Effect of Ice-Albedo Feedback on Global Sensitivity in a One-Dimensional Radiative-Convective Climate Model

WEI-CHYUNG WANG[1]

Goddard Institute for Space Studies, New York, NY 10025

PETER H. STONE

Department of Meteorology, Massachusetts Institute of Technology, Cambridge 02139

(Manuscript received 10 July 1979, in final form 19 October 1979)

ABSTRACT

The feedback between ice albedo and temperature is included in a one-dimensional radiative-convective climate model. The effect of this feedback on global sensitivity to changes in solar constant is studied for the current climate conditions. This ice-albedo feedback amplifies global sensitivity by 26 and 39%, respectively, for assumptions of fixed cloud altitude and fixed cloud temperature. The global sensitivity is not affected significantly if the latitudinal variations of mean solar zenith angle and cloud cover are included in the global model.

The differences in global sensitivity between one-dimensional radiative-convective models and energy balance models are examined. It is shown that the models are in close agreement when the same feedback mechanisms are included.

The one-dimensional radiative-convective model with ice-albedo feedback included is used to compute the equilibrium ice line as a function of solar constant. It is found that the fixed cloud temperature parameterization breaks down before the completely ice-covered earth instability sets in, i.e., the lowest cloud layer intersects the ground.

In addition, it is shown that the ice-albedo feedback has a similar amplification effect on the global warming caused by increase in atmospheric carbon dioxide concentration as in the case of solar constant change.

1. Introduction

One-dimensional radiative-convective (1-D RC) models which determine the thermal structure of the atmosphere by assuming a balance between the radiative flux and a parameterized convective flux are useful tools for climate studies (cf. Schneider and Dickinson, 1974). Such models, which can include realistic vertical distributions of radiatively important atmospheric constituents, can be used to examine the role that these constituents play in determining the global mean temperature structure (cf. Ramanathan and Coakley, 1978). One major drawback of these models is their lack of the ice albedo-temperature feedback which is of great importance because of the large difference in albedo between ice and ice-free areas (Wetherald and Manabe, 1975).

The original studies based on energy balance models (Budyko, 1969; Sellers, 1969) first indicated that global sensitivity was greatly enhanced by the ice-albedo feedback. More recently, Lian and Cess (1977) used more realistic radiation parameterizations consistent with climatological data for surface temperature and cloud cover, and found a much smaller enhancement of global sensitivity by this feedback. However, the energy balance models, unlike the 1-D RC models, do not explicitly treat the feedback between atmospheric vertical structure and radiation.

In this paper we use a simple parameterization of latitudinal temperature structure suggested by Stone (1978) to develop a simple method of incorporating the ice-albedo feedback in a 1-D RC climate model. We will use the 1-D RC model described by Wang *et al.* (1976) with the ice-albedo feedback included to study the effect of this feedback on the sensitivity of the earth's climate to changes in solar constant. The results will be compared with the sensitivity calculated with a Budyko-Sellers type of energy balance model. We will consider changes in both temperature and ice line as a function of solar constant. We will further study the effect on global surface temperature caused by increases in atmospheric carbon dioxide concentration.

[1] Present affiliation: Atmospheric and Environmental Research, Inc., Cambridge, MA 02139.

0022-4928/80/030545-08$06.00

2. Parameterization of ice-albedo feedback

The 1-D RC climate model described in Wang *et al.* (1976) is used in the present study. In this global mean model, the atmosphere is allowed to reach an equilibrium thermal structure with a time-marching computational procedure. For the assumed atmospheric compositions, cloud structure, global mean surface albedo and initial temperature distribution, the local radiative heating and cooling rates (averaged over the clear and cloudy regions) for solar and thermal radiation are computed at each altitude, with the net heating used to determine the local temperature to be used at the next time step. At any altitude where the computed temperature lapse rate is steeper than a preassigned maximum value (6.5°C km^{-1}) it is assumed that convection occurs with a vertical energy flux just sufficient to yield that preassigned maximum lapse rate. Interaction through the time-marching procedures is continued until energy balance is achieved at each level in the atmosphere. The relative humidity of the atmosphere is kept fixed. Thus, if the temperature increases during climate change, the absolute humidity also increases, causing a substantial positive feedback effect. In the 1-D RC model, we have included high-, middle- and low-level clouds since they have different radiative effects on the surface temperature (Manabe and Wetherald, 1964). Because of the model's inability to predict cloud feedback effects, the 1-D RC model assumes either fixed cloud altitude (FCA) or fixed cloud temperature (FCT) for climate studies. Comparison of empirical studies with model calculations (Cess, 1974, 1975, 1976) leaves open the question of which of these assumptions gives a more realistic parameterization. Without the ice-albedo feedback, the global mean surface albedo is a fixed value throughout the time-marching procedure.

In order to incorporate the ice-albedo feedback in a 1-D RC climate model, it is necessary to know the response of the global mean surface albedo to changes in the ice line as a result of changes in global mean surface temperature. We assume first the zonal mean surface albedo α_s has a simple form

$$\alpha_s = \begin{cases} a, & 0 \leq x < x_s \\ b, & x_s \leq x \leq 1 \end{cases} \tag{1}$$

where a and b are constants characterizing the ice-free and ice-covered surface albedos, respectively, x is the sine of the latitude and x_s the sine of the latitude of the ice-sheet edge. For the ice-sheet edge, the prescription by Budyko (1969) is adopted and slightly modified to better represent current annual mean Northern Hemisphere conditions, i.e., if and only if the zonal mean surface temperature T_s is less than −13°C, ice will be present. We will further use the simple formulation of the latitudinal temperature structure used by North (1975). In this formulation the annual mean latitudinal temperature structure is approximated by the first two Legendre polynomials in a series expansion, i.e.,

$$T_s(x) = \bar{T}_s + T_2 P_2(x), \tag{2}$$

where $P_2(x) = (3x^2 - 1)/2$. The first term $\bar{T}_s$ is the global mean surface temperature since the second term vanishes on integration from 0 to 1. This two-mode approximation yields a reasonably good latitudinal temperature distribution compared with observations (North, 1975).

In the atmosphere, T_2 varies with the tropospheric lapse rate because of the effect of large-scale eddies (Stone, 1978). Since the tropospheric lapse rate is essentially constant in the 1-D RC model, we can adopt the simple baroclinic adjustment parameterization suggested by Stone (1978) for taking into account the effect of the eddies on temperature structure in climate calculations, i.e., T_2 = constant. In fact, in Section 4 we will show that the model sensitivity is only weakly dependent upon the changes in T_2 that occur when different parameterizations are used in an energy balance model. This is because the dominant effect that has to be included in calculating changes in x_s is the latitudinal variation in T_s rather than the change in this variation. This can be shown by setting $x = x_s$, $T_s = -13$°C in Eq. (2) and calculating the relative changes in x_s caused by fractional changes in $\bar{T}_s$ and T_2. We find

$$\frac{T_2 \dfrac{\partial x_s}{\partial T_2}}{\bar{T}_s \dfrac{\partial x_s}{\partial \bar{T}_s}} = P_2(x_s) \frac{T_2}{\bar{T}_s} .$$

Since $T_2 \ll \bar{T}_s$ changes in x_s will be associated primarily with changes in $\bar{T}_s$. In the present study, $T_2 = -32.1$°C. This value is chosen so that $x_s = 0.95$ for the current climate. It is slightly different from the value used by North because of the different criterion adopted for the specification of the ice line. Then again setting $x = x_s$, $T_s = -13$°C in Eq. (2), we obtain

$$x_s = (0.6035 + 0.02078\bar{T}_s)^{1/2}. \tag{3}$$

This simple equation describes how the extent of the polar ice cap changes as the global mean surface temperature (°C) changes if the earth-atmosphere system is perturbed.

Next, we wish to determine the relationship between the global mean surface albedo $\bar{\alpha}_s$ and x_s. By definition

$$\bar{\alpha}_s = \int_0^1 S_s(x,x_s)\alpha_s(x,x_s)dx \Big/ \int_0^1 S_s(x,x_s)dx, \tag{4}$$

where S_s is the zonal mean solar radiation received at the surface. S_s is a complicated function, which depends on solar zenith angle and atmospheric transmissivity. In this study, it is assumed that S_s has the same latitudinal distribution as the solar insolation at the top of the atmosphere, $S(x)$. This approximation yields a global mean surface albedo which differs by only 0.002 from the value computed with the exact S_s. This is because the mean latitudinal changes in cloud cover and atmospheric albedo are small compared to latitudinal changes in $S(x)$. We consider this approximation to be adequate for our purpose. For $S_s(x)$ we use the annual average distribution of $S(x)$ as approximated by North (1975), i.e.,

$$S_s(x) = [1 + S_2P_2(x)]\bar{S}_s, \quad (5)$$

where $\bar{S}_s$ is the averaged value of $S_s(x)$ and $S_2 = -0.482$. Substituting Eqs. (1) and (5) into (4), we obtain the expression

$$\bar{\alpha}_s = [b(1 - x_s) + ax_s] + S_2(b - a)(x_s - x_s^3)/2, \quad (6)$$

where the first term is the geometrical surface albedo. Eqs. (3) and (6) when coupled give a highly nonlinear relation describing the feedback between ice albedo and temperature.

3. Sensitivity of global surface temperature to changes in solar constant

The condition that the current earth-atmosphere system is in a state of global radiative equilibrium, i.e., the net incoming solar radiation is in balance with the outgoing thermal radiation $\bar{F}$ at the top of the atmosphere, may be written as

$$\bar{S}(1 - \bar{\alpha}) = 4\bar{F}, \quad (7)$$

where $\bar{S}$ is the mean solar constant and $\bar{\alpha}$ the global earth-atmosphere albedo. In the calculations, the present value of the solar constant S_0 was taken to be 1365 W m^{-2} and the values of ice-free and ice-covered surface albedos, a and b, were chosen to be 0.087 and 0.55, respectively. The computed characteristics of the global model which correspond to the present climate are given in Table 1.

First we examine the sensitivity of the present global model with and without including the ice-albedo feedback. A convenient measure of the sensitivity of the global climate model is the parameter defined as

$$\beta = S_0 \frac{d\bar{T}_s}{d\bar{S}}\bigg|_{\bar{S}=S_0}. \quad (8)$$

We define β_0 to be the global sensitivity in the absence of the ice-albedo feedback. Values of β_0 and β for the present model and several other climate models are given in Table 2. The most accurate value of β in the energy balance models is probably that given by Lian and Cess (1977), whose calculations included the most realistic parameterization of the ice-albedo feedback. These models implicitly use empirical cloud parameterizations which are equivalent to the FCT parameterization (Cess, 1974, 1975, 1976). The 1-D RC model value of β with the FCT parameterization is in satisfactory agreement with that of Lian and Cess.

In general, the 1-D RC models yield smaller values of β_0 than the energy balance models. To see why, we take a closer look at the differences found between the 1-D RC models and the energy balance models. Combining Eqs. (7) and (8), we can express the global sensitivity in terms of the sensitivity of thermal radiation and global albedo to changes in surface temperature, i.e.,

$$\beta = \frac{\bar{F}}{\dfrac{d\bar{F}}{d\bar{T}_s} + \dfrac{\bar{S}}{4}\dfrac{d\bar{\alpha}}{d\bar{T}_s}}. \quad (9)$$

Consequently, the differences between the models can be studied by comparing the three terms in Eq. (9). However, values of $\bar{F}$ for different models are very close and differ at most by ~1%. Thus, in Table 3, we only present values of the other two terms for different models. We present results from calculations with three different versions of the 1-D RC model. Version I is the one described above, and versions II and III are described below.

The energy balance models fix the feedback be-

TABLE 1. Characteristics of the global model which correspond to present climate conditions.

Parameter	Value
Outgoing thermal radiation $\bar{F}$	236.5 W m^{-2}
Present-day solar constant S_0	1365 W m^{-2}
Sine of the latitude of the ice-sheet edge x_s	0.95
Global surface albedo $\bar{\alpha}_s$	0.10
Global earth-atmosphere albedo $\bar{\alpha}$	0.307
Global mean surface temperature $\bar{T}_s$	14.39°C

TABLE 2. Comparison of the global sensitivity parameters β_0 and β for several climate models.

Model	β_0 (°C)	β (°C)
A. Energy balance models		
Budyko (1969)	155	400
Sellers (1969)	150	326
Lian and Cess (1977)	147	184
Coakley (1979)	152	207
B. Radiative-convective models		
Wetherald and Manabe (1975) FCA*	114	
Present model		
FCA*	110	138
FCT*	135	188

* FCA and FCT denote fixed cloud altitude and fixed cloud temperature, respectively.

TABLE 3. Comparison of the model sensitivity of outgoing thermal radiation $\bar{F}$ (W m^{-2}) and global earth-atmosphere albedo $\bar{\alpha}$ to changes in surface temperature $\bar{T}_s$ (°C) for several climate models. For the present 1-D RC models, model version I is the one-zone global model while model versions II and III are based on a three-zone global model. Version II differs from version I in the sense that it includes the latitudinal variation of solar zenith angle. Version III includes latitudinal variations of both solar zenith angle and cloud cover.

Model	No ice-albedo feedback			Ice-albedo feedback		
	$\frac{d\bar{F}}{d\bar{T}_s}$	$\frac{\bar{S}}{4}\frac{d\bar{\alpha}}{d\bar{T}_s}$	β_0 (°C)	$\frac{d\bar{F}}{d\bar{T}_s}$	$\frac{\bar{S}}{4}\frac{d\bar{\alpha}}{d\bar{T}_s}$	β (°C)
A. Energy balance models						
Budyko (1969)	1.45	0	155	1.45	−0.87	400
Lian and Cess (1977)	1.63	0	147	1.63	−0.35	184
Coakley (1979)	1.55	0	152	1.55	−0.41	207
B. Present 1-D RC models (FCT)						
I	1.70	0.04	135	1.63	−0.37	188
II	1.66	0.06	139	1.61	−0.33	186
III	1.60	0.06	144	1.55	−0.26	185

tween $\bar{F}$ and $\bar{T}_s$ at a constant value, regardless of whether the ice-albedo feedback is present or not. However, in the 1-D RC model, this feedback is calculated explicitly by taking into account the changes in the thermal structure due to changes in surface temperature. The changes in the thermal structure are different when the surface albedo changes. Consequently, the strength of this feedback changes when the ice-albedo feedback is included. This accounts for most of the difference between the values of β_0. The remaining difference is due to a small negative feedback between global albedo and surface temperature resulting from noncompensating changes in both the cloud heights and the atmospheric water vapor absorption. These effects are not included in the energy balance models.

The good agreement between the Lian and Cess value of β and the 1-D RC model value is somewhat surprising. The latter model does not allow for latitudinal variations in mean solar zenith angle or in cloud cover, which could be expected to decrease β since these effects tend to shield surface albedos more in high latitudes than in low latitudes. To examine these effects we have made radiation calculations in the 1-D RC model with the earth's atmosphere divided into three equal-latitude zones (three-zone global model). In each zone the annual mean zonally averaged incoming solar radiation and the annual mean effective solar zenith angle are used for radiation calculations. At each time step, the global mean radiation fluxes which are used to calculate temperature are obtained by integrating over the values for the different zones.

First the solar zenith angle effect is studied. For this case we assume that the cloud characteristics, i.e., cloud cover and cloud vertical distribution in each zone, are the same as in the one-zone global model. Values of β_0 and β, along with the thermal radiation and global albedo sensitivity to surface temperature for this model (model version II), are given in Table 3. Next we study the combined

TABLE 4. Comparison of the global sensitivity to changes in solar constant between energy balance model and present 1-D RC model. For the energy balance model, results are obtained based on same feedbacks between surface temperature and both thermal radiation and global albedo as computed in the 1-D RC model. However, the water vapor feedback described in the text is not included in the energy balance model. *DA* and *BA* denote diffusive approximation and baroclinic adjustment parameterization, respectively.

Model	No ice-albedo feedback			Ice-albedo feedback		
	$\frac{d\bar{F}}{d\bar{T}_s}$	$\frac{\bar{S}}{4}\frac{d\bar{\alpha}}{d\bar{T}_s}$	β_0 (°C)	$\frac{d\bar{F}}{d\bar{T}_s}$	$\frac{S}{4}\frac{d\bar{\alpha}}{d\bar{T}_s}$	β (°C)
A. Energy balance model						
FCT *DA*	1.63	0	145	1.63	−0.50	209
FCT *BA*	1.63	0	145	1.63	−0.41	193
FCA *DA*	2.36	0	100	2.36	−0.45	124
FCA *BA*	2.36	0	100	2.36	−0.41	121
B. Present 1-D RC model						
FCT	1.70	0.04	135	1.63	−0.37	188
FCA	2.39	−0.24	110	2.36	−0.65	138

effects of solar zenith angle and high-latitude cloud shielding. Using the values of latitudinal cloud cover given by Cess (1976), we calculate the values of β and β_0 (model version III). For both models β hardly changes, because of compensating changes in sensitivity of thermal radiation and global albedo. We conclude that latitudinal variations in solar zenith angle and cloud cover have a negligible effect on global climate sensitivity. On the other hand, in the absence of the ice-albedo feedback there is nothing to compensate for the changes in the sensitivity of the thermal radiation, so that the values of β_0 do change. In fact, in model III, β_0 and β now agree well with those of Lian and Cess. For the FCA parameterization values of β_0 and β for model III are 113 and 133°C, respectively.

4. An "equivalent" energy balance model

With Eq. (3) it is possible to calculate from the 1-D RC model results the changes in position of the edge of the polar ice cap implied by the calculated changes in $\bar{T}_s$. We will compare these changes with changes calculated from an equivalent energy balance model, and we will take advantage of the same calculations to evaluate the sensitivity of the results to the use of the baroclinic adjustment parameterization. The energy balance model used for the calculations is North's (1975) two-mode version of Budyko's (1969) model. The model will be solved both with the linear diffusion parameterization used by North (1975) and with the baroclinic adjustment parameterization. The necessary modification of North's equations in the latter case is given by Stone (1978).

In order to make the energy balance model "equivalent" to the 1-D RC model, the parameter values were chosen so that the feedbacks included in both models had the same strengths for current climate conditions. The thermal radiation is parameterized by

$$\bar{F} = A + B\bar{T}_s. \tag{10}$$

The value of B was taken to be the value of $d\bar{F}/d\bar{T}_s$ calculated by version I of the 1-D RC model (values given in Tables 3 and 4). A was then chosen to give the value of $\bar{F}$ calculated by the 1-D RC model as given in Table 1.

The zonal earth-atmosphere albedo in the energy balance model is parameterized by

$$\alpha = \alpha_0 + \alpha_2 P_2(x) + \delta u(x - x_s), \tag{11}$$

where

$$u(x - x_s) = \begin{cases} 1, & x > x_s \\ 0, & x < x_s. \end{cases} \tag{12}$$

The feedback between ice albedo and temperature is dependent solely on δ and its value was chosen to match the feedback given by the 1-D RC model. In particular, for the latter model we can write

$$\frac{d\bar{\alpha}}{d\bar{T}_s} = \frac{\partial\bar{\alpha}}{\partial x_s}\frac{dx_s}{d\bar{T}_s} + \frac{\partial\bar{\alpha}}{\partial\bar{T}_s}. \tag{13}$$

The first term on the right represents the ice-albedo feedback mechanism, while the second term represents the global albedo-temperature feedback in the absence of the former. The values of the latter term correspond to the values of $d\bar{\alpha}/d\bar{T}_s$ given in Tables 3 and 4 in the absence of ice-albedo feedback. The values of $d\bar{\alpha}/d\bar{T}_s$ correspond to the values given in Tables 3 and 4 in the presence of ice-albedo feedback. The value of $dx_s/d\bar{T}_s$ for $x_s = 0.95$, calculated from Eq. (3), is 0.0110 (°C)$^{-1}$. Thus Eq. (13) can be used to calculate $\partial\bar{\alpha}/\partial x_s$ from the 1-D RC model results. For the energy balance model, from Eq. (12), we calculate

$$\bar{\alpha}(\delta) = \int_0^1 (1 + S_2P_2)(\alpha_0 + \alpha_2P_2 + \delta u)dx. \tag{14}$$

For $x_s = 0.95$, $S_2 = -0.482$, we find

$$\frac{\partial\bar{\alpha}}{\partial x_s} = -0.588\delta. \tag{15}$$

From this equation and the 1-D RC model results we find that the equivalent value of δ is 0.186. With δ known α_0 and α_2 are picked jointly so that $\bar{\alpha} = 0.307$ and $\bar{\alpha}_2$, the second moment of the albedo, is 0.480. $\bar{\alpha}_2$ is related to α_0 and α_2 by

$$\bar{\alpha}_2 = \int_0^1 (1 + S_2P_2)P_2(\alpha_0 + \alpha_2P_2 + \delta u)dx. \tag{16}$$

The exact value of $\bar{\alpha}_2$, 0.480, was found by using the annual mean zonal albedo and incident solar radiation distribution given by Ellis and Vonder Haar (1976). The resulting values are $\alpha_0 = 0.316$ and $\alpha_2 = 0.146$. Finally, the dynamical diffusion coefficient is calculated so that $T_s = -13$°C at $x_s = 0.95$ for the current value of the solar constant.

Values of β_0 and β for the 1-D RC model (version I) and the equivalent energy balance model are presented in Table 4, for both the FCT and FCA parameterization. Comparison between the results of the 1-D RC model and the results of the energy balance model with the baroclinic adjustment parameterization shows some differences, primarily because the latter does not include changes in global albedo due to changes both in cloud heights and atmospheric water vapor absorption. However, these differences are not large. For example, for the FCT parameterization, the values of β given by the two models differ by only 5°C (3%). We conclude that the energy balance models and the 1-D RC models are not inherently different, if the same physical mechanisms are included in each model.

Comparison between the results of the energy balance model with the baroclinic adjustment param-

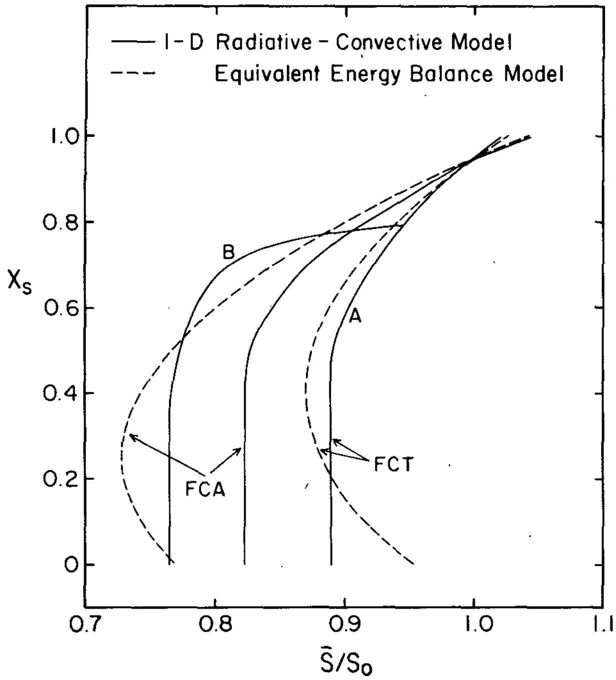

FIG. 1. Position of the edge of the polar ice-cap versus solar constant. In the one-dimensional radiative-convective model, as the solar constant is decreased, a point is reached at which the polar ice-cap becomes unstable and will grow spontaneously until it covers the whole earth. For the FCT parameterization, case A refers to clouds staying adjacent to surface after low-cloud base reaches surface at $\bar{S}/S_0 = 0.945$, while case B assumes that cloud dissipates. In the equivalent energy balance model, the dynamical transports are parameterized by the baroclinic adjustment process.

eterization and the results with the linear diffusion parameterization (Table 4) verifies that the results are not strongly dependent on the dynamical parameterization, i.e., on the neglect of variations in T_2. For example, the two values of β for the FCT parameterization differ by only 16°C (8%). In view of the fact that observations show that seasonal changes in the dynamical flux in midlatitudes are much larger than seasonal changes in the surface temperature gradient, we would in any case expect the baroclinic adjustment parameterization to be the more accurate of the two. For example, the best fit to the seasonal changes has the flux being proportional to the cube of the surface temperature gradient rather than to the first power (Stone and Miller, 1980).

5. Equilibrium ice line

Using the 1-D RC model (version I) with the ice-albedo feedback included, we calculate the changes in ice line as solar constant changes. The results are illustrated in Fig. 1. The results from the equivalent energy balance model are included for comparison. When the FCA parameterization is used, in both models a point is reached at which the ice cap is unstable and will grow spontaneously until it covers the whole earth. The unstable equilibrium state for the equivalent energy balance model can be calculated; it corresponds to that part of the curve of x_s vs S/S_0 for which $dx_s/d(S/S_0) < 0$. The corresponding unstable state in the 1-D RC model cannot be calculated because of the use of a time-marching procedure. Consequently the results from the 1-D RC model are plotted with the unstable transitions indicated by vertical lines. Large differences between the 1-D RC model and the equivalent energy balance model with the FCA parameterization occur for smaller values of the solar constant. These differences are caused primarily by the nonlinear feedback between the surface temperature and thermal radiation which are included in the former model but not in the latter.

The results with the FCT parameterization are also shown in Fig. 1. In this parameterization, as the surface temperature is decreased by the decrease in solar constant, there is a simultaneous decrease in cloud altitudes which further enhances the surface cooling (Lacis *et al.*, 1979). This positive feedback due to cloud altitude changes in the FCT parameterization yields a less stable climate as compared to the FCA parameterization. With the cloud altitudes continuing to decrease, the low-cloud base in the present 1-D RC model reaches the surface when the solar constant is reduced by ~5.5%. Once the cloud base reaches the planetary boundary layer, it is a matter of speculation as to what would happen next. In this study, we consider two extreme possibilities. In the first (case A) the cloud is assumed to be trapped inside the boundary layer, staying next to the surface with physical properties remaining unchanged. In the mean time, the middle and high level clouds continue to decrease until the middle-cloud base touches the top of the low cloud. After that a similar treatment is used for the middle cloud, and also for the high cloud if the high-cloud base reaches the top of the middle cloud. As we will see this parameterization mimics the behavior of the energy balance models which do not include clouds explicitly. The other case we will consider is motivated by the expectation that the liquid water content of the low-level clouds will decrease as the surface temperature decreases. In this case (case B), after the cloud base touches the surface, we assume that the cloud optical thickness decreases in proportion to the decrease of physical thickness, as the cloud top moves lower. Thus this case includes a negative feedback between global surface temperature and global albedo which will tend to offset the effect of the decrease in solar constant and prevent a completely ice-covered earth. In both cases, the cloud temperatures are not kept fixed, but change according to changes in surface temperature when the cloud reaches either the surface or another cloud. Cloud covers are unchanged. We do not propose that either of these two extremes are physically correct, but present the results solely for the purpose of illustrating the possible importance of cloud feedback in climate studies.

Fig. 1 depicts the calculations for the two cases. For case A, the qualitative behavior is similar to that found with the FCA parameterization. The ice cap becomes unstable and a completely ice-covered earth can occur if the solar constant is reduced by ~11%. This value is much smaller than the value obtained by using the FCA parameterization, but is close to the value of ~9% calculated by Coakley (1979) with an energy balance model which has an implicit FCT parameterization. For case B, after the low cloud touches the surface, further decreases in solar constant will decrease the surface temperature or ice line at a much slower pace as compared to case A simply because there is an additional negative feedback induced by the gradual removal of low cloud albedo. A completely ice-covered earth exists when solar constant is reduced by more than 23%. This value is more than twice as large as the reduction found for case A. These results indicate that cloud feedback may play an important role in climate change.

6. Sensitivity of global surface temperature to changes in atmospheric carbon dioxide concentration

Because of the importance of carbon dioxide in atmospheric radiative transfer and its role in global climate (Manabe and Wetherald, 1975), we also use the 1-D RC model (version I) to examine how doubling the present-day CO_2 concentration of the atmosphere affects global surface temperature when the ice-albedo feedback is included. At the same time we can determine how the result is affected by the parameterization of cloud altitudes.

Table 5 shows the change in $\bar{T}_s$ calculated with the present model when the nominal concentration of CO_2, 330 ppmv, is doubled. For comparison we include results from Manabe and Wetherald's (1967, 1975) calculations with a 1-D RC model. Their 1975 calculations had a more accurate radiation scheme than the 1967 calculation. Since their model had a nominal CO_2 concentration of 300 ppmv, the calculations with the present model were redone with this nominal concentration and the FCA parameterization as the Manabe and Wetherald model. The two models are in excellent agreement.

Table 5 also shows that the ice-albedo feedback in the present model enhances the surface temperature increase by 26% when the FCA parameterization is used. This appears to be in good agreement with Manabe and Wetherald's (1975) results. They

TABLE 5. Change in $\bar{T}_s$ (°C) caused by doubling the atmospheric carbon dioxide concentration in the 1-D RC models.

	No ice-albedo feedback		Ice-albedo feedback	
Model	FCA	FCT	FCA	FCT
A. 300 → 600 ppmv				
Manabe and Wetherald (1967)	2.36		2.93*	
Manabe and Wetherald (1975)	1.95			
Present model	1.95			
B. 330 → 660 ppmv				
Present model	2.00	3.00	2.51	4.20

* Calculated with a simplified three-dimensional general circulation model (Manabe and Wetherald, 1975), but with the radiation scheme of Manabe and Wetherald (1967).

found that the increase in $\bar{T}_s$ was 24% greater in their simplified three-dimensional general circulation model (which included ice-albedo feedback) than in their 1-D RC model (which did not include this feedback). However, the parameterization of ice-albedo feedback in the two models is quite different—e.g., the values of T_2 and of the critical T_s for ice formation are quite different—and so this agreement may be coincidental. Table 5 also shows that the ice-albedo feedback enhances the increase of surface temperature even more when the FCT parameterization is used, by 40% instead of 26%.

7. Conclusions

A simple method of incorporating the ice-albedo feedback in a one-dimensional radiative-convective model is presented. Using this model, we examine the global sensitivity of surface temperature to changes in incoming solar radiation with and without including the ice-albedo feedback. It is found that the amplification in global sensitivity due to ice-albedo feedback is 26 and 39%, respectively, for fixed cloud altitude and fixed cloud temperature parameterizations. The inclusion of latitudinal variations of solar zenith angle and cloudiness has very little effect on the global sensitivity with ice-albedo feedback included, because of compensatory changes in thermal radiation and global albedo. The global sensitivity for the fixed cloud temperature assumption agrees well with results obtained by Lian and Cess (1977).

The differences in global sensitivity between one-dimensional radiative-convective models and energy balance models are also studied. We find that the models are in good agreement if the feedbacks between surface temperature and global albedo and thermal radiation are included with similar strengths. We also find that the method for including the ice-albedo feedback in the 1-D RC model is not sensitive to the dynamical parameterization used.

The 1-D RC model is used to calculate the equilibrium ice line as a function of the solar constant. With the fixed cloud altitude parameterization this model is less stable than the equivalent energy balance model, because of the nonlinear feedback between surface temperature and thermal radiation. The fixed cloud temperature parameterization breaks down for decreases in solar constant $\geqslant 5\%$. The behavior for larger decreases depends on the assumed cloud feedbacks. For example, if the cloud is allowed to dissipate when it reaches the surface, the ice line stays at about ~40°N even with solar constant reduced by 20%. This is caused by the large negative feedback induced by the removal of low cloud albedo. These results suggest that proper treatment of cloud feedback in the climate model is crucial for climate studies.

We have also used the model to study the effect on surface temperature if atmospheric carbon dioxide concentration is increased by a factor of 2. The enhancement in surface temperature increase due to the ice-albedo feedback is 26 and 40%, respectively, when fixed cloud altitude and fixed cloud temperature are assumed. It is interesting to note that the amplification effect is the same as the case when the solar constant is perturbed.

Finally we conclude that the ice-albedo feedback can be included in a one-dimensional radiative-convective climate model in a realistic way. This addition represents a significant improvement in the ability of such models to simulate the effects of radiative perturbations on climate.

Acknowledgment. We thank J. E. Hansen for useful discussion. We are also indebted to one of the anonymous reviewers of the paper for several suggestions which have clarified the presentation.

REFERENCES

Budyko, M. I., 1969: The effect of solar radiation variations on the climate of the earth. *Tellus,* **21,** 611–619.

Cess, R. D., 1974: Radiative transfer due to atmospheric water vapor: global considerations of the earth's energy balance. *J. Quant. Spectros. Radiat. Transfer,* **14,** 861–871.

——, 1975: Global climate change: an investigation of atmospheric feedback mechanisms. *Tellus,* **27,** 193–198.

——, 1976: Climate change: an appraisal of atmospheric feedback mechanisms employing zonal climatology. *J. Atmos. Sci.,* **33,** 1831–1843.

Coakley, J. A., 1979: A study of climate sensitivity using a simple energy balance model. *J. Atmos. Sci.,* **36,** 260–269.

Ellis, J., and T. H. Vonder Haar, 1976: Zonal average earth radiation budget measurements from satellites for climate studies. Atmos. Sci. Pap. 240, Colorado State University, Fort Collins.

Lacis, A. A., W. C. Wang and J. E. Hansen, 1979: Correlated *K*-distributon method for radiative transfer in climate models: Application to effect of cirrus cloud on climate. NASA Publ. 2076, E. R. Kreins, Ed., 416 pp.

Lian, M. S., and R. D. Cess, 1977: Energy balance climate models: A reappraisal of ice-albedo feedback. *J. Atmos. Sci.,* **34,** 1058–1062.

Manabe, S., and R. F. Strickler, 1964: Thermal equilibrium of the atmosphere with a convective adjustment. *J. Atmos. Sci.,* **21,** 361–385.

——, and R. T. Wetherald, 1967: Thermal equilibrium of the atmosphere with a given distribution of relative humidity. *J. Atmos. Sci.,* **24,** 241–259.

——, and ——, 1975: The effects of doubling the CO_2 concentration on the climate of a general circulation model. *J. Atmos. Sci.,* **32,** 3–15.

North, G. R., 1975: Theory of energy-balance climate models. *J. Atmos. Sci.,* **32,** 2033–2043.

Ramanathan, V., and J. A. Coakley, 1978: Climate modeling through radiative-convective models. *Rev. Geophys. Space Phys.,* **16,** 465–489.

Schneider, S. H., and R. E. Dickinson, 1974: Climate modeling. *Rev. Geophys. Space Phys.,* **12,** 447–493.

Sellers, W. D., 1969: A global climate model based on the energy balance of the earth-atmosphere system. *J. Appl. Meteor.,* **8,** 392–400.

Stone, P. H., 1978: Baroclinic adjustment. *J. Atmos. Sci.,* **35,** 561–571.

Wang, W. C., and D. A. Miller, 1980: Empirical relations between seasonal changes in meridional temperature gradients and meridional fluxes of heat. Submitted to *J. Atmos. Sci.*

——, Y. L. Yung, A. A. Lacis, T. Mo and J. E. Hansen, 1976: Greenhouse effects due to man-made perturbations of trace gases. *Science,* **194,** 685–690.

Wetherald, R. T., and S. Manabe, 1975: The effects of changing the solar constant on the climate of a general circulation model. *J. Atmos. Sci.,* **32,** 2044–2059.