Excitation of Spiral Bands in Hurricanes by Interaction Between the Symmetric Mean Vortex and a Shearing Environmental Steering Current

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ABSTRACT

In hurricanes, linear, stationary, asymmetric, inertia-buoyancy oscillations can be resonantly forced at a radius where their tangential wavenumber times the orbital frequency of the mean swirling flow is equal to the local inertia frequency. If the hurricane is advected by an environmental geostrophic steering current without shear, the Coriolis force arising from the motion balances the environmental pressure gradient. However, if the motion of the storm differs from the geostrophic wind or if that wind has a horizontal shear, this balance is disrupted. For a uniform shear, resolution of the imbalance into radial and tangential components leads to a forcing that has a symmetric component and asymmetric components with tangential wavenumbers ±1 and ±2. The symmetric and wavenumber ±1 components have exponential horizontal structure, but the ±2 components have sinusoidal structure and are amplified by the wave action conservation mechanism of Willoughby (1978a,b). These waves resemble the spiral bands observed by research aircraft in Hurricane Anita of 1977.

As is generally the case for resonant forcing, the amplitude of the oscillations is sensitive to the dissipation rate, but for values of this quantity appropriate to cumulus friction and an environmental shear equal to 10% of the Coriolis parameter, the maximum horizontal velocity amplitude in the eye wall is several meters per second.

1. Introduction

Willoughby (1978a,b; here abbreviated I and II) has proposed that spiral bands in hurricanes arise from amplification of inertia-buoyancy waves that conserve wave action as they propagate inward from the storm's periphery and their intrinsic frequency is Doppler-shifted to higher values. In the first paper, waves on a barotropic vortex were assumed to be sinusoidal in time, height and azimuth. They were confined between reflecting planes at the surface and the tropopause, and their radial structure was obtained numerically. The model in II was similar, but the vortex was baroclinic; the waves were sinusoidal in time and height only, and a radiation boundary condition was imposed at the top of the domain. The numerical solutions described the band's structure as functions of both radius and height. In both models the amplitude of the wave was specified at the outer boundary and the model predicted amplification as the waves propagated into the center. Here we use a variant of the second model to develop one possible means for the band's excitation and to show that tangential wavenumber 2 and zero frequency with respect to the ground should define an important component of the forced wave field.

2. Model

The model of II has been recast in a system of translating, cylindrical height coordinates on a beta plane. As before, we consider small-amplitude, linear motions on an axisymmetric, baroclinic vortex in gradient balance. We further require that the variations of the Coriolis parameter across the domain be of the same order in comparison to the perturbation motions are in comparison to the mean flow. If there is a meridional pressure gradient in the storm's environment, it may be added to the symmetric pressure field of the mean vortex. The geostrophic wind associated with this gradient is chosen to be in the zonal direction and is not necessarily the same as the coordinate system's motion. Both the vortex motion and the geostrophic wind are of the same order as the perturbations. The nondimensional governing equations for linear motions with sinusoidal structure in azimuth and time are (see Appendix for list of symbols)

\[
\Omega u' - \xi v' + \frac{\partial p'}{\partial r} = F^*
\]

\[
= \rho^* \{ \beta r^* \cos \lambda + [C_2^* - f^* C_2^*] \sin \lambda - [C_2^* + f(C_2^* - U^*)] \cos \lambda \},
\]
\[ \Omega^* v' + \xi^* u' + S^* w' + \frac{1}{r^*} \frac{\partial p'}{\partial \lambda} = G^* \]
\[ = \rho^* \{ \dot{C}_x^* - f^* C_y^* \} \cos \lambda \]
\[ + \{ \dot{C}_u^* + f(C_x^* - U_y^*) \} \sin \lambda \}, \]
\[ \Omega^* w' - b' + \frac{\partial p'}{\partial z^*} = 0, \]
\[ \Omega^* b' + \frac{\partial b^*}{\partial r^*} - \frac{1}{r^*} u' + N^* z^* w' = 0, \]
\[ \frac{\partial u'}{\partial r^*} + u' + \frac{1}{r^*} \frac{\partial v'}{\partial \lambda} + \frac{\partial w'}{\partial z^*} = 0. \]

All lengths are nondimensionalized with 15 km, equal to both the radius of maximum wind and the depth of the troposphere; velocities with the maximum wind speed, chosen to be 84 m s\(^{-1}\); and frequencies with the ratio of the velocity to length scales. The maximum mean tangential wind thus occurs at \( r^* = 1 \) near the surface. Afloat the wind shear to zero in the lower stratosphere and it also decreases to zero between \( r^* = 1 \) and the center. The low mean pressure required to sustain this wind in gradient balance arises from a mean buoyancy anomaly in the vortex center.

The computational domain is \( (0 \leq z^* \leq 2) \) and \( (0 \leq r^* \leq 40) \). The horizontal and vertical resolution is 0.05 nondimensional unit or 0.75 km. In the parts of the domain where \( z^* > 1.5 \) or \( r^* > 30 \) the dissipation rate is increased to a large value using a hyperbolic tangent profile so that outward propagating waves incident on the outer edge or top of the domain are absorbed and not reflected back into the interior. The perturbation vertical velocity vanishes at the surface and \( p'(-\delta r^*, z^*, \lambda) = p'(\delta r^*, z^*, \lambda + \pi) \) at the center.

Eqs. (1) may be manipulated to yield a profusion of interactions among the vortex motions and the asymmetries. We disregard at the outset those components of the asymmetric motion with tangential wavenumbers 0 and \( \pm 1 \) which arise from \( \beta_0^* \) and the motion of the vortex center across the isobars. These waves have low apparent frequencies, and their Doppler-shifted frequencies are everywhere below the local inertia frequency \( (\Omega^* \xi^*)^{1/2} \), when \( v^* \) decreases with radius at a rate slower than \((r^*)^{-1/2}\). Willoughby (1977) showed that in hurricanes inertia-buoyancy waves in this frequency band have exponential rather than sinusoidal horizontal structure. In nature, tangential wavenumber \( \pm 1 \) will be strongly forced by the asymmetric frictional forces due to the storm's motion as well as by the inertial forces examined here. These perturbations constitute the principal asymmetry of the hurricane vortex. Rossby (1948) and Kuo (1950) have discussed their significance in relation to the vortex motion, but these perturbations do not have spiral geometry.

The action-conservation wave amplification mechanism requires that the waves be sinusoidal rather than evanescent and that their Doppler-shifted frequency increase in the direction of energy propagation. This happens only for \( |n| \geq 2 \). One possible mechanism for forcing the \( n = \pm 2 \) component involves horizontal shear of the environmental wind. If we let the environmental pressure gradient be constant with height and have its horizontal variation represented as a Taylor series in \( r^* \cos \lambda \),
\[ \rho^* f^* U_0^* = \rho^* \{ f_0^* U_0^*(0) \} r^* \cos \lambda + \ldots \}, \]
where \( U_0^* = \partial U_0^*/\partial y \). When (2) is substituted into (1) and \( C_x^* \) and \( C_y^* \) are assumed to be steady, the forcing becomes
\[ F^* = \rho^* \{ [\beta_0^* r^* v^* - f_0^* (C_x^* - U_y^*)] \} \cos \lambda \]
\[ - f_0^* C_y^* \sin \lambda - \frac{1}{2} \beta_0^* (C_x^* - U_y^*) \}
\[ - f_0^* U_x^* (0) r^* (1 + \cos 2\lambda) \]
\[ - \frac{1}{2} \beta_0^* r^* C_y^* \sin 2\lambda \}, \]
\[ G^* = \rho^* \{ f_0^* (C_x^* - U_y^*) \} \sin \lambda - f_0^* C_y^* \cos \lambda \]
\[ - \frac{1}{2} \beta_0^* C_y^* \sin 2\lambda \}
\[ - \frac{1}{2} \beta_0^* r^* C_y^* (1 + \cos 2\lambda) \}. \]

Both \( n = +2 \) and \( n = -2 \) components are thus forced. These solutions must be obtained independently and combined to generate the perturbation's structure.

If we eliminate \( w' \) and either \( u' \) or \( v' \) among the momentum equations, the expressions for the horizontal velocities in terms of the pressure and \( F_n^* \) and \( G_n^* \), the forcing of wavenumber \( n \), are
\[ M^* u' = -\Omega^* (\Omega^* + N^* z^*) \frac{\partial p'}{\partial r^*} - F_n^* \]
\[ + \xi^* (\Omega^* + N^* z^*) \frac{\partial p'}{r^*} + G_n^* \]
\[ + \Omega^* S^* \xi^* \frac{\partial p'}{\partial z^*}, \]
\[ M^* v' = \left[ (\Omega^* + N^* z^*) \xi^* - S^* \frac{\partial b^*}{\partial r^*} \right] \]
\[ \times \left( \frac{\partial p'}{\partial r^*} - F_n^* \right) + \Omega^* (\Omega^* + N^* z^*) \]
\[ \times \left( \frac{\partial p'}{r^*} + G_n^* \right) + S^* \Omega^* \xi^* \frac{\partial p'}{\partial z^*}, \]
where
\[ M^* = (\Omega^* + N^* z^*) (\Omega^* + \xi^* \xi^*) - S^* \xi^* \frac{\partial b^*}{\partial r^*}. \]
These equations, the corresponding relation for \( w' \) and mass continuity are then combined to obtain a single partial differential equation, analogous to Eq. (3) of II, which is integrated to obtain \( p'(r^*, z^*) \).

The perturbations of interest are forced in the following fashion: The mean vertical shear and horizontal buoyancy gradient in the lower troposphere are small, and at some radius, here termed the inertial resonance radius (IRR), \( \Omega^* \xi^* = \xi^* \xi^* \). Near the IRR, \( M^* \) will be nearly zero and \( F^* \) and \( G^* \) will act primarily to redistribute the mass field. However, any imbalance among the terms on the right in (2) will lead to large horizontal velocities and to resonant excitation of inertia-buoyancy waves.

Such perturbations have evanescent structure outside the IRR where \( \Omega^* \xi^* \) is much smaller than \( \xi^* \xi^* \) and propagating wavelike structure inside it. As they propagate inward, the wave's Doppler-shifted frequency increases, and in the lower troposphere near the eye wall reaches \( N^* \), so the waves are absorbed. In the upper troposphere, as we shall see, the situation is more complicated and the waves are reflected at the eye wall. It is the inward increase in frequency that leads to growth of the waves by conservation of wave action.

In the previous work, solutions were obtained to a homogeneous system by excitation at one boundary with a specified amplitude. Here, the boundary conditions are such that the solution would be everywhere zero except for the forcing in the interior. Thus, the amplitude is determined in terms of the steering current's shear and the storm's intensity and is no longer arbitrary. The model is tested by inserting reasonable values of these parameters and comparing the solutions with observed amplitudes and wave structures.

3. Computational results

The model described above is used to solve for \( p'(r^*, z^*) \) with the environmental shear fixed at 10% of the Coriolis parameter. For shears of this magnitude the terms containing \( \beta^* \) in the forcing of wavenumbers \( \pm 2 \) are small compared to the shear terms and are neglected. Figs. 1 and 2 show the radial and tangential perturbation winds computed from solutions for \( p' \) obtained with a dissipation rate of \( 3 \times 10^{-3} \), which is equivalent to \( 16.4 \text{ h}^{-1} \) in dimensional terms. The IRR is 22.5 times the radius of maximum winds, or 337 km, from the center. Near this radius the pressure field is nearly flat but horizontal motions of 1 or 2 m s\(^{-1} \) are excited. As the waves move inward they do not amplify appreciably until they reach \( r^* = 5 \). In the lower troposphere the waves are absorbed at the eye wall as previously described. However, in the upper troposphere the strongest wind at a given level is weaker, and \( \Omega^* \xi^* \) never reaches \( N^* \). Instead the waves encounter a turning point inside the eye where \( \xi^* \xi^* \) increases and becomes again equal to \( \Omega^* \). At the turning point, incident waves are reflected and the IRR excitation also operates. The excitation is much weaker at the inner IRR than at \( r^* = 22.5 \) because the forcing term is proportional to \( r^* \).

In the outer circulation the waves slope upward toward the center and phase propagates inward and down, but the reflected waves around the center slope and propagate outward. In general, the vertical wavelengths seem to be short. In dimensional terms, the vertical wavelength in the triangular region of short waves between \( r^* = 1 \) and \( r^* = 5 \) is about 1.5 km. This corresponds to the shortest wave the model can resolve. In the sloping wavefront which extends from \( r^* = 1.0 \) and \( z^* = 1.0 \) to the surface at \( r^* = 10.0 \) the vertical wavelength is longer, about 3 km.

Fig. 3 shows the horizontal and vertical fluxes of wave momentum, energy and buoyancy for this case. Well inside the IRR, the horizontal, vertically integrated wave energy and momentum fluxes are much the same as in II. The product of the wave momentum flux with the radius tends to remain constant and the wave energy flux increases dramatically toward the center. Both fluxes drop abruptly to zero in the eye wall. Near the IRR the momentum flux attains large positive values. It then decreases to zero and changes to a small inward flux somewhat beyond the IRR. The wave energy flux changes from inward within the IRR to outward beyond it and tails off toward zero at large radius. The relationship between the wave energy and momentum fluxes is wavelike outside the IRR. Since inertia-buoyancy waves do not propagate there, this may be a manifestation of the vortex-Rossby waves of McDonald (1968), which we were unable to simulate in Willoughby (1977).

The vertical fluxes are calculated at the tropopause (\( z^* = 1 \)). Outside \( r^* = 4 \) the waves are confined by a quasi-horizontal critical layer below the tropopause, but the critical layer slopes upward into the lower stratosphere around the vortex center and wave energy can pass through the tropopause there. Some downward flux of wave energy occurs near \( r^* = 1 \). This arises both from partial downward reflection of wave energy that has escaped from the troposphere and from IRR excitation at the vertical locus at \( \Omega^* \xi^* = \xi^* \xi^* \) surrounding the eye.

The amplitude of the solutions is sensitive to the dissipation rate. Since good empirical data are available at 700 mb, the maximum horizontal velocity at that level is adopted as a standard for comparison. The model was run using dissipation rates of \( 1 \times 10^{-2}, 3 \times 10^{-3}, 1 \times 10^{-3} \) and \( 3 \times 10^{-4} \). Fig. 4 illustrates the variation of maximum horizontal
velocity amplitude as a function of dissipation rate. Of the values considered, the amplitudes seem most reasonable—although perhaps a little large—for $\delta = 3 \times 10^{-3}$, equivalent to an $e^{-1}$ time of 16.4 h.

Merceret (1976) observed turbulent dissipation rates in Hurricane Caroline of 1975. If one assumes that the sum of the kinetic and available potential energies on the band scale is $10^{2} \text{ m}^{2} \text{ s}^{-2}$, the corresponding dissipation times range between 1 and 10 h. Stevens et al. (1977) cite times in this range as being appropriate to cumulus friction, in contrast to the several days required for radiative dissipation to act.

Figs. 5 and 6 compare synthetic radial profiles of the perturbation quantities computed from this model with those observed by research aircraft in Hurricane Anita of 1977. Both figures represent data at 3 km dimensional altitude and the axes are scaled so that comparable intervals on the axes represent approximately the same physical variations.

The synthetic profiles were created by adding
the linear solutions to the mean vortex. In Fig. 5 the solid lines are the sum of the mean with the coefficient of the cosine in the solution and the dashed lines with the coefficient of the sine. The empirical data depict four penetrations or exits along the same azimuth of the eye of Hurricane Anita by a research aircraft during the night of 2 September 1977. At that time the storm was weaker than the mean vortex in the model (65 compared to 90 m s⁻¹) and the radius of maximum wind was larger (20 compared to 15 km). Also, the mean vortex is less baroclinic at this level than Anita was.

In the synthetic profiles the perturbation pressure is so small that the solid line obscures the dashed, but for the other variables the two components are readily distinguishable. The relative amplitudes of the perturbations are much the same in both the model and the observations. The radial perturbation velocity is largest followed by the tangential and then the vertical, and the ratios among these quantities seem similar in both figures. Although
the absolute magnitudes in the synthetic profiles were adjusted by varying the dissipation, the ratios among the variables arose naturally from the model.

In Hurricane Anita substantial perturbation amplitude extended outward from the eye wall to beyond 100 km radius. But the model predicts realistic amplitudes only inside $r^* = 3$ (45 km). This distribution is inherent in the action-conservation mechanism and represents a serious defect in the theory. One possible explanation borrows heavily from the ideas of Stevens et al. (1977) and involves the nature of the dissipative process. It may be that far from the center the dissipation is primarily radiative and has a characteristic time of
several days. Near the center, cumulus friction or wave breaking may become more important and the dissipation time may decrease to a day or less. Thus, the waves could grow to reasonable amplitude at $r^* \geq 10$ and also be constrained from becoming unreasonably large near the center. This explanation may seem contrived, but the large dissipation rate necessary to limit the amplitude to reasonable values can arise only from these nonlinear mechanisms. Such processes manifestly increase toward the storm center as required in the foregoing argument.

A second, less serious difficulty involves the tangential propagation of the waves. The radar bands in Anita propagated slowly downwind, as did those in Hurricane Caroline described in I. The shear-interaction mode of excitation drives stationary waves, but the observed band's periods range from 6 to 12 h. It might be argued that time variations of the environmental shear in the moving, storm-centered coordinates could lead to the observed propagation. It is also possible that other modes of excitation, perhaps related to the trochoidal oscillation of the vortex center (observed period 5–6 h for Anita), may be present.

As a final comparison between the model and the Anita data, the normalized correlation coefficients corresponding to the fluxes in Fig. 3 were computed from the data. In Table 1, these are compared with similar quantities calculated from the solution both over the domain $0 \leq r^* \leq 7$ and $0 \leq z^* \leq 1$ and over $0 \leq r^* \leq 7$ at 3 km altitude. Since the wave-action mechanism and the Eliassen-Palm relation make specific predictions concerning the signs of the correlations, the theory may be tested by comparison of the signs of the correlations.

The magnitudes of the correlation coefficients and the number of effective degrees of freedom (taking into account the serial correlation of the variables) are such that the individual coefficients are not especially significant.

However, the agreement in sign between the empirical and theoretical correlations does support the model. If we compare the observed and model correlations at either 700 mb or averaged over the whole troposphere, agreement occurs in four out of five cases. Under the null hypothesis that either sign is equally probable in the empirical data, the odds of this happening by chance are $\sim 15\%$. Although the model is supported by these results, the reader should keep in mind that the empirical data contain a variety of phenomena in addition to the spiral-mode waves and that these data represent only a single case.

4. Discussion

Before summarizing the foregoing results we would like to speculate on the relationship between the mean tangential wind and radius. Riehl (1963) first suggested that the tangential wind outside the eye was inversely proportional to the square root of radius. The experience at the National Hurricane and Experimental Meteorology Laboratory supports this relation empirically for intense storms. As we showed earlier, for such a profile the inertia frequency is very nearly $v^*/r^*$ when the Rossby number is large. Thus, an azimuthally stationary $n = 1$ wave is close to resonance between the Doppler-shifted frequency and the inertia frequency at all radii. Wavenumber 1 is strongly forced by both inertial forces and asymmetric friction due to the storm's motion.

We now consider a situation in which the central pressure is falling in response to convective heating in the eye wall. This increases the maximum wind near the center, increases the magnitude of the (negative) horizontal shear, and reduces the inertia frequency. Eventually a region develops in which the inertia frequency is less than $v^*/r^*$ so the strongly forced wavenumber 1 waves can propagate and carry away the excess momentum. Such waves would be trapped between two IIR's and would be absorbed at a critical layer near the tropopause. The waves would reduce the vertical shear rather than act to radiate excess momentum away horizontally. This should eventually destroy the resonance by reducing the magnitude of the mean anticyclonic horizontal shear below the limiting magnitude of $v/2r$. Thus the circulation in the outer vortex is proportional to $(r^*)^{-m}$ where $m$ is limited to values $\leq 1/2$ by wave momentum transports.
Fig. 5. Synthetic profiles of the dynamic variables computed at 3 km altitude from the solutions in Figs. 1 and 2.
Fig. 6. Four radial profiles of the dynamic variables observed in Hurricane Anita of 1977 by research aircraft.
Table 1. Comparison of the correlations between dynamic variables as computed from observations in Hurricane Anita and from model results.

<table>
<thead>
<tr>
<th></th>
<th>$u'u'$</th>
<th>$u'p'$</th>
<th>$u'T'$</th>
<th>$w'p'$</th>
<th>$w'w'$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Anita ($z = 3$ km, $0 \leq r \leq 100$ km)</td>
<td>0.139</td>
<td>-0.059</td>
<td>0.205</td>
<td>-0.088</td>
<td>0.024</td>
</tr>
<tr>
<td>Model ($z^* = 0.2$, $0 \leq r^* \leq 7.0$)</td>
<td>0.182</td>
<td>-0.518</td>
<td>0.194</td>
<td>-0.124</td>
<td>-0.123</td>
</tr>
<tr>
<td>Model ($0 \leq z^* \leq 7.0$)</td>
<td>0.304</td>
<td>-0.536</td>
<td>0.232</td>
<td>-0.151</td>
<td>-0.103</td>
</tr>
</tbody>
</table>

By way of summarizing the computational results in the previous section, inward propagating inertia-buoyancy waves are excited by imbalances between the Coriolis force due to the storm’s motion and the pressure gradient force associated with a shearing environmental geostrophic wind, propagate into the storm center, and can grow through conservation of wave action to empirically reasonable amplitudes. However, the fast dissipation rates required to keep the amplitudes from becoming excessively large can arise only from nonlinear effects.

Both the signs of the wave fluxes and the relative magnitudes of the dependent variables agree with observations. However, large-amplitude waves are found only near the center and the phase lines are stationary rather than slowly propagating as in nature. Both of these discrepancies have plausible explanations in the context of the model, so the present mechanism may well be the cause of spiral bands in hurricanes.

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Appendix

List of Symbols

- $r^*$ radius
- $\lambda$ azimuth
- $z^*$ height
- $t^*$ time
- $u', v', w'$ density-weighted perturbation winds in the radial, tangential, and vertical directions
- $b'$ density-weighted perturbation buoyancy $[= \text{gravity} \times \text{density perturbation}]$
- $p'$ perturbation pressure
- $v^*$ mean-flow tangential wind
- $b^*$ mean-flow buoyancy
- $f^*$ Coriolis parameter $[= f_0^* + \beta^* r^* \cos \lambda]$
- $\zeta^*$ mean-flow vorticity $[= (\partial v^*/\partial r^* + v^*/r^* + f_0^*)]
- $\xi^*$ vertical shear of $v^*$
- $\Omega^*$ apparent frequency with respect to the ground
- $n^*$ tangential wavenumber
- $\sigma^*$ dissipation rate for Newtonian cooling and Rayleigh friction
- $N^*$ buoyancy frequency
- $\rho^*$ background density
- $C_x$ coordinate system’s motion in the $x^*$ direction $=-r^* \sin \lambda$
- $C_y$ coordinate system’s motion in the $y^*$ direction $=r^* \cos \lambda$
- $\dot{C}_x, \dot{C}_y$ time derivatives of the coordinate’s motion
- $-f^* \rho^* U_0^*$ environmental meridional pressure gradient
- $F^*$ forcing in the radial direction
- $G^*$ forcing in the tangential direction

References


