Resonant Planetary Waves in a Spherical Atmosphere

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ABSTRACT

A global model of planetary wave propagation in a spherical atmosphere is used to examine the spectrum of free or resonant planetary waves of the solstitial stratosphere. These free modes are located by forcing the model with a weak periodic vertical velocity along the lower boundary and looking for a resonant response in wave amplitude. The modes correspond to the natural traveling oscillations in the earth's atmosphere, of which the 5-day wave is the best known example.

The 15-day wave observed by Madden (1978) and others is found to be such a resonant mode. We find that the strong stratospheric winds cause the 15-day wave to become baroclinic by trapping the wave between the earth's surface and the strong winds at the stratopause. The strong winds effectively reduce the atmospheric damping which greatly reduces the amplitude of barotropic waves with periods > 10 days. The computed meridional structure of the 15-day wave is in reasonable agreement with Madden's (1978) observations at extratropical latitudes. Our results indicate that a mode resembling the H_3^1 Hough function represents the principal resonant component.

Other resonances at periods longer than 15 days for zonal harmonics 1, 2 and 3 are shown, and these modes are also baroclinic. At very long periods (50–100 days) broad resonant peaks are observed for all three zonal harmonics. These peaks indicate that the structure of stationary planetary waves is very sensitive to changes in the mean zonal wind (frequency changes in this model) as has been noted by other authors.

1. Introduction

During the last decade or so, a variety of interesting observational studies of traveling planetary-scale waves have been made. Their methodology was devised by Deland (1964) who suggested that the amplitude and direction of propagation of the transient waves could be found from the quadrature spectra of the Fourier sine and cosine components for a particular zonal wavenumber. Eliasen and Machenhauer (1965) later extended the method to spherical harmonic coefficients.

These numerous studies find that for zonal wavenumber 1, westward-propagating disturbances with a 5-day period commonly appear (Deland, 1964; Eliasen and Machenhauer, 1965; Madden and Julian, 1972; Rogers, 1976). More recently, waves of 14–16 day period have also been observed (Hirota, 1968; Eliasen and Machenhauer, 1969; Sato, 1977; Madden, 1978). A comprehensive review of the structure of the 5-day and 15-day wave by Madden (1978) indicates both have little westward phase tilt and their amplitudes grow with height. The 5-day wave achieves minimum amplitude in equatorial latitudes, is symmetric across the equator and appears in all seasons. The 15-day wave has maximum amplitudes between 60 and 70°N and in most noticeable in the winter and spring. Since Madden did not investigate its structure in the Southern Hemisphere, he could only speculate on its global properties. However, since its latitudinal structure bore a strong resemblance to the symmetric Hough function H_4^1, he surmised that it too was symmetric. Pratt and Wallace (1976) have observed a tropospheric counterpart to Madden's 15-day wave for zonal wavenumber 2. It propagates westward, has little westward phase tilt with height tilt, and appears to be an external wave in that it decays with height.

Sato (1977) has performed a power spectral analysis of pressure-height data from the winters of 1965–66 and 1966–67 for the Northern Hemisphere. He also found a spectral peak at 15 days for wavenumber 1 propagating westward at 30 and 300 mb. The power of the 30 mb wave was observed to be three times that of the 300 mb wave in 1966, suggesting that the wave is much stronger in the lower stratosphere than the troposphere. In 1967, how-

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ever, both the stratospheric and tropospheric components were a factor of 2 weaker and no clear peak in the power spectrum at 15 days was observed in the stratosphere. An analysis of the phase tilt of the 15 day waves by Sato showed that the westward propagating component had a slight tilt with height in both years. Sato also found peaks in the power spectra at 15–20 days for $m = 2$ for westward propagating waves.

Geisler and Dickinson (1976) performed a numerical investigation of the 5-day wave to determine its sensitivity to the mean zonal winds and temperature gradients at the lower boundary. They applied a time-dependent forcing at the lower boundary of the model atmosphere; by changing the frequency of the forcing until a resonant amplification of the response occurred, they found the free modes appropriate for a particular background zonal wind configuration used. They used a modified form of the primitive equations in spherical coordinates with a prognostic form of the divergence equation. Geisler and Dickinson found that the 5-day wave was largely unaffected by the combined effects of lower boundary temperature gradients and the background wind except at the solstice in the summer mesosphere where the background winds form a wave cavity and relatively large amplitudes can be achieved. Their computed wave structure agreed quite well with the limited observations of Madden and Julian (1972).

The success of the Geisler-Dickinson treatment of the 5-day wave has encouraged us to attempt a similar study of the 15-day wave observed by Madden (1978) and Sato (1977). Furthermore we have examined the entire low-frequency spectrum ($\tau > 5$ days) of free oscillations since knowledge of the period and structure of these free waves could provide valuable clues to the understanding of many large-scale atmospheric phenomena like stratospheric warmings and periodic variations of the zonal mean winds and temperatures (van Loon *et al.*, 1975, Madden, 1975).

A strong hint that the 15-day wave is most likely a free oscillation of the atmosphere is provided by the detailed analysis of eigensolutions and eigenfrequencies of the Laplace's tidal equation (LTE) by Longuet-Higgins (1968). With a motionless background atmosphere at 256 K, a barotropic, westward-propagating, zonal wavenumber 1 disturbance has eigenperiods of 5.0, 8.33, 12.5 and 15.6 days in addition to a multitude of lower frequency modes for each corresponding meridional Hough function.[2] The 5-day mode is the counterpart of the observed wave and is largely unaffected by background winds. The 12.5-day wave may be the counterpart of the observed 15-day wave in the troposphere because the inclusion of westerly background winds normally increases the periods of free westward-propagating mode by the Doppler effect. The Hough function corresponding to the 12.5-day oscillation is also symmetric across the equator. However, because of its slow phase speed it seems doubtful that the 12.5-day oscillation could remain unaffected by the strong mean zonal winds in the winter stratosphere.

In this study we use a form of the quasi-geostrophic equations developed by Matsuno (1970) for a spherical geometry. These are less general than the balance equations; however, we feel these equations are adequate since we are mainly interested in long-period oscillations.

In Section 2, a brief discussion of the equations and the method of solution is presented. The response of the model to various types of periodic forcing and the structures of the resonant solutions are given and compared with observations in Sections 3 and 4. The results are summarized and discussed in Section 5.

2. Theory

Since we are mainly concerned with periods which are at least twice that of the 5-day wave modeled by Geisler and Dickinson (1976), we shall assume horizontal motions are quasi-nondivergent. Geisler and Dickinson also used an initial value approach where forcing of a known frequency was slowly turned on at the lower boundary with a latitudinal structure approximately matching that of observed for the 5-day wave. Our approach is somewhat different: we apply a steady vertical velocity pattern at the bottom of the atmosphere and seek the response at the same frequency. By continuously varying the frequency of the forcing and by monitoring the amplitude of the response, we find relative maxima and minima. The maxima (or resonances) correspond to the natural free planetary-scale oscillations in the earth's atmosphere.

The equations used here are an adaption of those of Matsuno (1970). He used the quasi-geostrophic vorticity equation and thermodynamic equation derived by Dickinson (1968), with the important exception that for energetic consistency he allowed for the advection of planetary vorticity by the isallobaric component of the meridional wind v_i'. His vorticity and thermodynamic equations are

$$\left(\frac{\partial}{\partial t} + \bar{\omega}\frac{\partial}{\partial \lambda}\right)\zeta_g' + \frac{v_g'}{a}\frac{\partial \bar{Z}}{\partial \theta} + \frac{2\Omega\cos\theta}{a}v_i' - 2\Omega\sin\theta\left(\frac{\partial}{\partial z} - \frac{1}{H}\right)w' = 0, \quad (1)$$

[2] These periods were obtained from Longuet-Higgins' (1968) Fig. 2b assuming an equivalent depth of 9.8 km and a scale height of 7 km (see Geisler and Dickinson, 1976).

TABLE 1. Comparison of Lamb's parameter for Eq. (5) and LTE. τ = 15 days.

Mode	Lamb's parameter LTE	Eq. (5)
H_1^1	77,826	101,710
H_2^1	92.4	105.6
H_3^1	39.0	39.2
H_4^1	18.6	20.5
H_5^1	0.65	0.61

$$\left(\frac{\partial}{\partial t} + \bar{\omega}\frac{\partial}{\partial \lambda}\right)\frac{\partial \Phi'}{\partial z} - 2\Omega a \sin\theta \cos\theta \frac{\partial \bar{\omega}}{\partial z} v_g' + N^2 w' = 0, \quad (2)$$

where

$\bar{Z}$ absolute vorticity of background atmosphere
Φ' perturbation geopotential
$\bar{\omega}$ angular velocity of basic zonal flow [$=\bar{u}/a \cos\theta$]
ζ_g' perturbation geostrophic vorticity
v_g' geostrophic component of meridional wind
v_i' isallobaric component of meridional wind
H scale height (assumed constant)
Ω angular rotation rate of earth
a radius of earth
z $H \ln(p_0/p)$, where p_0 is a reference pressure
λ longitude
θ latitude
w' vertical velocity [$= -Hd(\ln p)/dt$]
t time
N^2 Brunt-Väisälä frequency (assumed constant).

A new dependent variable $\psi'(\theta,z)$ is introduced by the equation

$$\Phi'(\theta,z,\lambda,t) = e^{z/2H}\psi'(\theta,z)e^{i(m\lambda+\sigma t)}, \quad (3)$$

where m is a zonal wavenumber and σ a frequency. If w' is eliminated between (1) and (2), then it follows that

$$\frac{\partial^2\psi'}{\partial z^2} + \frac{1}{l^2 \cos\theta}\frac{\partial}{\partial\theta}\left(\frac{\cos\theta}{\sin^2\theta}\frac{\partial\psi'}{\partial\theta}\right) + \frac{1}{l^2 \sin^2\theta}\left\{\frac{m}{\sigma + m\bar{\omega}}\frac{1}{\cos\theta}\frac{\partial q}{\partial\theta} - \frac{l^2 \sin^2\theta}{4H^2} - \frac{m^2}{\cos^2\theta}\right\}\psi' = 0, \quad (4)$$

where

$$\frac{\partial q}{\partial\theta} \equiv \left[2(\Omega + \bar{\omega}) - \frac{\partial^2\bar{\omega}}{\partial\theta^2} + 3\tan\theta\frac{\partial\bar{\omega}}{\partial\theta} - l^2 \sin^2\theta\left(\frac{\partial^2\bar{\omega}}{\partial z^2} - \frac{1}{H}\frac{\partial\bar{\omega}}{\partial z}\right)\right]\cos\theta,$$

q is the zonally averaged potential vorticity and $l \equiv 2\Omega a/N$.

Eq. (4) is solved with a prescribed zonal wavenumber m and frequency σ subject to a radiation condition at the top and with an imposed vertical velocity at the bottom. The model extends from the ground (assumed horizontal) to 65 km which is the approximate altitude of the maximum observed wind speed of the polar night jet. A vertical grid distance of 2.5 km and meridional distance of 5° latitude is used. Solutions are obtained using the numerical scheme of Lindzen and Kuo (1969). In order to obtain realistic results equal and constant Newtonian cooling and Rayleigh friction terms were added to (4). This is equivalent to letting the frequency σ have a small imaginary part, i.e., $\sigma = \sigma_r + iK_r$, where $K_r < 0$. Although the wave equation (4) is singular at the equator, the inclusion of a small Rayleigh friction term in the vorticity Eq. (1) obviates this problem. The net result is that the terms $\sin^2\theta$ appearing in denominator of Eq. (1) become $\sin^2\theta + \epsilon$, where $\epsilon = (K_r^2/4\Omega^2)$ (Schoeberl and Strobel, 1978).

Matsuno (1970) applied his equations to a hemispheric model of the stratosphere; however, we wish to examine the global modes of atmosphere. To insure that both antisymmetric and symmetric global modes are simulated correctly by the modified equation (4), we have performed an eigenfunction analysis on the meridional structure equation for a motionless background atmosphere, i.e.,

$$\frac{1}{\cos\theta}\frac{\partial}{\partial\theta}\frac{\cos\theta}{\sin^2\theta + \Delta}\frac{\partial\psi'}{\partial\theta} + \frac{1}{\sin^2\theta}\left[\frac{2m\Omega}{\sigma}\right]\psi' - \frac{m^2}{\cos\theta^2 \sin^2\theta}\psi' = \lambda_0\psi \quad (5)$$

for $\sigma = 2\pi/15$ days and compared the solution with the eigenfunctions of LTE. The values of Lamb's parameter (λ_0) for Eq. (5) and the tidal equation are compared in Table 1. Fig. 1 shows a mode resulting from Eq. (5) and two modes from the tidal equation. The H_4^1 mode and the corresponding Eq. (5) mode are so close that we have plotted only the former. Clearly, no important differences exist in either the form of the eigenfunctions or value of Lamb's parameter between LTE and Eq. (5) at low frequencies.

3. A model wind field

In order to provide an illustrative example of the model and to test the sensitivity of response to antisymmetric and symmetric forcing functions, we use a wind field given by

$$\bar{u} = \sin 2\theta(0.8z) \text{ [m s}^{-1}\text{]}.$$

For the antisymmetric forcing we used $w' = w_0'$

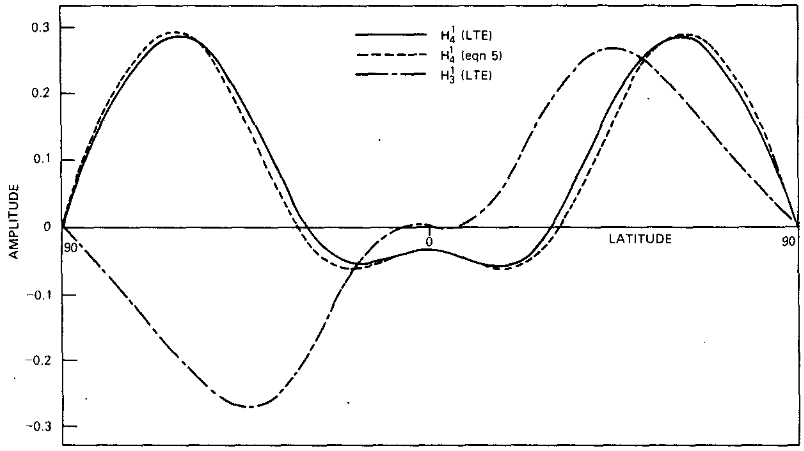

FIG. 1. Comparison of H_3^1 and H_4^1 eigenfunctions of Laplace's tidal equation and those associated with Eq. (5) (see text for details).

× sin2θ and the symmetric forcing we used $w' = w'_0 \times \cos\theta$; damping time was set to 1000 days.

The amplitude response of the model as a function of period is shown in Fig. 2. The amplitude shown is the norm of the wave amplitude within the model, i.e.,

$$N = \sum_{i,j}^{\mathrm{ALL}} |\psi'_{ij}|,$$

where i and j are indices indicating grid points. It is apparent that the different forcing functions have only slight effect on the model response and the overall magnitude shift in the amplitude can be attributed to the change in the input to the different model modes provided by the different forcing functions.

The peaks in the response function are caused by two different types of wave behavior within the model. For all cases resonant response (relative maxima in Fig. 2) occurs when the model solution in the interior correspond to $w' \approx 0$ in some forced region at the lower boundary. All external or barotropic waves have this response, and internal waves may also exhibit this behavior. With the exception of the external H_2^1 (5-day wave) and H_3^1 modes which have a relatively large phase speed, the shear in the model wind field traps the modes at some altitude which corresponds to the Charney-Drazin criti-

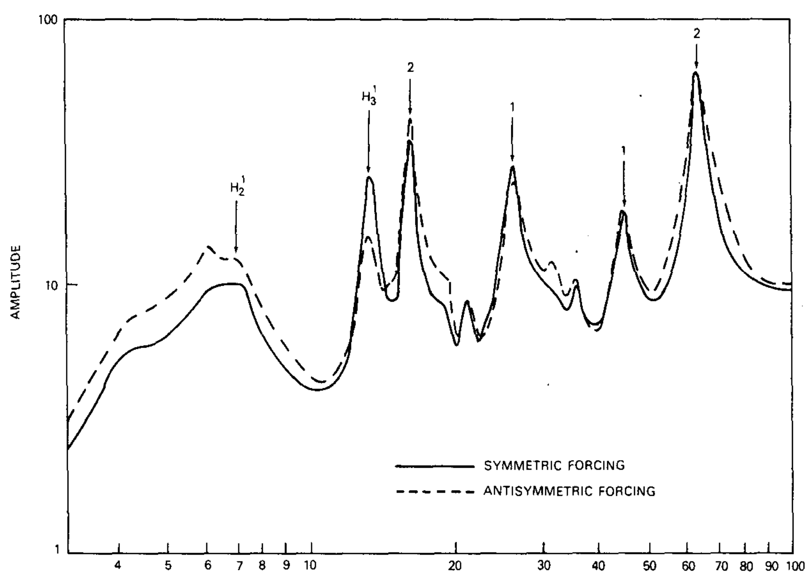

FIG. 2. Response of the norm of the model solution as a function of period in days using an analytical wind field and no damping. Amplitude is in geopotential meters (gpm); wave propagates westward.

cal velocity. The reflected wave which forms when upward propagating wave encounters a mean wind which exceeds this velocity combines with the upward propagating wave to produce a standing wave in height. When a node in the w' field coincides with the surface forcing region for w', resonance occurs. For Rossby waves propagating westward, the vertical wavelength decreases as the period increases. Thus as we move from short periods to longer ones, we first encounter the resonance of the barotropic mode which may exist only if the rotation speed of the atmosphere is smaller than ν_R where ν_R is the barotropic frequency of the wave in the barotropic phase speed of the wave. The next resonant solutions encountered as the period increases and the trapped or partially trapped internal modes where $w' \approx 0$ at the surface.

By examining the vertical structure of various solutions associated with the peaks in Fig. 3 we have noted some of the modes. Both the H_3^1 and H_2^1 modes have been identified because the ψ' solution decays with height typical of barotropic solutions. These modes are strongly present for both types of surface forcing. Other peaks are labeled only by the number of nodes in ψ' at 60°N since nonseparability of the equations prevents clear identification of these modes with the Hough function.

4. Realistic wind fields

We now examine the model response for more realistic wind fields. Since the 15-day wave is observed in winter, we will be mainly concerned with solstice condition. Fig. 3 shows the zonal wind representative of stratospheric-tropospheric distribution; the potential vorticity gradient computed from this wind distribution is positive everywhere. Figs. 4a, 4b and 4c show the amplitude response of the model for $m = 1, 2, 3$, respectively.

For weakest damping, a highly variable response as a function of period is evident for all wavenumbers as was also evident in Fig. 3. For $m = 1$, westward-propagating strong peaks appear at 5 and 15 days with other peaks at higher periods. When stronger damping is used, the long-period resonant peaks are greatly reduced in amplitude. For damping times between 30 and 100 days the 15- and 5-day waves still dominate with secondary resonances at 42, 65 and 80–90 days for $m = 1$. For $m = 2$, westward propagating, a 5.5- and 11-day wave are important. In addition, there are response peaks at 22, 35 and 50–80 days. This later peak appears to be the response of a very slowly changing mode, near resonance but nearly stationary. Similarly, for $m = 3$ the response maximizes at 6, 12, 26, 30, 48 and 60–80 days.

In the second experiment the wind field is that shown in Fig. 3 except that $|\bar{u}|$ is arbitrarily set to 10 m s^{-1} in the model when its value exceeds 10 m s^{-1}. The 15-day response is effectively eliminated suggesting strong winds are important in producing this mode. Note also that the response of the 5-day wave is reduced somewhat since damping is more effective in the weak wind field used in this case.

Fig. 5 shows the computed structure of ψ' for forcing periods of 5 days and Fig. 6 shows the computed structure for 14-day and 15-day forcing. The 5-day period solution compares reasonably well with Geisler and Dickinsons' (1976) results at midlatitudes even though our model equations are not as accurate as theirs for short periods. Because of the very low amount of damping used in the 5-day wave calculations, the mode is distorted at the equatorial grid point to a value much smaller than Geislers and Dickinson's. Both the 14- and 15-day solutions show an increase in amplitude with height in the winter hemisphere, characteristic of an internally reflected wave, and a decay with height in the summer hemisphere where trapping is not expected. The phase lines show that the solution has a phase reversal at low latitudes. Comparing the 14-day and 15-day solutions it is apparent that tremendous structural changes occur near resonance and the solution has very large vertical scale. Both

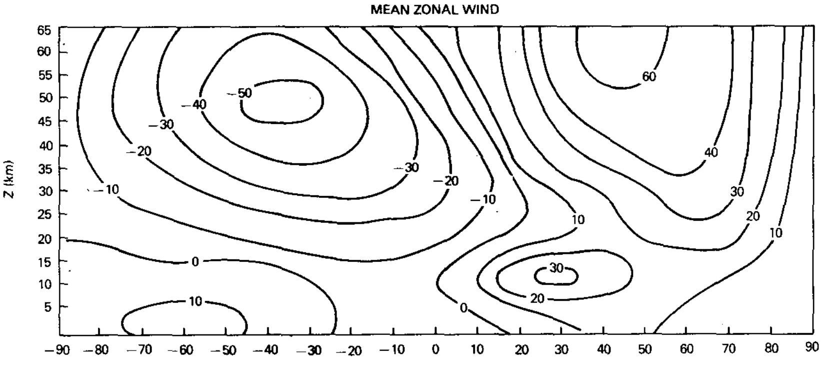

FIG. 3. Mean zonal wind distribution $\bar{u}$ (in m s^{-1}) used in this study.

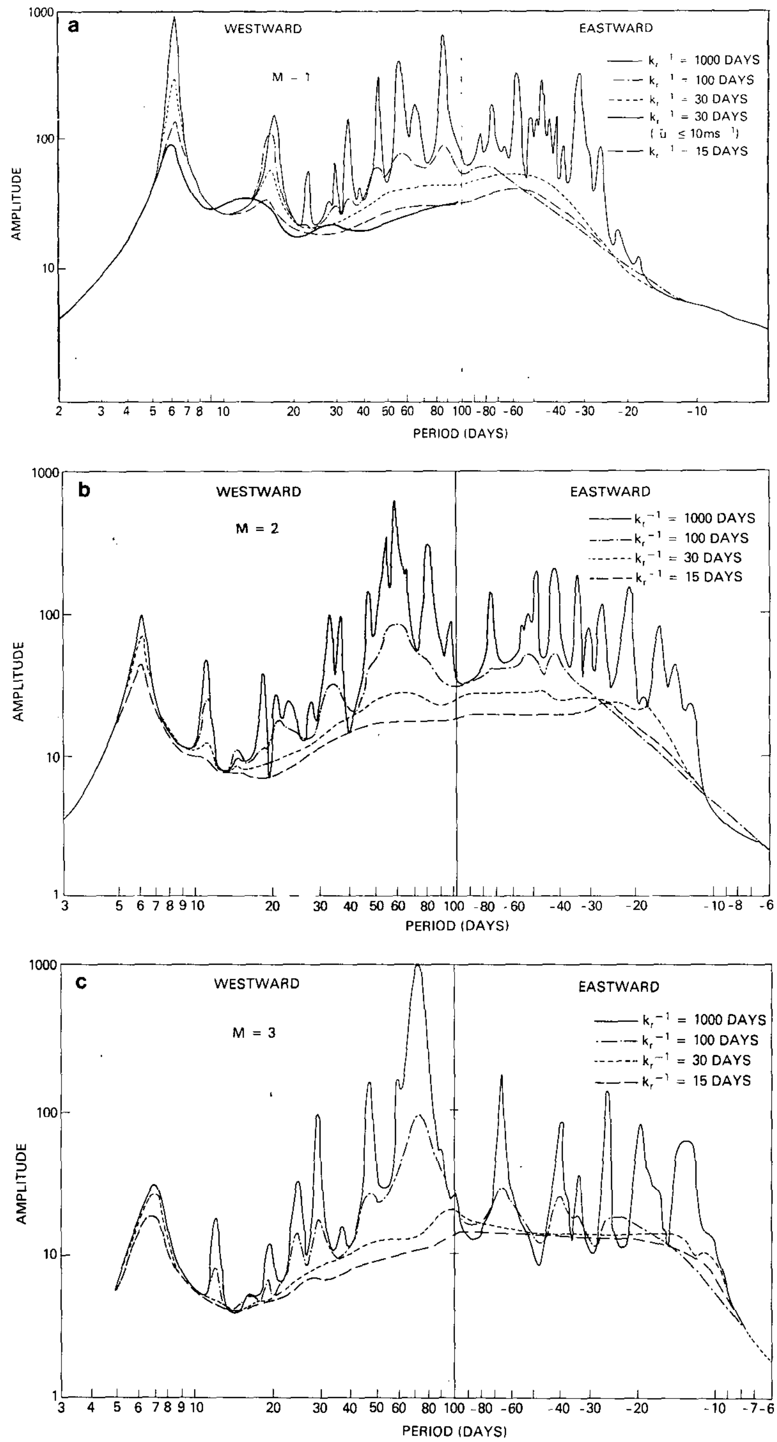

FIG. 4. Response of the norm of the model solution as in Fig. 2. Positive periods correspond to westward-propagating waves; negative periods to eastward-propagating waves. (a), (b) and (c) correspond to $m = 1$, 2 and 3, respectively.

solutions show little phase shift with heights; the 14-day wave tilts westward and the 15-day wave tilts eastward except near the upper boundaries where the radiation condition is imposed and both waves tilt westward. The hemispheric asymmetry and long vertical wavelength of the 14-day wave suggest the strong presence of the antisymmetric H_3^1 mode as identified in the last section.

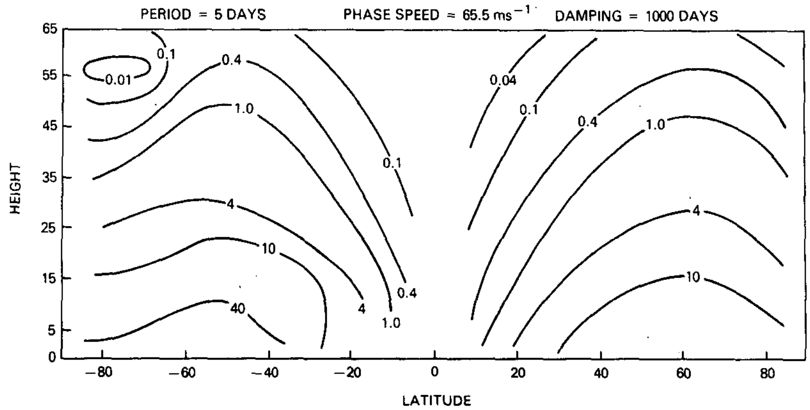

FIG. 5. Computed vertical structure for ψ' of 5-day wave with phase speed given at 45°N. Amplitude units are gpm although arbitrary normalization is used.

Fig. 5 compares the computed structure of the 15-day wave with observations reported by Madden (1978). Observations and model computations are normalized to the maximum amplitude of Madden's wave. At high latitudes reasonable agreement is obtained but at low altitudes agreement is poor. Unfortunately, the boundary forcing function used to excite the model solution also excites nonresonant modes which generally decay with height away from the lower boundary. These nonresonant modes contaminate the resonant solution at low altitudes and latitudes making meaningful comparison difficult. However, comparison at higher altitudes, where the resonant solution should more completely dominate the total solution, does show a little better agreement with observations. In making this

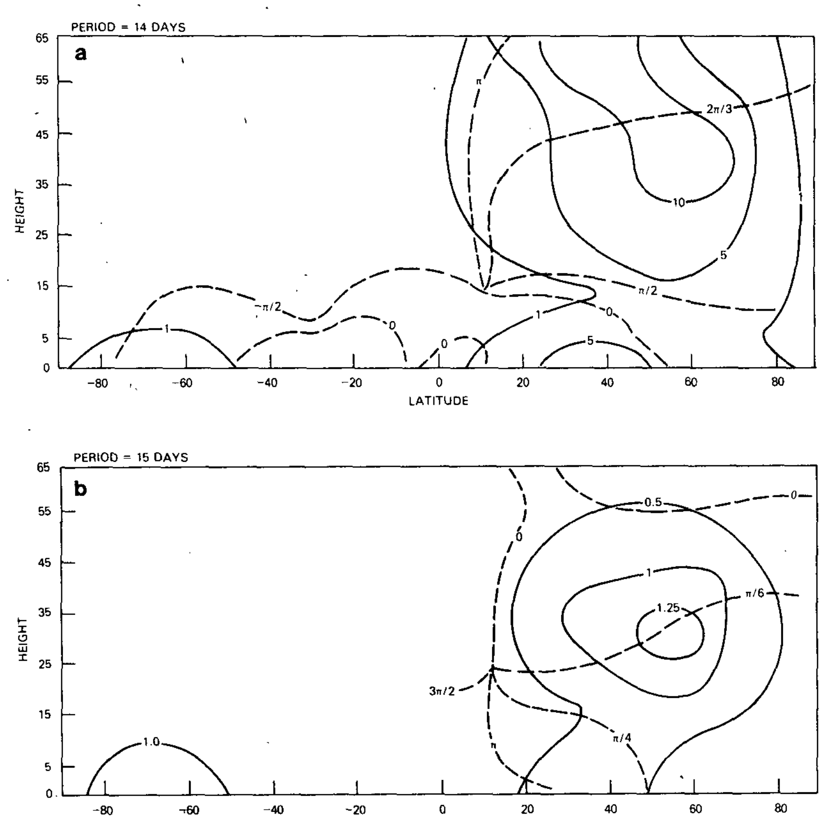

FIG. 6. Computed vertical structure for ψ' of 14-day (a) and 15-day (b) waves. Damping time is 30 days; arbitrary normalization.

kind of comparison, we have tacitly assumed that the meridional structure of the resonant mode varies weakly with altitude as is suggested by Fig. 6b.

The vertical structure of Madden's observation at 40°N with solutions with 14-, 15- and 16-day periods shown in Fig. 7 also does not show good agreement. Madden's 15-day wave decays with height while each of the calculated solutions eventually grow in amplitude because of the internal reflection. However, we note that Madden's 15-day wave is a composite, obtained from band-pass analysis of disturbances over a small period range from many years of data. Sato (1977) has shown that year-to-year variations of 15-day wave amplitude can be quite large. The large variation in the vertical structure of the resonant solution with period shown in Fig. 7 also suggests that an observationally obtained composite may have somewhat different growth rate with altitude than any computationally obtained solution. Finally, we note again that our boundary forcing plays some role in altering the solutions in the troposphere.

Figs. 4a–4c also show resonant response for eastward propagating modes at long periods. For $m = 1$, eastward, a broad maximum appears at 80–90 days with 100 days damping. Fig. 4b shows several small peaks at 37 and 50 days eastward for $m = 2$ for the same damping. Fig. 4c shows that $m = 3$ has several important maxima at 70, 43, 40 and 27 days. For very short-period, eastward-moving waves the response dies off rapidly with period. At the short periods shown the traveling planetary waves, if Doppler-shifted to stationary, would effectively see easterly winds everywhere and become evanescent. Thus the decay of the amplitude response of the model with period is expected.

5. Summary and discussion

Free planetary waves are solutions to Laplace's tidal equation which lie on the Rossby branch and have $w' = 0$ at the surface for some given frequency. For a resting atmosphere or one with a constant angular velocity these solutions are all barotropic. The most important free wave observed is the 5-day wave studied by Geisler and Dickinson (1976).

Using a linearized planetary wave propagation equation we have examined the spectrum of long-period planetary-scale oscillations for resonance during solsticial wind conditions. The stratospheric polar night jet is strong enough that only the fastest moving free modes such as the 5-day wave can exist relatively unaffected by the winds. Slower moving modes can be reflected and distorted by the polar night jet. The reflection from the strong winds occurs when a wave, propagating vertically, encounters Doppler-shifted westerly winds greater than its Charney-Drazin (1961) critical speed. A standing wave solution is set up below the reflection level with nodes and antinodes in w' occurring at various altitudes. If a node in w' coincides with the surface then resonance occurs, ψ' obtaining infinite values in the interior when a nonzero w' boundary condition is imposed. Pure resonance of the type described in the last sentence is removed by the introduction of damping which smoothes out the nodes and antinodes. While resonant response is now reduced, interior wave amplitude is much larger than that suggested by calculations which neglect the wind shear.

We have examined the response spectrum of zonal harmonics 1, 2 and 3. The 15-day wave reported by Madden (1978) and Sato (1977) appears to

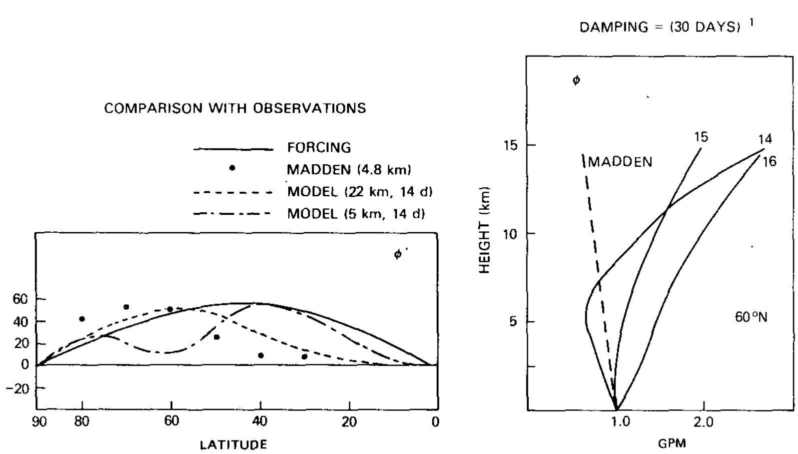

FIG. 7. Comparison of observations with computations. Left figure shows meridional structure of ϕ' normalized to the maximum amplitude for different altitudes and Madden's (1978) observations. Right figure shows vertical structure and observations of the wave at 60°N for observations and calculations at 14-, 15- and 16-day periods. The computations and observation were all normalized to 1 gpm at the surface.

be a natural resonant mode of the winter stratosphere; however, our model has not been entirely successful in reproducing the details of the observed structure of the 15-day wave. We attribute these difficulties to the rapid variability of the wave structure with period in our model, and the contamination of the resonant mode in the troposphere by nonresonant modes forced by our lower boundary condition. The former property has an observational counterpart in the wide year-to-year variability in the structure of the 15-day wave (Sato, 1977), and better agreement is obtained with observations if we compare with the resonant mode's meridional structure further away from the lower boundary.

The vertical structure of the 15-day wave indicates it is trapped by strong winds at the stratopause, and the amplitude of the model solution to 15-day periodic forcing is shown to be greatly reduced if there are no strong winds in the stratosphere. We have noted resonant responses for westward propagating waves for $m = 2$ at 5.5, 11, 22, 35 and 50–80 days and for $m = 3$ at 6, 12, 26, 30, 48 and 60–80. The 5.5- and 6-day solutions for $m = 2$ and 3 are mostly barotropic, the mean winds not being strong enough to affect them, but the other modes appear to be baroclinic like the 15-day wave. The wide variation in amplitude response at long periods is consistent with the results of Schoeberl and Geller (1977) who showed strong variations of stationary wave structure for $m = 1$ and 2 with polar night jet strengths. This is equivalent to frequency shifts at long periods in the steady wind field used in this study.

The extreme excursions of the norm of the wave amplitude for very low values of damping is partially a result of critical layer reflection in the summer hemisphere. In the absence of damping, steady-state numerical models treat critical layers as wave reflectors. Thus, baroclinic modes can exist in the summer hemisphere by reflection from the critical layer in the stratosphere. In addition, longer period modes with short vertical wavelengths are not effectively eliminated by long damping times. Both of these effects produce "noisy" response in the model for the damping times >100 days.

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