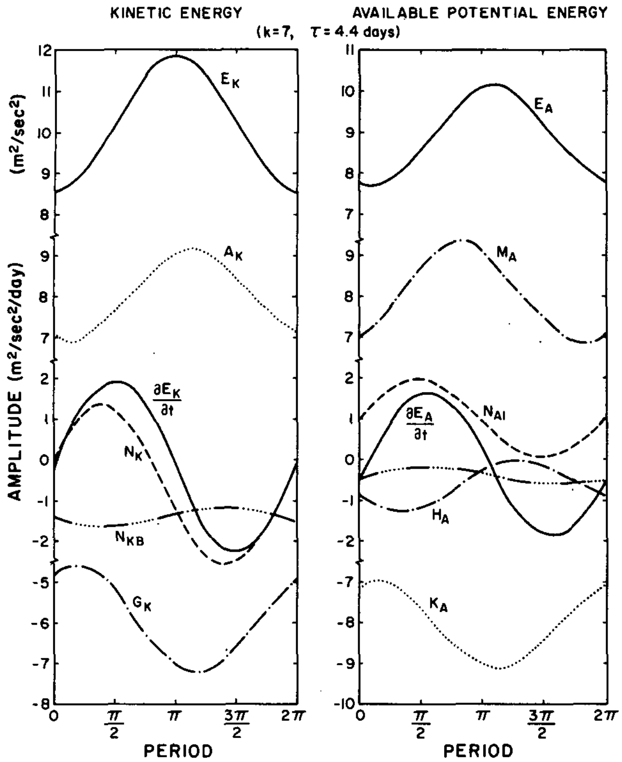

FIG. 6. As in Fig. 3 except for wavenumber 7, period 4.4 days.

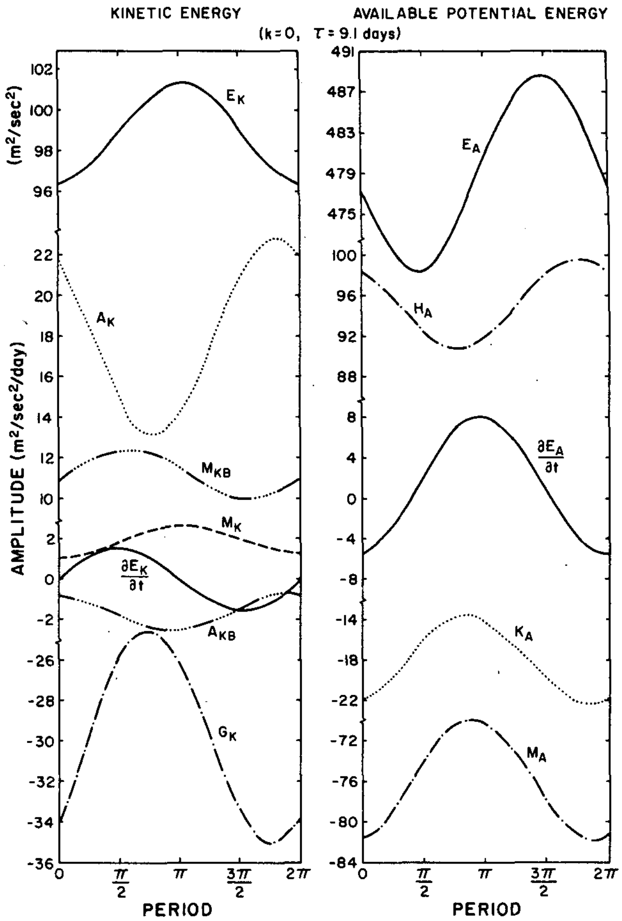

FIG. 7. As in Fig. 3 except for the zonal flow of period 9.1 days.

between the velocity wave and the gradient of the zonal mean temperature tends to balance the diabatic effect and energy conversion. Since such a transport of the available potential energy depends greatly on the intensity of the eddy kinetic energy, the growth and decay of the available potential energy become practically in phase with those of the kinetic energy of the waves.

Fig. 7 shows the growth and decay of the kinetic and available potential energies of the zonal flow (0,9.1). The following main characteristics of the energy evolution of the zonal flow may be noted:

1) Since the energy conversion A_K in the zonal flow is almost balanced by the effect of the Reynolds and frictional stresses (G_K), the growth and decay of the kinetic energy are primarily affected by the resultant interaction between the velocity waves and the gradient of the mean zonal flow (M_{KB},M_K):

2) While the diabatic heating H_A in the zonal flow is almost balanced by the energy conversion K_A, the growth and decay of the available potential energy in the zonal flow is essentially controlled by the resultant interaction of the velocity waves and the gradient of the mean zonal temperature (M_A):

3) There is approximately a 90° time lag between the rate of change in the available potential energy and that in the kinetic energy in the zonal flow; this is also the case for the time lag between the oscillation of the available potential and kinetic energies of the zonal flow (see also Table 3).

Acknowledgment. This research was partly supported by the National Science Foundation under Grant ATM-73-00188.

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