

Conditional Instability of the First Kind Upside-Down

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ABSTRACT

It is shown that entrainment leads to the generation of turbulence kinetic energy in a stratocumulus layer when the virtual temperature jump at the cloud top is weaker than a critical value. The critical value increases as the relative humidity of the air above cloud top decreases. This result is interpreted as a criterion for the instability of the layer cloud to penetrative downdrafts. The role of the instability in determining the subtropical and tropical distributions of boundary-layer cloudiness is assessed.

1. Introduction

Turbulent entrainment ordinarily tends to destroy turbulence kinetic energy, because the buoyancy force acts to oppose vertical motions in the statically stable layer which usually marks the edge of the turbulence. Under dry adiabatic processes, only the extraordinary circumstance of a statically unstable edge layer can permit entrainment to generate turbulence kinetic energy through the buoyancy force. But Lilly (1968) pointed out that the evaporative cooling of unsaturated air which has been entrained into a cloud can, under some conditions, cause the entrained air to sink unstably as a convective downdraft. Because this instability is somewhat similar to conditional instability of the first kind (cumulus instability), we call it conditional instability of the first kind upside-down (CIFKU). The purposes of this paper are to determine the criterion for the onset of CIFKU, to suggest a simple theory of the effects of CIFKU on a stratocumulus layer, and to assess the role of CIFKU in determining the global distribution of boundary-layer cloudiness.

2. The stability criterion

We consider a turbulent, uniformly saturated cloud mass separated from an overlying, quiet, unsaturated layer by a thin transition layer, across which the temperature, mixing ratio and turbulence intensity may change sharply (Fig. 1). The cloud layer may lie within the boundary layer, or it may be a free elevated deck (perhaps in a frontal zone), or it may even be an anvil produced by deep cumulus convection (although, for simplicity, we ignore the ice phase in this paper). In this section, our purpose is to determine the conditions under which entrainment across the transition layer can lead to buoyant convection in the cloud layer.

We measure the relative buoyancy of fluid particles using the virtual dry static energy

$$s_v = c_p T_v + gz, \tag{2.1}$$

where the virtual temperature

$$T_v = T(1 + \delta q - l) \tag{2.2}$$

is defined in such a way as to take into account the effects of both vapor (with mixing ratio q) and liquid (with mixing ratio l). Here $\delta = 0.608$. As moist (and dry) conservative variables, we use the total water mixing ratio ($q + l$) and the moist static energy

$$h \equiv c_p T + gz + Lq. \tag{2.3}$$

Also, we define the saturation moist static energy

$$h^* \equiv c_p T + gz + Lq^*, \tag{2.4}$$

where q^* is the saturation mixing ratio, and the following useful thermodynamic coefficients:

$$\epsilon \equiv \frac{c_p T}{L}, \tag{2.5}$$

$$\gamma \equiv \frac{L}{c_p} \left(\frac{\partial q^*}{\partial T} \right)_p, \tag{2.6}$$

$$\beta \equiv \frac{1 + (1 + \delta)\gamma\epsilon}{1 + \gamma}. \tag{2.7}$$

For convenience, the variations of γ and β with T and p are plotted in Fig. 2. We can use (2.1)–(2.3) and (2.5) to write

$$s_v = h - [1 - (1 + \delta)\epsilon]Lq - \epsilon L(q + l). \tag{2.8}$$

We let ∂_p represent a differential at constant pressure, so that

$$L\partial_p q^* = \gamma\partial_p(c_p T) = \frac{\gamma\partial_p h^*}{1 + \gamma}. \tag{2.9}$$

With this preparation, we can now write

$$\begin{aligned} \partial_p s_v &= \beta \partial_p h + [1 - (1 + \delta)\epsilon] \left(\frac{\gamma \partial_p h}{1 + \gamma} - L \partial_p q \right) - \epsilon L \partial_p (q + l) \\ &= \beta \partial_p h + [1 - (1 + \delta)\epsilon] \left[\left(\frac{\gamma}{1 + \gamma} \right) \partial_p (h - h^*) + L \partial_p (q^* - q) \right] - \epsilon L \partial_p (q + l) \\ &= \beta \partial_p h + \left[\frac{1 - (1 + \delta)\epsilon}{1 + \gamma} \right] L \partial_p (q^* - q) - \epsilon L \partial_p (q + l), \end{aligned} \tag{2.10}$$

from which it follows that the turbulent fluxes of s_v , $q^* - q$ and $q + l$ across isobaric surfaces are related by

$$F_{s_v} = \beta F_h + \left[\frac{1 - (1 + \delta)\epsilon}{1 + \gamma} \right] L F_{q^*-q} - \epsilon L F_{q+l}. \tag{2.11}$$

In uniformly saturated air q^* and q are everywhere equal, so that (2.11) simply reduces to

$$F_{s_v} = \beta F_h - \epsilon L F_{q+l}. \tag{2.12}$$

We now ask how entrainment influences F_{s_v} . As indicated in Fig. 1, we let the subscripts B and $B+$ denote levels just below and just above the cloud top level, respectively. Assuming that the layer between B and $B+$ is too thin to accommodate significant storage of h or $(q + l)$, and using the fact that the turbulent fluxes vanish at level $B+$, we can express the conservation laws for h and $(q + l)$ as

$$(F_h)_B = -E\Delta h + \Delta R, \tag{2.13}$$

$$(F_{q+l})_B = -E\Delta(q + l), \tag{2.14}$$

respectively, where E is the entrainment mass flux, R the radiation flux, and

$$\Delta(\) \equiv (\)_{B+} - (\)_B. \tag{2.15}$$

From (2.12)–(2.14), we find that in uniformly cloudy air,

$$(F_{s_v})_B = -E[\beta\Delta h - \epsilon L\Delta(q + l)]. \tag{2.16}$$

We first note that, as pointed out by Lilly (1968), cloud top radiative cooling tends to drive convection in the cloud layer (see also Randall, 1980). This

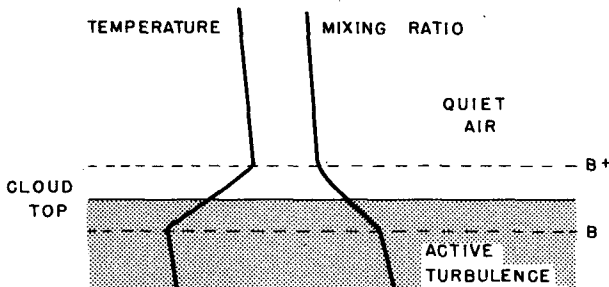


FIG. 1. Schematic diagram of the distributions of temperature and mixing ratio near cloud top. The stippled area represents cloudy air.

largely explains why cloud layers are almost always turbulent. But our purpose here is to determine under what conditions *entrainment itself* promotes convection [i.e., tends to increase $(F_{s_v})_B$]. According to (2.16), the requirement is

$$\beta\Delta h - \epsilon L\Delta(q + l) < 0; \tag{2.17}$$

this is the criterion for the onset of CIFKU. Assuming that the layer between B and $B+$ is infinitesimally thin, we can apply (2.10) to relate the various

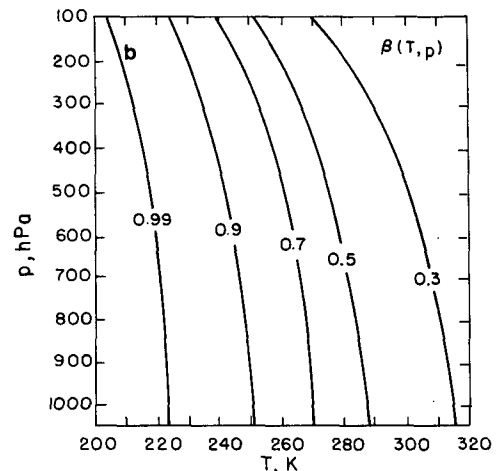
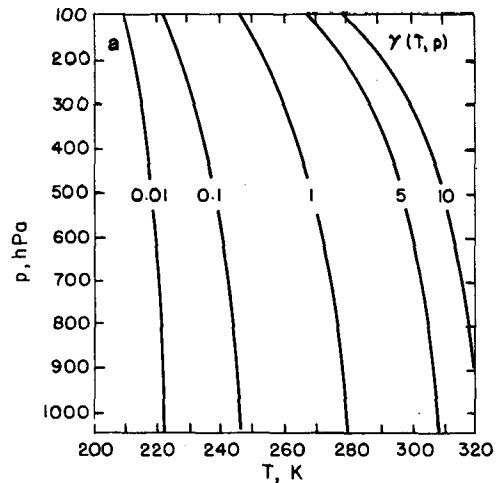


FIG. 2. Distributions of (a) $\gamma(T, p)$ and (b) $\beta(T, p)$.

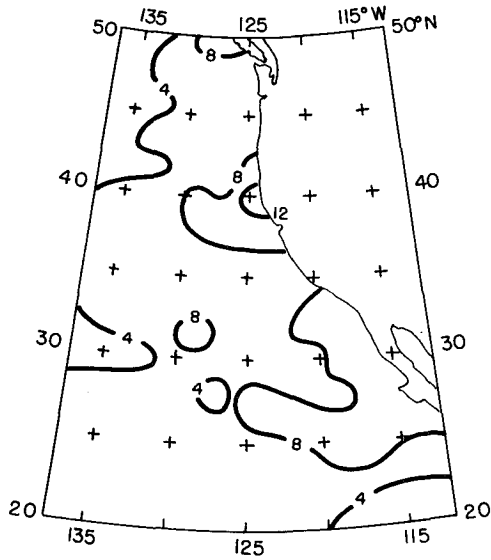


FIG. 3. The distribution of $(\Delta s_v)_{crit}/c_p$ [K], off the coast of California, from the data of Neiburger (1960).

discontinuities across the cloud top. In this way, we find that

$$\beta \Delta h - \epsilon L \Delta(q + l) = \Delta s_v - (\Delta s_v)_{crit}, \quad (2.18)$$

where

$$(\Delta s_v)_{crit} = \left[\frac{1 - (1 + \delta)\epsilon}{1 + \gamma} \right] L(q_{B+}^* - q_{B+}). \quad (2.19)$$

In view of (2.18), we can rewrite (2.17), the criterion for the onset of CIFKU, as

$$\Delta s_v < (\Delta s_v)_{crit}. \quad (2.20)$$

This shows clearly that CIFKU occurs when Δs_v is less than a positive critical value which increases as the relative humidity of the air above cloud top decreases. Fig. 3 shows the distribution of $(\Delta s_v)_{crit}/c_p$ for average summer conditions over the eastern North Pacific, based on the temperature,

pressure and relative humidity distributions at the inversion top, as reported by Neiburger (1960). Values in excess of 5 K occur, particularly near the California coast.

We can also give the following ‘‘parcel theory’’ interpretation of CIFKU. When unsaturated air (for which $(\Delta s_v)_{crit} > 0$) is entrained into a cloud, it is cooled and moistened by the evaporation of cloud droplets. As a result, its virtual dry static energy is decreased from $(s_v)_{B+}$ to $(s_v)_{B+} - (\Delta s_v)_{crit}$. If the latter value is less than $(s_v)_B$, i.e., if (2.18) is satisfied, then the entrained air is negatively buoyant with respect to its cloudy environment, and so it sinks unstably through the cloud. This process is schematically illustrated in Fig. 4.

Lilly (1968) assumed that the criterion for CIFKU is that the wet bulb potential temperature decreases upward at cloud top; this is equivalent to the condition $\Delta h < 0$. But from (2.17) we see that $\Delta s_v - (\Delta s_v)_{crit} < 0$ means $\Delta h < \epsilon L \Delta(q + l) / \beta < 0$, so that CIFKU actually does not begin until Δh is appreciably negative. For $T = 273$ K and $p = 1000$ hPa, we find that $\Delta h/c_p \approx 0.4 \Delta(q + l)$ K, where $\Delta(q + l)$ is expressed in $g \text{ kg}^{-1}$. Since $\Delta(q + l) \approx -5 \text{ g kg}^{-1}$ can occur, our instability criterion is significantly more stringent than Lilly’s. If we repeat our derivation but ignore the virtual temperature correction or ignore the liquid water component of the virtual temperature correction, our stability criterion reduces to $\Delta h > 0$. This illustrates the importance of including the liquid water correction in the definition of virtual temperature.

This importance may seem strange, since $l \ll q$. But what matters is that F_l can be as large as or even larger than F_q . Imagine, for example, that fluctuations in $(q + l)$ occur on an isobaric surface where T is homogeneous. Then, since there can be no fluctuations of $q^*(T, p)$, q' must be zero, and so $|l'|$ is ‘‘infinitely larger’’ than $|q'|$. On the other hand, if fluctuations in T occur on an isobaric surface where $(q + l)$ is homogeneous, then fluctuations of

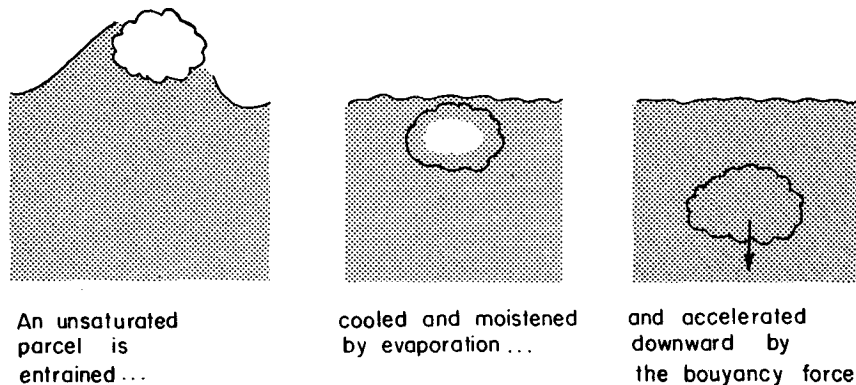


FIG. 4. A schematic illustration of the ‘‘parcel’’ interpretation of CIFKU. The stippled area represents cloudy air.

q^* (and q) must occur, so that $l' = -q'$, or $|l'| = |q'|$. In real situations, in which fluctuations of both T and $(q + l)$ occur, we therefore anticipate that $|l'|$ will often be at least as large as $|q'|$.

As mentioned above, the stability criterion reduces to $\Delta h > 0$ if we ignore the virtual temperature correction or its liquid water component. This implies that the water vapor component of the virtual temperature correction does not matter for the criterion. The reason is that, as shown by (2.9), inside the cloud the fluxes of vapor, sensible heat and moist static energy are related to each other by positive factors of proportionality, so that they must all have the same sign, and increase or decrease together.

3. The nature of the convective downdrafts

It is interesting to compare the convective downdrafts of CIFKU with the convective updrafts of CIFK (cumulus updrafts). The most obvious differences are in the directions of the buoyancy forces and the resulting motions. While cumulus updrafts are driven by condensation heating, CIFKU downdrafts are maintained by evaporative cooling. The updraft heating extends upward from the PBL top throughout the conditionally unstable layer, which may fill the troposphere. But downdraft cooling is confined to the cloud layer, which in many cases is only a few hundred meters deep. As is well known, the entrainment of environmental air into cumulus updrafts has a debilitating effect, through dilution of the cloud mass with dry, low-energy air and also through the frictional effects of momentum exchange. Although entrainment into CIFKU downdrafts also tends to retard their motion through momentum exchange, it is essential, nevertheless, to their existence, because it supplies the liquid water which, by evaporation, maintains their negative buoyancy. This suggests that the downdrafts will prefer a small scale, perhaps on the order of a

few hundred meters, so that mixing can be highly effective.

4. The roles of CIFKU in the marine boundary layer

The ability of CIFKU sharply to reduce the fractional cloudiness within a cloud layer suggests that the instability is responsible for the breakup of the subtropical marine stratocumulus regimes at their westward and equatorward boundaries. The marine stratocumulus regimes have been discussed by Lilly (1978) and Schubert *et al.* (1979a,b), while the neighboring tradewind cumulus regimes have recently been studied by Albrecht *et al.* (1979) and Albrecht (1979).

Fig. 5 summarizes the envisioned roles of the marine stratocumulus layers and CIFKU in the Hadley circulation. The right side of the figure represents a marine stratocumulus regime, and the left side represents tradewind cumulus and ITCZ regimes. From right to left, the sea surface temperature increases and the large-scale subsidence decreases. Separating the marine stratocumulus regime of large fractional cloudiness from the tradewind cumulus regime of small fractional cloudiness is a "transition zone." By definition, the poleward boundary of the transition zone marks the onset of CIFKU; the stratocumulus deck is therefore destroyed as it is advected into the transition zone, in the sense that the fractional cloudiness is reduced below 100%.

Some cloudy fragments of the stratus layer may escape evaporation, and may persist as the upper portions of moist plumes rising through the marine layer. Are these cumulus clouds? They are certainly associated with buoyant convection, since CIFKU is by its nature a convective instability. But in order for the cloud fragments to qualify as cumulus clouds, they must derive their kinetic energy primarily from CIFK, i.e., from buoyancy maintained by the release of latent heat in saturated updrafts embedded

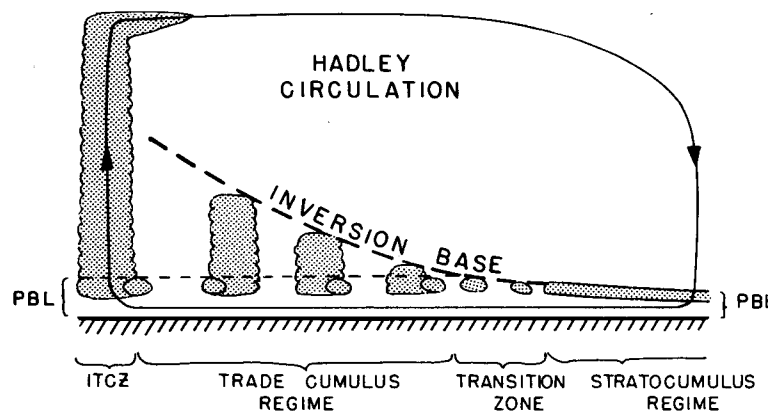


FIG. 5. A schematic illustration of the role of CIFKU in determining the tropical and subtropical distributions of cloudiness. Details are given in the text.

in a dry adiabatically subsiding environment. The convective thermals of the (pre-CIFKU) stratocumulus layer certainly do not qualify as cumulus clouds, and since (2.20) is not the criterion for the onset of CIFK, neither do the (post-CIFKU) cloud fragments.

Instead, the cloud fragments can be described as cumulus humilis clouds, which generate a significant fraction of their kinetic energy during their rise as unsaturated thermals in the subcloud layer. Esbensen (1978) has termed these "boundary-layer cumuli," and has shown that their impact on the large-scale environment is very different from that of deeper cumuli. The eventual release of latent heat, with its attendant cloudiness, does not dominate the dynamics of these thermals; it only serves to make them visible near the upper limits of their trajectories. For this reason, boundary-layer cumuli are more closely related to dry convective turbulence than to deeper cumuli.

As argued by Betts (1973), nonprecipitating saturated thermals which overshoot their level of neutral buoyancy and then sink back, experiencing dilution with their environment at all times, produce a net downward transport of thermodynamic energy, and so cool the upper portion of the cloud layer. In some cases this cooling near cloud top may be accelerated radiatively. The resulting destabilization favors an increase in the depth of the cloud layer with time. In this way, cumulus humilis clouds may significantly intensify the process of mass entrainment at the top of the transition zone's convective layer.

Further downstream, warming, moistening and deepening of the cumulus humilis layer will lead to the onset of trade cumulus convection. The cumulus humilis clouds may continue to exist, but they will be inconspicuous among their towering neighbors. We define the equatorward boundary of the transition zone as the location of the onset of CIFK.

The cloud base level of the trade cumulus layer can be regarded as the equatorward extension of the midlatitude stratocumulus cloud base. Similarly, the trade inversion, which generally marks the top of the tradewind cumulus layer, can be regarded as the equatorward extension of the inversion at the top of the midlatitude stratus deck. We thus have a continuous evolution of the cloud layer from midlatitudes into the tropics. But a conceptual problem nevertheless arises, since the top of the PBL is usually taken to coincide with the cloud top in the marine stratocumulus regime, and with the cloud base in the trade cumulus regime. Is it possible that the PBL top descends through the cloud layer, from the stratocumulus cloud-top level in midlatitudes to the cumulus cloud base level in the trades?

In my opinion, the cleanest solution to this problem is to consider cumulus humilis clouds as lying within the PBL, and to define the PBL top as the

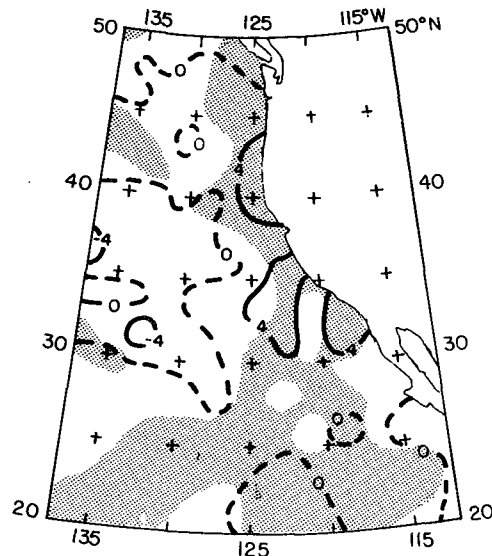


FIG. 6. The distribution of $[\Delta s_p - (\Delta s_p)_{crit}]/c_p$ [K], off the coast of California, from the data of Neiburger (1960). Stippled regions are those in which the relative humidity at the inversion base exceeded 90%.

highest level reached by the cumulus humilis convection, when such convection is active. When deeper cumuli coexist with the cumulus humilis clouds, the bases of the deeper clouds will lie within the PBL, but in any case these deep clouds have subcloud inflow layers which can extend to the earth's surface, and so it is impossible to define the PBL top in such a way that the circulations of deep cumuli lie entirely above it.

Since the equatorward boundary of the transition zone marks the onset of CIFK, it also marks the separation of the PBL top from the cloud top level. Further toward the equator, the PBL top lies at the top of the cumulus humilis layer. We have thus concluded that the PBL top and the cloud top coincide throughout the transition zone, but that the cloud top lifts above the PBL top further toward the equator. This is the situation summarized in Fig. 5.

Although an extensive observational study is needed to verify this picture of the interrelations between the PBL stratus regime, the transition zone and the trade cumulus regime, we can obtain some preliminary observational support from the data of Neiburger (1960). The data were obtained during a series of cruises in the eastern north Pacific, during the summer months. In Fig. 6, the shaded regions are those in which the average relative humidity at the inversion base (which, we assume, coincided with the PBL top) exceeded 90%. The observed average values of $\Delta s_p - (\Delta s_p)_{crit}$, also shown in the figure, were mainly positive in the humid regions where stratus layers must have occurred frequently. This is consistent with our analysis, which shows

that such stratus layers would have been stable. In those regions where the low average relative humidity suggests that the PBL was usually cloud-free, the figure shows that $\Delta s_v - (\Delta s_v)_{\text{crit}}$ was mainly negative. This is also consistent with our analysis, which shows that stratus layers could not have been stable in these regions.

5. Conclusions and discussion

The central points of this study are that cloud masses can be unstable with respect to penetration by the overlying air, that the instability can lead to the partial destruction of the cloud, and that the distribution of cloudiness in the subtropical planetary boundary layer is partly determined by this mechanism. The first priority of further research along these lines should be more complete verification of these results, by both observational studies and detailed numerical simulations.

Several questions must be raised. First, given that CIFKU occurs, how can its effects be parameterized for numerical models? The fractional cloudiness, the rate of entrainment at cloud top and the turbulent exchanges associated with CIFKU must all be determined in an adequate parameterization theory. Is it possible for CIFKU to continue indefinitely without greatly reducing the fractional cloudiness, under some synoptic conditions?

We must also ask whether CIFKU influences the mesoscale circulations which are so prominent in satellite photos of stratocumulus layers. Closed cellular convection (e.g., Agee *et al.*, 1973) seems particularly likely to be modified by CIFKU.

The role of CIFKU in determining the distributions of upper level cloudiness, including altostratus and cirrus, must be assessed. For this purpose, it will be necessary to generalize the theory to include the ice phase.

Finally, it seems likely that CIFKU is partly responsible for the rapid entrainment and intense turbulence near the tops of cumulus clouds (see, e.g., Warner, 1970). The study of CIFKU may therefore improve our understanding of CIFK.

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