Boundary-Layer Forcing as a Possible Trigger to a Squall-Line Formation

W.-Y. Sun and Y. Ogura

Laboratory for Atmospheric Research, University of Illinois, Urbana 61801

(Manuscript received 16 January 1978, in final form 27 October 1978)

ABSTRACT

The study of the life history of the 8 June 1966 squall line by Ogura and Chen (1977) indicates that a well-defined narrow band of horizontal convergence was present at low levels prior to the appearance of first radar echoes. A two-dimensional model for the dry planetary boundary layer is developed and applied to this case in order to test the hypothesis that the prestorm convergence is produced by boundary-layer processes in association with a strong horizontal temperature gradient. The level 3 turbulence closure approximation by Mellor and Yamada (1974) is incorporated into the model as well as the similarity hypothesis of Businger et al. (1971) for the lowest constant flux layer. The basic driving mechanism is the diurnal variation of the temperature contrast across the observed dry line. Air on the northwest side was warm, while on the southeast side it was cool. The temperature contrast was introduced into the model as a lower boundary condition for the potential temperature.

The model is integrated, starting from the early morning conditions through the late afternoon. The results indicate the development of ascending-descending motions as soon as a horizontal temperature gradient is established. In time, the mixed layer also develops and its depth increases. As expected, it increases faster on the warm side than on the cool side. The intensity of the upward motion increases at a rate larger than that of the downward motion and the center of the ascending motion remains at a certain level (~800 mb). The center of the downward motion moves up in time. Thus, the upward motion is concentrated in the mixed layer at the location of the sharp gradient in the inversion. The major observed features in the velocity and temperature fields in the prestorm situation are well simulated by the model.

Further, the result of a sensitivity test for a different initial wind field indicates that the location and intensity of the resulting ascending motion is rather sensitive to the initial wind field. It is concluded that, if the synoptic-scale low-level wind blows in the right direction (offshore in sea-breeze terminology), a low-level horizontal temperature gradient of the magnitude observed in the 8 June 1966 squall line case is capable of generating upward motion with sufficient intensity to release the potential instability.

1. Introduction

It is well known that the immediate vicinity of the dry line is a preferred zone for thunderstorm development and squall line organization over the Great Plains. A dry line is defined here as the surface boundary between maritime tropical air masses from the Gulf of Mexico and dry air masses from the desert southwest. According to Rhea (1966), 60% of new radar echoes were within 200 mi of the dry line which later evolved into the squall line during the spring months from 1959 through 1962.

In a recent article, Ogura and Chen (1977, hereafter referred to as OC) analyzed data gathered from the upper air mesonetwork established by the National Severe Storms Laboratory (NSSL) for a squall line that occurred along the dry line on 8 June 1966. This episode of severe weather has become celebrated by virtue of the excellent set of data. The first radar echoes were detected at about 1600 CST and a nearly continuous band had formed by 1830. At 2000 the intense squall line became disorganized and dissipated by 2300.

Serial soundings were started at 1100 and continued until 2300. Thus data were collected several hours before storm development as well as after development.

One of their findings which is relevant to the present work is that a well-defined narrow band of horizontal wind convergence (and consequently ascending motion) with a width of ~ 100 km was present at low levels prior to the appearance of first radar echoes. The location and orientation of the line of maximum ascending motion and of the subsequent squall-line formation agree well. Ogura and Chen conclude that this low-level convergence triggered the release of the potential instability.

The question as to possible cause(s) of this low-level prestorm mesoscale convergence remained unanswered in OC. However, they have speculated that the formation of the prestorm convergence might be associated with boundary-layer processes. This speculation was based on the following findings:

1) The maximum low-level ascending motion remained near 800 mb while increasing in intensity until 1830 CST.
2) A strong horizontal temperature gradient was
observed in the mixed layer across the line of the subsequent storm formation. The temperature gradient was approximately 4 K over a distance of 100 km at 1430 CST.

3) Associated with this horizontal temperature gradient, a distinctive difference was observed in the development of the mixed layer across this line dividing northwest dry warm air and southeast moist cool air. The top of the mixed layer on the dry air side reached 650 mb level at 1530 CST, whereas it remained at the 800 mb level on the moist air side. This difference was apparently caused by nonuniform surface heating.

4) The location of the prestorm ascending motion coincided with the location at a sharp gradient in the height of the mixed layer.

We further note that, since the dry line and the associated squall line were nearly stationary, this case is different from those dry line cases studied by Schaeffer (1974). For the same reason, it is unlikely that the development of the squall was associated with the propagating gravity waves, observed by Matsumoto et al. (1967) and Uccellini (1975).

A sea-breeze circulation is a well-known mesoscale vertical circulation caused by a strong horizontal temperature gradient. Literature dealing with observational and modeling studies of the sea breeze includes Defant (1951), Estoque (1962), Neuman and Mahrer (1974), Physick (1976) and Mahrer and Pielke (1977). In most sea-breeze models, however, the height of the model domain was taken to be 2–3 km, which is not deep enough to consider the development of the deep mixed layer as observed in OC. A height-independent mean wind, which is usually assumed in sea-breeze modeling, is not appropriate to our case. Above all, many sea-breeze modelings assume a discontinuous temperature gradient across the shoreline. In contrast, the width of the strong horizontal temperature gradient zone in our case was ~80 km and the center of the prestorm ascending motion was located inside this zone.

In a recent article, Anthes (1978) applied a two-dimensional mesoscale model to study the evolution of strong sea breeze on a stagnant base state. In contrast to previous studies, he considered the relationship of the planetary boundary layer, the thermodynamic structure and the vertical circulation associated with the sea breeze in detail. Even though he assumed a discontinuous surface temperature gradient across the shoreline under zero mean wind conditions, the result thermodynamic structure and the vertical circulation are found to be similar in many aspects to those simulated in this article (Section 4).

The purpose of this paper is to develop a two-dimensional numerical model to simulate the prestorm vertical circulation observed in OC. The planetary boundary-layer model will be integrated, starting from the early morning conditions, in order to investigate the temporal evolutions of the mixed layer and associated vertical circulations. In doing so, the horizontal distribution of the ground surface temperature will be prescribed as a function of time so that a strong horizontal temperature gradient is established. In the model, the similarity equations proposed by Businger et al. (1971) are applied to the constant flux layer (i.e., a layer from the surface to the 10 m level). The level 3 turbulence closure model approximations proposed by Mellor and Yamada (1974) is applied to the layer above the constant flux layer. We note that André et al. (1976) and Zeman and Lumley (1976) pointed out that simple gradient models such as those proposed by Mellor and Yamada cannot represent turbulent diffusion processes realistically in the whole mixed layer. It was felt that an application of Mellor and Yamada's formulations to a mesoscale modeling was a natural and essential step between an application of a simple K theory and an attempt to apply more sophisticated formulations proposed by André et al. (1976) and Zeman and Lumley (1976) in the future.

It will be shown that the major observed features in the velocity and temperature fields in the prestorm situation are well simulated by the planetary boundary-layer model. A sensitivity test of the synoptic-scale wind has also been made. It is concluded the prestorm vertical circulation observed on 8 June 1968 was associated with the differential development of the mixed layer across the dry line and was driven by the horizontal temperature gradient.

2. Governing equation

Fig. 1 shows the location and orientation of the squall line under consideration for 1700 CST 8 June 1966, together with the NSSL upper air mesonet network in 1966. The line AB is drawn perpendicular to the squall line which was oriented in northeast to southwest direction.
In OC, most of the analyzed meteorological fields were presented for the vertical cross section through the line AB. In the following analysis, therefore, we shall take the x axis along the line AB, with the positive x axis pointing from northwest to southeast. The y axis is perpendicular to the x axis and the positive z axis is pointing vertically upward.

We shall use the anelastic approximation instead of Boussinesq approximation, because the vertical dimension of our model is not small compared to the scale height of the atmosphere (Ogura and Charney, 1962; Ogura and Phillips, 1962). Further, we assume that the flow is hydrostatic since the horizontal scale of motion under consideration is of the order of 100 km, much larger than the vertical scale. We divide all meteorological variables into two components, mean values and turbulence components. The equations for mass continuity, mean motion (denoted by U, V and W) and the first law of thermodynamics for mean potential temperature (denoted by Θ) are

\[
\frac{\partial U}{\partial x} + \frac{\partial W}{\partial z} - \frac{\partial U}{\partial z} = \frac{\partial}{\partial z} \left( -\frac{W}{\rho} \right)
\]  

(2.1)

\[
\frac{\partial U}{\partial t} = -U \frac{\partial U}{\partial x} - W \frac{\partial U}{\partial z} + \frac{\partial}{\partial z} \left( \frac{U w'}{\rho} + f \frac{\partial \pi}{\partial y} \right)
\]

(2.2)

\[
\frac{\partial V}{\partial t} = -U \frac{\partial V}{\partial x} - W \frac{\partial V}{\partial z} + \frac{\partial}{\partial z} \left( \frac{V w'}{\rho} - f \frac{\partial \pi}{\partial y} \right)
\]

(2.3)

\[
\frac{\partial \Theta}{\partial z} = -k
\]

(2.4)

\[
\frac{\partial \Theta}{\partial t} = -U \frac{\partial \Theta}{\partial x} - V \frac{\partial \Theta}{\partial y} - W \frac{\partial \Theta}{\partial z} + \frac{\partial}{\partial z} \left( \frac{\partial w'}{\rho} - \frac{w'}{\rho} \frac{\partial \pi}{\partial z} \right)
\]

(2.5)

Here f is the Coriolis parameter (8.47 × 10^{-5} s^{-1}), π the nondimensional pressure defined by \( \pi = (p/p_0) R \theta / \rho \) [where p is pressure, \( p_0 = 1000 \text{ mb} \), R the gas constant and \( c_p \) the specific heat of dry air at constant pressure], \( \rho \) is the air density, and \( u', v', w' \) and \( \theta' \) are the turbulence components of the velocity and the potential temperature. In this two-dimensional modeling study, all variations for mean and turbulent variables along the y direction are assumed to vanish except for \( \Theta \) and \( \pi \). We assume that \( \partial \pi / \partial y \) in Eq. (2.3) and \( \partial \Theta / \partial y \) in Eq. (2.5) are related to satisfy the geostrophic and thermal wind relations all the times, i.e.,

\[
f U = -c_p \Theta \frac{\partial \pi}{\partial y}
\]

(2.6)

\[
\frac{\partial \Theta}{\partial z} = -f \frac{\partial U}{\partial z}
\]

(2.7)

where \( \Theta \) is the potential temperature averaged over the domain. We further assume that \( U \) is a time-independent prescribed function of \( z \).

To close the above set of equations, we need to express the turbulent transport of momentum and heat in terms of mean variables. We use the level 3 approximations of Mellor and Yamada (1974) in which there are two prognostic equations, one for the turbulent kinetic energy \( (\theta')^2 \) and the other for the temperature variance \( (\theta^2) \). The reader is referred to the original paper for these prognostic equations and the diagnostic equations for other turbulence variables.

3. Model equations

a. The finite-difference grid and numerical integration

The finite-difference version of the second-moment equations yields the fact that the vertical increment (\( \Delta z \)) generally appears in the form of \( \Delta z/l \), where \( l \) is a constant away from the surface but tends toward a constant away from the surface. Therefore, we shall introduce a new vertical coordinate \( \xi \) defined by

\[
\xi = \frac{z}{z_a} + \frac{z_a}{z_c} \ln \left( \frac{z + z_a}{z_c} \right)
\]

(3.1)

In the current calculation we have chosen \( z_a = 1.5 \times 10^4 \) cm, \( z_c = 2.5 \), \( z_a = 1.0 \times 10^6 \) cm.

We take 25 levels in the vertical. The height of the first level is at 10 m above the ground surface and the highest level (H) is at 6738.5 m. The horizontal width \( L \) of the integration domain is 400 km and the horizontal increment \( \Delta x \) is 10 km. The grid arrangement in the new coordinate is vertically staggered. All mean variables, \( U, V, W, \Theta \) and \( \pi \), are located at the same level, while all turbulence variables are defined at the midpoint between these levels. The second column (denoted by Z) and the third column (denoted by ZH) in Table 1 show the height of levels where the mean variables and turbulence variables are defined in the model.

We use the forward time and the computationally simple and stable upstream advective approximations. All other spatial derivatives are approximated by centered finite differences. It will be shown in Section 4 that an implicit damping or diffusion associated with the upstream treatment of advective terms does not appear to be critical in this particular problem because
Table 1. Stretched vertical coordinate in the model. Mean variables are specified at height ZH. \( \pi \) is the nondimensional pressure.

<table>
<thead>
<tr>
<th>Level</th>
<th>Z(m)</th>
<th>ZH(m)</th>
<th>( \pi )</th>
</tr>
</thead>
<tbody>
<tr>
<td>25</td>
<td>6738</td>
<td>6543</td>
<td>0.778</td>
</tr>
<tr>
<td>24</td>
<td>6348</td>
<td>6154</td>
<td>0.789</td>
</tr>
<tr>
<td>23</td>
<td>5960</td>
<td>5767</td>
<td>0.801</td>
</tr>
<tr>
<td>22</td>
<td>5574</td>
<td>5382</td>
<td>0.813</td>
</tr>
<tr>
<td>21</td>
<td>5191</td>
<td>5001</td>
<td>0.824</td>
</tr>
<tr>
<td>20</td>
<td>4810</td>
<td>4622</td>
<td>0.836</td>
</tr>
<tr>
<td>19</td>
<td>4433</td>
<td>4247</td>
<td>0.848</td>
</tr>
<tr>
<td>18</td>
<td>4060</td>
<td>3876</td>
<td>0.859</td>
</tr>
<tr>
<td>17</td>
<td>3691</td>
<td>3509</td>
<td>0.871</td>
</tr>
<tr>
<td>16</td>
<td>3327</td>
<td>3149</td>
<td>0.882</td>
</tr>
<tr>
<td>15</td>
<td>2970</td>
<td>2794</td>
<td>0.893</td>
</tr>
<tr>
<td>14</td>
<td>2619</td>
<td>2311</td>
<td>0.901</td>
</tr>
<tr>
<td>13</td>
<td>2272</td>
<td>2111</td>
<td>0.915</td>
</tr>
<tr>
<td>12</td>
<td>1945</td>
<td>1785</td>
<td>0.925</td>
</tr>
<tr>
<td>11</td>
<td>1626</td>
<td>1474</td>
<td>0.935</td>
</tr>
<tr>
<td>10</td>
<td>1322</td>
<td>1181</td>
<td>0.945</td>
</tr>
<tr>
<td>9</td>
<td>1040</td>
<td>911</td>
<td>0.954</td>
</tr>
<tr>
<td>8</td>
<td>783</td>
<td>670</td>
<td>0.962</td>
</tr>
<tr>
<td>7</td>
<td>557</td>
<td>464</td>
<td>0.969</td>
</tr>
<tr>
<td>6</td>
<td>371</td>
<td>299</td>
<td>0.975</td>
</tr>
<tr>
<td>5</td>
<td>227</td>
<td>178</td>
<td>0.980</td>
</tr>
<tr>
<td>4</td>
<td>128</td>
<td>96</td>
<td>0.983</td>
</tr>
<tr>
<td>3</td>
<td>65</td>
<td>47</td>
<td>0.985</td>
</tr>
<tr>
<td>2</td>
<td>29</td>
<td>19</td>
<td>0.986</td>
</tr>
<tr>
<td>1</td>
<td>10</td>
<td>5</td>
<td>0.987</td>
</tr>
</tbody>
</table>

The explicit diffusion dominates the implicit diffusion. The time increment used is about 10–15 s.²

b. Boundary conditions

The lateral boundaries (x = 0 and x = L) are open in the sense that \( W \) and the horizontal gradients of \( U, V, \) \( \Theta \) and all turbulence variables are assumed to vanish. At the top of the domain (z = H), the horizontal velocities \( (U_H, V_H) \) and potential temperature \( (\Theta_H) \) are assumed to be constants. The pressure is then prescribed at \( z = H \) in order to satisfy the geostrophic wind relation

\[
\frac{\partial \Phi}{\partial x} \frac{1}{c_p \Theta_H} = \frac{\partial U_H}{\partial y} = \frac{f U_H}{c_p \Theta_H}.
\]

With these boundary conditions, \( W \) at \( z = H \) does not necessarily vanish when determined by integrating the mass continuity equation upward from the ground. The turbulence variables, \( \bar{u} \bar{w}, \bar{w} \bar{w}, \bar{u} \bar{v}, \bar{u} \bar{w}, \bar{v} \bar{w}, \bar{u} \bar{\theta}, \bar{w} \bar{\theta}, \bar{q} \) and \( \bar{\theta} \) are assumed to vanish at \( z = H \).

The basic driving mechanism of the vertical circulation considered here is the diurnal variation of the temperature contrast across a line dividing warm and cool air. This is conveniently introduced into this model as a lower boundary condition for the potential temperature. The potential temperature at the ground surface is given by

\[
\Theta_s(x,t) = \Theta_0 + \left[ \theta_a + \theta_b \tanh \left( \frac{x - L/2}{80 \text{ km}} \right) \right] \sin \left( \frac{2\pi t}{26 \text{ h}} \right).
\]

We assign the following values in this experiment: \( \Theta_0 = 301.9 \text{ K}, \theta_a = 19.5 \text{ K} \) and \( \theta_b = 5 \text{ K} \). The reason for choosing 26 h, rather than 24 h, in the above expression is that the diurnal variation of the ground-surface temperature is better represented in this way for the period we consider, i.e., from the early morning through the late afternoon. We indicate in Eq. (3.3) that this temperature contrast is concentrated in a zone with a width of 80 km. At the ground surface \( (z = 0) \), both the horizontal and vertical velocities are assumed to be zero.

As indicated earlier, the first grid point in our grid array is located at 10 m above the ground surface. The similarity equations proposed by Businger et al. (1971) are applied to relate the most important turbulence variables, \( \bar{u} \bar{w}, \bar{w} \bar{w}, \bar{w} \bar{w} \) to the mean variables \( U, V \) and \( \Theta \) in this constant flux layer. The equations may be found also in Mahrer and Fiehle’s (1977) paper. Other turbulence variables are calculated from Mellor and Yamada’s closure approximations.

c. External parameters

Our model of the planetary boundary layer includes the external parameters \( \partial \Phi / \partial y \) and \( \partial \Theta / \partial y \) or, according to Eqs. (2.6) and (2.7), \( U_a \) which is assumed to be independent of time. The early morning upper air sounding was made inside the NSSL mesonet at Altus, Oklahoma (LTS), at 0548 CST. The wind observations at that and subsequent times through afternoon indicates a rather large variation of wind with time. Fig. 2 shows the \( U_a \) we used in cases I and III. This profile is close to the average over the two observations at 1133 and 1351 CST at LTS in the layer above 700 mb, close to the average over the three observations at 0548, 1133 and 1351 CST in the layer between 700 and 800 mb. The positive \( U_a \) near the ground surface was selected based on the surface pressure analysis at 0600 CST by Eisen (1972) and from the geostrophic relation. The wind profile for case II will be described later.

d. Initial conditions

The model integration with respect to time starts from 0630 CST. The initial potential temperature distribution is assumed to be uniform in the x direction and its vertical structure is shown in Fig. 3. In contrast to the variability in the wind, no large variation in the potential temperature with time was observed that day in the layer above the inversion height. The profile shown in Fig. 3 is close to that observed at LTS at 0548 CST.
We assume in this experiment that the initial wind field and the potential temperature field are balanced geostrophically. Consequently, we shall take $U_0$ determined in Section 3c as the initial condition for $U$ in our first experiment. We shall also consider the sensitivity of the model to a different initial $U$ profile.

As to the initial condition for $V$ (denoted by $V_0$), $V_0$ is required to be independent of height because the initial potential temperature distribution is homogeneous in the $x$ direction. The choice of $V_0$ is not trivial.
because our physical system is driven by the differential surface heating which is in turn controlled by the wind speed near the ground surface. We selected $V_s=7$ m s$^{-1}$, as shown in Fig. 2. This value corresponds to the $y$ component velocity observed at LTS at 0548 CST and averaged over the lowest 50 mb layer.

The initial distribution of the pressure at the midpoint of the integration domain is calculated from the hydrostatic equation (2.4). The remaining initial pressure field is then determined from the geostrophic wind relations at 0630 CST.

For the convenience of discussion, we introduce the
pressure deviation ($\Delta\pi$) from the horizontal average pressure at each time, i.e.,

$$\Delta\pi = \pi - L^{-1} \int_0^L \pi dx. \quad (3.5)$$

The initial distribution of $\Delta\pi$ is shown in Fig. 4. In this figure the symbols A and B along the horizontal axis denote the corresponding locations in Fig. 1. The symbol M in Fig. 4 denotes the midpoint between A and B. The vertical coordinate in Fig. 4 is stretched according to Table 1. The initial vertical distribution of $\pi$ is also shown in Table 1. Fig. 1 shows that the initial pressure gradient is pointing toward the negative $x$. This pressure force is balanced with the positive $V_x^*$. Thus our system stands still until the differential surface heating is imposed.

Mellor and Yamada's closure approximations require
the nonvanishing initial values of $q^4$ and $\bar{\theta}^3$. No observational data are available to specify these values in the case under consideration. We select rather arbitrarily that the initial distribution of $\bar{\theta}^3$ is given by

$$\bar{\theta}^3(x,z) = 0.5 \times 10^{-5} + 0.5 \times 10^{-4} \exp[-|z-4 \times 10^3|/1.5 \times 10^4],$$

(3.6)

where $\bar{\theta}^3$ is measured in units of K$^2$ and $z$ is measured in units of cm. The initial distribution of $q^4$ is given by

$$q^4(x,z) = 40 + 10^8 \exp[-|z-4 \times 10^3|/1.5 \times 10^4],$$

(3.7)

where $q^4$ is measured in units of cm$^2$s$^{-2}$. The magnitude of $\bar{\theta}^3$ and $q^4$ given in Eqs. (3.6) and (3.7) are very small compared to those which subsequently develop in the mixed layer. Some preliminary simulations indicated that the model result is insensitive to the choice of the initial conditions for $\bar{\theta}^3$ and $q^4$ as far as their values are small.

4. Results and discussions

Three separate runs were made. Case I may be regarded as a control simulation for which all specifications are as discarded in the preceding section. Case II differs from case I in the specification of the initial $x$ component of the mean velocity ($U_p$). This case was run in order to investigate the sensitivity of model results to a different prevailing mean wind. Case III was run using the level 1 turbulence closure approximation by Mellor and Yamada, with all other specifications identical to those in case I. All modeling integrations were performed starting from the initial conditions at 0630 CST. Extensive comparisons will be made with the observation as analyzed in OC. In the interest of conserving space, however, figures from OC will not be produced here. To facilitate the comparison, the reader is recalled that the domain between A and B in Fig. 5 and many figures which follow corresponds to the domain where the results of data analysis were displayed in OC.

a. Case I

Fig. 5 shows the distributions of mean potential temperature $\Theta$, pressure perturbation $\Delta p$ and three components of mean velocity $U$, $V$ and $W$ at 1040 CST on the vertical cross sections. We observe that the height of the inversion layer on the warm side (left side of $M$) is about 815 mb, only slightly higher than that on the cool side (right side of $M$) at this time. However, a horizontal temperature difference of about 2 K has been established in the mixed layer by this time. It is easy to understand from the hydrostatic relation that this temperature distribution strengthened the initial horizontal pressure gradient pointing toward the negative $x$ in the mixed layer (Fig. 5b). Hence a negative $x$ component velocity is developing at the location of $M$ (Fig. 5c). Corresponding to this development, upward motion is increasing on the warm side and the downward motion on the cool side (see Fig. 5e). The vertical circulation is quite symmetric, with the maximum upward motion of 1.6 cm s$^{-1}$ and the maximum downward motion of $-1.8$ cm s$^{-1}$. The center of this circulation is located at 820 mb.

It is of interest to note that some significant changes
occur with time in meteorological variables not only in the mixed layer but also in the layer above the inversion. These changes are generated by the ascending and descending motions which extend into the stable layer above the inversion. First, the temperature on the left side of M is slightly lower than that on the right side in the layer just above the inversion (this tendency is more evident in Fig. 6a). Obviously this is caused by adiabatic cooling (warming) associated with the ascending (descending) motion. Corresponding to this temperature distribution, the horizontal pressure gradient pointing toward the negative x decreases in the layer.
just above the inversion. Hence $U$ velocity is intensified and a local maximum of $U$ is observed at 790 mb over the mid-point $M$.

The initial variations of the meteorological variables described above continue to intensify and become more pronounced in the next several hours. Fig. 6 shows the distributions of mean variables at 1420 CST. Fig. 6a indicates that the height of the inversion layer is about 810 mb on the cool air side and 710 mb on the warm side. A strong horizontal temperature gradient exists in the mixed layer near $M$ in which the temperature difference across the AB region is $\sim 4$ K. Further, as described above, temperature in the layer just above the inversion is lower on the left side of $M$ than on the right side. All these features in the potential temperature distribution agree well, not only qualitatively but also quantitatively, with observational analysis shown in Fig. 11 of OC.

Fig. 6b shows the development of a strong horizontal pressure gradient in the mixed layer and the weakening
of the horizontal pressure gradient in the layer above the inversion.

Corresponding to this development in the pressure field, the $U$ field develops the structure shown in Fig. 6c. The structure is characterized by the following features.

1) $U$ remains positive on the warm side of M whereas a strong negative $U$ develops on the cool side in the mixed layer. It is noteworthy that the negative $U$ extends throughout nearly the entire depth of the mixed layer.

2) The positive $U$ has a local maximum at approximately 750 mb over M and has a local minimum at approximately 700 mb on the cool side.

3) The $U$ in the layer above 500 mb in the region between A and M is stronger than that in the region between M and B. The horizontal variation of $U$ mentioned in (2) and (3) is due mainly to the horizontal distribution of pressure associated with the vertical circulation. Note that the line connecting the local maximum of $U$ in the layer above 750 mb coincides roughly with the line of zero $W$ shown in Fig. 6e.

4) Fig. 6c shows that the vertical wind shear is quite strong at the inversion level on the cool side, whereas it is weak on the warm side. This is understandable because $U$ is negative in the mixed layer and positive in the layer above the inversion on the cool side. All these characteristics in the structure of the $U$ field agree well with observational analysis shown in Fig. 14 of OC.3

Fig. 6d shows the distribution of the $y$ component velocity $V$ at 1420 CST. Recall that the initial distribution of $V$ at 630 CST had a constant value of 7 m s$^{-1}$ everywhere and that the height-and-time-independent pressure gradient force is imposed all the time in the $y$ direction. Consequently, $V$ changes in response to the variation of $U$ with time. By 1420 CST a low-level jet

---

8 We realize that caution should be exercised in comparison of our numerical results with the observations. This is because our numerical experiment is being carried out under several simplifications and assumptions such as (i) two-dimensionality, (ii) the time-independence imposed on $\partial e/\partial y$ and $\partial H/\partial y$, and (iii) the time-independence imposed on $U$ and $V$ at the top of the integration domain. Further, Ogura and Chen noted that some portions of the temporal variation of wind observed in the NSSL mesonetwork were associated with those processes which were apparently not related to boundary layer processes, such as the slow movements of the surface wind shift line and a short wave upper trough. Consequently, agreement (iii) may be just coincidental.
has developed, located over M and 875 mb level. We also note that $V$ is weaker on the warm side of M than on the cool side in the mixed layer. This tendency is reversed in the stable layer above the mixed layer. This behavior is similar to the observational analysis shown in Fig. 7 (here the result of data analysis is included since it was omitted in OC). However, the average vertical gradient of $V$ from our experiment is much weaker than that observed as a whole. This is probably due to our height-independent initial profile of $V$.

Fig. 6e shows that an asymmetry in the $W$ field becomes evident at 1420 CST. The upward motion has a local maximum of 14.9 cm s$^{-1}$, while the downward motion has a maximum of $-8.7$ cm s$^{-1}$. The center of the upward motion remains at the 810 mb level, as it was at 1040 CST, whereas that of the downward motion has moved up to the 730 mb level. The observational analysis shown at 1430 CST in Fig. 6 of OC shows the upward motion of 15 cm s$^{-1}$ at 800 mb, in good agreement with the numerical result. However, the downward motion shown in Fig. 6 of OC is not as strong as that in our experiment. In this regard, we note that the analysis of observed data in OC was based on the upper air sounding data taken from the mesonetwork where the average separation between two neighboring stations was $\sim 85$ km. Moreover, the analysis scheme applied in OC tends to smooth the analyzed fields and has a relatively large analysis error near the boundary of the analysis domain. This may partially account for some discrepancy between the observational analysis and the experimental result. For the same reason we feel that the ascending motion generated in our experiment is somewhat weaker than the observation. We further note that the vertical circulation was not uniform along the squall line in the real atmosphere. Hence the vertical circulation observed on a particular vertical cross section could be larger than that obtained in this two-dimensional model.

Figs. 5e and 6e further show that the vertical circulation is confined and anchored within the AB domain where the large horizontal surface temperature gradient is imposed and subsequently the sharp gradient of the top height of the mixed layer has developed. The propagation of the sea-breeze front observed in the results of numerical modeling studies by Physick (1976) and others is not seen here. It is also interesting to note that Figs. 6c and 6e show that the ascending motion is fed almost exclusively by the air in the mixed layer on the cool side. Even though our model does not include moisture, the air on the cool side is moist in the real situation. Therefore, the configuration of $U$ and $W$ is quite favorable for the formation and maintenance of a squall line.

In order to gain a further insight into the physical processes responsible for the temporal variations of $U$ and $V$ in the mixed layer, we shall estimate the magnitudes of terms involved in the horizontal momentum equations. We first look at $U$ near M at 900 mb (Fig. 6c). The magnitudes of the vertical eddy transport, the pressure gradient and the Coriolis force in Eq. (2.2) at this location as estimated from our result are $-\frac{\partial w'}{\partial z} = 1.8 \times 10^{-1}$ cm s$^{-1}$, $-\frac{\partial \mathcal{V}}{\partial x} = -2.5 \times 10^{-1}$ cm s$^{-1}$, $fV = 5.8 \times 10^{-2}$ cm s$^{-2}$. Therefore, a strong pressure gradient, which is in turn associated with the strong horizontal temperature gradient, is primarily responsible for generating the strong negative $U$ in the mixed layer and consequently the vertical circulation discussed.

We next consider the low-level jet located at 850 mb in Fig. 6d. The magnitudes of the vertical eddy transport and the ageostrophic force in Eq. (2.3) are $-\frac{\partial w'}{\partial z} = -1.2 \times 10^{-2}$ cm s$^{-1}$ and $f(U_0 - U) = 5.9 \times 10^{-2}$ cm s$^{-2}$. Therefore, the low-level jet is generated when $U$ becomes subgeostrophic due to the process described above. The vertical eddy transport of momentum has a negative effect and tends to reduce $V$ in the particular situation we considered.

The distributions of $\Theta$ and $W$ at 1630 CST are presented in Fig. 8. Fig. 8a shows that the inversion layer is at 670 mb on the warm side of M, whereas it remains near the 810 mb level on the cool side. The general patterns of $U$ and $V$ (figures are not shown) are similar to the corresponding distributions at 1420 CST, but the magnitudes of perturbations are amplified. Fig. 8b shows that the center of the downward motion has continued to move up, reaching 670 mb level at this time. The maximum upward motion is $\sim 30$ cm s$^{-1}$, close to that observed (Fig. 8b of OC), whereas that of the downward motion is $-16$ cm s$^{-1}$.

The differences in the intensity and location of the upward and downward motions are related to the difference in the development of the mixed layer between the warm and cool sides. The major portion of the ascending motion is confined in the layer below 670 mb, the height of the inversion. On the other hand, the thermal stratification in the layer between 800 and 670 mb on the cool side is very stable and, consequently, the descending air tends to spread horizontally in the layer above the inversion. This can be seen from the vertical tilt of the axis of the descending motion which is much larger than that of the ascending motion (Figs. 6e and 8b). The descending motion is weaker than the ascending motion because the descending motion is decelerated by the stable stratification in the layer above the inversion. The difference in intensity between the ascending and descending motions is a familiar feature in simulated sea-breeze circulations. Another interesting feature in Fig. 8b is that the center of the ascending motion is located 20 km to the left of M. This is also consistent with the observation. Apparently the wind blowing from right is stronger than that from left in the mixed layer.

The thermodynamic structure and the vertical circulation shown in Figs. 6 and 8 are quite similar to those in the sea-breeze case in the mature stage of the
Fig. 8. Distributions at 1630 CST of (a) potential temperature (K) and (b) vertical velocity (cm s\(^{-1}\)) with a contour interval of 8 cm s\(^{-1}\).

development as simulated by Anthes (1978). He also showed that the solenoid term dominated in the development of the vertical circulation during the 8 h following the beginning of the heating cycle. Because he considered only zero mean wind conditions, the distributions of \(U\) and \(V\) are different from those in our case.

Results from the model integration at 1730 CST indicated that the maximum upward motion is 35 cm s\(^{-1}\) at 810 mb. The persistence of the level of the maximum upward motion at this level is one of the most interesting behaviors of the vertical circulation and agrees well with the observation. The low-level jet at this time is located at the same location as in Fig. 6d, but its maximum speed is now 11.5 m s\(^{-1}\). Further, a new inversion layer has already started developing above the ground surface. As mentioned earlier, the first radar echoes on 8 June 1966 were detected at about 1600 CST
and a nearly continuous band was formed by 1830 CST. Since our dry model cannot describe the motion field in which the release of latent heat plays a dominant role, we have discontinued the model integration at 1730 CST.

Fig. 9 summarizes the temporal variations of some key variables. It is evident that the maximum heat flux at the ground surface occurs near 1230 CST. However, the potential temperature at 10 m height keeps increasing until 1630 CST because the surface heat flux is positive during the period. Both the maximum upward motion ($W_{\text{max}}$) and the height of the inversion of the warm side ($h_a$) increase monotonically during the entire period of the numerical integration. The most important feature shown in Fig. 9 is the rapid increases of both $W_{\text{max}}$ and $h_a$ after 1300 CST, in a sharp contrast to the nearly steady state attained by $h_b$ and the temperature difference between $\Theta_a$ and $\Theta_b$. The deep mixed layer means a deep layer through which an air parcel is able to ascend without being decelerated by the stable stratification. This ascending motion would further help raise the inversion layer. On the other hand, the compensating descending motion would tend to push down the inversion layer on the cool side. The resultant difference in $h_a$ and $h_b$ would increase the horizontal pressure gradient, thus accelerating the development of the vertical circulation.

There is one feature in Fig. 9 which we are not able to resolve at present. The peak value of the surface heat flux on the warm side resulted in our experiment is 54 K cm s$^{-1}$. This value is extremely large compared to those found in the literature. For example, this value is about twice that was observed in Wangara experiment (Clark et al., 1971). This value may not be unrealistic in view of the fact that the surface temperature was observed to exceed 38°C in the afternoon in the western part of the NSSSL mesonetwork and the surface wind was about 10 m s$^{-1}$ in the real situation. However no observed data are available to compare with the numerical result. As an independent check, we have also applied Tennekes’ (1973) one-dimensional mixed-layer model with our initial condition. The result indicates that the maximum heat flux should be approximately 55 K cm s$^{-1}$ to produce the height of the mixed layer and the potential temperature in the mixed layer observed in the warm side in our case. Thus the extreme surface heat flux does not appear to be a consequence of the particular turbulence model we adopted.

We do not include any figures showing the fields of turbulence variables. One reason for this is that no observed data are available for comparison in the case under consideration. Another reason is that, as mentioned in Section 1, Mellow and Yamada's level 3 turbulence closure model is known to have some shortcomings. We feel that the present two-dimensional boundary-layer model based on their approximations was able to simulate quite realistically all gross features of the observed planetary boundary layers, including the height of the mixed layer. However, we feel that the turbulent transport across the inversion layer is unrealistically small. This is evident in the unrealistically strong vertical wind shear near the inversion layer on the cool side (Figs. 5c and 6c). The downward heat flux near the inversion is also found to be too weak compared with the results of the other numerical experiments for the developing mixed layer or the observation in other occasions. The vertical profile of the computed $\theta^2$ does not appear to be realistic in that the second maximum near the inversion is missing (Lenschow, 1970; Willis and Deardorff, 1974; Kuo and Sun, 1976; Zeman and Lumley, 1976; André et al., 1976).

Some of these unrealistic features in the turbulence fields are also evident in the results of Yamada and Mellow (1975). Apparently these shortcomings stem from two sources. One is the incorrect diffusion term (André et al., 1976; Zeman and Lumley, 1976). The other is the assumption that turbulence vanishes in the region where the Richardson number is larger than 0.21. In our experiment, Richardson number is usually larger than 0.21 in the inversion layer. We are currently working on improving the Mellor-Yamada model by incorporating the more sophisticated representations for turbulence transport terms proposed by Zeman and Lumley (1976).

Before closing this subsection, a remark will be made on the computational diffusion effect which is implicitly involved in the upstream differencing scheme we have applied in this experiment. The implicit diffusion terms in the $z$ and $x$ directions may be written as

$$\text{DIF}_z = \frac{1}{2} \left[ W \frac{\partial U}{\partial z} \right], \quad \text{DIF}_x = \frac{1}{2} \left[ U \frac{\partial U}{\partial x} \right].$$

To estimate the possible magnitude of this term at 1420 CST, we select the location at the 792 mb over M where the vertical wind shear is the maximum and the
vertical velocity is about $-3 \text{ cm s}^{-1}$. The magnitude of DIF$_x$ is estimated as $1.2 \times 10^{-2} \text{ cm s}^{-2}$ and that of DIF$_z$ is $5.2 \times 10^{-3} \text{ cm s}^{-2}$. On the other hand, the magnitude of the explicit stress term $-\frac{\partial w^2}{\partial z}$ is $6.6 \times 10^{-2}$ at the same location. DIF$_x$ is about $4.0 \times 10^{-3} \text{ cm s}^{-2}$ at the center of the ascending motion where $W$ is $15 \text{ cm s}^{-1}$, whereas $\frac{\partial w}{\partial z}$ is $2.4 \times 10^{-3} \text{ cm s}^{-1}$. This would suggest that the factitious diffusion may not seriously affect the accuracy of our numerical integration in the particular situation under consideration where the explicit turbulent transport processes dominate over the advection process.

b. Case II

As mentioned in Section 3, there was some uncertainty in determining the initial condition for the $x$ component of velocity $U_x$ from observations, although the $U_x$ profile we specified in case I appears to be reasonable in that the predicted fields of velocity and potential temperature in the afternoon agree well with observations. Here we shall take a different profile of $U_x$ to test the sensitivity of the model result to the $U_x$ profile. The $U_y$ profile we selected is exactly the same as that in case I in the layer above $2600 \text{ m}$ ($\sim 700 \text{ mb}$). Below this level $\frac{\partial U_y}{\partial z}$ is constant and equal to the $U_y$ gradient in the layer between 700 and 500 mb. Then $U_y$ at the ground surface is $-1.31 \text{ m s}^{-1}$. All other parameters are identical with those in case I.

The main reason for selecting this negative $U_y$ in the layer below nearly $850 \text{ mb}$ is the following. The earlier numerical experiments of the sea-breeze circulation (e.g., Estoque, 1962) indicate that the location and shape as well as intensity of the sea-breeze circulation are strongly affected by the direction of the prevailing wind, i.e., the upward motion associated with the sea breeze is stronger in an offshore wind than that in an onshore wind.

Fig. 10 shows the predicted results in case II at 1430 CST. We immediately observe that the distributions of all elements are significantly different from case I. In comparison with Fig. 6, we note that the maximum horizontal temperature gradient in the mixed layer is weaker in case II than in case I, even though the overall horizontal differences in the mixed layer between A and B are nearly the same ($\sim 4 \text{ K}$) in two cases. More importantly, the isentropic lines tilt toward the warm side in case II. The consequence of this potential temperature distribution is that the horizontal pressure perturbation calculated by the hydrostatic equation is wider spread in case II than in case I and, as can be seen in Fig. 10b, the core of negative $U$ in the mixed layer is less concentrated. In fact, no positive $U$ is observed in the entire mixed layer. Consequently, the maximum upward motion in case II is only $6.2 \text{ cm s}^{-1}$ (Fig. 10d), less than a half of that in case I. Another noteworthy feature is that the maximum upward motion occurs on the warm side, $50 \text{ km}$ away from M.

Ogura and Chen mentioned that the synoptic situation on 22 May 1966 was quite similar to that on 8 June 1966. In fact, a local severe storm warning was issued in the morning. Nevertheless, no severe convective activity was reported over the NSSL network. It would be interesting to point out that on 22 May, the wind in the mixed layer came from the cool side. This alone indicates that the situation on 22 May 1966 was less favorable for the formation of a squall line. The wide spread of the ascending motion observed on that day is also consistent to what we have obtained in case II here.

c. Case III

In this case Mellor and Yamada's level 1 turbulence closure approximations are used. All other conditions are identical to those in case I. The motivation for applying the level 1 model is the following. We are interested in extending the present study into a three-dimensional moist model. In such a modeling study it is highly desirable to use a simple turbulence model to relax the prohibitive computer requirements. We are, therefore, interested in knowing to what extent the vertical circulation depicted in case I can be reproduced using a simple level 1 model.

Fig. 11 shows the distribution of $W$ at 1430 CST in case III. The maximum upward velocity is $13.6 \text{ cm s}^{-1}$, very close to $14.8 \text{ cm s}^{-1}$ in case I. The figures for the distributions of $U$, $V$ and $\Theta$ are not shown here, but they are again surprisingly similar to those in case I. The noticeable difference is in the height of the inversion on the warm side: it is $\sim 40 \text{ mb}$ lower than in case I at 1430 CST and $70 \text{ mb}$ lower at 1630 CST. The potential temperature is also $\sim 1 \text{ K}$ lower in the mixed layer.

The difficulty we found in the application of the level 1 model is that irregular small-scale patterns appear in the turbulence fields on the warm side. These patterns are associated with the temporal variations with the period of $2\Delta t$. Apparently these oscillations are caused by "overshooting" of turbulence variables when they try to adjust to their new environment. This was not observed in case I because turbulence variables represented by the level 3 model can maintain their persistencies.

5. Summary and concluding remarks

One of the authors (Ogura, 1975) discussed the close relationship between the low-level convergence and the formation of organized deep convective systems observed both in the tropical regions and middle latitudes. During the past few years more evidence has been presented by many authors to indicate this relationship, including Ulanski and Garstang (1978a,b) and OC. In the present work we have attempted to identify the
basic mechanism responsible for the generation of the prestorm mesoscale convergence observed in OC. The conclusion of this investigation is that, if the synoptic-scale low-level wind blows in the right direction (offshore in sea breeze terminology), the low-level horizontal temperature gradient of the magnitude observed in the 8 June 1966 squall-line case is capable of generating upward motion with sufficient intensity to release the potential instability.

In this connection, we may refer to the work of Weiss and Purdom (1974). They demonstrated that the formation and maintenance of thunderstorms in a moving squall line is greatly influenced by the low-level temperature field through which it moves. In a case they
studied, the early morning cloudy area was 5–8 K cooler at 1200 CST than the clear area, with most of the temperature gradient concentrated along the original cloud boundary. Thunderstorms began to form over or near the area of large temperature gradient.

The basic physical processes involved in the formation of the prestorm low-level convergence, as revealed by the numerical experiment in case I, may be summarized as follows. As a point of reference, Fig. 12 shows the schematic streamlines in the mid-afternoon. At early morning, the temperature, pressure and wind fields are assumed to be balanced geostrophically and the temperature is homogeneous in the x direction. As soon as the temperature in the mixed layer on the left side of M becomes higher than that on the right side due to the imposed differential surface heating, the horizontal pressure gradient pointing toward the negative x direction is intensified. This generates the wind blowing from the cool side to the warm side (an onshore air flow) at the center of the temperature gradient zone.
Fig. 11. Distributions at 1430 CST in case III of vertical velocity with a contour interval of 4 cm s$^{-1}$.

Fig. 12. Schematic vertical cross section across the dry line at 1630 LT in case I. J, U and D denote the locations of the cores of the jet in the y direction, the upward motion and the downward motion, respectively. The dashed lines are the contour lines of 2 cm s$^{-1}$ for the vertical velocity.
To satisfy the mass continuity, the ascending motion develops on the warm side and the descending motion on the cool side. At the early stage of the development, the vertical circulation is almost symmetric with respect to the vertical at \( M \) in that the pattern and intensity of the ascending and descending motions are nearly the same.

As time progresses, the mixed layer on the warm side becomes deeper than that on the cool side. The vertical circulation is also intensified to extend into the stably stratified layer above the inversion. The upward expansion of the isobaric surfaces produces the horizontal pressure gradient pointed toward the positive \( x \) direction (an offshore acceleration) in the layer above the inversion. Thus a local maximum of the \( x \) component velocity (and convergence on the cool side) is established in that layer. Also the descending air flowing out from the top of the ascending branch of the vertical circulation (a return flow) tends to spread along the inversion. Through these processes, the core of descending motion moves up to just above the inversion, leaving the widespread weak downward motion in the mixed layer on the cool side. Meanwhile, the core of the ascending motion remains in the mixed layer with a concentrated pattern. Thus the air flow shown schematically in Fig. 12 is established. We emphasize in Fig. 12 that the ascending motion is fed almost exclusively by the air in the mixed layer on the cool (and moist, in a real situation) side. Therefore, the configuration of the vertical circulation is quite favorable for the formation and maintenance of a squall line.

Throughout this sequence of events, the vertical eddy transport of momentum is found to be of secondary importance in our case. The vertical eddy heat transport is the one which drives the entire circulation. Fig. 12 also shows the core of the low-level jet in the \( y \) direction located on the right of the ascending motion when we look toward the downstream. This low-level jet is generated by the unbalance between the Coriolis force and pressure gradient force. Thus both the ascending motion and the low-level jet are the end products of the variation of the \( x \) component of velocity with time which in turn is generated by the varying horizontal pressure gradient force in the \( x \) direction.

The present numerical experiment was performed assuming the important motions could be captured in two dimensions. Furthermore, we had some difficulty in specifying the initial conditions of the model. This difficulty arose because only one station (LTS) made an early morning sounding in the mesonet network that day and because a surface front (or more precisely a wind shift line) was slowly moving into the mesonet network. Consequently, the representativeness of the low-level observations at LTS at 0548 CST are uncertain. It would be desirable to gather the mesoscale planetary boundary layer data covering at least the major part of its diurnal variation for modeling purposes.

The maximum upward motion predicted in the model agrees well with that observed. Nevertheless, we feel that the intensity of the predicted ascending velocity was underestimated in comparison with the observation when we consider the difference in the horizontal width of the ascending motion area. It is \( \sim 40 \) km in the experiment and \( 90 \) km in the observational analysis. The latter may not represent the real situation since it is strongly affected by the horizontal resolution determined by the average station separation (85 km in this case). Further, it is subject to the smoothing effect of the objective analysis scheme applied in OC. We have pointed out in the text that a shortcoming of the turbulence parameterization used is its failure to produce strong turbulence mixing around the inversion layer. With an improved parameterization, a stronger transfer of westerly momentum into the mixed layer and enhancement of the low-level convergence would be expected.

Acknowledgments. The authors wish to thank Dr. Su-Tzai Soong for offering valuable comments during the course of this study and Dr. Robert Wilhelmson for improving the manuscript. The main part of the numerical computation was done at the National Center for Atmospheric Research which is supported by the National Science Foundation. The Research Board of the University of Illinois also provided computer support. This work was supported by the Environmental Research Laboratories, National Oceanic and Atmospheric Administration, under Grant 04-7-022-44026 and by the National Science Foundation under Grant ATM 75-19336.

REFERENCES


Melior, G. L., and T. Yamada, 1974: A hierarchy of turbulence


