A Study of Climate Sensitivity Using a Simple Energy Balance Model

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ABSTRACT

The results of simple zonal energy balance climate models are rather sensitive to the parameterizations used to calculate the fluxes of solar radiation absorbed, thermal radiation emitted and energy transported by the atmosphere and oceans. For this reason results are examined for North's (1975a) constant coefficient diffusion model using climatologically consistent radiation parameterizations. With these radiation parameterizations, the calculated climate is less sensitive to changes in the incident solar radiation than was previously found using other parameterizations. In addition, how the model's results are influenced by the biofeedback mechanism recently proposed by Cess (1978) is studied. This feedback accounts for changes in the surface albedo caused by changes in the vegetation that might accompany climate change. Based on the model results, this feedback could be an important link between the climate and the earth's orbit around the sun.

1. Introduction

Budyko (1969), Sellers (1969) and North (1975a,b) have constructed simple energy balance models of the earth's climate. In these models the longitudinally averaged annual mean surface temperature for a particular latitude belt is obtained by balancing the fluxes of the solar radiation absorbed, the thermal radiation emitted and the energy transported by the atmosphere and ocean. These fluxes in turn are taken to be simple functions of latitude and surface temperature. The models have been used primarily to study the sensitivity of the earth's climate to changes in the solar constant. Changes in the calculated annual mean global-average surface temperature and in the position of the ice line, which represents the equatorward extent of the polar ice caps, have served as indicators of the climate change.

It is well known, however, that the results of such simple models are rather sensitive to the parameterizations used to express the energy fluxes as functions of latitude and surface temperature. Recently, Cess (1976) and Lian and Cess (1977) have shown that the large decreases in the global mean surface temperature calculated for a 1% decrease in the solar constant had been based on parameterizations for the absorbed solar radiation and for the emitted thermal radiation, which are inconsistent with climatologies of surface temperature, cloud cover and satellite observed radiative fluxes. Whereas the models produced a decrease of 4 to 5°C, Lian and Cess, using radiation parameterizations based on the climatologies, obtained a decrease of only 1.84°C. The purpose of this paper is to investigate how, in addition to their sensitivities, some of the other results obtained with such models might be affected if we use radiation parameterizations which are consistent with the climatologies.

The results to be studied are the curve which gives the equilibrium position of the ice line as a function of solar constant and the shift in the ice line caused by changes in the earth's obliquity. The curve will be examined to determine the reduction in solar constant which is sufficient for the earth to become completely ice covered. Past studies have indicated that this reduction might be as small as 1.6–4%. How the model's climate responds to changes in the earth's obliquity will be examined to learn if the model would predict ice ages as a result of changes in the earth's orbit, as was suggested by Milankovitch. In addition, we will study how these results are affected if the albedo parameterization is adjusted to allow for changes in vegetation which might have accompanied the 18 000 YBP[2] glaciation (CLIMAP, 1976). Cess (1978) has recently suggested that because of its effect on the albedo, such changes in vegetation represent a sizable feedback in the climate system.

For this study, we will use North's (1975a) model in which the meridional transport of energy is modeled by diffusion with a constant diffusion coefficient. We will use Cess's climatologically derived parameterization for the emission of thermal radiation, and we will use the albedo parameterization that is developed in the next section for the absorption of solar radiation. The

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[2] Years before present.

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albedo parameterization will be based on Budyko's (1969) concept of an ice line which delineates the equatorward extent of permanent snow and ice.

Recently, Oerlemans and Van den Dool (1978) performed a study similar to that described in this paper. They found that, with climatologically consistent radiation parameterizations, results of the energy balance model were less sensitive to changes in the solar constant than was previously found. The results presented in this paper agree with their findings, but the radiation parameterizations are quite different. How some of the differences influence the behavior of the model is examined in Section 3.

Before proceeding, we should keep in mind that the results presented in this paper apply only for the diffusion model used to calculate the meridional energy transport. If we had used some of the other models proposed for this transport, the results might have been different (Lindzen and Farrell, 1977). In view of the wide range of possibilities obtainable from these other models, however, it is probably best for this study to limit ourselves to the diffusion model.

2. Description of model and radiation parameterizations

The model to be used was developed by North (1975a). It is based on an energy balance equation which is given by

$$-D\frac{d}{dX}(1-X^2)\frac{d}{dX}T(X) = QS(X)[1-\alpha(X,X_s)]-I[X,T(X)], \quad (1)$$

where Q is the solar constant divided by 4 (=340 W m^{-2}); $S(X)$ is the annual average fraction of Q received by latitude θ for which $X=\sin\theta$; $\alpha(X,X_s)$ is the albedo at latitude θ, where the edge of the polar ice cap is at latitude $\theta_s(X_s=\sin\theta_s)$; $I(X,T(X))$ is the flux of thermal radiation emitted at latitude θ; $T(X)$ is the surface temperature; and D is the diffusion coefficient which is taken to be constant. For simplicity, we will consider (as did North) an idealized model in which the Southern Hemisphere is identical to the Northern Hemisphere. For such a model the variables in (1) are even functions of X.

The parameterizations used for $I(X,T(X))$ was developed by Cess (1976). Using climatologies of the annual mean zonal average surface temperature and fractional cloud cover, he demonstrated that the flux of thermal radiation fits closely the functional form proposed by Budyko (1969), i.e.,

$$I(X,T(X))=A_1+B_1T(X)+[A_2+B_2T(X)]A_c(X), \quad (2)$$

where $A_c(X)$ is the fractional cloud cover at X. The constants that gave the best fit to the observed fluxes for the Northern Hemisphere were $A_1=257$ W m^{-2}, $B_1=1.63$ W m^{-2}°C^{-1}, $A_2=-91$ W m^{-2} and $B_2=-0.11$ W m^{-2}°C^{-1}. With these constants the largest difference between the observed annual mean flux and that calculated using (2) was only 1.2% of the observed flux for any 10° latitude belt.

For the parameterization of the albedo, the longitudinally averaged annual mean albedo at any latitude is assumed to be given by

$$\alpha(X,X_s)=A_c(X)\alpha_c(X,X_s)+[1-A_c(X)]\alpha_s(X,X_s), \quad (3)$$

where $\alpha_c(X,X_s)$ is the average albedo for the cloud-covered areas and $\alpha_s(X,X_s)$ the average albedo for the clear-sky areas. According to Lian and Cess (1977), the albedo of the cloud-covered areas is given by

$$\alpha_c(X,X_s)=0.641-0.494\mu+0.258\alpha_s(X,X_s), \quad (4)$$

where μ is the cosine of the solar zenith angle. As was the parameterization for the flux of thermal radiation, Eq. (4) was obtained from climatologies of cloud cover and of satellite-observed albedos for the Northern Hemisphere.

In the present model the clear-sky albedo $\alpha_s(X,X_s)$ is calculated in the following manner: First, the zonal mean surface albedo is obtained by area weighting the albedos of ocean and land surfaces. The albedo of an ice-free ocean surface is given by

$$\alpha_W=\frac{0.05}{(\mu+0.15)}. \quad (5)$$

The albedo α_L of an ice-free land surface is taken to be 0.25. These choices for α_L and α_W are in fair agreement with the snow and ice-free continental and oceanic surface albedos given by Kondratyev (1969).

For a given surface albedo, we calculate the clear-sky albedo by linearly interpolating between the clear-sky albedos obtained for surface albedos 0.1 and 0.8. These clear-sky albedos are given by

$$\left.\begin{aligned}\alpha_{0.1}&=\frac{0.15}{\mu+0.58}\\ \alpha_{0.8}&=-\frac{7.7}{\mu-12}\end{aligned}\right\}. \quad (6)$$

These expressions were derived from reflectivities calculated by Braslau and Dave (1973) for a clear-sky model of the earth's atmosphere. As is shown in Fig. 1, this procedure gives clear-sky albedos which agree well with the reflectivities obtained by Braslau and Dave for surface albedos $0.1\leqslant\alpha_0\leqslant0.8$, and for $0.17\leqslant\mu\leqslant1.0$ or solar zenith angles $0°\leqslant\theta\leqslant80°$.

Finally, the surface albedo of snow- and ice-covered surfaces is taken to be 0.63. This value gives the observed clear-sky albedo, 0.52, for the 10° latitude belt centered at 85°N (Vonder Haar and Ellis, 1975; Cess,

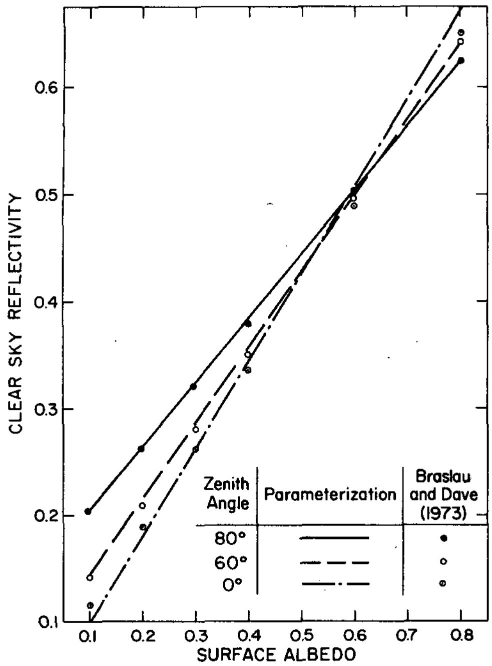

FIG. 1. Clear-sky reflectivities calculated with the parameterization and clear-sky reflectivities calculated by Braslau and Dave (1973).

1976). Snow and ice almost always cover the surfaces of this latitude belt (Kukla and Kukla, 1974).

To define an ice line which marks the equatorward extent of the polar ice caps, we take 10×10^6 km^2 as the area of extent of the Northern Hemisphere covered by permanent ice and snow (Kukla and Kukla, 1974), and we uniformly distribute this area over polar latitudes. We then obtain an ice line at 74°N—not significantly different from 72°N as was originally suggested by Budyko (1969).

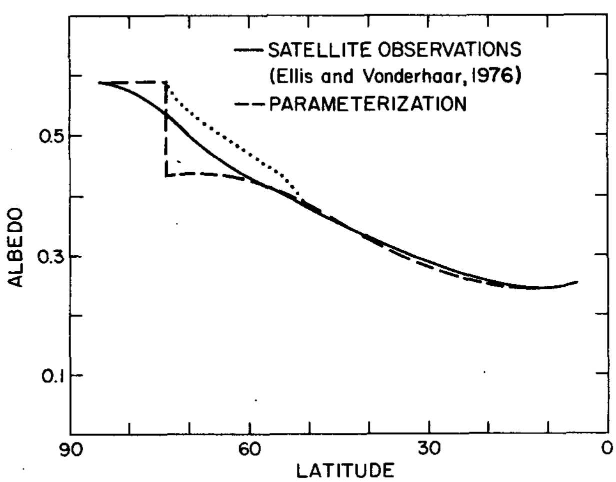

FIG. 2. Albedo obtained from parameterization (dashed curve) and albedo observed from satellites (solid curve) (Ellis and Vonder Haar, 1976). Discontinuity of parameterized albedo is caused by ice line at 74°N. A simple model for the seasonal variation of ice and snow cover gives a smooth parameterized albedo (dotted curve).

Fig. 2 shows the zonal average annual mean albedos for the Northern Hemisphere obtained from satellite observations (solid curve) [after Ellis and Vonder Haar (1976)] along with those obtained from the parameterization with the ice line at 74°N (dashed curve). For the parameterized albedos, the cloud cover fractions were taken from London (1957), as listed by Cess (1976), and the ocean area fractions were taken from Sellers (1965). The cloud cover fraction was assumed to be the same for both land and ocean areas. The fraction $S(X)$ of solar radiation incident at each latitude was evaluated using the earth's orbital parameters and was found to be approximately given by

$$S(X)=1.0-0.477P_2(X)-0.045P_4(X)+0.008P_6(X)+0.014P_8(X), \quad (7)$$

where $P_n(X)$ is the Legendre polynomial of order n. The largest difference between the actual $S(X)$ and that given by (7) occurs at the poles, $X=\pm1$, where the difference is about 1% of the actual value. The annual mean cosine of the solar zenith angle was taken to be $S(X)/2$. The cloud cover fractions, ocean area fractions and solar zenith angles used to calculate the albedo are listed in Table 1.

In Fig. 2 we see that south of 50°N, where on a zonal average basis snow and ice cover is infrequent, the agreement between the parameterized and the observed albedos is excellent. This agreement should be expected since the albedos of the cloud-covered areas, and therefore of the largest contributors to the zonal albedos, were adjusted to fit the observations (Lian and Cess, 1977). Nevertheless, with such features as the distribution of land and ocean areas and zenith angle dependencies for clear-sky and ocean surface albedos, the parameterization gave better agreement with the observations than it would have achieved if, for example, a fixed albedo had been assigned for clear skies regardless of zenith angle and surface type.

The existence of an ice line causes a discontinuity in the parameterized albedo. Such a discontinuity, of

TABLE 1. Annual mean zonal average cloud cover fractions A_c [from London (1957) as listed by Cess (1976)], ocean area fractions A_w (from Sellers, 1965), and cosines μ of the solar zenith angle used in the albedo parameterization.

Latitude	A_c	A_w	μ
5	0.51	0.772	0.609
15	0.44	0.736	0.591
25	0.41	0.624	0.558
35	0.47	0.572	0.512
45	0.57	0.475	0.451
55	0.64	0.428	0.381
65	0.64	0.294	0.316
75	0.61	0.713	0.272
85	0.55	0.934	0.252

course, is not observed. Snow and ice cover are nonuniform in space and time and this nonuniform distribution smooths the latitudinal variation of the zonal mean annual average surface albedo. How the discontinuity in the parameterized albedo affects the results of the energy balance model is examined in the next section.

To solve the energy balance equation (1) given the ice line position, we expand $T(X)$ in a series of Legendre polynomials $P_n(X)$:

$$T(X)=\Sigma_n T_n P_n(X). \tag{8}$$

Because of the assumed symmetry between the Northern and Southern Hemispheres, the amplitudes of the odd Legendre polynomials in (8) are zero. With this representation for $T(X)$, the energy balance equation may be rewritten (see Appendix) as an infinite set of coupled algebraic equations for the amplitudes T_n. Of these amplitudes, only those associated with low-order modes, and therefore with large spatial and temporal scales, are to be considered (North, 1975a). Fortunately, the T_n rapidly approach zero as n increases. Consequently, the high-order modes have little effect on the low-order modes and the amplitudes of the high-order modes may be set to zero. In this study only modes with $n \leqslant 8$ were included. Once truncated, the set of coupled equations may be solved algebraically after a value has been assigned to the diffusion coefficient D.

To insure that the model produces a reasonable solution $T(X)$ for the present climate, we simultaneously adjust A_1 [in (2)] and D to give the observed T_0 and T_2. For the Northern Hemisphere $T_0=14.9$°C and $T_2=-28.0$°C. These amplitudes are derived from the surface temperatures given by Crutcher and Meserve (1970). Only these amplitudes are used to adjust A_1 and D because the next largest amplitude, $T_4=-3.5$°C, is considerably smaller than both T_0 and T_2. The values which gave the observed T_0 and T_2 were $A_1=260.3$ W m^{-2} and $D=0.611$ W m^{-2} °C^{-1}.

3. Results

The results of the model calculations are summarized in terms of the equilibrium position X_s of the ice line which is shown in Fig. 3 as a function of the solar constant. For these results, the ice line is assumed to be at the −10°C isotherm, as was suggested by Budyko (1969), and the X_s vs Q curve is obtained following the methods outlined by North (1975a) and described in the Appendix. With the constants given in the previous section, the model places the ice line at 73°, very close to its position estimated from observations. For comparison, the results obtained using the radiation parameterizations just described (dashed curve) along with those obtained using the radiation parameterizations adopted by North (1975a) (solid curve) and the position of the −10°C isotherm of the lowest model level of Wetherald and Manabe's (1975) general

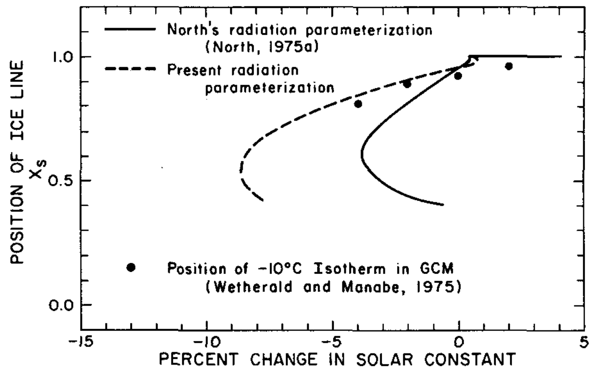

FIG. 3. Equilibrium position of ice line (−10°C isotherm) X_s as a function of solar constant. Results obtained with the radiation parameterizations described in Section 2 are shown as the dashed curve; results obtained with the radiation parameterizations adopted by North (1975a) are shown as the solid curve, and those using the −10°C isotherm of lowest GCM level from calculations performed by Wetherald and Manabe (1975) by points.

circulation model (GCM) calculations are shown in the figure. As the figure shows, the results obtained with the parameterizations described here agree with those obtained with the GCM much better than do the results obtained with the parameterizations adopted by North.

a. Change in T_0 for 1% decrease in solar constant

The change in the annual average global mean surface temperature T_0 calculated for a 1% decrease in the solar constant is proportional to the slope of the X_s vs Q curve at the present value of Q. In the present model T_0 decreases 2.07°C. This decrease is only slightly larger than the 1.84°C decrease obtained by Lian and Cess (1977). Nevertheless, it is instructive to investigate the reasons for the difference.

The decrease in T_0 caused by a 1% decrease in Q may be evaluated from the change in T_0 per fractional change in Q, QdT_0/dQ. This quantity is derivable from the condition of radiative equilibrium for the earth-atmosphere system which is expressed by

$$I_0=Q(1-\alpha_0), \tag{9}$$

where I_0 is the global long-term average of the flux of thermal radiation emitted by the system and α_0 the planetary albedo of the system. Note that the planetary albedo is given by

$$\alpha_0=\frac{1}{2}\int_{-1}^{1} S(X)\alpha(X)dx.$$

From (9) we see that QdT_0/dQ is given by

$$QdT_0/dQ=\frac{I_0}{dI_0/dT_0+Qd\alpha_0/dT_0}. \tag{10}$$

Thus, the change in T_0 for a given change in Q is affected by I_0, dI_0/dT_0 and $d\alpha_0/dT_0$.

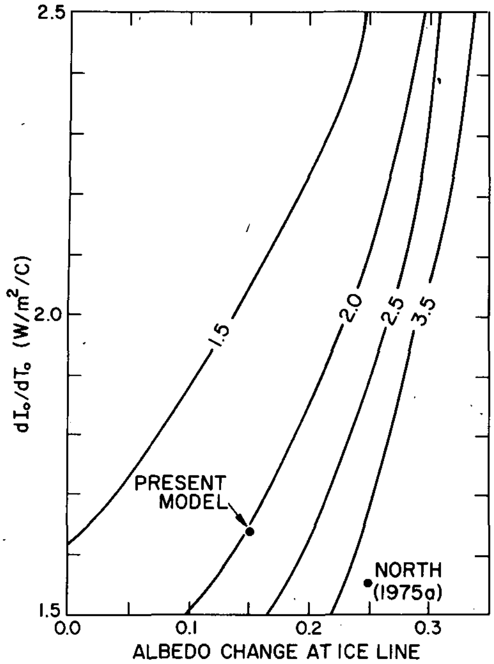

FIG. 4. Isopleths of decrease in global average surface temperature calculated for 1% decrease in solar constant.

Differences between the temperature changes can now be attributed to differences in these three factors. Lian and Cess (1977) used (2) with $B_2=0.0$ to calculate I_0 and dI_0/dT_0. With $B_2=0.0$ and for fixed cloud amount, $dI_0/dT_0=1.63$ W m^{-2} °C^{-1}. Setting $B_2=0.0$ in the present model reduces the decrease to 1.97°C and brings it closer to that obtained by Lian and Cess. The remaining discrepancy may be attributed to differences in the albedo-temperature feedback $d\alpha_0/dT_0$.

To model the change in albedo for a given change in surface temperature, Lian and Cess used the observed latitudinal changes in the annual mean zonal average albedos and surface temperatures. These changes yielded a feedback, $Qd\alpha_0/dT_0=-0.32$ W m^{-2} °C^{-1}. Using the parameterization described in the previous section, we obtain $Qd\alpha_0/dT_0=-0.41$ W m^{-2} °C^{-1}. Thus, the parameterization produces a feedback which is 27% larger than the feedback derived by Lian and Cess from observations. This difference is reduced when the parameterization described in the previous section is modified so that the albedo changes smoothly with latitude.

A simple method for smoothing the albedo is to allow for seasonal excursions of ice and snow. We do this by taking the fraction of snow and ice cover at a particular latitude to be equal to the fraction of time that the snow and ice line is equatorward of the latitude. Kukla and Kukla (1974) give the area covered by snow and ice as a function of time. For the Northern Hemisphere, they give 10×10^6 km^2 as being permanently covered and as much as 60×10^6 km^2 as being covered at the peak of winter. Assuming, as a first approximation, that the area covered has a sinusoidal time dependence, we find that the fractional cover at latitude X is given by

$$f(X)=1-\pi^{-1}\arccos[(X-X_s+A)/A], \quad (11)$$

where, from Kukla and Kukla's data, we obtain $X_s=0.961$ ($\theta_s=74°$) and $A=0.098$. Using (11) for the fractional cover of snow and ice, we obtain a smooth distribution of albedos across the ice line, as is indicated by the dotted curve in Fig. 2. Further smoothing and presumably better agreement with observations could be obtained if we had allowed for the longitudinal dependence of snow and ice cover. As will be shown, however, smoothing the albedo has little influence on the behavior of the model.

With the smoothed albedo and with $B_2=0.0$, the model constants are readjusted to give the observed amplitudes T_0 and T_2 as before. With $A_1=256.4$ W m^{-2} and $D=0.652$ W m^{-2} °C^{-1}, the observed amplitudes are recovered. With these constants the ice line is at 70°. The difference between this position and the 73° position obtained without smoothing is caused by differences in the amplitudes of the temperature modes with $n>2$. With these changes the model gives a temperature decrease of 1.85°C for a 1% decrease in solar constant. This decrease now agrees with that obtained by Lian and Cess. We note that, by smoothing the albedo, we have only slightly modified the sensitivity of T_0 to changes in Q.

The decrease in T_0 obtained with the present model should be compared with the 4.48°C decrease obtained using the albedo and thermal radiative flux parameterizations adopted by North (1975a). The difference in the decrease is reflected in the different slopes of the two curves shown in Fig. 3. As is illustrated in Fig. 4, the difference is primarily the result of the different albedo parameterizations. Fig. 4 shows isopleths of the decrease in the global average surface temperature caused by a 1% decrease in solar constant as a function of dI_0/dT_0 and of the change in albedo at the current ice line as the surface changes from ice-free to ice-covered. In the present model, this albedo change is 0.15, while in the parameterization used by North the change is 0.25. The isopleths in the figure were computed using the parameterizations, $I(X)=A+BT(X)$ and

$$\alpha(X,X_s)=\begin{cases} b_0, & X>X_s \\ a_0+a_2P_2(X), & X<X_s. \end{cases} \quad (12)$$

The albedo parameterization is taken from North (1975a). For each value of $B=dI_0/dT_0$, A and D [in (1)] were adjusted to give the observed amplitudes T_0 and T_2 [in (8)] for $Q=340$ W m^{-2} and the observed amplitudes of the $n=0$ and $n=2$ Legendre modes for the fraction of solar radiation absorbed. For the data given by Ellis and Vonder Haar (1976), these amplitudes are $H_0=0.710$ and $H_2=-0.474$. Also, for a given change in the albedo at the ice line, $b_0-[a_0+a_2P_2(X_s)]$, the parameters b_0, a_0 and a_2 were adjusted to give the observed H_0 and H_2. The present position of the ice line was taken to be at $X_s=0.961$ ($\theta_s=74°$). To obtain the results shown in the figure, the model was solved using only the first two Legendre modes.

The 2.07°C decrease obtained with the present parameterization should also be compared with the results of recent energy balance model calculations performed by Oerlemans and Van den Dool (1978). Using radiation parameterizations which were consistent with climatology, they obtained a decrease in the global mean surface temperature of 1.5°C for a 1% decrease in the solar constant. Despite differences between the models, the temperature decrease they obtained is similar in magnitude to that given here. We should recognize, however, that the agreement is due to compensating differences in the radiation parameterizations.

To obtain a relationship between surface temperature and the flux of emitted infrared radiation, Oerlemans and Van den Dool, like Cess (1976), relied on the annual mean latitudinal variation of the surface temperature and infrared flux. They obtained

$$I(X)=A(X)+BT(X), \qquad (13)$$

where A was made a function of latitude to allow for the zonal average height of continents above sea level. Using the data that Cess used, they obtained the best fit of (13) with observations for $B=2.23$ W m^{-2} °C^{-1}. This value differed from that obtained by Cess primarily because Oerlemans and Van den Dool did not allow explicitly for the effects of clouds on the emitted radiation as did Cess. Instead, the effects of clouds are implicitly included in their parameterization inasmuch as both the annual mean temperature and cloud cover are related through their distributions with latitude.

We see from Fig. 4 and we can deduce from (10) that large values of dI_0/dT_0 lead to small decreases in the global mean surface temperature for a 1% decrease in solar constant. From the 1.5°C decrease obtained by Oerlemans and Van den Dool we can also deduce from (10) that their larger value of B is partially compensated by a larger albedo-temperature feedback. They obtained $Qd\alpha_0/dT_0=-0.62$ W m^{-2} °C^{-1}, 50% larger than the feedback obtained with the parameterization described in the previous section and nearly twice the feedback obtained by Lian and Cess (1977).

We should recognize that the albedo-temperature feedback in these energy balance models is affected by the parameterization used to calculate the meridional transport of energy and by the value of dI_0/dT_0. Together these govern the latitudinal distribution of changes in the surface temperature, and thereby they govern the distribution of changes in the surface albedo. The parameterization used by Oerlemans and Van den Dool to calculate the meridional transport of energy is essentially the same as that used in the present model. Therefore, it cannot be the cause of the different magnitudes obtained for $d\alpha_0/dT_0$. From the results used to construct Fig. 4 we also find that for a given albedo parameterization $|d\alpha_0/dT_0|$ decreases by only 10% as dI_0/dT_0 increases from 1.5 to 2.5 W m^{-2} °C^{-1}. Thus, the different values adopted for dI_0/dT_0 cannot be the cause of the different magnitudes. Differences in the albedo parameterizations appear to be the primary cause.

In their parameterization of the albedo, as in their parameterization of the infrared radiative flux, Oerlemans and Van den Dool did not allow explicitly for the effects of clouds. Instead, the effects were included implicitly. Not allowing explicitly for such effects, however, can lead to unrealistic results.

Oerlemans and Van den Dool developed their albedo parameterization by extracting albedos from maps based on satellite observations. They categorized the observed albedos according to surface type. For example, they assigned an albedo of 0.31 to continents with surfaces free of ice and snow and 0.61 to continents with surfaces covered by snow. The differences between the albedos of snow-free and snow-covered continents is due mainly to differences in clear-sky albedos. If we allow for the presence of clouds, these differences, according to Lian and Cess, are given by $\Delta\alpha=(1-0.74A_c)\Delta\alpha_s$; or for $A_c=0.5$, $\Delta\alpha=0.63\Delta\alpha_s$. So, for $A_c=0.5$, to obtain the albedo change reported by Oerlemans and Van den Dool, $\Delta\alpha_s=0.48$. This change in the clear-sky albedo is much larger than $\Delta\alpha_s\approx0.25$ calculated using the parameterization described here. Furthermore, based on the results shown in Fig. 1, the large change in the clear-sky albedo, $\Delta\alpha_s=0.48$, seems unreasonable because, for solar zenith angles between 60° and 80°, such a change would require a surface albedo change >0.7. Such a change seems too large. So we see that because they did not explicitly account for the cloud contribution to the observed albedos, the albedos and albedo differences reported by Oerlemans and Van den Dool may have been contaminated by variations in cloud cover and cloud reflectivities.

The curves in Fig. 3 have portions for which $dX_s/dQ<0$ near $X_s\approx1$. Previous studies have shown that such branches of the solution are unstable to small perturbations (North, 1975b; Drazin and Griffel, 1977); therefore, we conclude that, according to the model, small polar ice caps are unstable.

As is the sensitivity of the surface temperature to changes in the solar constant, however, this result is also affected by each of the parameterizations used for the three components of the energy balance equation. Indeed, dX_s/dQ becomes positive near $X_s\approx1$ when the albedo is smoothed using the procedure described earlier. We should recognize, however, that negative slopes are not necessarily removed when we use a smooth albedo, as parameterizations which give smooth albedos can also give $dX_s/dQ<0$ near $X_s\approx1$.

b. Reduction in Q sufficient to cause an ice-covered earth

As shown in Fig. 3, using the parameterizations adopted by North (1975a), the model predicts an ice-covered earth when the solar constant is reduced by only 4%. On the other hand, with the parameterizations described in Section 2, the model predicts an ice-covered earth when the solar constant is reduced between 8 and 9%. The smoothed albedo does not alter this result. Although the reduction in the solar constant required to cause an ice-covered earth is now found to be more than twice as large as the reduction which North had found, the solar constant for which the earth becomes ice-covered is still considerably larger than those which, according to theories of stellar evolution, existed during the life of the earth. According to these theories, the solar radiative flux may have been as small as 50–70% of the present flux during the life of the earth (Sagan and Mullen, 1972). Geological evidence indicates, however, that the earth has never been completely covered by ice. Clearly, the model results are inconsistent with these theories of stellar evolution and the geological record. Some have reconciled such inconsistencies by suggesting that the composition of the earth's atmosphere billions of years ago was such that high surface temperatures were maintained through the greenhouse effect despite the low radiative output of the sun (Sagan and Mullen, 1972; Hart, 1978).

c. Implications for Milankovitch's theory of ice ages

Past changes in the obliquity and the longitude of perihelion have caused changes in the latitudinal and seasonal distribution of the incident solar radiation. Of these, changes in the longitude of perihelion affects only the length of the seasons and therefore only the seasonally averaged rate at which solar radiation is received. However, they do not influence the latitudinal distribution of the incident radiation received annually and, therefore, they do not influence the results of these simple annual mean energy balance models (North and Coakley, 1978). Only changes in obliquity affect the results of these models.

At high latitudes changes in the orbit can cause perturbations to the seasonally averaged indicent radiation which are several times the perturbations in the annual mean. Because of this difference, Budyko (1969) has suggested that to test Milankovitch's hypothesis, we should use models which include a seasonal cycle. Indeed, we would expect the responses of seasonal and annual mean models to differ if the extent of ice cover is governed through ablation by the incident solar radiation. Such a model has been developed by Pollard (1978). North and Coakley (1978), however, have shown that for models like that under consideration, in which the ice line is solely determined by the temperature field, the response to orbital changes is nearly the same with or without a seasonal cycle. Consequently, only the effects due to changes in obliquity need be considered.

Because of the albedo-temperature feedback, the model should produce large polar ice caps when the obliquity is near a minimum. The earth's obliquity ranges from 22.1° to 24.5°, and it is currently 23.45° (Vernekar, 1971). Orbits with a smaller obliquity have, on an annual basis, less solar radiation incident at polar latitudes and more incident at equatorial latitudes. This redistribution of the zonally averaged annual mean incident solar radiation causes the polar ice caps to advance.

The last glaciation, which peaked at about 18 000 YBP, is thought to have been triggered by the small obliquity which existed 25 000 YBP. At that time, the obliquity was ~22.2°. The resulting distribution of the incident solar radiation may be approximated by

$$S(X)=1.0-0.491P_2(X)-0.052P_4(X)+0.004P_6(X)+0.013P_8(X). \quad (14)$$

Previous studies by Budyko (1969) and by Suarez and Held (1976) have indicated that the difference between the latitudes of the ice line predicted by energy balance models for the present and for the 25 000 YBP obliquity is less than 4°. This change is considerably smaller than the change which is thought to have taken place. To estimate the change which occurred, we take 41×10^6 km^2 as being the area of the Northern Hemisphere covered by permanent snow and ice during the glacial maximum (Gates, 1976). This gives 57°N as the latitude of the ice line or a change of 17° from its present position.

As are the other model results, however, the latitudinal change of the ice line is also sensitive to the parameterizations used for the components of the energy balance equation. For example, if we use the parameterizations adopted by North, we find that for the same change in obliquity the model predicts almost a 5° change in the latitude of the ice line. However, if we use the parameterization described in Section 2, we find that the model predicts only a 2° change. The same result was obtained when the albedo was smoothed. A second comparison between the results of the model and observations is obtained by using surface temperatures. Using ocean cores from the midlatitudes of the Southern Hemisphere, Hays *et al.* (1976) concluded that the amplitude of the surface temperature change which might be linked to changes in obliquity ranges from 1°C to several degrees. The model, on the other hand, predicts a change of 0.3°C for midlatitudes. Furthermore, as was noted by Cess (private communication), at low latitudes where a decrease in obliquity causes the incident solar radiation to increase the model predicts higher surface temperatures. Data from the last glacial maximum, however, suggest that all latitudes were cooler (CLIMAP, 1976). Because of the discrepancies between the observations and the model predictions, we conclude that these simple energy balance models fail to explain the ice ages as a result of changes in the earth's orbit.

These findings raise the question, how do we explain the existence of the last ice age? Clearly, if we assume that the radiative output of the sun has remained constant, then we find that the simple models will not allow such large polar ice caps as occurred to be in equilibrium; i.e., such ice caps could not be subject to the equation of state imposed by the model. Using the parameterizations described in Section 2, the energy balance model gives an ice line temperature of $-6°C$ when the latitude of the ice line is 60°. Because this temperature is considerably below the freezing point, we might expect that the stress on this ice cap, according to the model, would be small. If in addition to this result we take into consideration the large time constants associated with large ice caps (Birchfield, 1977), it seems reasonable that such ice caps could have survived for long periods without being in equilibrium. Of course, we are left with the problem of explaining how such a large ice cap formed.

d. *Allowing for changes in vegetation*

Reconstructions of the last ice age indicate that it was accompanied by substantial changes in the vegetation which covered the ice-free continents (CLIMAP, 1976). Because of these changes, the surface albedos of the ice-free continents differed from those of today. Cess (1978) has suggested that this albedo change represents a biospheric feedback on the climate. As a first approximation, he assumed that the changes in vegetation were controlled by changes in the surface temperature. Using estimates of the albedo change and of the surface temperature change obtained from GCM simulations of the present climate and of the climate for the 18 000 YBP reconstruction (Gates, 1976), he found that the rate of change of the zonal albedo with the zonal surface temperature was about $d\alpha/dT = -0.0025$. If we assume that only the surface albedo of the continents changed, then we obtain for the Northern Hemisphere $d\alpha_L/dT = -0.0062$.

To include this feedback in the albedo parameterization described in Section 2, we take the ice-free albedos of the continents to be given by

$$\alpha_L(X) = 0.25 + (d\alpha_L/dT)[T(X) - T_p(X)], \quad 0.25 \leqslant \alpha_L \leqslant 0.30, \quad (15)$$

where $T_p(X)$ represents the present distribution of surface temperatures with latitude. The upper limit imposed on α_L is the same as that imposed in the GCM for major deserts (Gates, 1976).

Applying the albedo feedback to latitudes equatorward of 50°, as did Cess (1978), gives a global mean temperature decrease of 3.73°C for a 1% decrease in the solar constant. If we allow for the effects of the nonzero B_2 and the albedo discontinuity, we find that this decrease agrees well with the 3.27°C decrease obtained by Cess. Thus, the temperature decrease calculated with the biofeedback is about 80% larger than that calculated without. On the other hand, with the biofeedback the reduction in Q required to cause an ice-covered earth remains about 8% and the latitudinal change in the ice line caused by the 23.45° to 22.2° change in obliquity becomes 2.5°. Thus, although the biofeedback as proposed by Cess (1978) greatly enhances the sensitivity of the model to changes in the solar constant, it has little influence on the other findings.

Although the biofeedback mechanism fails to affect the sensitivity of the model climate to changes in the earth's obliquity, we note that had we changed α_L from 0.25 to 0.30 in place of using (15), we would have found that the latitude of the ice line moved almost 7°. Furthermore, the temperature decreased at all latitudes and it decreased several degrees at midlatitudes. Obviously, such an albedo change could have been at least partly responsible for the formation of the large ice sheets of the 18 000 YBP glaciation. We note, however, that to achieve the albedo change that would cause such a large shift in the latitude of the ice line, we would have to use a value of $d\alpha_L/dT$ larger in magnitude than that proposed by Cess. To be sure, the change in vegetation was probably not caused solely by changes in surface temperature. It probably resulted from a combination of factors. Nevertheless, we should also recognize that the value of $d\alpha_L/dT$ given above was derived using the temperature change calculated with a GCM and since this temperature change was in response to the prescribed albedo change rather than the albedo change being in response to the temperature change, we cannot rule out the possibility that the magnitude of $d\alpha_L/dT$ might, in fact, be larger than Cess proposed. As a result, the influence of this biofeedback mechanism on the climate may be stronger than has been suggested thus far.

4. Conclusions

Using radiation parameterizations which are consistent with observations, we have examined the results of North's (1975a) diffusive transport model with constant diffusion coefficient. For the emission of thermal radiation, we used the parameterization developed by Cess (1976) and for the absorption of solar radiation we used the albedo parameterization developed in Section 2. Since the albedo parameterization was based on Budyko's (1969) concept of an ice line which separates latitudes with permanent ice cover from those without, the albedo obtained from the parameterization is discontinuous at the ice line. This discontinuity, however, hardly affected the model results.

With the observationally consistent radiation parameterizations, the energy balance model is less sensitive to changes in the incident solar radiation than it is with the radiation parameterizations used in previous studies (Budyko, 1969; Sellers, 1969; North, 1975a). The results obtained with the observationally consistent radiation parameterizations also agree more closely with those obtained with a GCM. Cess (1976) and Lian and Cess (1977) arrived at these conclusions in their studies and the results of this paper support their findings. Oerlemans and Van den Dool (1978) also arrived at similar conclusions but several differences were noted between their parameterizations and those described here. The agreement between model results was shown to arise from a compensation of differences between the infrared and albedo parameterizations.

The addition of a biofeedback as proposed by Cess (1978), which—through changes in vegetation—links changes in the surface temperatures to changes in the albedo, increases the decrease in the surface temperature calculated for a 1% decrease in the solar constant by a factor of 1.8. This result is also in agreement with those reported by Cess (1978).

The climate calculated with the energy balance model is only slightly affected by changes in the earth's obliquity. Even with the biofeedback of Cess the response of the model to changes in obliquity is small compared to the differences thought to exist between glacial maxima and minima. However, if biofeedback proves to be stronger than was estimated by Cess, then it could greatly amplify the sensitivity of the model to changes in obliquity.

As was mentioned in the Introduction, however, we should keep in mind that the results presented in this paper were obtained using North's (1975a) constant coefficient diffusion model to calculate the meridional flux of energy transported by the atmosphere and the oceans. The results might have been different had we used some of the other models that have been proposed for this energy flux. Due to the wide range of possibilities, however, how these other models might have affected the results was not investigated.

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APPENDIX

Solving the Energy Balance Equation

Like the temperature, each term of (1) may be expanded in a series of Legendre polynomials. When we perform such an expansion and take advantage of the analytic properties of the Legendre polynomials, we find that (1) becomes a set of coupled algebraic equations for the amplitudes of the temperature modes. This set of equations may be represented by

$$\Sigma_n B_{mn} T_n = Q H_m(X_s) + A_m, \tag{A1}$$

where

$$B_{mn} = [n(n+1)D + B_1]\delta_{mn}$$

$$+ B_2 T_n \left[\tfrac{1}{2}(2m+1) \int_{-1}^{1} dx P_m(X) P_n(X) A_c(X) \right], \tag{A2}$$

$$H_m(X_s) = \tfrac{1}{2}(2m+1) \int_{-1}^{1} dx P_m(X) S(X) \times [1-\alpha(X,X_s)], \tag{A3}$$

$$A_m = A_1 \delta_{m0} + A_2 \left[\tfrac{1}{2}(2m+1) \int_{-1}^{1} dx P_m(X) A_c(X) \right]. \tag{A4}$$

The amplitudes T_n are obtained by solving (A1) in the usual manner.

To find the solar constant Q for which the ice line X_s is in equilibrium at the $-10°C$ isotherm, we note that

$$\Sigma_n T_n P_n(X_s) = T_s \equiv -10°\mathrm{C} \tag{A5}$$

and therefore

$$Q = \frac{T_s - \Sigma_{nm} A_m B_{mn}^{-1} P_n(X_s)}{\Sigma_{nm} H_m(X_s) B_{mn}^{-1} P_n(X_s)},$$

where B_{mn}^{-1} are elements in the inverse of matrix **B**.

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