On the Mechanism of the Monsoonal Mid-Tropospheric Cyclone Formation

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(Manuscript received 18 January 1978, in final form 31 March 1978)

ABSTRACT

The radiosonde data collected prior to the development of a well-documented mid-tropospheric cyclone (MTC) over the west coast of India is used to determine the actual basic state. Such observed basic state is in turn used as the basis of a general quasi-geostrophic instability eigenvalue analysis, whereby one can check whether or not the hypothesis suggested in Mak's (1975) analysis is relevant in a real setting. We will discuss the degree of similarities and differences between the observed basic state and the idealized basic state. The incompleteness of Mak's analysis is noted and a supplementary analysis is given. The results using the observed state show that there does exist a dominantly most unstable mode which in its overall structure and characteristics resembles that of the observed MTC. This lends further support to the idea that MTC initially arises from a baroclinic instability of the broad southwest monsoonal flow, of which the direction as well as the magnitude vary significantly with height. The energetics of this genesis mechanism are clarified. The dynamical roles of each element in this basic state are also investigated.

The main shortcomings of the model results are that the growth rate is somewhat too small and the level of maximum intensity of the disturbance is somewhat too low. Such discrepancy strongly suggests the plausible importance of moist convection.

1. Introduction

There is a very recent review article (Carr, 1977b) on the various aspects of the important mid-tropospheric cyclones (MTC) in the southwest monsoonal circulation. For our discussion it suffices to say that the question on the dynamic origin of MTC is still unresolved. Although three suggestions have been offered in the literature, there is no general consensus as to what the most likely cyclogenesis mechanism is. One suggestion is that MTC over northeastern Arabian Sea could be merely a locally modified disturbance that originally comes from the Bay of Bengal (Miller and Keshavamurthy, 1968). The second suggestion is that it might result from an export of vorticity due to the heat low at the northwest of India in conjunction with a buildup of moisture over western India (Ramage, 1966). The third suggestion is that it might result from a dynamic instability of the broad southwest monsoonal flow, of which the direction as well as the magnitude vary significantly with height in the lower troposphere (Mak, 1975; hereafter referred to as M75).

Carr (1977b) has raised valid criticisms against each of these three suggestions. In regard to the first one, he noted that more often than not MTC do develop in situ over the northeastern Arabian Sea. As a counter-argument of Ramage's suggestion, he pointed out that it remains to be shown that a maximum in the vorticity export from the heat low is highly correlated with MTC formation. Furthermore, it has not been shown how the envisioned processes would dynamically give rise to a cyclone with the observed gross characteristics and structure. The criticism of Mak's analysis is that the required value for the crucial element of that model is unrealistically large. That element is the meridional component of the basic flow.

The basic state used in M75 was chosen mainly on the basis of intuition and physical reasoning. It was representable by simple functional forms. It was deliberately chosen so for its simplicity in a first theoretical attempt. In spite of the obvious oversimplifications in the model, the results were quite encouraging. It calls for a followup investigation using an actual basic state which is to be deduced from real data. By doing so, one should be able to tell whether the earlier results were fortuitous. This article reports the results of such an investigation.

It should be emphasized that we strive to find out the extent to which one can account for the initial development of a MTC without considering moist convection. At a later stage of development moist convection must play an increasingly important role in dictating the evolution of the storm (Carr, 1977a). The question about the possible importance of moist convection in the initial development is left for a separate investigation.

A theoretical oceanographic study by Schulman (1967) is quite relevant to this analysis. He attempted

1 This was brought to our attention by a reviewer.
to explain the intermediate-scale fluctuations in deep waters as a consequence of instability of the mid-ocean circulation. He took note of the fact that the density field of the thermocline has not only a pronounced vertical gradient but also a measurable longitudinal gradient. Using such a density field as the driving force of the basic flow, he analyzed the instability of the latter for quasi-geostrophic disturbances. His chosen basic density field was such that the basic flow is essentially on the meridional plane. His basic horizontal flow is therefore essentially unidirectional at different elevations. Consequently, his most unstable mode only has a finite north-south length scale and an essentially infinite east-west length scale. As will be seen later, our basic flow is non-unidirectional which has far-reaching consequences in the instability characteristics and in the structure of the unstable modes.

The data analysis is reported in Section 2. The generalized instability analysis is formulated in Section 3. We will note that the analysis in M75 was incomplete. A supplementary analysis is presented in Section 4. The results will show that the earlier model by itself is not enough to make a strong conclusion about MTC initiation because of two shortcomings in the theoretical results. However, the overall resemblance between the theoretical model and the observed MTC suggests that the basic hypothesis on the cyclogenesis is dynamically sound. The results based on the observed basic state are presented in Section 5. These results will show that a much stronger case can be made in favor of our basic hypothesis for the initial cyclogenesis of MTC. The roles of the individual features of the basic state are examined in Section 6.

2. Determination of the observed basic state

An arrangement was made with Dr. C. Ramage of the University of Hawaii to obtain a limited amount of upper air data collected in the summer of 1963 during the International Indian Ocean Expedition. Figs. 1 and 2 show the time variations of the geopotential height of nine standard pressure surfaces and of the surface pressure over Bombay. This is a simple-minded way of illustrating the distinct mid-tropospheric character of the system first documented by Miller and Keshavamurthy (1968). It is worth noticing that according to this unsmoothed time series the cyclone...
show the first signs of development in the 800–900 mb layer. This level is substantially lower than that of the maximum intensity at the mature stage. According to Dr. Ramage, the Indian radiosonde data has hysteresis errors and large radiation-caused errors. As a result the wind data are the most reliable, followed in order by temperature, pressure-height and dew point. Thus, we do not attach too much meaning to the rapid fluctuations in the time series of the geopotential.

The observed basic state which will be required for our subsequent theoretical analysis consists of three elements: the zonal velocity component \( \bar{u}(p) \), the meridional velocity component \( \bar{v}(p) \), and the static stability \( \sigma(p) = -\frac{(R/\rho)[(\partial T/\partial p) - (\partial T/\partial \rho)]}{(\partial T/\partial p)} \). The average value of \( \sigma \) in June at three stations far apart on the subcontinent—Bombay, Nagpur and Madras—are very similar. Fig. 3 shows the values of \( \sigma \) at Bombay with one standard deviation about the mean for the month of June 1963. An inspection of the wind data suggests that for about two weeks prior to the onset of the MTC on 26 June the flow field over west-central India showed little significant variations (Brode, 1977).

We therefore computed the average value of \( \bar{u}(p) \) and \( \bar{v}(p) \) for the period from 12 to 25 June. Fig. 4 shows the \( \bar{u} \) and \( \bar{v} \) fields and their variability over Bombay obtained from wind data. The dashed lines show the smoothed wind profiles which will be used in the subsequent instability analysis. The smoothing is expected to be less consequential at higher levels since the static stability increases rapidly with height. The horizontal temperature gradient is to be related to the vertical shear in \( \bar{u} \) and \( \bar{v} \) according to the thermal wind equations.

There are certain significant differences as well as similarities between the observed profiles of \( \bar{u} \), \( \bar{v} \) and \( \sigma \) and the corresponding profiles used in M75. The observed \( \bar{u} \) profile has mainly an easterly shear as that used in M75. However it has a fairly thick layer of westerly flow and a westerly shear in the lowest 100 mb. Carr (1977a, Fig. 16a) found a similar structure. The observed \( \bar{v} \) profile differs greatly from that used in M75. However, the magnitudes of \( \bar{v} \) are by no means negligible, suggesting that we are dealing with a non-unidirectional baroclinic basic flow. Furthermore, the vertical shear in \( \bar{v} \) in the lower-middle troposphere is large and is of the same sense as that used in M75. The observed \( \sigma \) increases greatly with height, whereas \( \sigma \) was treated as a constant in M75. The structure of the basic state to be used later on in our instability analysis is shown in Fig. 5.

It might be helpful to briefly comment on the conceptual meaning of a basic state in an instability study like this one. We feel that if a system in fact arises from dynamic instability, the proper basic state should be deduced from the data collected shortly before the phenomenon occurs. Because of the presence of noise in the data, one has to average the atmospheric state over a certain time interval when one attempts to deduce the basic state. The averaging interval should be long enough that one can obtain a statistically reliable estimate; and it should also be short enough that one does not lose the unique information of the true basic state. We chose 15 days as our averaging interval mostly on the basis of data inspection. The proper interpretation of such a basic state is that this is most probably close to the actual state just prior to the time when a MTC was observed to develop. It would not be correct to construe that this is the basic state that persists for the averaging interval of 15 days.

3. Formulation of the instability analysis

Following M75 we assume that, for the purpose of testing the relevance of the basic hypothesis of MTC cyclogenesis, it is enough to consider quasi-geostrophic
dynamics. The notations for the velocity components in $x$, $y$ and $p$ directions are $u, v$ and $\omega$; those for temperature and geopotential are $T$ and $\phi$. Subscripts stand for partial derivatives. Using the same set of non-dimensionalization constants as in M75, we obtained the following dimensionless perturbation equations in the $p$-coordinate system:

$$\begin{align*}
\text{Ro}(u_t + u u_x + v u_y + u \omega_p) &= -\phi_x + v, \\
\text{Ro}(v_t + v v_x + v v_y + v \omega_p) &= -\phi_y - u, \\
0 &= -\phi_p - T/p, \\
\text{Ro}(T_t + u T_x + v T_y - u \phi_p v + v \phi_p u - \omega_p S) &= 0,
\end{align*}$$

where $S = P_e/(f L)^2 \phi$ and $\text{Ro} = U/fL$ with $U$ being the value of the basic zonal flow at the top of the model and $L$ the length scale of the perturbation in $x$ direction. When we examined the instability characteristics, we scanned through a range of values of $L$ compatible with the quasi-geostrophic assumption. In doing so, $S$ would also take on different values.

The usual power series expansion for each dependent variable using $\text{Ro}$ as the small expansion parameter would lead to a set of zeroth-order equations and a set of first-order equations. By combining these two sets of equations (see M75 for details of this procedure), we finally obtained one equation that governs the zeroth-order quantity of the geopotential field $\phi_{(0)}$, as

$$\begin{align*}
\left(\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right)
\left[\frac{\partial^2 \phi_{(0)}}{\partial x^2} + \frac{\partial^2 \phi_{(0)}}{\partial y^2} + \frac{\partial}{\partial \frac{\partial \phi_{(0)}}{\partial p}} \frac{1}{S \frac{\partial \phi_{(0)}}{\partial p}} \right] \\
- \left[\frac{d}{dp} \left(\frac{1}{dA} \right) \right] \frac{\partial \phi_{(0)}}{\partial x} - \left[\frac{d}{dp} \left(\frac{1}{dA} \right) \right] \frac{\partial \phi_{(0)}}{\partial y} &= 0,
\end{align*}$$

where $\bar{u}, \bar{v},$ and $S$ are prescribed functions of $p$. The first-order term of the $p$ velocity is related to $\phi_{(0)}$ by

$$\omega_{(1)} = -\frac{1}{S} \left(\frac{\partial \phi_{(0)}}{\partial x} + \frac{\partial \phi_{(0)}}{\partial y} \right) + \frac{1}{S} \left(\frac{d \bar{u}}{dp} \frac{\partial \phi_{(0)}}{\partial x} + \frac{d \bar{v}}{dp} \frac{\partial \phi_{(0)}}{\partial y} \right).$$

Using $\omega = 0$ at $p = 0$ and 1 as the boundary conditions we then have

$$\left[\frac{\partial}{\partial t} + \frac{\partial}{\partial x} + \frac{\partial}{\partial y}\right] \frac{\partial \phi_{(0)}}{\partial \frac{\partial \phi_{(0)}}{\partial p}} \frac{d \bar{u}}{dp} + \frac{d \bar{v}}{dp} + \frac{d \bar{\phi}_{(0)}}{dp} = 0$$

at $p = 0, 1$. The normal mode solution of (2) is

$$\phi_{(0)} = \text{Re}\{A(p)e^{i(\gamma S - c t)}\},$$

where $\gamma$ is the nondimensional wavenumber in the $y$ direction and $c$ the eigenvalue and $A(p)$ eigenfunction of the problem. Substituting (5) into (2) we obtain

$$\left[\frac{d^2}{dp^2} - \frac{d}{dp} \frac{d \ln S}{dp} \right] + c (1 + \gamma^2 - M) A = 0,$$

where

$$M = [\bar{u}'' + \gamma \bar{v}'' - (\bar{u}' + \gamma \bar{v}')(\ln S)'] / (\bar{u} + \gamma \bar{v} - c).$$

Each prime indicates a derivative with respect to $p$. The boundary conditions (4) become

$$\frac{dA}{dp} = \frac{\bar{A}'}{\bar{\gamma} \bar{\phi} + c}.$$
Eqs. (6), (7) and (8) are the generalized counterparts of Eqs. (10), (11) and (12) in M75. They are to be solved numerically with the iterative scheme used in M75. One hundred and one grid points were used in the computations to insure adequate resolution. The observed profiles of $u$, $v$ and $S$ are very accurately represented by a combination of sinusoidal and exponential curves. Both the eigenvalue and eigenfunction $A(p)$ are determined simultaneously in this method. The instability characteristics are embodied in $c$. The zeroth-order fields $\phi_{(0)}$, $T_{(0)}$, as well as the horizontal divergence $\partial \phi_{(1)}/\partial p$ and the vertical velocity $\omega_{(1)}$ fields are related to $A(p)$ as follows:

$$
\phi_{(0)} = A_r \cos x - A_i \sin x
$$

$$
T_{(0)} = -p \left( \frac{dA_r}{dp} \cos x - \frac{dA_i}{dp} \sin x \right)
$$

$$
\frac{\partial \phi_{(1)}}{\partial p} = (1+\gamma^2)[A_r(u+\gamma v-c)+A_i c] \cos x
$$

$$
+ (1+\gamma^2)[A_r(u+\gamma v-c)+A_i c] \sin x
$$

$$
\omega_{(1)} = \int_0^p \frac{\partial \phi_{(1)}}{\partial p} dp
$$

where $A_r = A_r(p) + iA_i(p)$ and $c = c_r + ic_i$. The common exponentially amplifying factor $e^{\varepsilon t}$ is not shown in (9). In using (9) we will specifically examine the vertical structure of the wave on an $(x, p)$ plane at a latitude $\gamma_0$ and at a time $t_0$ such that $\gamma_0 - c_0 t_0 = 2\pi$.

It is also instructive to examine the instability mechanism from the point of view of energy conversion processes. The energy equations can be written as

$$
\frac{dK_R}{dt} = \frac{1}{2} e^{2\varepsilon t} \left[ \frac{d}{dp} \left( W_r \Phi_r + W_i \Phi_i \right) \right]
$$

$$
+ \left( W_r \frac{d\Phi_r}{dp} + W_i \frac{d\Phi_i}{dp} \right),
$$

(10)

$$
\frac{dA_R}{dt} = \frac{1}{2pS} \left[ (U_r T_r + U_i T_i) \frac{d\bar{u}}{dp} - (V_r T_r + V_i T_i) \frac{d\bar{v}}{dp} \right]
$$

$$
- \frac{1}{2} e^{2\varepsilon t} \left[ W_r \frac{d\Phi_r}{dp} + W_i \frac{d\Phi_i}{dp} \right],
$$

(11)

where $K_R$ and $A_R$, respectively, stand for the eddy kinetic energy and eddy available potential energy averaged over an $x, y$ wavelength domain. The subscripts $r$ and $i$ for $W$, $\Phi$, $U$, $V$ and $T$ stand for the real and imaginary parts of those quantities, respectively; $U$ and $V$ are determined from $\Phi$ geostrophically. The first term on the right side of (10) represents the convergence of wave energy flux in the vertical and the second term is the conversion from $A_R$ to $K_R$. The first two terms on the right side of (11) stand for the two components of conversion of the available potential energy of the basic state to $A_R$ [i.e., $-\bar{u}^T (\partial \bar{T}/\partial x)$ and $-\bar{v}^T (\partial \bar{T}/\partial y)$, where perturbation variables are indicated by primes].

4. Supplementary analysis for the idealized basic state used in Mak (1975)

Only one value of $\gamma$ was considered in M75; it was equal to 1. This amounts to only determining the most unstable mode among a subset of modes which have equal length scales in both $x$ and $y$ directions. The analysis was incomplete from a theoretical point of view. We now present the supplementary part of that analysis. The eigenvalues and corresponding eigenfunctions were examined for $-10 < \gamma < 10$ and $500 \leq L \leq 2500$ km. These values adequately cover the relevant range of values of $\gamma$ and $L$. We first show in Fig. 6 the dependence of the growth rate on $\gamma$ and $L$ for a special case which has only a linear easterly shear in the basic flow (Eady's case). The numerical values of the $e$-folding time $\tau = L/c_i U$ in this study are based on $U = 24$ m s$^{-1}$. As expected, the variations of the growth rate in this unidirectional baroclinic flow are symmetric about the $\gamma = 0$ axis, and the growth rate has a local maximum for a length scale of the order of 1000 km ($L = 1200$ km) on the $\gamma = 0$ axis, $L$ is to be interpreted as about a quarter of the wavelength in $x$-direction. The nondimensional phase speed is equal to $-0.5$ for all unstable modes.

Fig. 7 shows the growth rates on the $(\gamma, L)$ plane for the idealized basic state used in M75 with $\epsilon = -1.0$ where $|\epsilon|$ measures the relative strength of the maximum $\bar{u}$ with respect to the maximum $\bar{u}$. In contrast to Fig. 6, Fig. 7 has no symmetry about the $\gamma = 0$ axis. There are instead two local maxima at each value of $L$. One is located at a positive value of $\gamma$ and the other
at a negative value of $\gamma$. They belong to two distinct subsets of unstable modes with a small region of overlapping on the $\gamma, L$ plane. We distinguish them in Fig. 6 by using solid contours vs dashed contours. We will refer to them as P modes and N modes respectively. The N modes were overlooked in M75. The effective wavelength associated with a horizontal wavenumber vector is

$$\mathcal{L} = \frac{L_x L_y}{(L_x^2 + L_y^2)^{(1/2)}} \left(1 + \gamma^2\right)^{-1/2} \tag{12}$$

where $L_x = L$ and $L_y$ are the wavelengths in $x$ and $y$ directions, respectively. The line in Fig. 7 joining the P modes that have largest growth rate for each value of $L$ is very close to a straight line. It passes through the points $L = 500$ km, $\gamma = 1$ and $L = 1500$ km, $\gamma = 2.9$. It follows that $L_y$ is virtually the same along this dash-dotted line and is about 500 km. According to (12), $\mathcal{L}$ is approximately equal to the smaller one of the $L_x$ and $L_y$ when one is much larger than the other. Hence, the asymptotically most unstable mode among the subset of P modes is the one that has $L_y \approx 500$ km, $L_x \gg L_y$, with an effective length scale $\mathcal{L} \approx 500$ km. The structure of this mode is essentially the same as the one examined closely in M75. It has a good resemblance to that of an MTC. Fig. 7 also shows that the N modes have larger growth rates. However, we do not feel that much physical significance should be attached to these modes for the following reason. Similar to the P modes these modes also have an asymptotically most unstable one with a unique effective length scale. The line that joins the largest growth rate for each value of $L$ is also almost a straight line and passes through the point $L = 1500$ km, $\gamma = -6.0$. It follows that the asymptotic effective length scale is $\sim 250$ km. Modes with such small length scale are clearly not very meaningful within the framework of quasi-geostrophic dynamics. Hence, what was overlooked in M75 does not invalidate the overall conclusion in that study.

There is, however, one undesirable feature about the P mode for this idealized basic state. It is concerned with the variations of the phase speed $c$, with $\gamma$ and $L$ shown in Fig. 8. Here we see that the asymptotically most unstable P mode has a very large phase speed. This is in stark contrast with observation. The agreement with observation is good only for those modes whose $L_x$ is comparable to $L_y$ as noted in M75. This must be associated with the unrepresentative elements in the idealized basic state. It highlights the necessity of examining the instability problem with an observed basic state.

As a summary of this section, we conclude that the most unstable among the P modes for the idealized basic state considered in M75 is expected to prevail within the framework of quasi-geostrophic dynamics. This mode has many features quite reminiscent to the observed MTC. There are however two shortcomings: first, its phase space is much too large and second, the idealized basic flow must have a rather large meridional component. The exaggerated meridional component of the basic flow has such an overwhelming influence that it gives rise to the asymptotically most unstable modes with essentially infinite east–west length scale. This situation is similar to that encountered by Schulman (1967).

5. Instability characteristics of the observed basic state

The observed basic state also gives rise to two subsets of unstable modes on the $\gamma, L$ plane. They may also be appropriately referred to as P modes and N modes since one subset is primarily associated with positive values of $\gamma$, the other with negative values of $\gamma$. The overlapping region on the $\gamma, L$ plane of these two subsets of modes is more extensive than that in the case of the idealized basic state. Fig. 9 shows the variations with $\gamma$ and $L$ of the growth rate for the P modes. There is a well-defined maximum located at $L = 1080$ km and $\gamma = 0.7$. By (12), we found that the effective wavelength $\mathcal{L}$ is 900 km. The corresponding observed quantity of the MTC would be the radius of

Fig. 7. As in Fig. 6 except for the idealized basic state used in M75.

Fig. 8. Variations of the dimensionless phase speed $c$ on the $\gamma, L$ plane for the idealized basic state used in M75.
the cyclone, on the order of 750 km (Carr, 1977a). Hence the agreement is good. The values in Fig. 9 were computed as those in Fig. 6 using $U = 24$ m s$^{-1}$. If one uses a more representative value of $U = 12$ m s$^{-1}$, the $e$-folding time of the most unstable mode would be 5.5 days. Thus the magnitude of the theoretical growth rate is too slow. It appears, then, that moist convection might be needed in order to get a more realistic growth rate. A twofold increase in the growth rate of baroclinic disturbances with moist convection over a dry model might be expected (Gall, 1976). However it is not expected to be an easy task to design an adequate moist model for a MTC study because such a disturbance is known to depend sensitively on the parameterization procedure in a numerical model (Carr, 1977a).

Fig. 10 shows the phase speed $c_e$ of the $P$ modes. The dash-dotted line indicates the positions of the maximum growth rate for each value of $L$. We found that the nondimensional phase speed of the most unstable mode is about 0.23, amounting to less than 3 m s$^{-1}$. This agrees much better with observation than that of the asymptotically most unstable mode for the idealized basic state. The near stationarity of the observed MTC in June 1963 was perhaps partly also due to the topographic effect of the adjacent Western Ghats. The growth rate of the $N$ modes is shown in Fig. 11. The $P$ modes grow about twice as fast as the $N$ modes in the region of the $\gamma, L$ plane under consideration. The most unstable $N$ mode evidently has a small $\gamma$ and a value of $L$ much smaller than 500 km. It therefore has very little physical meaning in a quasi-geostrophic framework. No attempt was made to examine it in detail.

We next examine the structure of the most unstable mode ($L = 1080$ km, $\gamma = 0.7$). That $\gamma$ is equal to 0.7 implies a horizontal structure characterized by a tilt by 35º to the west of the north-south direction. No observation of such a tilt has been reported. We feel that we should reserve our judgment on this aspect of the disturbance until more observational and theoretical investigations are completed.

The vertical structure is examined in terms of $\phi_0$, $T_0$, $\partial \omega_0 / \partial \psi$ and $\omega_1$ on an $x, p$ plane presented in Fig. 12. These results are computed from a normalized complex eigenfunction $A(p)$ whose maximum magnitude is set to unity. The prominent features of the structure may be summarized as follows. The geopotential field has a gross mid-tropospheric character. The vertical tilting is westward at the lower portion and eastward at the upper portion. The relative intensity of the perturbation is more concentrated in the mid-troposphere than in the case of the idealized basic state. This is in better agreement with observation. The temperature field has a cold core below and a warm core above the level of maximum geopotential perturbation. The relative phase between $\phi_0$ and $T_0$ is such that the temperature structure described above is located at about $\frac{1}{3}$ of a wavelength to the west of the trough. A layer of convergence extending up to 700 mb is found with a maximum at about $\frac{1}{3}$ of a wavelength to the west of the trough. There is a weaker but deeper layer of divergence.

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**Fig. 9.** As in Fig. 6 except for the $P$ modes associated with the observed basic state.

**Fig. 10.** As in Fig. 8 except for the $P$ modes associated with the observed basic state.

**Fig. 11.** As in Fig. 6 except for the $N$ modes associated with the observed basic state.
Fig. 12. Cross-sectional structure of the most unstable mode for the observed basic state in terms of the geopotential $\Phi_0$, the temperature $T_0$, the horizontal convergence $\partial u_{(1)}/\partial p$ and the $\rho$-velocity $\omega_{(1)}$.

above 700 mb. This divergence pattern results in a broad and deep layer of upward motion, with its maximum located somewhat to the west of the trough. Carr (1977a) however found that by 1 July this MTC has its upward motion maximum slightly to the east of the cyclone. This disagreement probably reflects the limitation of this model. The overall resemblance between this most unstable mode and the observed MTC is good. The agreement is much better than that on the basis of the idealized basic state.

We do notice that the theoretical mode has its maximum intensity at a somewhat lower level than the observed MTC, 850 mb vs 650 mb. To assess the implication of this discrepancy, we should bear in mind that this dry model can at best tell us about the initial state of development. As pointed out in Section 2, we have detected from the geopotential data that the lower levels, say 850 and 900 mb, do appear to show development at an earlier time than the upper levels, say 600 and 700 mb. On the basis of this observational information in conjunction with our theoretical result, we feel that it is justifiable to make the following suggestion. The MTC apparently develops first at a lower midtropospheric level due to the instability of a non-unidirectional baroclinic flow. This dynamic mechanism gives rise to a large-scale vertical motion field which is conducive to vigorous moist convection. The latter would in turn significantly increase the growth rate and progressively raise the level of maximum intensity higher to the midtroposphere as a result of vorticity transport by the clouds.

We now examine the energetics of this most unstable mode as computed according to Eqs. (10) and (11). Fig. 13 shows the vertical distribution of the two terms on the right side of (10). It shows that eddy kinetic energy is generated from eddy available potential energy in the 875–575 mb layer. There is actually a destruction of $K_\rho$ both below and above. The vertical wave energy flux is such that we find a vertical divergence in the layer 850–600 mb, a strong convergence below 850 mb and a weaker convergence above 600 mb. Now we can explain why the unstable mode has a midtropospheric character. The intensity at the surface is relatively weak in spite of the strong convergence of wave energy flux, because there is a significant destruction of kinetic energy associated with the rising motion through the cold core. The intensity at the upper troposphere is weak for two reasons: first, the convergence of wave energy flux is weak due to the strong static stability; second, there is also a small rate of destruction of kinetic energy associated with the upward motion through a secondary cold region. Fig. 14 shows

Fig. 13. Vertical variations of the components in the eddy kinetic energy equation.

Fig. 14. Vertical variations of the two conversion processes from zonal available potential energy (APE) to eddy APE.
that both processes of generating eddy available potential energy, $-\overline{u'\overline{T}'(\partial \overline{T}/\partial x)}$ and $-\overline{v'\overline{T}'(\partial \overline{T}/\partial y)}$, are comparable in magnitude and reinforce one another. The second process is slightly larger. The cold core at the trough region in the lowest 150 mb arises from negative contribution from both advection terms $-\overline{u'\overline{T}_x}$ and $-\overline{v'\overline{T}_y}$. From 850 to 600 mb, there is a warm advection because the $\overline{T}_x$ and $\overline{T}_y$ reverse their signs.

It has been proposed in the literature that the MTC thermal structure is due to evaporation of raindrops in the low levels and release of latent heat at higher levels (e.g., Miller and Keshavamurthy, 1968). One would expect this to be an important factor at a later stage of development of a MTC. But our results indicate that the moist convective process is not an indispensable factor in explaining the thermal structure at the initial state.

6. Roles of the individual elements in the observed basic state

In this section we report our attempts to delineate as precisely as feasible the role played by each of the elements in the observed basic state, namely $a$, $b$, and $\sigma$. We did so by examining the instability characteristics of several basic states which differ from the observed state in having one of the elements greatly modified. This would give rise to qualitative differences in the structure of the disturbance as well as differences in the phase speed, growth rate and length scale of the most unstable modes. From such information, we attempted to inductively establish the causal relationships among the various features of the basic state and the characteristics of the disturbance. To achieve this goal, it is not necessary to scan through the whole $\gamma, L$ parameter plane. It should be sufficient just to compare

Table 1. Variations of the instability characteristics of the most unstable mode for $\gamma = 1.0$ with four profiles of static stability: (a) constant ($\sigma = 1 \times 10^{-4}$ mb s$^{-1}$ mb$^{-2}$), (b) linearly increasing with height, (c) exponentially increasing with height, (d) observed distribution.

<table>
<thead>
<tr>
<th>$a$ profile</th>
<th>$c_v$</th>
<th>$c_v/L$ (10$^4$ m s$^{-1}$)</th>
<th>$L = L/\sqrt{2}$ (km)</th>
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</thead>
<tbody>
<tr>
<td>(a)</td>
<td>0.160</td>
<td>0.220</td>
<td>622</td>
</tr>
<tr>
<td>(b)</td>
<td>0.277</td>
<td>0.160</td>
<td>767</td>
</tr>
<tr>
<td>(c)</td>
<td>0.260</td>
<td>0.178</td>
<td>728</td>
</tr>
<tr>
<td>(d)</td>
<td>0.231</td>
<td>0.168</td>
<td>834</td>
</tr>
</tbody>
</table>

the most unstable mode discussed in Section 5 with the most unstable mode for a reasonable value of $\gamma$, say $\gamma = 1.0$.

We first examined the unstable waves associated with four profiles of static stability, keeping the observed $a$ and $b$. Fig. 15 shows the four profiles used for this purpose, with a full description in the legend. Profiles (b) and (c) were simple functions chosen to approximate the observed $\sigma$ in two different aspects. Profile (b) is closer to the observed profile in terms of absolute values and slope in the lower troposphere, whereas profile (c) is closer in terms of the overall increase with height throughout the troposphere. Profile (a) is the one used in M75. Table 1 summarizes the instability characteristics of the most unstable mode in each case for the purpose of making intercomparison.

First of all, the growth rate for profile (a) has the largest value as expected. The reason is simply that the vertical average values of static stability for the other three profiles are larger. The growth rate and length scale for both profiles (b) and (c) approximate the observed case to within 5–10%. Agreement in terms of phase speed is not as good, the values being 10–20% larger than for the observed case. The differences between the observed case and profile (a) are significantly larger, about 30% for each characteristic. We interpret these results as demonstrating that whereas the details in the observed profile of static stability are of only minor importance, the gross features of the observed profile are very important in determining the instability characteristics of the disturbance. The most important feature of the observed $\sigma$ profile is its large increase with height.

There are several effects of a vertically increasing static stability on the characteristics of the unstable waves. Compared to case (a), the geopotential field in each of the cases of vertically increasing static stability has a much more pronounced maximum intensity in the lower mid-troposphere. This can be attributed to the suppressive effect of the large static stability on the energetics in the upper tropospheric region as discussed in Section 5 for the observed case.

The phase speed $c_v$ of the disturbance is much larger in cases (b), (c) and (d) than in case (a). We interpret this result as follows. The movement of the disturbance
Fig. 16. The structure of $\phi_{(0)}$ of the most unstable mode for a basic state using $\theta = 0$ with the observed $\bar{u}$ and $\bar{\theta}$.

Fig. 17. As in Fig. 16 except for a basic state using $\bar{u} = 0$ with the observed $\bar{\theta}$ and $\bar{\varphi}$. 
is mostly subjected to steering by roughly the vertical average basic flow in the layer where its amplitude is most significant. Since the effective steering current in cases (b) and (c) is larger than in case (a) due to a smaller role played by the upper tropospheric easterlies, it follows that the phase speed is larger.

Fig. 16 shows the perturbation geopotential field for a basic state consisting of the observed \( \bar{u} \) and \( \theta \), with \( \bar{v} \) altogether suppressed to zero. We see that there is virtually no prominent maximum intensity at the mid-tropospheric levels. The length scale in this case is very large (\( \mathcal{L} = 2470 \) km). On the other hand, when \( \bar{u} \) is suppressed instead, the structure still somewhat resembles a MTC, although it becomes shallower and has an exaggerated vertical tilt (Fig. 17). The length scale becomes quite small in this case (\( \mathcal{L} = 560 \) km). The growth rate in the case without \( \bar{v} \) is greater than that in the case without \( \bar{u} \) by about 50%. The large difference in the length scale in these two cases can be partially accounted for in terms of the difference in the effective static stability. When \( \bar{v} \) is zero, the influence of \( \bar{u} \) extends throughout the whole troposphere. Hence the effective static stability is roughly the vertical average for the whole model atmosphere, which is very large. On the other hand, when \( \bar{u} \) is zero the profile of \( \bar{v} \) is such that only the lower mid-tropospheric region is important. It follows that the effective static stability is roughly the average of \( \theta \) over that region and is considerably smaller. This alone, however, is not the whole explanation. We find that when we replace the observed static stability by a constant \( \theta = 1 \times 10^{-2} \) m² s⁻² mb⁻² the length scale in these two cases is still very different, \( \mathcal{L} = 1237 \) km for the case with \( \bar{v} = 0 \) and \( \mathcal{L} = 371 \) km for the case with \( \bar{u} = 0 \). Since the difference here cannot be attributed to any difference in effective static stability, we can conclude that there is some other factor related to the vertical structure of the basic-state wind which plays a very important role in determining the dominant length scale. As a suggestion of what this factor might be, we note that the length scale tends to decrease as the mid-tropospheric maximum intensity becomes more pronounced. This effect is also seen in the analysis of M75 which showed that as the strength of \( \bar{v} \) is increased the mid-tropospheric maximum becomes more pronounced and the dominant length scale becomes smaller. It seems plausible that the two effects are dynamically related since an increase in the degree of the mid-tropospheric concentration is similar to decreasing the effective vertical length scale, and the tendency to maintain a constant aspect ratio would lead to a smaller horizontal length scale.

We also tried to find out whether the low-level westerly shear in \( \bar{u} \) below 800 mb plays an indispensable role. A series of computations were made with that feature replaced by a linear extrapolation of the upper level easterly shear to the surface. We found that the resulting perturbation has its maximum intensity at the surface (Fig. 18). The reason is that our modification

![Fig. 18. As in Fig. 16 except for a modified \( \bar{u} \) with the observed \( \theta \) and \( \theta \).](image-url)
in $\bar{u}$ amounts to reversing the sign of $\partial T / \partial y$ there. This gives rise to a low level warm advection by $-\partial (\partial T / \partial y)$. Even though the low-level $-\bar{u}'(\partial T / \partial x)$ still results in a cold advection, it is too small to counteract the effect of warm advection. Consequently, we have a warm core perturbation. It follows from the consideration of energetics that the perturbation would have a maximum amplitude at the surface.

The three series of computations described above reveal the following aspects of the roles of $\bar{v}$ and $\bar{u}$. The basic meridional wind component is responsible for the relatively large amplitude of the disturbance at the mid-tropospheric levels. It is also a controlling factor in determining the length scale of the disturbance. The large easterly shear in $\bar{u}$ contributes greatly to the growth rate. At the very low levels, both $\bar{u}$ and $\bar{v}$ are important in that their vertical shear, particularly the $\bar{u}'s$, give rise to a cold advection. This accounts for the cold core and the relatively weak amplitude of the disturbance at the surface. Hence, we may conclude that each of the $\bar{u}$ and $\bar{v}$ components of the southwest monsoon plays a different but complementary role in sustaining the structure of the MTC.

7. Concluding remarks

This analysis may be taken as a strong evidence in support of a simple self-contained explanation for the formation of a mid-tropospheric cyclone. By applying the general theory of quasi-geostrophic baroclinic instability to the concrete conditions of the flow field and stratification over the northeastern Arabian Sea, we have demonstrated that the formation of a MTC is a logical necessity. This is at least a partial explanation in the sense that moist convection could also be a contributing factor. The latter has yet to be demonstrated theoretically.

Some researchers may choose to use the name MTC for describing a much broader class of cyclones than we intend here. We are particularly interested in the almost synoptic-scale cyclones embedded in the large-scale summer monsoonal flow. Within this context, we believe that broad environmental influences are unique and play a pivotal role in determining the fate of weak disturbances in this region of the world.

This analysis also shows that the set of unstable baroclinic modes in the classical study (Eady, 1949) actually consists of two overlapping subsets. They coincide in wavenumber space because there is a symmetry associated with the use of a unidirectional zonal flow. When the direction of a baroclinic flow varies with height, the two subsets of modes become distinct. For the observed monsoonal basic state the most unstable mode in one of these subsets is that which most resembles the MTC. The other subset of modes have much shorter length scale and may be more readily influenced by nongeostrophic effects.

Acknowledgments. The suggestions from the reviewers were quite useful and much appreciated. Part of the results reported here was presented in the 1977 IUTAM/IUGG Symposium on Monsoon Dynamics, New Delhi, India. This research was supported by the Global Atmospheric Research Program, Climatic Dynamics Research Section, National Science Foundation, under Grant ATM74-01188.

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