The Transport of Trace Chemicals by Planetary Waves in the Stratosphere. Part 1: Steady Waves

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ABSTRACT

When dissipation is present a forced planetary wave will interact with the zonal flow in the stratosphere and cause acceleration of the zonal wind, mean-zonal diabatic heating and net transport of trace chemicals. Quasi-geostrophic scaling is used to derive expressions relating the above quantities to each other and to wave statistics for steady-state conditions and Newtonian cooling.

Numerical calculations are presented for stationary wavenumber 1 which indicate that in the stratosphere the effect of the wave on the mean-zonal quantities is 1/5 to 1/20 that required by observations and models. In the mesosphere the calculated value is roughly 1/20 the required size. Reasons for this discrepancy are discussed.

1. Introduction

It has been recognized for many years that the distribution of trace gases in the stratosphere depends on dynamics as well as on chemical sources and sinks. The realization that some anthropogenic trace chemicals may threaten the earth's ozone shield has heightened interest in stratospheric trace chemicals, but most research has been devoted to the chemical rather than the dynamical aspects of the problem. This emphasis is mainly due to the complexity of the transport process, particularly when coupled with photochemistry.

No attempt will be made here to summarize the literature on the transport of trace chemicals in the stratosphere. The reader is referred to papers by Dunkerton (1978), Mahlman and Moxim (1978) and Holton (1980) for discussion and references on this subject. Discussion in this paper will be limited to the relationship between current thinking and the research to be described.

Unlike the troposphere, which is characterized by a large spectrum of waves, the extratropical stratospheric circulation is dominated by long (wavenumbers < 4) planetary-Rossby waves which owe their existence mainly to thermal and orographic forcing in the troposphere and can be assumed to be linear and inviscid to first approximation (Holton, 1975). The dynamical importance of these waves and the accompanying mean meridional circulation suggests an equal importance in the transport of trace chemicals.

The basic framework for the study of the effects of these planetary waves on the zonal-mean state is the non-acceleration theorem (Charney and Drazin, 1961; Boyd, 1976; Andrews and McIntyre, 1976) which states that linear, dissipationless waves with nonzero Doppler-shifted frequency will have no permanent effect on the mean-zonal wind and temperature. There are several important implications of the theorem for the present research. First, the non-acceleration theorem applies to temporary (i.e., the wave amplitude returns to zero after passage of the wave) as well as steady (constant amplitude) waves (Andrews and McIntyre, 1978). The latter case will be considered in this paper and the former in Part II. Second, although the theorem has been most often applied to the non-acceleration of the mean-zonal (zonally averaged) wind by the waves, it also applies to the mean-zonal temperature (by the thermal wind law) and can be extended to show that waves satisfying the above conditions will have no permanent effect on the mean-zonal trace chemical mixing ratio as well. Third, since an eddy flux of heat and trace chemicals will in general occur concurrently with the vertical propagation of planetary waves the theorem implies the existence of a wave-induced mean-meridional circulation whose transport exactly cancels that from the waves. The tendency for the effects of the wave and mean motions to cancel each other has also been observed in tracer studies with general circulation models (Hunt and Manabe, 1968; Mahlman and Moxim, 1978) which implies that the non-acceleration

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theorem should at least be the starting point for the study of trace chemical transport. It also underscores the need to include the coupling between planetary waves and the zonal flow when modeling transport. This last implication is important because it suggests that net transport will tend to be a small difference between large and opposing contributions from the eddies and mean motions.

As the above statements suggest, when the non-acceleration theorem is valid there will be no acceleration of the zonal flow by the waves, no wave-induced mean-zonal diabatic heating and no net transport of inert trace chemicals. On the other hand, when one or more of the conditions of the theorem are violated we can expect acceleration of the zonal flow, mean diabatic heating and tracer transport to occur together.

One may consider violations of the non-acceleration theorem in terms of the physical mechanism involved (e.g., radiative, photochemical or viscous damping; critical level formation; flow instability) or in terms of wave type (e.g., planetary-Rossby, mixed Rossby-gravity, gravity, tidal, etc.). The obvious complexity of transport requires the application of general circulation models to obtain more accurate solutions to the problem. These models, however, are expensive to operate and difficult to interpret, which suggests the use of simpler models to gain insight into the mechanisms involved. Steady-state models are particularly useful in this regard because simple analytic relationships can be derived between wave forcing, diabatic heating and net transport. In this paper the simplest possible violation of the non-acceleration theorem will be considered, i.e., radiative dissipation of the wave by Newtonian cooling. We shall assume quasi-geostrophic scaling and steady-state conditions. A key assumption is that the *net* acceleration of the zonal flow is zero, i.e., that the accelerations caused by eddy radiative dissipation and (indirectly) by mean-zonal diabatic heating are balanced. Thus, we shall refer to the acceleration of the mean flow by the dissipating wave in the context of an assumed balance with a mean-zonal diabatic forcing.

It can be argued that the abovementioned balance is also present in the real stratosphere for time scales $\gtrsim$ 2 weeks. This conclusion is consistent with the relatively short radiative relaxation time in the stratosphere (20 days at 15 km, decreasing to 3 days at 50 km) compared with the seasonal time scale (~90 days). Furthermore, as will be shown in this paper, the calculated mean-flow acceleration (e.g., -0.5 m s^{-1} day^{-1} at 50 km) is large enough to cause the observed seasonal change in the zonal wind but much smaller than that believed to be present in the real stratosphere. This suggests that the mean state represents an approximate balance between large opposing dissipative forces in the eddies and mean-zonal state. Moreover, the above argument also implies that the mean-zonal diabatic heating is largely the *result* of eddy dissipation. This viewpoint is in contrast to the conventional one in which the mean-zonal diabatic heating is assumed to be *externally* imposed and to then "drive" equator-to-pole and summer-to-winter circulations. The extent to which eddy and mean-zonal dissipation are balanced in the real atmosphere is of interest but not central to the present study. What is essential is the assumption that the magnitude of the change in the thermodynamic and kinetic mean-zonal state is proportional to the nonconservative wave-mean flow interaction taking place.

We have applied the results to the special case of stationary wavenumber 1 in the winter stratosphere and, in particular, determined the acceleration of the zonal wind, mean-zonal diabatic heating and net transport of an inert ozone-like tracer for a wave with dissipation by Newtonian cooling.

The purposes of this paper are twofold: first, to relate zonal-flow forcing, mean-zonal diabatic heating and net tracer transport to wave properties and Newtonian cooling; second, to estimate the contribution from steady-state wavenumber 1 to the total observed forcing, diabatic heating and tracer transport.

2. Acceleration of the zonal flow

In this section we shall derive the acceleration of the mean-zonal flow by a radiatively dissipated planetary wave. It should be emphasized that this acceleration is not simply a function of the eddy momentum flux convergence since the latter quantity will, if the non-acceleration theorem is applicable, be cancelled by an acceleration due to a north-south mean flow Coriolis torque. Rather, it is an acceleration which depends explicitly on the wave dissipation, i.e., it would be zero if the flow were adiabatic. Furthermore, it has been shown (e.g., Dickinson, 1969) that this mean-flow acceleration occurs concurrently with a nonzero eddy flux of potential vorticity—a result which will be evident in the discussion to follow.

We shall first derive the acceleration of the mean flow in terms of the eddy potential vorticity flux and the mean-zonal diabatic heating and then use the eddy perturbation equations to express the former in terms of the wave properties and Newtonian cooling rate. The presentation follows that of Holton (1974) except that diabatic effects are included and $z = -H \ln p/p_0$ coordinates are employed.

The mean-zonal momentum, thermodynamic and continuity equations are

$$\bar{u}_t = f\bar{v} - (\overline{u_0'v_0'})_y, \tag{1}$$

$$\bar{\phi}_{zt} + (\overline{v_0'\phi_{0z}'})_y + S\bar{w} + \bar{Q} = 0, \tag{2}$$

$$\bar{v}_y + \bar{w}_z - \bar{w}H^{-1} = 0. \quad (3)$$

Even though we shall assume steady-state conditions, the time-dependent terms $\bar{u}_t$ and $\bar{\phi}_{zt}$ also have been included in (1) and (2) so that in the analysis to follow the dependence of mean-zonal quantities on dissipation terms can be shown explicitly.

The eddy perturbation equations are

$$\bar{u}u'_{0x} + \bar{u}_y v_0' - fv_1' + \phi'_{1x} = 0, \quad (4)$$

$$\bar{u}v'_{0x} + fu_1' + \phi'_{1y} = 0, \quad (5)$$

$$\alpha\phi'_{0z} + \bar{u}\phi'_{0zx} - f\bar{u}_z v_0' + Sw_1' = 0, \quad (6)$$

$$u'_{1x} + v'_{1y} + w'_{1z} - w_1'H^{-1} = 0, \quad (7)$$

where the subscripts 0 and 1 denote zeroth and first order in Rossby number perturbation quantities, and

$$fv_0' = \phi'_{0x},\ fu_0' = -\phi'_{0y},\ w_0' = 0. \quad (8)$$

The geopotential ϕ is related to temperature by $\partial\phi/\partial z = RT/H$, where H is the scale height. The static stability S is given by

$$S = \frac{R}{H}\left(\frac{dT}{dz} + \frac{KT}{H}\right).$$

If we define density as

$$\rho_s = \rho_0 \exp(-z/H), \quad (9)$$

then $\bar{w}$ and $\bar{v}$ can be eliminated from (1)–(3) to yield

$$\bar{u}_t - \frac{f}{\rho_s}\int\left(\frac{\rho_s}{S}\bar{\phi}_{zt}\right)_z dy = -(\overline{u_0'v_0'})_y + \frac{f}{\rho_s}\left(\frac{\rho_s}{S}\overline{v'\phi'_{0z}}\right)_z + \frac{f}{\rho_s}\left(\frac{\rho_s}{S}\int\bar{Q}dy\right)_z = \overline{v_0'q_0'} + \frac{f}{\rho_s}\left(\frac{\rho_s}{S}\int\bar{Q}dy\right)_z, \quad (10)$$

where $\overline{v_0'q_0'}$ is the eddy flux of potential vorticity. Eq. (10) when differentiated with respect to y is the usual mean-zonal potential vorticity equation (Holton, 1975, pp. 50–51) except that in (10) the Coriolis parameter f has not been replaced by a constant value f_0. For adiabatic conditions stationarity of the flow requires that the potential vorticity flux equal zero (Dickinson, 1969; Holton, 1974).

The potential vorticity flux can be related to eddy dissipation by means of the wave energy equation which is obtained by multiplying (4), (5), (6) and (7) by u_0', v_0', ϕ'_{0z}/S and ϕ_0' respectively, zonally averaging and summing the equations:

$$(\bar{u}\overline{u_0'v_0'} + \overline{v_1'\phi_0'} + \overline{v_0'\phi_1'})_y + \frac{1}{\rho_s}\left(\rho_s\overline{w_1'\phi_0'} - \frac{\overline{f\bar{u}v_0'\phi'_{0z}}}{S}\right)_z + \frac{\alpha}{S}\overline{\phi'^2_{0z}} = \bar{u}\left[(\overline{u_0'v_0'})_y - f\left(\frac{\overline{v_0'\phi'_{0z}}}{S}\right)_z\right]. \quad (11)$$

When (4) and (6) are multiplied by ϕ_0' and zonally averaged, we have

$$\bar{u}\overline{u_0'v_0'} + \overline{v_1'\phi_0'} + \overline{v_0'\phi_1'} = 0, \quad (12)$$

$$\alpha\overline{\phi'_{0z}\phi_0'} - f\bar{u}\overline{v_0'\phi'_{0z}} + S\overline{w_1'\phi_0'} = 0. \quad (13)$$

Substituting (12) and (13) into (11) yields

$$\frac{\alpha\overline{\phi'^2_{0z}}}{S} - \frac{1}{\rho_s}\left(\frac{\alpha\rho_s}{S}\overline{\phi_0'\phi'_{0z}}\right)_z = \bar{u}\left[(\overline{u_0'v_0'})_y - \frac{f}{\rho_s}\left(\frac{\rho_s}{S}\overline{v_0'\phi'_{0z}}\right)_z\right] = -\bar{u}\overline{v_0'q_0'}. \quad (14)$$

Eq. (14) is equivalent to that of Dickinson [1969, Eq. (19)] although the derivation is different. When the Newtonian cooling is zero the potential vorticity flux will vanish and no acceleration of the zonal flow will result. On the other hand, when radiative dissipation is present the wave will decelerate the zonal flow with a rate which depends on the temperature variance and vertical gradient of the correlation between temperature and geopotential. Eqs. (1) and (14) can be regarded as diagnostic relationships and can be used to determine the acceleration of the zonal flow which results from dissipating planetary waves or indirectly from mean-zonal diabatic heating. The left side of (14) is particularly easy to evaluate since it requires wave structure information at only one latitude.

The acceleration of the mean flow can be found by differentiating (10) twice with respect to y, i.e.,

$$\bar{u}_{yyt} - \frac{2}{\rho_s}\beta\left(\frac{\rho_s}{S}\bar{\phi}_{zt}\right)_z - \frac{f}{\rho_s}\left(\frac{\rho_s}{S}\bar{\phi}_{zt}\right)_{yz} - \frac{1}{\rho_s}\beta_y\int\left(\frac{\rho_s}{S}\bar{\phi}_{zt}\right)_z dy = (\overline{v_0'q_0'})_{yy} + \left[\frac{f}{\rho_s}\left(\frac{\rho_s}{S}\int\bar{Q}dy\right)_z\right]_{yy}, \quad (15)$$

where $\beta = \partial f/\partial y$.

To find the contribution to the total acceleration from the dissipating wave we set $\bar{Q}$ in (15) to zero, neglect the term containing the second derivative of f, and use $\bar{\phi}_y = -f\bar{u}$, yielding

$$\bar{u}_{yyt} + \frac{f^2}{S}\left(\bar{u}_{zzt} - \frac{\bar{u}_{zt}}{H}\right) - \frac{2\beta}{f}\bar{u}_{yt} + 2\frac{\beta^2}{f^2}\bar{u}_t = (\overline{v_0'q_0'})_{yy} - \frac{2\beta}{f}\left[(\overline{v_0'q_0'})_y - \frac{\beta}{f}\overline{v_0'q_0'}\right], \quad (16)$$

where Eq. (10) and $\partial/\partial y$ [Eq. (10)] with $\bar{Q}$ set to zero have been used to replace terms in (15).

The wave forcing can be found with (14) and (16) if solutions to the wave propagation equations are available for a given zonal wind distribution $\bar{u}$, static

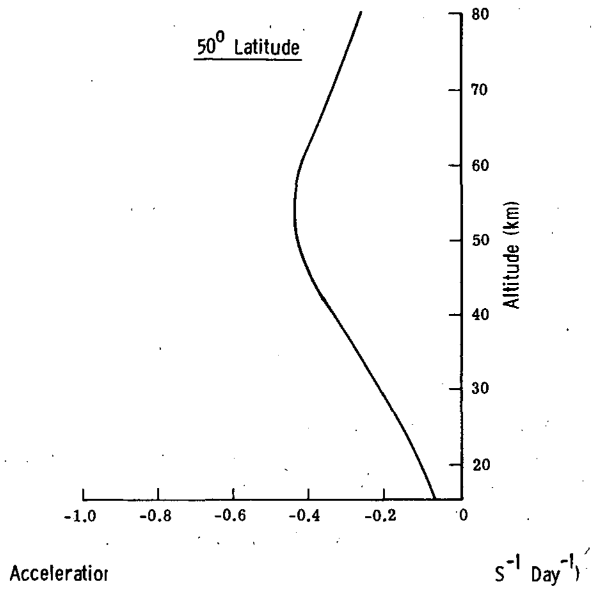

FIG. 1. Acceleration of the mean-zonal wind at 50°N by the dissipating planetary wave. Units: m s^{-1} day^{-1}.

stability S, and Newtonian cooling coefficient α. For this purpose we have used the wave structure results from the theoretical study of Schoeberl *et al.* (1979) [see also Schoeberl and Geller (1977)]. Model results have two advantages over observed data for our purpose; information is available to much higher altitudes than with observational data and the dissipation parameters (Newtonian cooling and Rayleigh friction) used in the calculation are known exactly. It should be noted that Eqs. (4)–(7) include wave dissipation only in the form of Newtonian cooling, whereas Schoeberl *et al.* included Rayleigh friction as well. However, the size of their Rayleigh friction coefficient was too small to have a significant effect on the results (Schoeberl, personal communication). Thus the wave damping in their calculations is almost entirely the result of Newtonian cooling.

The "wind model II," wavenumber 1, "slow" Newtonian cooling results of Schoeberl *et al.* (1979, Fig. 1b) were used in the calculation after the terms in Eq. (16) were replaced by finite-difference approximations. The results of Schoeberl *et al.* (mean-zonal winds, wave amplitudes and phases) were then interpolated to a two-dimensional grid with north–south and vertical spacings of 10° and 1 km, respectively. For simplicity the static stability S and scale height H were assumed to be constants with values of 4×10^{-4} S^{-2} and 7 km, respectively. Furthermore, Schoeberl *et al.* employed spherical geometry while Eq. (16) assumes Cartesian geometry. The boundary conditions used to obtain solutions for (16) were that $\partial\bar{u}/\partial t = 0$ at the bottom (10 km), top (100 km) and sides (0 and 90° latitude) of the domain.

The zonal wind acceleration was assumed to be zero at the lower boundary (10 km) because both the wave amplitude and Newtonian cooling coefficient are small for the scale height above 10 km. A supplementary calculation with $\partial\bar{u}/\partial t$ set equal to -0.1 m s^{-1} day^{-1} revealed little sensitivity ($<1\%$) of the calculated acceleration above 15 km to the lower boundary condition.

The results of the calculation for 50° latitude is shown in Fig. 1. As this figure indicates radiative damping of the wave decelerates the zonal wind at all altitudes with a maximum near the stratopause. It is interesting to compare the zonal wind drag obtained in this calculation with the model values of Schoeberl and Strobel (1978). They found that realistic stratospheric and mesospheric jets could be obtained with Rayleigh friction coefficients of $k_r^{-1} \geq 40$ days in the lower stratosphere and $k_r^{-1} \approx 10$ days above the stratopause. In contrast, the Rayleigh friction coefficient (k_r^{-1}) deduced from Fig. 1 (in conjunction with the zonal winds in Schoeberl *et al.*) is ~200 days at all altitudes between 20 and 70 km. Thus, the computed coefficients are between 1/5 and 1/20 the "required" size. We shall discuss the significance of this result in more detail in Section 5.

3. Mean-zonal diabatic heating

In the last section it was shown that radiative dissipation of a planetary wave will decelerate the zonal flow. This result is consistent with the models of Leovy (1964) and Schoeberl and Strobel (1978) who found that deceleration of the zonal flow by the eddies was necessary to balance the indirect acceleration of the flow by the mean-zonal diabatic heating. The above papers did not consider the kind of wave dissipation or the nature of the waves involved. In the extratropical stratosphere, planetary waves are probably more important in this respect than gravity waves or tides. In the mesosphere, however, the latter may assume a more important role.

Analysis of the models of Leovy and also of Schoeberl and Strobel suggests the need for a dynamical relationship between mean-zonal diabatic heating and eddy dissipation. The need for this relationship is best illustrated by considering the acceleration of the mean-zonal flow which results from the north–south component of "thermally driven" cells (i.e., meridional cells induced in some sense by a north–south gradient in diabatic heating) and the source of the counterbalancing acceleration which is required to maintain quasi-steady-state conditions. In the absence of critical levels or momentum transport by small-scale waves, this counterbalancing torque must be the result of nonconservative effects in the planetary-scale waves.

Thus, a plausible starting point is the assumption that the drag on the zonal flow exerted by the dissipating planetary wave is balanced exactly (and indirectly) by the mean-zonal diabatic heating and that the net acceleration of the mean flow is zero.

Alternately, this balance appears in the mean-zonal potential vorticity equation [y derivative of Eq. (10)] in terms of a potential vorticity flux divergence and a z derivative of the mean-zonal diabatic heating. Hence, when the acceleration of the mean flow is zero [left side of (15) equals zero] the mean-zonal diabatic heating can be found explicitly in terms of the eddy dissipation. Thus (15) yields

$$f\left(\frac{\rho_s}{S}\int \bar{Q}dy\right)_z = \frac{1}{\bar{u}}\left[\frac{\alpha\rho_s\overline{\phi_z'^2}}{S} - \left(\frac{\alpha\rho_s}{S}\,\overline{\phi'\phi_z'}\right)_z\right]. \quad (17)$$

Differentiation of (17) with respect to y yields

$$f\left(\frac{\rho_s\bar{Q}}{S}\right)_z = A_y - B_{zy} - C_y - \frac{\beta}{f}(A - B_z - C), \quad (18)$$

where

$$\left.\begin{aligned} A &= \frac{\alpha\rho_s\overline{\phi_z'^2}}{S\bar{u}} \\ B &= \left(\frac{\alpha\rho_s\overline{\phi'\phi_z'}}{S\bar{u}}\right) \\ C &= \frac{\alpha\rho_s\overline{\phi'\phi_z'}}{S\bar{u}^2}\,\bar{u}_z \end{aligned}\right\}.$$

To find $\bar{Q}$, Eq. (18) was integrated downward from an upper boundary Z_B to the height Z. We assumed that $\bar{Q}$ and α at Z_B are vanishingly small or that Z_B is well removed from the region of interest so that ρ_s at Z_B is small. Then the diabatic heating at Z is

$$\bar{Q} = \bar{Q}_1 + \bar{Q}_2, \quad (19)$$

where

$$\bar{Q}_1 = \frac{S}{f\rho_s}\left(-B_y + \frac{\beta}{f}B\right),$$

$$\bar{Q}_2 = \frac{S}{f\rho_s}\int_{Z_B}^{Z}\left[A_y - C_y - \frac{\beta}{f}(A - C)\right]dz.$$

Before calculating the values of $\bar{Q}_1$ and $\bar{Q}_2$ from the wave solutions of Schoeberl *et al.*, it is enlightening to discuss the importance of these two quantities in the Lagrangian-mean description of fluid motion (Andrews and McIntyre, 1978). Lagrangian-mean velocities are more physically meaningful than conventional Eulerian means in the study of tracer transport because they define the motion of the center of mass of the fluid particles.

The Lagrangian-mean of a quantity equals the Eulerian-mean plus a Stokes correction, i.e.,

$$\bar{Q}^L = \bar{Q} + \bar{Q}^S \quad (20)$$

and is found by taking the average value of Q over particles displaced from the positions they would occupy if the wave were absent. The reader is referred to Andrews and McIntyre (1978) for a complete discussion of Lagrangian-mean theory. For our purposes, however, it is sufficient to note that the Lagrangian-mean thermodynamic and continuity equations are (Andrews and McIntyre, 1978)

$$\bar{\theta}_t^L + \bar{w}^L\bar{\theta}_z^L + \bar{v}^L\bar{\theta}_y^L + \bar{Q}^L = 0, \quad (21)$$

$$\bar{v}_y^L + \bar{w}_z^L - \frac{\bar{w}^L}{H} = 0 \text{ (steady waves).} \quad (22)$$

Thus, for steady-state conditions since $\bar{\theta}_z^L \approx S$ and $\bar{\theta}_y^L$ is small, $\bar{w}^L \approx -\bar{Q}^L/S$.

We shall show that $\bar{Q}_1$ in (19) is the Stokes correction $\bar{Q}^S$ and also that $\bar{Q}_2$ is the Lagrangian-mean heating $\bar{Q}^L$. To show the former, we note that [from Eqs. (6) and (8)]

$$Q' = \alpha\phi_z',\ \phi' = f\bar{u}\eta',$$

where η' is the north–south particle displacement, i.e., $D_t\eta' = v'$, $D_t = (\partial/\partial t + \bar{u}\partial/\partial x) = \bar{u}\partial/\partial x$ (steady-state). The quantities B and B_y in (19) can then be rewritten as

$$\left.\begin{aligned} B &= \rho_s f\overline{\eta'Q'}S^{-1} \\ \text{and}\quad B_y &= \rho_s S^{-1}[f(\overline{\eta'Q'})_y + \overline{\eta'Q'}\beta] \\ \bar{Q}_1 &= -(\overline{\eta'Q'})_y. \end{aligned}\right\}. \quad (23)$$

However, the Stokes correction to the mean-zonal heating is $\bar{Q}^S \approx (\overline{\eta'Q'})_y$ (Andrews and McIntyre, 1978). Thus $\bar{Q}_1 = -\bar{Q}^S$ and from (19), (20) and (23)

$$\bar{Q}^L = \bar{Q} + \bar{Q}^S = \bar{Q} + (\overline{\eta'Q'})_y = \bar{Q}_2. \quad (24)$$

Thus, the Lagrangian-mean vertical velocity is

$$\bar{w}^L \approx -\frac{\bar{Q}^L}{S} = -\frac{\bar{Q}_2}{S} \quad (25)$$

and $\bar{v}^L$ can be found from $\bar{w}^L$ with Eq. (22).

It is also informative to show how the static balance between the eddy and mean-zonal thermal dissipation given by Eq. (15) with $\partial\bar{u}/\partial t = 0$ can be expressed in the Lagrangian-mean formulation.

We start by noting that the potential vorticity flux can be written as

$$\overline{v'q'} = \frac{f}{\rho_s}\left(\frac{\rho_s}{S}\,\overline{v'\phi_z'}\right)_z - (\overline{u'v'})_y$$

$$= \frac{f}{\rho_s}\left(\frac{\rho_s}{S}\,\overline{v'\phi_z'}\right)_z - f\bar{v}. \quad (26)$$

Since

$$\left.\begin{aligned}\bar{w}^L &\approx \bar{w} + (\overline{\eta' w'})_y \\ \bar{v}^L &\approx \bar{v} - \frac{1}{\rho_s}(\rho_s \overline{\eta' w'})_z\end{aligned}\right\},$$

Eq. (26) can be rewritten as

$$\begin{aligned}\overline{v'q'} &= \frac{f}{\rho_s}\left(\frac{\rho_s}{S}\overline{v'\phi_z'}\right)_z - f\bar{v}^L - \frac{f}{\rho_s}(\rho_s\overline{\eta' w'})_z \\ &= -f\bar{v}^L + \frac{f}{\rho_s}\left(\frac{\rho_s}{S}\overline{\eta' Q'}\right)_z, \qquad (27)\end{aligned}$$

where we have used

$$\overline{\eta' w'} = (\overline{v'\phi_z'} - \overline{\eta' Q'})S^{-1}, \qquad (28)$$

which can be obtained by multiplying the eddy thermodynamic equation in the form

$$D_t\phi_z' - f\bar{u}_z D_t\eta' + SD_t\zeta' + Q' = 0 \qquad (29)$$

by η', averaging zonally, and making use of Andrews and McIntyre's (1976) expression (A2) as it applies to steady waves, i.e., $\phi D_t\Psi = -\Psi D_t\phi$.

However, we also know from (14) that (with $\phi' = f\bar{u}\eta'$)

$$-\bar{u}\overline{v'q'} = +\frac{1}{S}(f\overline{\eta' Q' u_z} + f\overline{\eta_z' Q'\bar{u}}) - \frac{1}{\rho_s}\left(\frac{\rho_s}{S}f\bar{u}\overline{\eta' Q'}\right)_z. \qquad (30)$$

Substituting (30) into (27) for $\overline{v'q'}$ yields

$$\bar{v}^L = \frac{\overline{\eta_z' Q'}}{S} \quad \text{and} \quad \bar{w}^L = -\frac{1}{\rho_s}\int_{Z_B}^{Z}\frac{\rho_s}{S}(\overline{\eta_z' Q'})_y dz \qquad (31)$$

with the aid of (22) ($\bar{w}^L = 0$ at Z_B).

Thus, by setting $\bar{u}_t = 0$ in (10) we are assuming a balance between the eddy flux of potential vorticity and generation of mean-zonal potential vorticity by diabatic heating or, alternately, that the torque on the Lagrangian-mean flow $f\bar{v}^L$ is balanced by a radiation stress divergence proportioinal to $\overline{\eta_z' Q'}/S$, i.e.,

$$f\bar{v}^L = f\overline{\eta_z' Q'}S^{-1}. \qquad (32)$$

Furthermore, since $\bar{w}^L = -\bar{Q}^L/S$, Eq. (31) implies

$$\bar{Q}^L = \frac{S}{\rho_s}\int_{Z_B}^{Z}\frac{\rho_s}{S}(\overline{\eta' Q'})_y dz. \qquad (33)$$

It also can be shown by direct substitution of $\phi' = f\bar{u}\eta'$ and $\alpha\phi_z' = Q'$ into the expression for $\bar{Q}_2$ of (19) that $\bar{Q}_2$ equals the right-hand side of (33), i.e., $\bar{Q}^L = \bar{Q}_2$.

The diabatic heating rates $\bar{Q}_1$ $(-\bar{Q}^S)$ and $\bar{Q}_2(\bar{Q}^L)$ have been calculated with the planetary wave results of Schoeberl *et al.* (1979). Our primary goal is a second assessment of the relative importance of a radiatively dissipated steady planetary wave on the mean-zonal average state of the winter stratosphere. Recall from Section 2 that deceleration of the zonal flow by the dissipating wave was $1/5 \rightarrow 1/20$ the size required by models. A comparison of $\bar{Q} = \bar{Q}_1 + \bar{Q}_2$ with heating rates inferred from observations will permit a second test of the importance of this dissipative mechanism. It is also of interest to compare the relative sizes of $\bar{Q}^L$ and $\bar{Q}^S$ to see whether the approximation suggested by Dunkerton (1978), i.e., $\bar{Q}^L \approx \bar{Q}$ is justified for the particular case being considered here.

The values for $\bar{Q}_1$ and $\bar{Q}_2$ were found by replacing the spatial derivations in (19) with finite differences. As noted previously, our results are approximate because a constant lapse rate and scale height were used along with Cartesian instead of spherical geometry.

The quantity $\bar{Q}_2$ was found with a downward integration from 100 km where α and $\bar{Q}$ were assumed to be zero. The sensitivity of the solutions to the upper boundary condition was tested by repeating the calculation but with α (70–100 km) set equal to α (70 km). The two solutions differed by less than 7% at 65 km and less than 1% at 60 km and we therefore conclude that the solutions are sensitive to diabatic heating only within a scale height above the point in question.

As Fig. 2 indicates there is net heating at low latitudes and cooling at high latitudes which is consistent with results obtained for the real atmosphere (Vincent, 1968). This pattern of heating and cooling is also consistent with the physical picture discussed above. Thus, when the radiation stress divergence balances the Coriolis torque from the Lagrangian-mean northward velocity [Eq. (32)], mass continuity requires downward motion at high latitudes and upward motion at low latitudes. For steady-state conditions the former must be compensated thermodynamically by cooling and the latter by heating.

The calculated cooling rates at 75°N and 25 and 40 km are 0.06 and 0.10 K day^{-1}, respectively, and should be compared with rates inferred for the real atmosphere of 0.8 K day^{-1} (Vincent, 1968) and 5.0 K day^{-1} (Hartmann, 1976, Southern Hemisphere). The present cooling rates are thus $\sim 1/15$–$1/50$ the size of values inferred from observations. This ratio is comparable to that obtained for the zonal wind deceleration in Section 2.

Fig. 2 also indicates, for the particular case considered here the error encountered when the vertical Lagrangian-mean velocity is inferred from the *Eulerian* mean-zonal diabatic heating (Dunkerton, 1978). At both latitudes the Stokes correction $\bar{Q}^S$ $(-\bar{Q}_1)$ is usually smaller than $\bar{Q}^L$, but certainly not negligible compared with it. The same conclusion applies to the heating rates at other latitudes (not shown).

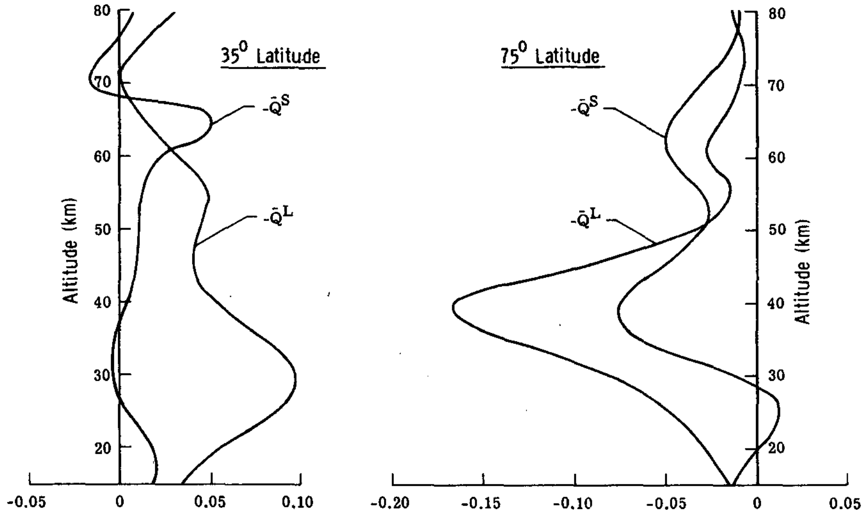

FIG. 2. Mean-zonal diabatic heating at 35 and 75°N. $-\bar{Q}^S$ ($=\bar{Q}_1$) is the Stokes correction while $-\bar{Q}^L$ ($= -\bar{Q}_2$) is the Lagrangian-mean diabatic heating. Units: K day^{-1}.

4. Transport of a conservative ozone-like tracer

In this section we shall derive the time rate of change of a conservative ozone-like tracer when $\partial\bar{u}/\partial t = 0$ and when the deceleration of the zonal flow due to eddy dissipation and to the indirect effect of mean-zonal diabatic heating are balanced. We shall first derive a general expression for the rate of tracer change and then evaluate the expression with the results of Schoeberl *et al.* (1979).

The rate of tracer change can be found from the mean-zonal tracer continuity equation if the eddy fluxes of heat and the tracer can be expressed in terms of particle displacements. The tracer flux can be obtained in the same way as the heat flux [Eq. (28)]. Thus, the integrated eddy tracer continuity equation

$$r' + \eta'\bar{r}_y + \zeta'\bar{r}_z = 0 \qquad (34)$$

when multiplied by v' and zonally averaged yields

$$\overline{v'r'} - \overline{\eta'w'}\bar{r}_z = 0. \qquad (35)$$

When D_t (31) is multiplied by r' and zonally averaged we have (for steady waves)

$$\overline{v'r'}\bar{r}_y + \overline{w'r'}\bar{r}_z = 0. \qquad (36)$$

The velocities $\bar{w}$ and $\bar{v}$ can be expressed in terms of particle displacement with (28), (2) and (3) and substituted into the mean-zonal continuity equation

$$\bar{r}_t + \bar{v}\,\bar{r}_y + \bar{w}\,\bar{r}_z + (\overline{v'r'})_y + (\overline{w'r'})_z - (\overline{w'r'})H^{-1} = 0, \qquad (37)$$

along with the tracer flux relations (35) and (36), to give

$$\bar{r}_t - \bar{r}_z[\bar{Q} + (\overline{\eta'Q'})_y] + \frac{\bar{r}_y}{\rho_s S}\left[\rho_s \int \bar{Q}dy + \rho_s\overline{\eta'Q'}\right]_z = 0. \qquad (38)$$

Eqs. (22), (24) and (25) can now be used to rewrite (38) as

$$\bar{r}_t = \bar{r}_t^L = -\bar{w}^L\bar{r}_z - \bar{v}^L\bar{r}_y. \qquad (39)$$

The first equality of (39) follows from

$$\bar{r}_t^L \approx \bar{r}_t + (\overline{\eta'r'})_{yt} = \bar{r}_t \text{ (steady wave)}. \qquad (40)$$

The result (39) is not unexpected. The tracer change is caused by advection from a Lagrangian-mean circulation.

In this paper we have assumed that the mean-zonal diabatic heating is caused only by the radiative dissipation of a single, steady planetary wave. In general, however, the total mean-zonal diabatic heating also may be the result of other kinds of waves or other forms of dissipation. In this case, the Lagrangian-mean heating $\bar{Q}^L$ could be evaluated separately for the specific wave in question and compared to $\bar{Q}^L_{\text{Total}}$ to determine its relative importance.

The remainder of this section is devoted to a calculation of the rate of change of an inert ozone-like tracer for the wave used in the calculations in Sections 2 and 3. For highly stratified tracers such as ozone, the contribution to the rate of change from horizontal advection should be less than from vertical advection, i.e.,

$$\bar{r}_t \approx -\bar{w}^L\bar{r}_z = \bar{Q}_2 S^{-1}\bar{r}_z. \qquad (41)$$

Eq. (41) was evaluated at 75° with the heating rate shown in Fig. 2 $\bar{Q}_2(\bar{Q}^L)$ and the ozone profile shown in Fig. 3.

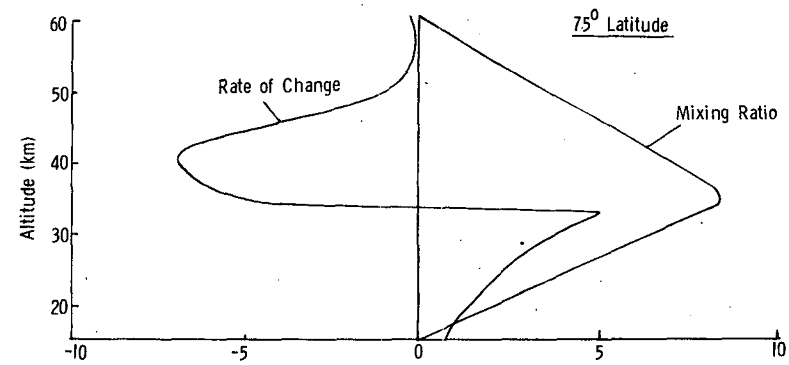

FIG. 3. Ozone mixing ratio (ppm) and rate of change of ozone (10^{-8} ppm s^{-1}) at 75°N latitude.

The rate of change of the ozone-like tracer shown in Fig. 3 should be compared with observations only in the lower stratosphere where ozone is quasi-conservative. At 23 km and 75°N Hering and Borden[1] (1967) found a fall to spring ozone increase at 75°N of ~1 ppm compared with the calculated increase at 23 km of 2.2×10^{-8} ppm $s^{-1} \times 180$ days $= 0.3$ ppm (Fig. 3). Thus the calculated increase is one-third the observed. This is a more favorable comparison than obtained for the mean-zonal diabatic heating and eddy acceleration but may be fortuitous since photochemistry, which was not included in the model, will oppose any high-latitude increases caused by dynamics.

The many limitations in the model make detailed quantitative comparisons with observations unwarranted. We can, however, conclude that the net transport from steady radiatively damped waves is the correct order of magnitude compared with observations. The results shown in Fig. 3 also will be of benefit for comparison with the transient wave studied in Part 2.

5. Discussion

In this paper we have derived the zonal wind deceleration, mean-zonal diabatic heating, and rate of change of an inert trace chemical for a radiatively dissipated planetary wave. The derivations have shown that when the eddy acceleration of the zonal flow balances the (indirect) acceleration due to the mean-zonal diabatic heating, the above three quantities can be simply related to the Newtonian cooling coefficient and eddy statistics.

It was shown that the eddy flux convergence and mean-motion terms, which appear in the mean-zonal tracer continuity equation, generally cancel each other and the net transport is due to a mean meridional cell defined by part of the mean-zonal diabatic heating. This part ($\bar{Q}_2$) was shown to be the Lagrangian-mean diabatic heating $\bar{Q}^L$ and was also shown to be proportional to the vertical integral of eddy dissipation terms.

The fact that $\bar{Q}_2$ depends on vertical integrals of eddy statistics implies that net transport can occur in a dissipationless region if dissipation is present in a neighboring region. The reason for this is that for steady-state conditions eddy drag at one level requires a counterbalancing Coriolis torque from a meridional velocity. This meridional velocity, in turn, implies vertical motions to satisfy mass continuity. Thus, adjacent layers will be coupled together.

Calculations for wavenumber 1 at 50°N indicate that in the mesosphere the effects of wave dissipation on the mean-zonal flow are ~1/20 the value required by Schoeberl and Strobel's (1978) model. This suggests that either much stronger damping of planetary-scale waves is required or that dissipation of other kinds of waves may be important.

In the stratosphere the effects of wave dissipation on the mean-zonal flow and diabatic heating were between 1/5 and 1/20 the size required by observations and models. There are several possible reasons for this difference: 1) Dissipation of planetary waves by other mechanisms in addition to radiative damping may be important. 2) Parameters used in the calculations may be incorrect. For example, radiative-photochemical coupling may increase the effective relaxation rate. 3) Critical levels may be important; particularly at the equinoxes when zonal winds are weak. 4) Wave transience when coupled with dissipation may substantially increase the effect of the wave on the zonal flow. This question will be treated in Part 2. 5) The calculation does not include higher wavenumbers or travelling waves.

More complete models than the one used here are required to investigate further the reasons advanced above to explain the smaller effect of the dissipating

[1] Hering, W. L., and T. Borden, 1967: Ozonesonde observations over North America, Vol. 4, *Environ. Res. Pap.*, No. 279, AFCRL, Bedford, MA [NTIS AD-666436].

TABLE 1. Heat flux and mean-zonal wind deceleration at 50°N, and cooling rate at 75°N for wave amplitudes ϕ and $2 \times \phi$ at 25 and 40 km. Observed values are from Vincent (1968) and Hartmann (1976).

	25 km			40 km		
	Observed	ϕ	$2 \times \phi$	Observed	ϕ	$2 \times \phi$
Heat flux, 50°N (K m s^{-1})	75	15	59	60	32	130
Cooling rate, 75°N (K day^{-1})	0.8	0.06	0.10	5.0	0.24	0.30
Deceleration, 50°N (m s^{-1} day^{-1})		0.13	0.50		0.40	1.60

wave on the mean-zonal state than apparently required by observations. It is, however, of interest to investigate the sensitivity of the results to the input parameters. This was accomplished by redoing the calculation with different choices for the mean-zonal wind, Newtonian cooling, and wave amplitude and phase. It is of particular interest to study a case where the wave amplitude decreases with height above 60 km. This property is characteristic of observational data but was not present in the model results of Schoeberl *et al.* (1979).

Barnett (1980) has presented wavenumber 1 data up to 90 km averaged over December 1975–February 1976. The use of these data resulted in mean flow deceleration in the middle stratosphere at high latitudes and acceleration at low latitudes. The opposite behavior was found in the mesosphere, i.e., mean flow acceleration at high latitudes and deceleration at low latitudes. This behavior is in sharp contrast to the deceleration found over almost the entire domain with Schoeberl *et al.*'s (1979) results. The magnitude of the heating and cooling rates and mean-zonal accelerations were comparable, however, i.e., maximum heating and cooling rates of ± 0.5 K day^{-1} and accelerations of ± 0.5 m s^{-1} day^{-1}.

It is also of interest to inquire whether the addition of higher frequency and/or traveling waves to the model discussed here would improve the agreement with observations. Such a comparison would require theoretical or observational wave structure information for the other waves. Lacking such information, we can still make a rough estimate of the effects of other waves by simply increasing the amplitude of the wave considered here until the eddy properties (e.g., total heat flux, eddy kinetic energy, etc.) are comparable in size to those observed in the real atmosphere.

Table 1 lists the heat flux, diabatic heating and eddy deceleration of the zonal wind for the wave structure data considered in the previous sections, for a 2× (amplitude) wave, along with values inferred from observations.

As can be seen from this table, doubling the wave amplitude increased the heat flux fourfold—to a size comparable to the observed flux—while the cooling rate at 75° was increased by a factor of 4–5 and the zonal flow deceleration increased by a factor of 3–4 at 25 and 45 km. Since the diabatic heating and zonal wind deceleration for the original wave amplitude were 1/5 to 1/20 the size required by observations, the results for the (2× amplitude) wave are roughly 1/2 to 1/5 the required size. Therefore, we conclude that even when the amplitude of the steady wave is increased to a size large enough to make the heat flux comparable to the total observed heat flux, the calculated heating rate and eddy drag is still somewhat less than indicated by observations and models. This implies the need to consider additional factors which affect wave dissipation, such as wave transience and critical levels.

As was pointed out in the Introduction the calculation presented here is not appropriate for the purpose of quantitative investigations of wave-mean flow interactions in the extratropical stratosphere. Two obvious improvements in the calculation would be the use of spherical coordinates and a more realistic radiative calculation. These improvements would no doubt lead to altered acceleration rates, diabatic heating rates and ozone transport. However, it is not likely that they would materially alter our conclusions.

The results also provide insight into the relationship between the transport and dynamic properties of more complicated models. For example, a well-known property of both models and observations is the channeling of wave energy into the vicinity of the jet core. This in turn implies, by Eq. (18), that the latitudinal dependence of the mean-zonal diabatic heating, and hence the location of the maximum in the downward Lagrangian mean motion, will be dependent on the jet location. If, for example, the location of the core of the jet, as simulated by a numerical model, is too far south, we would expect the maximum in the vertical Lagrangian mean motion to also be too far south, which would have important implications for the transport properties of the model.

In this study the mean-zonal diabatic heating was calculated under the assumption that it arises from the eddy-induced deceleration of the mean-zonal flow by radiative dissipation. In contrast, more customary approaches assume that differential diabatic heating is externally imposed and, in turn, "drives" winter-to-summer and equator-to-pole meridional circulations. The two lines of thought are not inconsistent, however, since, in general, some forms of eddy drag is required to balance the Coriolis torque from the mean meridional circulation. As was shown here, it may be more enlightening in some cases to treat the eddy dissipation as the fundamental driving force and to consider the mean-zonal diabatic heating to be a result rather than a cause.

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