NOTES AND CORRESPONDENCE

A Space-Time Analysis of Tropospheric Planetary Waves in the Northern Hemisphere

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ABSTRACT

The propagation of planetary waves at the 500 mb level is investigated by applying an extended version of wavenumber-frequency analysis. By separating the variance attributable to wave disturbances from the total variance, one may account for the noisy character of atmospheric motions. Several possible representations of zonal wave motions are discussed. The global morphology of the spectra of geopotential is given using a separation into westward and eastward traveling waves, and the differences to previous analyses are elucidated. It is shown that the traveling character of the waves dominates for nearly all latitudes and wavenumbers. The latitudinal and wavenumber bands where westward or eastward traveling waves prevail can be easily distinguished from those bands where wave disturbances are very small compared to the noise.

1. Introduction

Space-time analyses have been described by several authors (e.g., Deland, 1964; Eliasen and Machenhauer, 1965; Kao, 1968; Hayashi, 1971; Willson, 1975; Pratt, 1976; Fraederich and Böttger, 1978). Most of these spectra of geopotential data are based on relatively short time periods and are restricted to a narrow latitudinal band. This investigation uses a time series of the height of the 500 mb level extending from 1949 to 1975 (9861 days). The daily pressure level data are given at grid points spaced 5° in latitude (from 15–90°N) and 10° in longitude and were provided by the German Weather Service. In order to separate the "noisy" variance from the variance of wave motions, one may apply an analysis similar to that employed in optics to determine the degree and type of polarization of light (e.g., Born and Wolf, 1975). Thus it is ensured that the sum of westward and eastward traveling variances (or traveling and standing variances) is equal to the total variance resulting from wave disturbances, whereas in previous formulations the sum is equal to the total space-time variance including the noise. In addition, the ratio of wave variance to total space-time variance can be calculated as a measure of the relative strength of the waves.

The method of calculating the westward and eastward traveling (or traveling and standing) part of waves in the geopotential height after eliminating the noisy fractions from the total variance and a comparison with previously used techniques are presented in the second section. The third section gives a survey of the wavenumber-frequency behavior of the planetary waves with zonal wavenumbers up to $m=6$ over the Northern Hemisphere from 15–85°N.

2. Method of calculation

The pressure level data for each latitudinal zone are first separated into modes by a Fourier analysis for each day, as was proposed by Hayashi (1971). The higher order noisy modes are thus eliminated from the modes with lowest wavenumbers which describe the global behavior of the pressure and lead to planetary waves. The new time series with daily values for the sine and cosine terms [or the real and imaginary parts of a rotary vector, as in Hayashi (1978)] for each mode and each latitude, denoted respectively by S_m and C_m, are used to compute the power spectral estimates $P_\omega(S_m)$ and $P_\omega(C_m)$. The calculation procedure for autospectral and cross-spectral estimates is described by Jenkins and Watts (1968). The spectral estimates, based on samples of 9861 days and a Parzen lag window of length 120 days have a frequency spacing of 0.0042 day^{-1} and a spectral bandwidth of 0.015 day^{-1}. The 95%-confidence limits for the spectral density P are $(0.9, 1.2)\cdot P$ and the corresponding confidence limits for the squared coherency estimates are ± 0.013. Such narrow confidence limits are achieved only if the time series are long enough. As expected, the spectral estimates $P_\omega(S_m)$ and $P_\omega(C_m)$ (not shown here) have a marked red

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character with excessive power at the one-year period, but they generally do not show isolated peaks surpassing the confidence limits. It should be mentioned that the time series have been high-pass prefiltered in order to avoid leaking of excessive variance into higher frequencies. The spectra were then rectified by the corresponding frequency response function.

The next step of the analysis is the computation of the cross-amplitude spectral estimates $A(\omega,m)$ and the cross-phase spectral estimates $F(\omega,m)$ between S_m and C_m, using the same smoothing procedure as above. This representation which is not invariant against a zonal translation of the coordinate system is equivalent to the separation into coquadrature and quadrature estimates $L(\omega,m)$ and $Q(\omega,m)$, i.e.,

$$A(\omega,m)^2=L(\omega,m)^2+Q(\omega,m)^2 = \mathrm{coh}_\omega^2(C_m,S_m)P_\omega(C_m)P_\omega(S_m), \quad (2.1)$$

$$F(\omega,m)=\arctan[-Q(\omega,m)/L(\omega,m)]. \quad (2.2)$$

In contrast to previous formulations (e.g., Pratt, 1976), the propagating variance is not defined here by the quadrature spectrum directly. Instead, each cross-amplitude spectral value is considered as the square root of the product of two variance densities. The first factor in this product is the fraction $W_\omega(S_m)$ of the variance density $P_\omega(S_m)$ which belongs to a wave motion, and the second is the corresponding fraction $W_\omega(C_m)$ of $P_\omega(C_m)$. The basic assumption for this analysis is that the zonal sine and cosine coefficients of wave disturbances are coherent with each other. This means that there is a single origin for the different types of waves (westward and eastward traveling or traveling and standing waves). In contrast, the sine and cosine coefficients of noise are incoherent. Because of the fact that only the product of the two variance fractions $W_\omega(S_m)$ and $W_\omega(C_m)$ is given by $A(\omega,m)^2$, one must determine the ratio of the residual "noisy" variances of $P_\omega(S_m)$ and $P_\omega(C_m)$ in order to determine $W_\omega(S_m)$ and $W_\omega(C_m)$. According to the nature of random noise (incoherence of sine and cosine coefficients) and in order to maintain invariance of incoherence against a zonal translation of the coordinates, this ratio has to be 1.0. The variances are then

$$W_\omega(S_m)=\{A(\omega,m)^2+\tfrac{1}{4}[P_\omega(S_m)-P_\omega(C_m)]^2\}^{\frac{1}{2}} +\tfrac{1}{2}[P_\omega(S_m)-P_\omega(C_m)], \quad (2.3)$$

$$W_\omega(C_m)=\{A(\omega,m)^2+\tfrac{1}{4}[P_\omega(S_m)-P_\omega(C_m)]^2\}^{\frac{1}{2}} -\tfrac{1}{2}[P_\omega(S_m)-P_\omega(C_m)]. \quad (2.4)$$

It is seen that, if there is a pure wave motion without noise and $A(\omega,m)^2=P_\omega(S_m)P_\omega(C_m)$, the wave variances $W_\omega(S_m)$ and $W_\omega(C_m)$ become equal to $P_\omega(S_m)$ and $P_\omega(C_m)$. In the opposite case, if $A(\omega,m)=0.0$ and $P_\omega(S_m)=P_\omega(C_m)$, both wave variances vanish. Because equal noise fractions cancel out, the difference $W_\omega(S_m)-W_\omega(C_m)$ is equal to $P_\omega(S_m)-P_\omega(C_m)$. The transition to wave amplitudes is made by assuming that the portion of variance in each frequency interval is produced by a wave at the concerned frequency with an amplitude equal to the square root of the variance within the interval. Special parts of the spectra can be examined by a variation of the frequency resolution. The character of the zonal waves is determined by the amplitudes corresponding to the wave variances $W_\omega(S_m)$ and $W_\omega(C_m)$ and by the phase difference $F(\omega,m)$. A rotated ellipse is defined in this way and the state of the wave is determined by a vector rotating in it with the concerned frequency ω. The maximum values on the sine and cosine axis (or the tangent lines on the ellipse parallel to the axes) are defined by the amplitudes, and the phase angle $F(\omega,m)$ is the angle formed by the vectors to the tangent points. Now the computation of the miror axis $b(\omega,m)$ and the major axis $a(\omega,m)$ as well as the rotation angle $R(\omega,m)$ of the ellipse can be carried out easily:

$$a(\omega,m)^2=\tfrac{1}{2}[W_\omega(S_m)+W_\omega(C_m)+\mathrm{RT}], \quad (2.5)$$

$$b(\omega,m)^2=\tfrac{1}{2}[W_\omega(S_m)+W_\omega(C_m)-\mathrm{RT}], \quad (2.6)$$

where

$$\mathrm{RT}=\{W_\omega(S_m)^2+W_\omega(C_m)^2 +2W_\omega(S_m)W_\omega(C_m)\cos[2F(\omega,m)]\}^{\frac{1}{2}} \quad (2.7)$$
$$=\{[P\omega(S_m)-P_\omega(C_m)]^2+4L(\omega,m)^2\}^{\frac{1}{2}}.$$

$$2R(\omega,m)=\arctan\{2\cos[F(\omega,m)W_\omega(S_m)^{\frac{1}{2}}W_\omega(C_m)^{\frac{1}{2}} \div[W_\omega(C_m)-W_\omega(S_m)]\} \quad (2.8)$$
$$=\arctan\{2L(\omega,m)/[P_\omega(C_m)-P_\omega(S_m)]\}.$$

The total wave variance $[W_\omega(S_m)+W_\omega(C_m)]/2$ {as well as the total space-time variance $[P_\omega(S_m)+P_\omega(C_m)]/2$} is invariant against a zonal translation of the coordinates since

$$[W_\omega(C_m)+W_\omega(S_m)]/2 =[a(\omega,m)^2+b(\omega,m)^2]/2=\text{constant}. \quad (2.9)$$

The decomposition of a rotated ellipse is possible in different ways. One may assume two standing waves (oscillating vectors) with the corresponding phase difference. Another way is to define a traveling wave (rotating vector) and a standing wave (oscillating vector). However, the phase between both must be defined, so that this representation is not unique. If the phase is chosen to be 0°, the total wave variance (2.9) can be divided into two parts. The first part is assigned to the rotating vector $b(\omega,m)$, i.e.,

$$\mathrm{VR}(\omega,m)=b(\omega,m)^2. \quad (2.10)$$

$\mathrm{VR}(\omega,m)$ can be interpreted as the power of two oscillating vectors [each $\tfrac{1}{2}b(\omega,m)^2$] which are 90° out of phase and parallel to the axes. The power of the

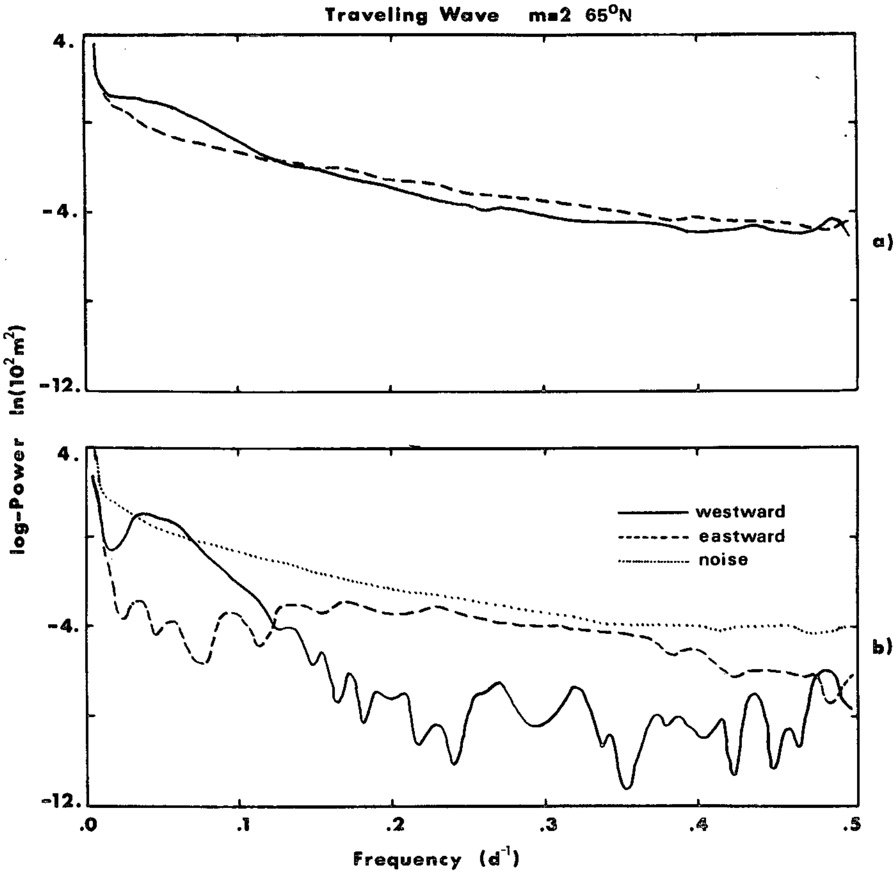

FIG. 1. Power spectra of westward and eastward traveling waves using two different methods of calculation (see text).

oscillating vector $a(\omega,m)$ is

$$\tfrac{1}{2}a(\omega,m)^2=\tfrac{1}{2}\{b(\omega,m)+[a(\omega,m)-b(\omega,m)]\}^2$$
$$=\tfrac{1}{2}b(\omega,m)^2+\tfrac{1}{2}[a(\omega,m)^2-b(\omega,m)^2].$$

The first part of this sum $[\tfrac{1}{2}b(\omega,m)^2]$ is already included in (2.10), so that the remaining part of (2.9) is

$$\mathrm{VS}(\omega,m)=\tfrac{1}{2}[a(\omega,m)^2-b(\omega,m)^2]=\tfrac{1}{2}\mathrm{RT}, \quad (2.11)$$

which is also obtained by subtracting (2.10) from (2.9). If either $b(\omega,m)=0$ or $a(\omega,m)=b(\omega,m)$ the parts (2.10) and (2.11) represent pure traveling or pure standing waves, respectively. The angular position of the standing wave is given by (2.8). It is noted that (2.11) corresponds to previous formulations of Pratt [1976, Eq. (12)] or Hayashi [1977, Eq. (4.9)] who used different approaches and denoted this quantity as standing variance. The nonunique partition of space-time power spectra into standing and traveling parts has been discussed, e.g., by Deland (1972) or Hayashi (1977). Finally, one can introduce a decomposition into a left-hand and a right-hand circulating wave (rotating vectors) so that the left-hand (westward) traveling wave variance is $[a(\omega,m)+b(\omega,m)]^2/4$ and the right-hand (eastward) traveling variance is $[a(\omega,m)-b(\omega,m)]^2/4$:

$$\tfrac{1}{4}[a(\omega,m)\pm b(\omega,m)]^2$$
$$=\tfrac{1}{2}\{A(\omega,m)^2+\tfrac{1}{4}[P_\omega(S_m)-P_\omega(C_m)]^2\}^{\frac{1}{2}}$$
$$\pm\tfrac{1}{2}Q_\omega(C_m,S_m)$$
$$=\tfrac{1}{4}[W_\omega(S_m)+W_\omega(C_m)]\pm\tfrac{1}{2}Q_\omega(C_m,S_m). \quad (2.12)$$

Again, in the case of vanishing noise, the expression (2.12) corresponds to previous formulation (Hayashi, 1971). The sum of westward and eastward traveling variances, the total wave variance, becomes $[a(\omega,m)^2+b(\omega,m)^2]/2=[W_\omega(S_m)+W_\omega(C_m)]/2$. If the traveling variance is defined as the difference between westward and eastward traveling variances (e.g., Deland, 1964; Fraederich and Böttger, 1978) one obtains the well-known formulation

$$\mathrm{TR}(\omega,m)=|Q_\omega(C_m,S_m)|. \quad (2.13)$$

The decomposition into westward and eastward traveling waves is preferred here since it needs no additional assumptions. It is pointed out that both waves result only from wave motions and that the sum of them is always less than or equal to the total space-time variance. The method enables one to specify that fraction of the total space-time variance which belongs to westward and eastward traveling waves as a measure of the relative strength of the waves (analogous to the degree of polarization in optics), and it avoids an overestimation of either part of wave variance. This fraction coincides with the maximum value of the coherence between the cosine and sine coefficients when the coordinate system is translated (Hayashi, 1978). It is obvious from (2.12), (2.1) and (2.9) that both westward and eastward traveling variances are non-negative as well as invariant with a zonal translation of the coordinate system.

3. Results

Fig. 1 illustrates the principal characteristics of this analysis. As an example, westward and eastward traveling variances are shown for the frequency range 0.0 day^{-1} to 0.5 day^{-1} (Nyquist frequency) and zonal mode $m=2$ at 65°N using the known formulation [e.g., Hayashi, 1977, Eq. (2.3)] in Fig. 1a and using the present concept [Eq. (2.12)] in Fig. 1b. One notices that a considerable amount of red noise without statistically significant peaks is superimposed on both waves in case a which is removed in case b. This noisy part is given by the difference between the total space-time variance $[P_\omega(S_m)+P_\omega(C_m)]/2$ and the total wave variance (2.9). The fluctuations of the very small variances in case b (exaggerated by the logarithmic representation) arise from the statistical character of the cross-spectral estimates (2.1). Even if both time series are random noise, the expected value of the squared coherency (corresponding to the 50% confidence limit) does not vanish but depends on the length of the time series and on the lag window, as mentioned earlier. Therefore, the expected value

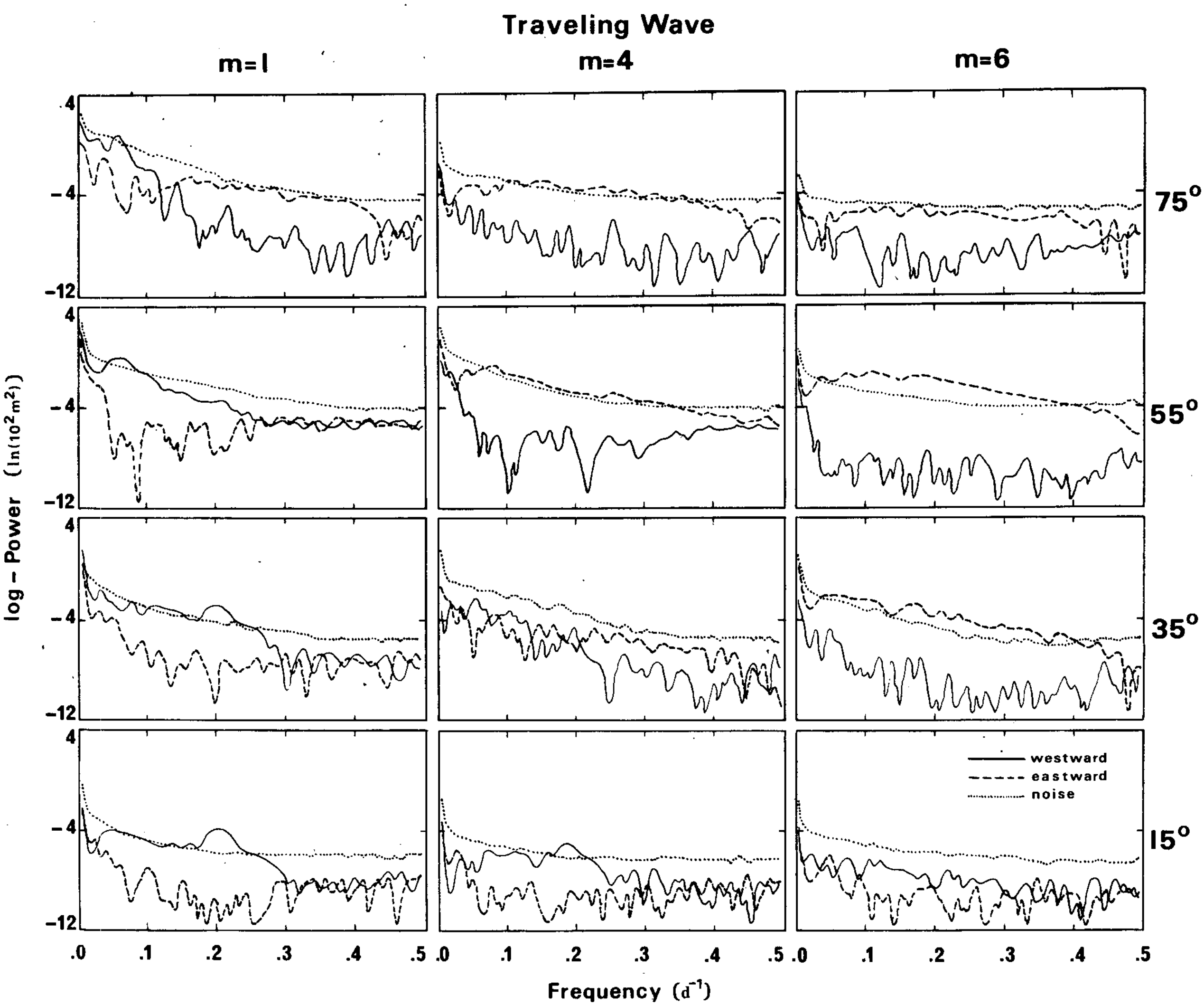

FIG. 2. Some spectra of westward and eastward traveling waves and of the "noise" for zonal modes $m=1$, 4, 6 at latitudes 15, 35, 55 and 75°N.

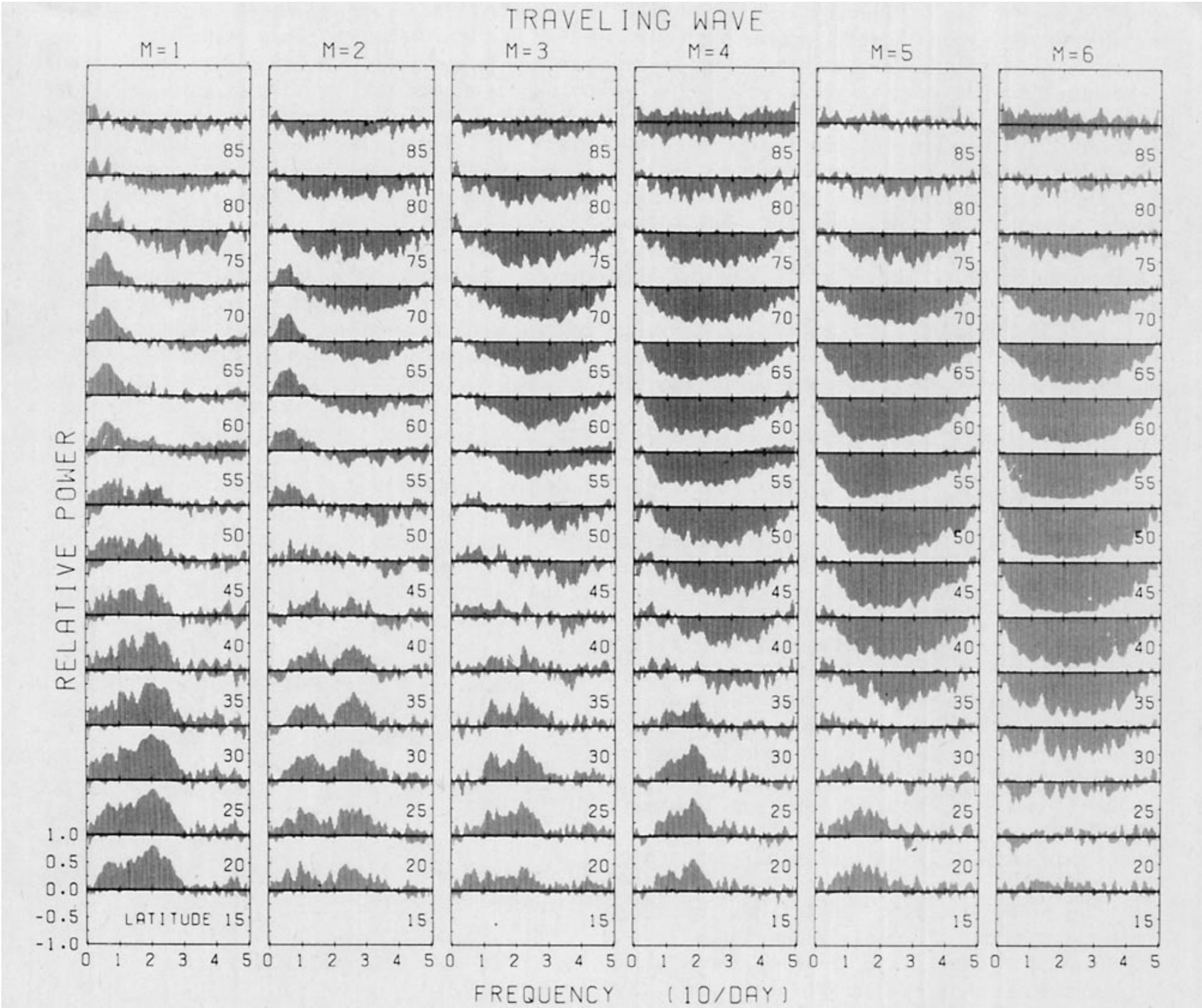

FIG. 3. Relative power of traveling waves up to zonal mode number $m=6$ at latitudes 15 to 85°N. Westward traveling waves are defined positive, eastward traveling waves negative.

for the estimate (2.1) cannot be smaller than a constant fraction of $P_\omega(C_m)P_\omega(S_m)$ and the wave variances fluctuate due to the random nature of the estimates. This also suggests that the very large increase of the wave variances at the lowest frequencies is partly a statistical effect. To show this, one can calculate the ratio of both wave variances to the total space-time variance, obtaining the relative powers of the waves. Whereas the sum of both relative powers is always 1.0 in case a, this sum lies between 0.0 and 1.0 for case b depending on what fraction of the total power belongs to wave disturbances. The wave variances in case a can be obtained by summing the corresponding wave variance and half of the noise in case b. The expression (2.13) requires that the difference of both wave variances be the same in both cases.

In Fig. 2 some more examples of wave and noise spectra are given. One notices that the predominance of either type of waves depends on latitude as well as on wavenumber. In all cases, there is a steep ascent of power at the lowest frequencies. In order to avoid the effect of large amplitudes due to the red spectrum and to show the strength of wave motions compared to the noise, relative powers of the first six modes of westward and eastward traveling waves for latitudes between 15 and 85°N are plotted in Fig. 3. Westward (eastward) traveling waves are represented here by a positive (negative) value. A striking feature in this plot are the large eastward traveling waves at higher modes and the large westward traveling waves at lower modes which can also be observed in Fig. 2. These regions are separated from each other by an approximate diagonal line reaching from 65°N, $m=1$

to 15°N, $m=6$. The latitudinal range of eastward traveling waves becomes larger at higher modes and the maximum amplitudes grow and shift from higher to midlatitudes. This behavior reflects synoptic-scale Rossby-Haurwitz waves (e.g., Holton, 1975). Westward traveling waves are prominent on the lower left side of this diagonal. The largest amplitudes for the $m=1$ mode occur at lower latitudes with a maximum around a period of 5 days, and at midlatitudes (55–70°N) with periods around 18 days. These features are also valid for $m=2$, the difference being that the arrangement at lower latitudes has split up into two parts near 10 days and 4 days. Westward traveling waves at lowest latitudes and periods around 5 days and somewhat larger can be observed all the way up to mode $m=5$. This result agrees well with that of Madden (1978) for the $m=1$ wave, who calculated coherencies and phases between the $m=1$ sine and cosine series [see Eqs. (2.1) and (2.2)]. The 5-day wave is not only present at the 500 mb level, but it prevails (at least for $m=1$) at all heights between sea level and 100 mb (and above). One notices that the power values become very small at some latitudes near the diagonal line and near the pole. This indicates that the noise is the prevailing fraction of the total variance in these regions. It sometimes happens for these spectra that the relative powers of the westward and eastward traveling part become nearly equal. This indicates the presence of pure standing waves [see Eqs. (2.12) and (2.9); the small axis $b(\omega,m)$ vanishes]. However, the amplitudes are relatively small and fluctuating, so that these little disturbances cannot be regarded as belonging to one dominating sort of waves.

On the whole, it can be said that pure standing waves dominate only at some latitudes and for certain wavenumbers, whereas pure traveling waves with large amplitudes are present in broad latitudinal and zonal mode bands. The variance ellipses are therefore almost circular in these bands and generally unsettled in the regions with small amplitudes, so that the rotation angles $R(\omega,m)$ [the angular positions of the standing waves (2.8), the antinodes, are given by $[R(\omega,m)+2\pi n)/m$, where $n=0, \pm1, \pm2 \ldots$] are rather randomly distributed over the range $-90°$ to $+90°$ for both cases.

4. Conclusion

Using the concept of a rotated ellipse which is determined exclusively by wave disturbances, one defines a decomposition of the spectra of planetary waves into westward and eastward traveling parts for each zonal mode. In this way a partition of the total space-time variance into a noisy fraction and a fraction belonging to wave motions is made possible so that it is no longer necessary to add the noisy part to the traveling waves, or, in the case of a partition into traveling and standing waves, to add the noise either to the traveling or to the standing part. The partition into standing and traveling waves is based on the assumption that both are of single origin and coherent and that the phase difference between both waves is 0°. These assumptions differ from those of Hayashi (1977) who assumed incoherence of traveling and standing waves. The broad latitudinal range from 15 to 85°N and the long time series of 9861 days used for this analysis lend confidence to the significance of the results. It is shown that pure westward or eastward traveling waves dominate in broad latitudinal and zonal mode bands. Standing waves, defined by equal amplitudes of westward and eastward traveling waves, dominate only in a few cases. While westward traveling waves prevail at low latitudes and smaller modes, eastward traveling waves are dominant at higher latitudes and higher modes. The boundary between the two domains can be easily established. Near this boundary and also near the pole, the amplitudes of wave motions become very small compared to the noise, so that the separation of the noise is necessary for a realistic impression of the wave power.

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