The Evolution of Droplet Spectra and the Rate of Production of Embryonic Raindrops in Small Cumulus Clouds

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ABSTRACT

Calculations have been made of the evolution of droplet spectra within small cumulus clouds which are entraining undersaturated environmental air. The mixing process is assumed to be highly inhomogeneous. In the extreme situation considered, environmental air is entrained in discrete blobs or parcels, causing some droplets of all sizes to be completely removed from the condensate spectrum, while others do not change in size. This model, which is based on laboratory experiments, corresponds to a situation in which the time constant for droplet evaporation is small relative to that for turbulent mixing; in the classical (homogeneous) model, which has been used by other workers, the reverse applies. The calculations produce spectral shapes which agree well with those observed in cumulus by Warner (1969), and they indicate that favored droplets may grow very much faster through the condensate spectrum than is predicted classically.

1. Introduction

Despite considerable effort (e.g., Warner 1969, 1973; Mason 1971; Mason and Jonas 1974; Lee and Pruppacher 1977) and some progress, no generally accepted solution has been provided to two important problems in cloud physics involving the population of cloud droplets produced by condensation. The first is that the measured times required to produce raindrops in water clouds may be appreciably shorter than values calculated on the basis of classical theory for growth by condensation followed by stochastic coalescence. The second is that the predicted size distributions of cloud droplets within the condensational stage of growth are inconsistent with those observed in cumulus clouds. In this note we outline the results of some recent calculations, based on a new model of inhomogeneous mixing, which appear to offer a solution to both of these problems.

The basic idea, based on laboratory experiments, has already been reported by Latham and Reed (1977). It is that when undersaturated environmental air is entrained into a growing cumulus some cloud droplets are greatly reduced in size, while others—at the same level, but more remote from the blobs or filaments of entrained air—are not directly influenced. This is clearly distinguished from the homogeneous description of entrainment employed by other workers, where it is assumed that the reduction in supersaturation produced by entrainment is, at any level, the same at all points.

A prediction of our inhomogeneous model is that natural clouds should contain adjacent regions of strongly different water content but similar mean droplet size and dispersion. Confirmatory evidence for this prediction has been obtained in several field studies (Knollenberg 1976; Corbin *et al.* 1977; Rodi 1978), and more have recently been obtained in experiments conducted at the UMIST station on Great Dun Fell, in Cumbria. The calculations described below were based on a very simple two-stage picture of inhomogeneous mixing. Undersaturated air is drawn into a growing cloud in "blobs" or filaments, of arbitrarily chosen but fixed macroscopic volume V_0. Entrainment is accompanied by instantaneous evaporation of all the droplets in a volume V_1 of cloudy air surrounding the blob, where V_1 is that volume which can supply sufficient vapor to cause the relative humidity in V_0 to rise to 100%. The entrainment and evaporation processes are immediately followed by mixing of the volumes V_0 and V_1 with the rest of the cloud at that level.

2. Time constants

Despite the support provided by the laboratory experiments of Latham and Reed (1977), the above description of the mixing process is clearly simplistic. The extent to which it may be acceptable will depend on certain rate processes associated with the mixing process within cumulus clouds. These are turbulent diffusion of the entrained air into the cloud, molecular diffusion at the interface between a blob and the surrounding cloudy air, and the evaporation of a droplet of radius r in an undersaturated environment. The characteristic times governing these three processes are defined as τ_T, τ_D and τ_r, respectively.

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If τ_T or τ_D is much less than τ_r, any inhomogeneities created by the mixing process will be substantially smoothed out before significant droplet evaporation can occur, and the mixing will approach the classical description employed by other workers. On the other hand, if $\tau_r \ll \tau_T$, τ_D, the mixing process will approximate to the inhomogeneous description employed in our calculations.

We let X(cm) be a linear scale characteristic of the entrained blobs (corresponding, for example, to the cube root of V_0 in our calculation), ϵ (cm^2 s^{-3}) the rate of kinetic energy dissipation via turbulent mixing, D (cm^2 s^{-1}) the molecular diffusion coefficient, S (%) the supersaturation, α ($\sim$5 μm) the length associated with the condensation coefficient (Fukuta and Walter, 1970), and write

$$\tau_T \sim (X^2/\epsilon)^{1/3},$$

$$\tau_D \sim X^2/D,$$

$$\tau_r \sim 10^{-6}(\rho_w/\rho_\infty)((r+\alpha)^2 - \alpha^2)/DS,$$

where r is measured in microns and ρ_w/ρ_∞ ($\sim 10^5$) is the ratio of the densities of liquid water and saturated vapor. Taking $\epsilon \sim 100$ cm^2 s^{-3} and $D \sim 0.25$ cm^2 s^{-1} we find that $\tau_T < \tau_D$ for $X > 0.11$ cm. Thus, the nature of the mixing will depend in general on the relative values of τ_T and τ_r.

For the conditions quoted τ_T possesses values of 0.22, 1 and 20 s for $X = 1$, 10 and 1000 cm, respectively. The values of τ_r corresponding to $r = 1$, 3, 10 and 20 μm are 0.2, 0.8, 4 and 12 s for $S = -20\%$. Because values of τ_r are sensitive to r, conditions for inhomogeneous mixing will occur on somewhat smaller scales for smaller droplets than for larger ones. But generally speaking, these calculations suggest that this new model may have reasonable validity for scales in excess of $\sim$1 m. It may be relevant to note that Warner (1979) has reported that inhomogeneities in droplet concentrations occur on a scale of $\sim$1 m in cumulus, but not on lower scales; however, we cannot explain his finding of no inhomogeneities on scales of $\sim$10 m. More detailed examination of these rate processes is being performed.

3. Model calculations

The calculations were based on those of Warner (1973). An updraft of constant speed U produces a cloud of base temperature T_B in an environment of constant relative humidity 80% and constant lapse rate $\Gamma = -7.5$°C km^{-1}. Condensation occurs on a distribution of NaCl nuclei consisting of N particles per cubic centimeter in n mass classes [based on Warner (1973) and Lee and Pruppacher (1977)]. The subsequent evolution of the cloud as it moves upward at speed U (m s^{-1}) and cools was calculated from the standard equations (Warner, 1973) for three different models which we label H, I and A. H is an homogeneous case, in which entrainment of outside air occurs steadily and uniformly in the manner assumed by other workers (Warner, 1973; Lee and Pruppacher, 1977). This is the classical picture described earlier. I is an inhomogeneous case, already outlined, in which undersaturated blobs of constant size V_0 are drawn into the cloud, either at random intervals or regularly [at a rate λ (s^{-1})] and completely evaporate droplets (of all size classes) until the humidity rises to 100%. A is an adiabatic case in which the cloud does not interact with its environment. In models H and I the entrained air contains nuclei of the same activity spectrum as those at cloud base. Two models were run for the mean infiltration frequency λ in I. In the first $\lambda(t)$ is set equal to $\mu U(v(t)/V_0)$, where $v(t)$ is the cloud volume at time t, and in the second $\lambda(t) = \lambda_0 = \mu v_0 U/V_0$, where v_0 is the original cloud volume. In both, the effective entrainment parameter μ is always equal to that assumed in the homogeneous model. The results from the two were very similar. To date, calculations have been made for $U = 1$, 2 m s^{-1}; $T_B = 5$, 10, 15°C; $N = 100$, 200, 300, 600 cm^{-3}; $n = 5$, 38; and $\mu = 5 \times 10^{-4}$, 10^{-3} m^{-1}.

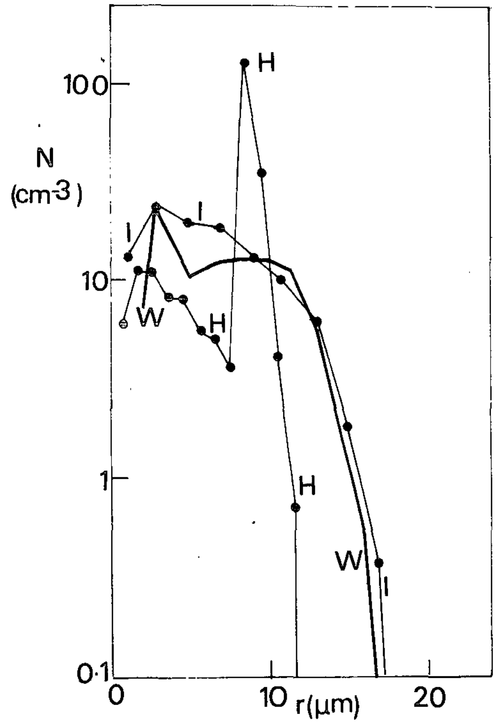

FIG. 1. Size distribution in cumulus. W, measured by Warner (1969) (note that he could not detect droplets of radius $r < 2$ μm); H, calculated on the homogeneous model; I, calculated on the inhomogeneous model. $N = 200$ cm^{-3}, $n = 6$, $\lambda(0)^{-1} = 10$ s, $L_H = 0.45$ g m^{-3}, $L_I = 0.42$ g m^{-3}, $U = 1$ m s^{-1}, $\mu = 10^{-3}$ m^{-1}.

4. Results

Fig. 1 shows a typical size distribution observed in cumulus by Warner (1969) at a stage where the

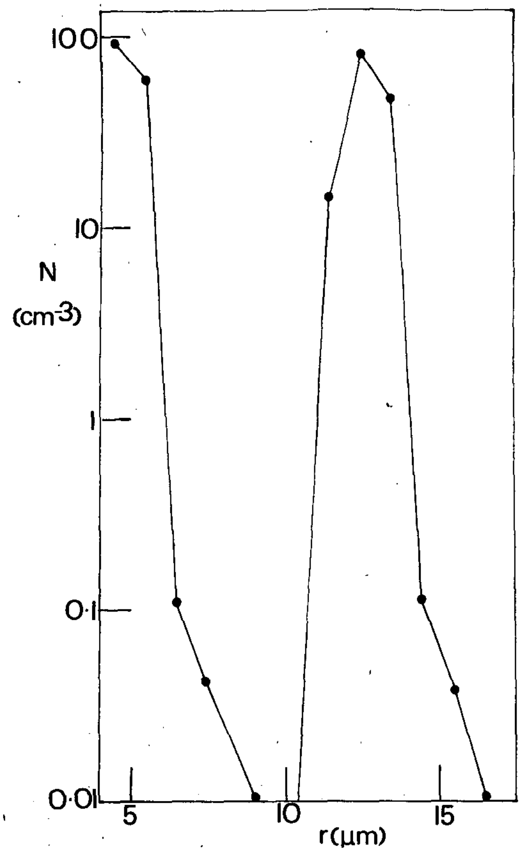

FIG. 2. Bimodal spectrum produced on the inhomogeneous model. $N = 300$ cm^{-3}, $n = 38$, $\lambda(0)^{-1} = 200$ s, $L_I = 1.87$ g m^{-3}, $U = 1$ m s^{-1}, $\mu = 5 \times 10^{-4}$ m^{-1}.

liquid water content was around 0.4 g m^{-3}. It also displays two calculated spectra. H is based on the homogeneous model, and is very similar in shape to those calculated by Warner (1973) on his classical model of mixing. It is seen to bear little resemblance to the observed spectrum (W). However, the size distribution I, based on our inhomogeneous model with $[\lambda(0)]^{-1} = 10$ s, is seen to agree closely with that observed.

Another problem on which Warner has focused attention is the frequent occurrence of bimodal spectra in cumulus clouds. Spectra containing more than one mode are produced by the inhomogeneous model when λ, the frequency of infiltration of the cloud by blobs, is low. An example of a bimodal spectrum produced by the model is presented in Fig. 2. It is seen to resemble ones measured by Warner (1969). Field research is required in order to establish ranges of values of λ applicable to natural clouds.

Table 1 presents values of liquid water content L_H, L_A, L_I, supersaturation S_H, S_A, S_I and maximum radius of droplets in the spectrum R_H, R_A, R_I, after various growth times t for the homogeneous, adiabatic and inhomogeneous models, respectively. Also presented are values of the concentration N_T of droplets of maximum radius R_T. The striking observation is that the largest droplets grow much faster in the inhomogeneous model than in either the homogeneous or adiabatic models—even though, in the latter case, the liquid water content is about twice as great. For example, we see that about 150 s are required in the inhomogeneous model for the largest drops to achieve a radius of 13 μm, while about 400 and 500 s are required on the adiabatic and homogeneous models, respectively. For $R_T = 15$ μm the figures are about 200 s for the inhomogeneous model, 700 s on the adiabatic model and 800 s on the homogeneous. We see that in our inhomogeneous description of the entrainment process the largest drops move through the condensational stage about three times as fast as is predicted classically. Therefore, it appears that this finding may resolve the longstanding question,

TABLE 1. Calculated values, at various times t, of liquid water content (L), supersaturation (S), maximum drop radius (R) and concentration (N_T) of droplets of radius R. The subscripts H, A and I refer respectively to the homogeneous, adiabatic and inhomogeneous models. $[\lambda(0)]^{-1} = 10$ s. Calculations on the homogeneous model were not made beyond 700 s.

t (s)	100	200	300	400	500	600	700	800	900	1000
T_c (°C)	14.4	13.7	13.0	12.3	11.5	10.8	10.0	9.3	8.5	7.7
T_e (°C)	14.2	13.5	12.7	12.0	11.2	10.5	9.7	9.0	8.2	7.5
R_H (μm)	8.5	10.2	11.3	12.2	13.0	13.8	14.5	—	—	—
R_A (μm)	9.3	11.0	12.1	12.9	13.6	14.2	14.8	15.3	15.8	16.2
R_I (μm)	10.5	15.2	17.6	19.4	20.5	22.2	23.3	24.3	25.2	26.1
N_T (ℓ^{-1})	58	6.3	2.6	1.4	0.57	0.53	0.36	0.26	0.19	0.15
L_H (g m^{-3})	0.10	0.26	0.39	0.51	0.62	0.72	0.80	—	—	—
L_A (g m^{-3})	0.20	0.46	0.71	0.94	1.18	1.41	1.64	1.86	2.09	2.31
L_I (g m^{-3})	0.06	0.27	0.41	0.52	0.62	0.71	0.79	0.88	0.93	1.00
S_H (%)	0.37	0.20	0.17	0.16	0.15	0.14	0.14	—	—	—
S_A (%)	0.37	0.21	0.17	0.15	0.13	0.12	0.11	0.10	0.10	0.09
S_I (%)	1.12	0.55	0.42	0.41	0.38	0.39	0.38	0.33	0.38	0.34

referred to earlier, of the rate at which raindrops can be produced in cumulus. In this connection, it is interesting to note that the values of N_T ($\sim 1\ \ell^{-1}$) are of the right order of magnitude for raindrop concentrations.

The reason for the greatly enhanced growth rates in the inhomogeneous model is apparent from the inspection of Table 1—the values of supersaturation are much greater. This is because in the inhomogeneous model more droplets are completely evaporated, and the newly activated droplets that replace them cannot compete so effectively for the available water vapor. Thus the supersaturation rises above that for the homogeneous case and those drops which do not encounter the infiltrating blob will grow faster.

The absolute growth rates of droplets will depend on factors which may vary greatly from cloud to cloud, but we believe that the difference in growth rates (between homogeneous and inhomogeneous mechanisms) will generally be close to those presented in Table 1. Our major conclusions were found to be insensitive to variations in N, U, T_B, λ and μ and also to the choice of CCN spectrum. It appears to us to be sensible, as the next stage in this study, to devote our principal effort, through field, laboratory and theoretical work, to establishing more precisely the processes involved in mixing. This will involve consideration of the various time constants discussed earlier.

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