

Wave Transience in a Compressible Atmosphere. Part II: Transient Equatorial Waves in the Quasi-Biennial Oscillation¹

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ABSTRACT

The Holton-Lindzen (1972) theory of the quasi-biennial oscillation is reevaluated in the light of some recent developments in the theory of wave, mean-flow interaction due to Andrews and McIntyre (1976a,b). These developments suggest that wave transience should be regarded as the primary cause of the oscillation, in a chronological sense, with wave absorption providing an essential, but chronologically secondary cause. The spontaneous formation and descent of shear zones anticipated in the analytic theory of Part I is here applied to the theory of the oscillation, by extending the numerical calculations of that paper to include equatorial waves of the type observed in the equatorial stratosphere. Some remarks are also made concerning probable causes of the observed QBO variability.

1. The quasi-biennial oscillation

The quasi-biennial oscillation (QBO) of zonal mean wind in the tropical stratosphere was first observed by Reed *et al.* (1961) and Veryard and Ebdon (1961). A review of this oscillation is given in Wallace (1973), and the period of observation has recently been extended by Coy (1979).

The QBO is so named because while the oscillation period is close to two years, there have been significant departures from this period so as to cause the average period to be about 27 months. These departures have included a few anomalously short cycles (1959–60 and 1972–73) and several anomalously long ones (1962–69 and 1975–77). As a result the average cycle length exceeds 24 months by a significant amount.

That the oscillation is not purely biennial is important in that it is strongly suggested that the QBO is not related in any fundamental way to the annual cycle. The theory of Holton and Lindzen (1972) which seeks to explain this oscillation as due to the upward propagating observed Kelvin and Rossby-gravity waves (Wallace and Kousky, 1968; Yanai and Maruyama, 1966) now appears to be well established. The theory does not depend on any kind of periodicity in external forcing, but rather indicates that the QBO is an oscillatory mean flow response to constant wave stresses at the tropopause. The resultant 27-month period is to be regarded as coincidental with the annual cycle at best, and in no way is derived from it.

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Some support for the Holton-Lindzen theory has recently been provided by the laboratory experiment of Plumb and McEwan (1978). The alternating formation and descent of mean flow shear zones in the nonrotating annular tank closely paralleled the gross features of the stratospheric oscillation. Plumb and McEwan also demonstrated that the laboratory oscillation could be predicted numerically by a set of approximate governing equations very similar to those used by Holton and Lindzen (1972). In Plumb and McEwan's analysis the mean flow acceleration is assumed to be governed by an equation of the form

$$\frac{\partial \bar{u}}{\partial t} = - \sum \frac{\partial B_i}{\partial z} + \dots, \quad (1)$$

where the summation represents the total vertical gradient of wave radiation stresses, and additional terms describe the effect of viscosity. The result (1) is very similar to that of Holton and Lindzen, except that in the latter paper all quantities are integrated across the equatorial waveguide so as to eliminate meridional wave radiation stress gradients.

Now it is well known from the theory of wave, mean-flow interaction that the integrated vertical gradients of wave radiation stress are, in fact, identically zero for steady and conservative waves (Andrews and McIntyre, 1976a, and references). In the laboratory experiment it was recognized that wave absorption provides the mechanism by which the mean flow is accelerated. Holton and Lindzen (1972) suggested that a type of wave absorption (radiative damping) is also relevant to the

atmospheric oscillation. Their formalism [and that of Plumb (1977) and Plumb and McEwan, (1978)] was derived from the two-scaling theory of Lindzen (1971). Lindzen investigated the effect of weak damping and shear on equatorial waves through the use of the slowly varying, linear, and *steady* waves approximations. The resulting gradients in wave radiation stress (for equal mechanical and thermal dissipation) are evaluated through formulas of the form

$$-\frac{\partial B_i}{\partial z} \propto \frac{\alpha B_i}{W_i}, \quad (2)$$

where α is the damping rate and W_i is the vertical group velocity of the i th wave component.

As recognized by Plumb (1977), an important feature of the atmospheric oscillation is that its amplitude is very nearly independent of height above 23 km. This is apparently due to the decrease in atmospheric density with height. Plumb's (1977) numerical integrations suggested that when the damping rate was made sufficiently small, the atmospheric oscillation was characterized by "open-ended" jets whose amplitude was independent of height above a certain level. This atmospheric solution was rather different from the laboratory oscillation, in which the mean flow response was very small at upper levels.

Now it has recently become apparent that the formulas (2) cannot accurately describe the wave radiation stress gradients for the atmospheric solution. The reason for this lies in the neglect in (2) of wave transience. As analytically and numerically described in Part I (Dunkerton, 1981), under certain conditions wave transience can lead to the spontaneous formation and descent of regions of mean wind shear in an atmosphere. Lindzen's (1971) formalism has recently been extended by Andrews and McIntyre (1976a,b) to include wave transience, and the analysis in this study is based upon this new Lagrangian-mean formalism (the bulk of this study, of course, utilizes the *linearized* version of the theory). The purpose of this paper is to extend the standing-wave numerical calculations of the previous paper to include three-dimensional, transient equatorial waves.

On the basis of these new results we are led to believe that the primary cause of the atmospheric oscillation is wave transience, and not wave absorption, in the sense that transience is *chronologically* more important. Wave absorption is still seen as necessary to the oscillation, but chronologically secondary. (The various means by which transience and dissipation contribute to the acceleration will be clarified below.) While this is in itself an important conceptual result, it is perhaps of more significance that the sensitivity of the oscillation

period to variations in parameters is thereby significantly altered from what would have been otherwise deduced from the Boussinesq, damped wave problem (Plumb, 1977). As a result it becomes possible to assess how the various irregularities in the observed oscillation period could have been caused by changes in the relevant parameters.

Needless to add, it is apparent that a fundamental difference exists between the QBO and its laboratory analog. While this does not detract at all from the merit of the laboratory experiment, it does suggest that caution should be exercised in comparing the two oscillations.

2. Theoretical development

There are two methods by which the theoretical formalism may be developed. In the first, the equations of motion are separated into mean and perturbation parts, and following linearization are then manipulated to form an approximate equation describing mean flow acceleration, and an approximate Eliassen-Palm relation. This method is extensively utilized in Andrews and McIntyre (1976a,b) and results in a generalization of Lindzen's (1971) theory.

The second approach derives the mean flow and wave action equations in exact finite-amplitude form. The approximate equations are then derived via the slow varying, linear waves approximations. Andrews and McIntyre (1978a,b) have developed the exact equations, and on the basis of their work Dunkerton (1980) has reduced these equations to approximate form. This method was also sketched in the first paper for the simpler two-dimensional, internal wave problem.

Both approaches yield the same set of approximate equations. These equations may then be integrated across the equatorial waveguide, and if latitudinal mean wind shear is ignored, the mean flow acceleration becomes

$$\frac{\partial \bar{u}}{\partial t} = \frac{\overline{u'X'} + \overline{v'Y'} + \overline{\phi_z'Q'}N^{-2}}{c - \bar{u}} + \frac{1}{2} \frac{\partial}{\partial t} \frac{\overline{u'^2} + \overline{v'^2} + \overline{\phi_z'^2}N^{-2}}{c - \bar{u}}, \quad (3)$$

where \bar{u} is the latitudinally integrated mean zonal wind; u' , v' , ϕ_z' , X' , Y' and Q' are perturbation zonal velocity, meridional velocity, thickness, zonal friction, meridional friction and diabatic cooling, respectively; N^2 is the static stability, and c is wave phase speed. An overbar represents a zonal and latitudinal average.

The result (3) is the approximate form of Kelvin's circulation theorem. As discussed in Part I, there is a wave contribution to the circulation contained in the second term on the right (sometimes referred to as the approximate minus wave *pseudo-*

momentum). The first term represents the change in the circulation due to departures from conservative motion.

The second theoretical result is an approximate equatorial wave action equation which assumes the familiar form (Andrews and McIntyre, 1976b)

$$\frac{1}{2} \frac{\partial}{\partial t} \frac{\overline{u'^2} + \overline{v'^2} + \overline{\phi_z'^2} N^{-2}}{c - \bar{u}} + \frac{1}{\rho_0} \frac{\partial}{\partial z} \rho_0 W \frac{\overline{u'^2} + \overline{v'^2} + \overline{\phi_z'^2} N^{-2}}{c - \bar{u}} + \frac{\overline{u'X'} + \overline{v'Y'} + \overline{\phi_z'Q'} N^{-2}}{c - \bar{u}} = 0, \quad (4)$$

in which appear (in conservation form) the latitudinally integrated wave action density A , flux B , and dissipation F , respectively. The quantity W is the vertical component of group velocity, which may be derived from the dispersion relation for equatorial waves:

$$\epsilon^{-1/2} (\epsilon \hat{\omega}^2 - \beta k \hat{\omega}^{-1} - k^2) = \beta(2n + 1), \quad (5)$$

where $\hat{\omega} = k(c - \bar{u})$ and ϵ is the eigenvalue, such that

$$\epsilon = \frac{m^2}{N^2}, \quad (6)$$

where m is the vertical wavenumber. We find that

$$W \equiv \frac{\partial \hat{\omega}}{\partial m} = \pm \frac{\hat{\omega}}{m} \frac{\epsilon \hat{\omega}^2 + \beta k \hat{\omega}^{-1} + k^2}{2\epsilon \hat{\omega}^2 + \beta k \hat{\omega}^{-1}}. \quad (7)$$

It is possible to verify from the wave fields themselves that W is also the vertical wave action transfer speed, but we omit the derivation here.

The approximation on which (3), (4) and (7) are based are conveniently summarized as follows: 1) linearization of wave fields; 2) slowly-varying approximation in time and height; 3) weak damping of waves and no mean damping; 4) neglect of latitudinal shear and integration over waveguide; and 5) neglect of the "residual" mean meridional circulation \bar{v}^* as discussed in Andrews and McIntyre (1976a).

3. Analytic solutions

Numerical integration of the approximate equations with wave transience neglected has been performed by Holton and Lindzen (1972), Plumb (1977), and Dunkerton (1979). In the following sections we present some numerical solutions with the transience terms included. As recognized in the previous paper, inclusion of the transience terms leads in some cases to an analytic solution describing mean flow evolution. To arrive at these solutions we neglect damping altogether and consider an initial condition of no motion, so that

$$\bar{u} = A \quad (8)$$

uniformly. There results the single quasi-linear equation

$$\frac{\partial A}{\partial t} + \frac{1}{\rho_0} \frac{\partial}{\partial z} \rho_0 W(A) A = 0 \quad (9)$$

which may be written in the alternate form

$$\frac{\partial A}{\partial t} + \frac{\partial}{\partial z} W(A) A = \frac{W(A) A}{H}, \quad (10)$$

where H is the density scale height. This equation defines a set of characteristic curves

$$t + c_1 = H \int dA/[W(A)A], \quad (11a)$$

$$z + c_2 = H \ln[W(A)A], \quad (11b)$$

where c_1 and c_2 are constants determined by the initial and boundary conditions.

The analytic solutions of the previous paper were found to predict the formation and descent of a shock or discontinuity, in zonal mean wind. The inclusion of a small viscosity, designed to parameterize the effects of subgrid-scale diffusion, suggested that in reality this shock would assume the form of a descending region of strong mean wind shear. The effect of wave damping was also described there, and it was found that as the damping was increased past a critical value, the solutions asymptotically approached the Boussinesq, damped wave solutions of Plumb (1977).

Although we omit the details here, it may be remarked that the transient equatorial Rossby-gravity wave problem also possesses shock solutions under the same conditions as the transient Kelvin wave problem examined previously. The resulting solutions differ insofar as the group velocity is cubic in A instead of quadratic, and the numerical values of the wave parameters are changed.

4. Simulated quasi-biennial oscillations

a. The numerical model

The prototype, standing-wave model of the previous paper is now extended to include equatorial waves. For simplicity, only two modes are allowed, although it has occasionally been suggested that other *unobserved* modes may be present (Lindzen and Tsay, 1975; Andrews and McIntyre, 1976a). In the following, the subscripts K and RG refer to the observed Kelvin and Rossby-gravity waves, respectively. From (5) and (7) it follows that

$$W_K = \frac{k(c - \bar{u})^2}{N}, \quad (12a)$$

$$W_{RG} = \frac{k(c - \bar{u})^2}{N} \left[\frac{2\beta}{k^2(\bar{u} - c)} - 1 \right]^{-1}, \quad (12b)$$

where $W > 0$ in both cases. The wave action dissipation $F = 2\alpha A$, where α is the rate coefficient for equal mechanical-thermal damping (Part I). When mechanical damping is set to zero, however, it follows that

$$F_K = \alpha_T A_K, \tag{13a}$$

$$F_{RG} = \frac{\alpha_T N}{k(\bar{u} - c)^2} \left[\frac{\beta}{k^2(\bar{u} - c)} - 1 \right] B_{RG} \tag{13b}$$

(Holton and Lindzen, 1972; Plumb, 1977; Dunkerton, 1979). Here α_T is the Newtonian cooling coefficient.

The numerical model is governed by the three coupled quasi-linear equations

$$\frac{\partial \bar{u}}{\partial t} = \nu \frac{\partial^2 \bar{u}}{\partial z^2} - \frac{1}{\rho_0} \frac{\partial}{\partial z} \rho_0 B_K - \frac{1}{\rho_0} \frac{\partial}{\partial z} \rho_0 B_{RG}, \tag{14}$$

$$\frac{\partial A_K}{\partial t} + \frac{1}{\rho_0} \frac{\partial}{\partial z} \rho_0 B_K + F_K = 0, \tag{15a}$$

$$\frac{\partial A_{RG}}{\partial t} + \frac{1}{\rho_0} \frac{\partial}{\partial z} \rho_0 B_{RG} + F_{RG} = 0, \tag{15b}$$

subject to the boundary conditions

$$\bar{u}(z = 17) = 0, \tag{16}$$

$$A_K(z = 17) = A_B, \tag{17a}$$

$$A_{RG}(z = 17) = -A_B. \tag{17b}$$

A suitable initial zonal wind profile is chosen:

$$\bar{u}(z, 0) = c(z - 17)/20. \tag{18}$$

Here the tropopause height is taken to be 17 km. The model extends upward to 36 km on a grid with forward differencing in height and time, and

$$\Delta z = 250 \text{ m}, \tag{19a}$$

$$\Delta t = 864 \text{ s}. \tag{19b}$$

Model parameters are

$$H = 7 \text{ km}, \tag{20a}$$

$$\nu = 0.3 \text{ m}^2 \text{ s}^{-1}, \tag{20b}$$

$$\beta = 2.29 \times 10^{-11} \text{ m}^{-1} \text{ s}^{-1}, \tag{20c}$$

$$N = 2 \times 10^{-2} \text{ s}^{-1}, \tag{20d}$$

$$k_K = (6.37 \times 10^6 \text{ m})^{-1}, \tag{20e}$$

$$k_{RG} = 4k_K, \tag{20f}$$

$$c_K = 30 \text{ m s}^{-1}, \tag{20g}$$

$$c_{RG} = -c_K, \tag{20h}$$

$$A_B = 1 \text{ m s}^{-1}. \tag{20i}$$

b. Simulation with damping independent of height

As previously noted by Plumb (1977) and Dunkerton (1979), there is a critical value of Newtonian cooling coefficient

$$\alpha_c = kc^2/NH \tag{21}$$

which separates two regimes of mean flow evolution. When α_T is greater than α_c the wave is absorbed at a faster rate than its growth due to the density stratification, and as a consequence the maximum mean flow response is found at the lowest levels. This was apparently the case in the laboratory experiment of Plumb and McEwan (1978), since in their effectively Boussinesq simulation α_c is zero. On the other hand, when α_T is less than α_c the wave is able to propagate to very high levels, and as a result the mean flow response tends to be independent of height at these levels, much like the observed oscillation. In this regime, however, we expect that wave transience plays a major role in the mean flow acceleration, instead of wave damping. That this, in fact, is true will be clarified now.²

To demonstrate this point, Fig. 1 displays a quasi-biennial oscillation for the case

$$\alpha_T = 0.4 \times 10^{-6} \text{ s}^{-1}. \tag{22}$$

This value is less than the critical damping rate

$$\alpha_c = 1.0 \times 10^{-6} \text{ s}^{-1} \tag{23}$$

and as a consequence the mean flow response is very nearly independent of height over the range of the model. Actually, the period of this simulated oscillation is just under two years, which is less than that of the observed oscillation. However, the object of these simulations is a qualitative description of the oscillation, and small discrepancies in period should not be taken seriously, since these can easily be accounted for by slight changes in the model parameters.

In contrast to the symmetric standing-wave experiments of the preceding paper, the realistic QBO simulation is characterized by significant asymmetry which is due to the differing vertical group velocities and dissipation rates of the two modes considered. This asymmetry is apparent in the overall magnitude of mean wind shear which is somewhat larger in the descending easterlies (unlike the observed oscillation). There is also a difference in the relative length of individual easterly and westerly phases at various levels. For example, at 30 km the respective length of these phases is 0.9 and 1.1 years, while at 20 km it is 1.1 and 0.9 years, respectively.

² As emphasized in Part I, wave transience also plays an important role in the Boussinesq regime $\alpha > \alpha_c$.

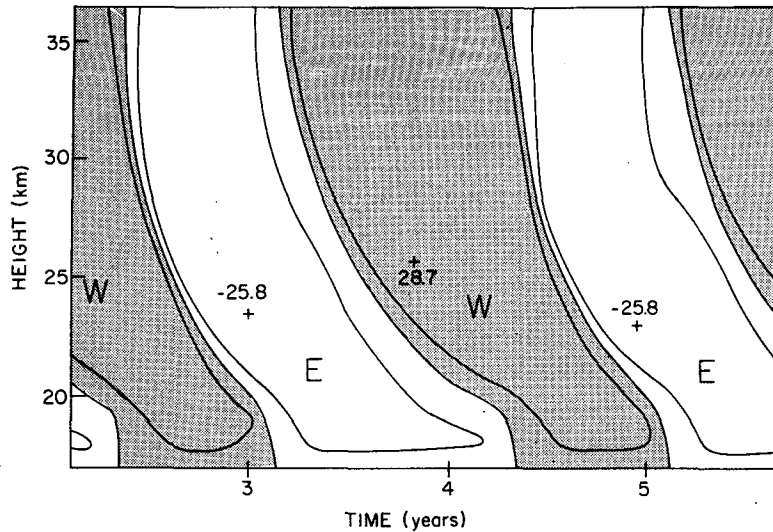


FIG. 1. Simulated quasi-biennial zonal wind oscillation with damping independent of height. Contour interval is 20 m s^{-1} .

The role played by wave transience in this simulation is indicated in Figs. 2a–2c, which display a sequence of mean flow and wave action profiles during a period of descending easterlies. In the first profile, an easterly shear zone lies atop the westerly jet. Wave action for both modes is concentrated in their respective shear zones (due to the convergence of the vertical group velocity in these regions). In both cases the wave action density is of the same order of magnitude as the mean flow maxima. However, because the thermal damping becomes prominent when the intrinsic frequency vanishes, there is a relatively rapid decay of wave action as the waves propagate into their respective shear zones.

This wave absorption has the effect of making the mean flow change locally “permanent” since when each wave decays in its shear zone the mean flow is not required to decay also (as would be the case for a purely transient wave). In other words, the effect of transient wave decay is identically cancelled by the absorption, so that there is no reverse gradient of wave action flux set up, and hence no mean flow deceleration.

It is clear, however, that damping is not to be regarded as the primary cause, in a chronological sense, of the formation and descent of each shear zone. This is evident from Fig. 2b. As the easterlies have continued to descend, mean flow viscosity has eroded away the westerly jet and once again the Kelvin wave is able to propagate freely to upper levels. As it propagates vertically, however, the Kelvin wave is able to induce a significant mean flow acceleration at the leading edge of the wave packet, which because of the density stratification is largest near 35 km. The spontaneous formation and descent of the new westerly shear zone is

somewhat akin to the analytic solutions of the previous paper. However, it should be noticed that in the present case the induced mean flow is not equal to the wave action density, as it was in Part I. Although because of damping and viscosity it is not possible to relate \bar{u} to A_K directly, it is “temporarily” true at intermediate levels that

$$\bar{u} \approx A_K - c \quad (24)$$

(in effect one could take $\bar{u} = -c$ as the initial condition). The vertical group velocity therefore is approximately four times as large in the present problem as it was in the initial stages of the analytic solutions of the previous paper, in which $\bar{u} = A$ exactly. As a result the westerly shear zone forms more rapidly than would have otherwise been expected from the analytic solutions of Part I.

This in effect explains why Plumb (1977) and Holton and Lindzen (1972) were able to simulate an apparent atmospheric solution with wave transience neglected. The formula (2) used by those authors assumes an *instantaneous* propagation of wave action to upper levels, which together with the damping is able to mimic the real solution rather well. On physical grounds, however, such an instantaneous response is impossible. When wave transience is included, it is seen that damping no longer causes the formation of the shear zone, but rather by the time wave absorption has become operative, the shear zone has already formed, and as already discussed the effect of damping is to make the induced mean flow acceleration “permanent”.

c. Simulation with realistic damping

In Part I it was shown that variations in damping have a very slight effect on the period of the

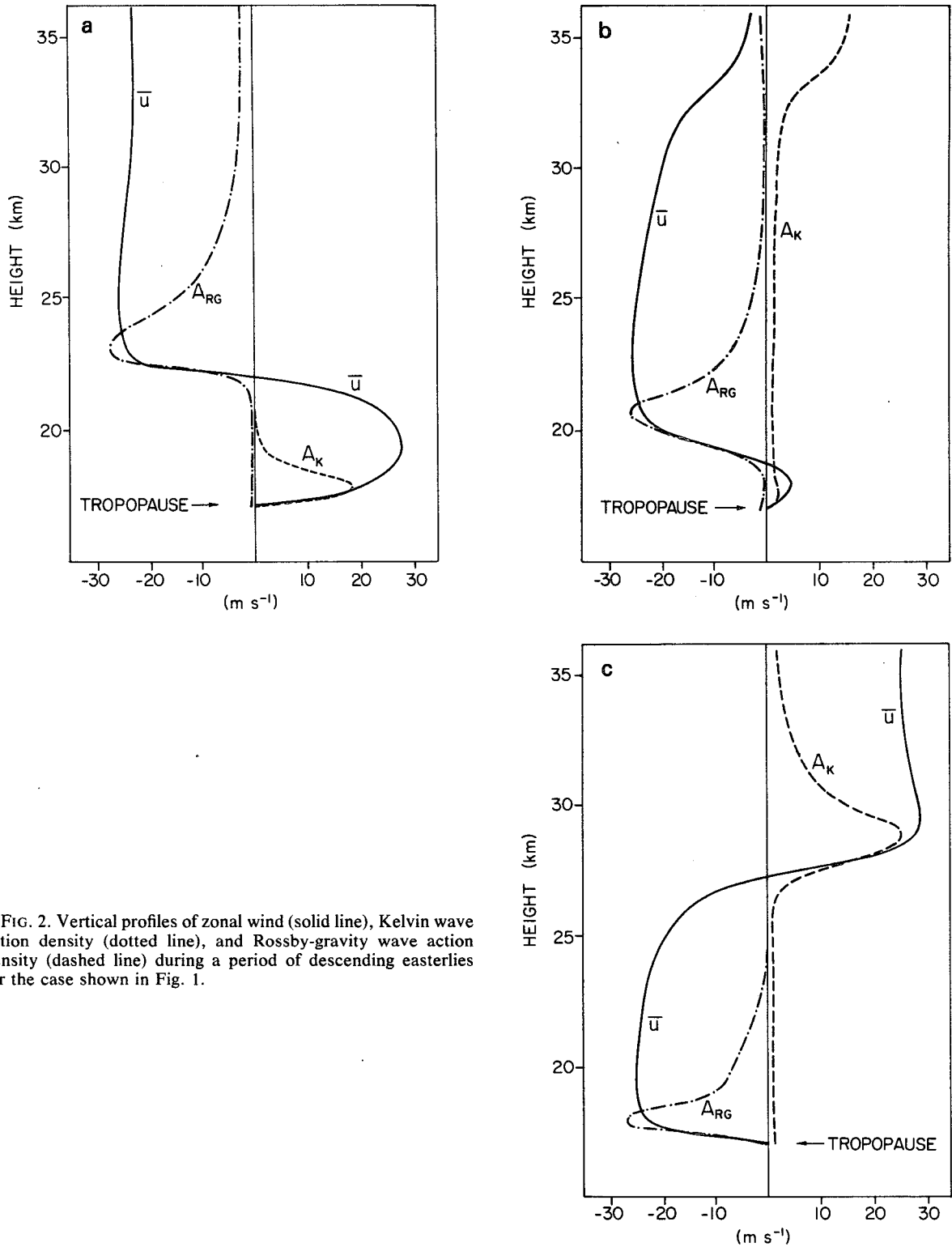


FIG. 2. Vertical profiles of zonal wind (solid line), Kelvin wave action density (dotted line), and Rossby-gravity wave action density (dashed line) during a period of descending easterlies for the case shown in Fig. 1.

atmospheric oscillation. A similar result was found in the equatorial wave simulations. Fig. 3 displays an oscillatory solution for the case

$$\alpha_T = 0.4 \times 10^{-6} \text{ s}^{-1} [1 + 3(z - 16)/20], \quad (25)$$

with all other parameters held constant. This profile of Newtonian cooling coefficient is an attempt to model the expected rapid increase in α_T with height in the stratosphere (Dickinson, 1973; Blake and Lindzen, 1973).

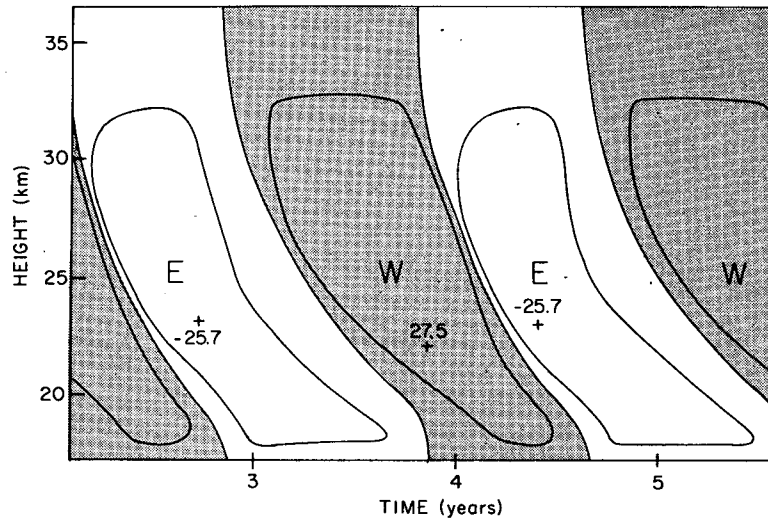


FIG. 3. As in Fig. 1 except for the linear damping profile case.

The simulated oscillation is in most respects identical to the previous one insofar as the zonal mean wind is concerned. There is a slight reduction in the sharpness of the westerly shear zone. The oscillation period, however, is unchanged from its previous value.

The most important differences are (1) a reduction in the maximum amplitude of A_k at upper levels, and (2) a somewhat faster absorption of wave action in each shear zone. Figs. 4a–4c display a sequence of wave action profiles analogous to those of Fig. 2. It is apparent that wave transience is instrumental in the formation of the new westerly shear zone, but now wave absorption makes a significant contribution to the acceleration, also. For the Rossby-gravity wave the situation is much the same as before, and wave transience is dominant.

There is a certain ambiguity as to whether we should regard wave transience as the primary cause of the oscillation in this case, because of the larger contribution from the damping. Clearly, there exists a continuous transition from transience to damping-dominated cases, and in this simulation we are approaching the transition point. Nevertheless, the general insensitivity of the oscillation period and magnitude to α_T supports the view taken here that transience is again to be regarded as the primary mechanism, with absorption playing an essential, but chronologically secondary, role.

5. Variations in oscillation period

a. Relevant parameters

Plumb's (1977) lucid analysis of the QBO phenomenon revealed that the period of the oscillation was governed by a simple relation of the form

$$T = \eta \frac{kc^3}{N\alpha B_0}, \quad (26)$$

where η is a dimensionless number dependent on the details of the problem. Plumb's numerical experiments suggested that $\eta \approx 14$ in a Boussinesq fluid but rapidly decreases in an atmosphere when the critical damping is approached.

This large variation in η in the atmosphere case suggests that the scaling used to derive (26) is inappropriate for small values of damping. That formula is based on the expectation that

$$T \propto \frac{c}{B_0} Z, \quad (27)$$

where Z is the height scale. Plumb correctly adopted the "dissipation scale height"

$$Z = \frac{kc^2}{N\alpha} \quad (28)$$

for the Boussinesq problem. Now while it is *acceptable* for some purposes to adopt this formula for the atmospheric case also, it is desirable to incorporate the large variation in η directly into the formula. One way of accomplishing this would be to adopt the density scale height as the height scale, so that

$$T = \eta' \frac{cH}{B_0} \quad (29)$$

The principal advantage of this formula is that it allows us to visualize directly the influence of changes in H on the period T .

It will be noticed in (29) that the sensitivity of T to variations in c , k and N is apparently reduced. Of course, the sensitivity of T to variations in α is reduced, as our experiments have indicated. (Interestingly, the experiments described in Part I actually implied that T is *slightly reduced* when the damping is reduced below its critical value.)

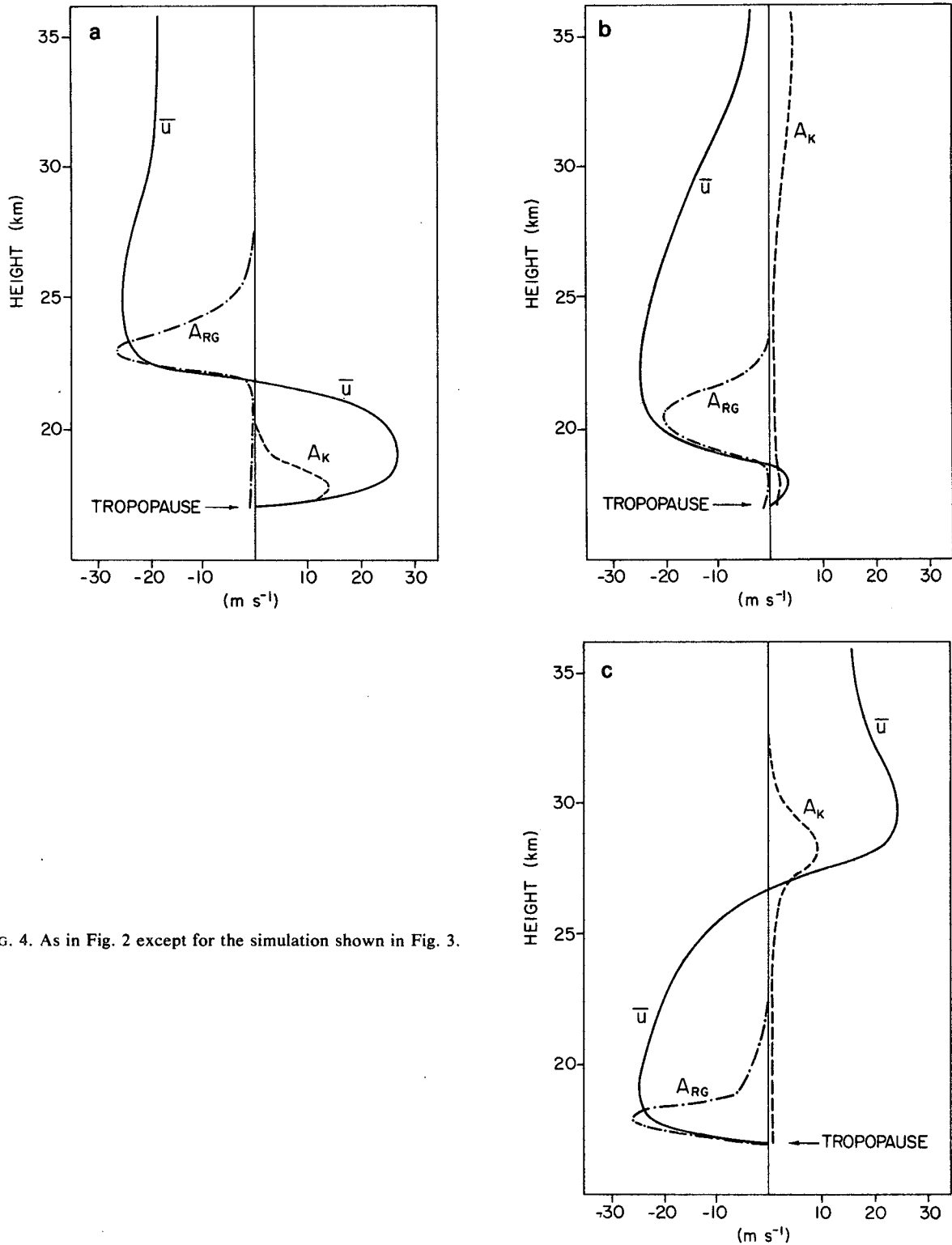


FIG. 4. As in Fig. 2 except for the simulation shown in Fig. 3.

b. Irregularities in the observed QBO

The discovery of the QBO provoked a multitude of suggestions as to how such an oscillation could result from periodicities in external forcing. At-

tempts have also occasionally been made to explain irregularities in the observed oscillation in terms of variations in external forcing. One such suggestion came from Newell (1970), namely, that the biennial cycle was interrupted in 1963 with the eruption of

Mt. Agung in March of that year. Newell speculated that the effect of the resultant dust cloud was a significant warming at the base of the stratosphere, which apparently represented a departure from the biennial temperature pattern prior to that time. As already noted, the series of cycles beginning about 1962 and extending into 1969 were all anomalously long (i.e., lying outside one standard deviation of the average). While it is certainly incorrect to regard a biennial cycle as interrupted by this event, the possibility of a causal, or influential, relationship merits some attention.

Presumably the effect of the eruption was an increased heating rate (Newell, 1970) which would either (i) alter the wave propagation in the lower stratosphere or (ii) cause an overall adjustment in the circulation or both. Now it is clear from (29) that increases in H may lead to a lengthening of the oscillation period. Such changes, however, would be based on changes in absolute temperature, and it is difficult to conceive that such a change could have equalled that observed $\sim 25\%$ increase in oscillation period in the years 1962–69.

On the other hand, it is not yet possible to rule out mechanism (ii). However, the answer depends in part on whether Newell's estimate of the heating rate is correct, still a matter of some uncertainty. It is in fact possible that the apparently anomalous increase in stratospheric temperature observed by Newell may have been due to the QBO itself, as westerlies were descending at the time. (Nevertheless, the question posed here might merit further study, especially if it can be shown that an anomalously large heating rate was present at this time. A source of mean heating might act to retard the descent of shear zones by generating a compensat-

ing vertical component of the residual mean meridional circulation.)

c. Simulated variations

It is of interest to note that the oscillation period is inversely related to the tropopause wave stress (B_0). While variations in the observed oscillation period could be attributed to a variety of causes, we are led to believe that variations in B_0 may provide the most important means of QBO variability. Changes in monsoonal activity and other convective features might lead to changes in B_0 which, though small, could substantially affect the oscillation period. It is to be noted that B_0 is a quadratic quantity whose variability may be significantly greater than otherwise expected from changes in tropospheric features. (Incidentally, the contention of Nastrom and Belmont (1979) of an apparent correlation between QBO period and solar radiation is not inconsistent with our hypothesis—noting, however, that the arrow of causality enters the troposphere *first!* Indeed, it may be of some interest to investigate the possibility of solar-related changes in tropical tropospheric activity.)

To explore further the effects of variations in B_0 a numerical simulation was performed in which all model parameters were held constant except $A_{RC}(z = 17)$. The latter was reduced by a factor of 2. The linear damping profile was again employed, and Fig. 5 displays the solution. A number of interesting features are evident. First, the oscillation period is now 2.35 years (22% increase). Second, the sharpness of easterly and westerly shear zones is now approximately equal. Third, the decrease in $A_{RC}(z = 17)$ has affected the relative length of both

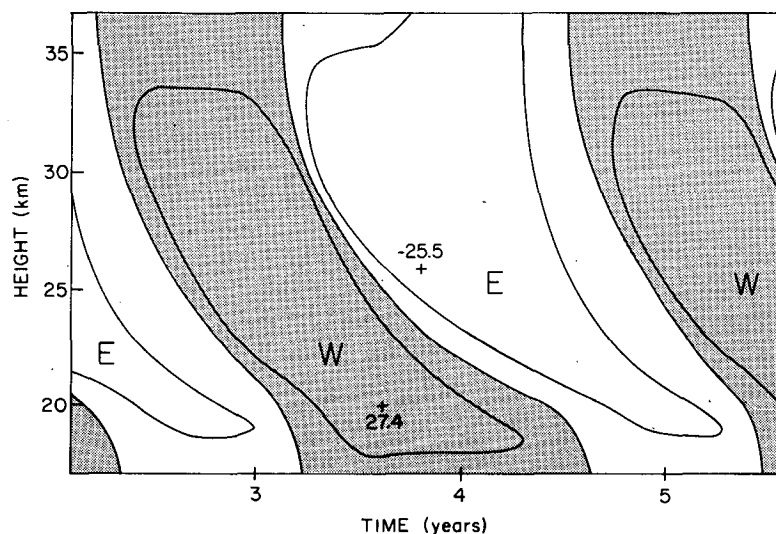


FIG. 5. As in Fig. 3 except with $A_{RC}(z = 17) = 0.5 \text{ m s}^{-1}$.

the easterly and westerly phases. Both phases have been increased in length, and furthermore the easterly phase dominates the westerly phase at 30 km (1.4 vs 0.9 years, respectively) while at 20 km the reverse is true (0.9 vs. 1.3 years, respectively). This pattern is just the opposite of the one observed in the previous two simulations, and also bears a striking resemblance to the observed oscillation during the years 1962–69 in which the oscillation exhibited a series of anomalously long cycles.

It is of some significance that observed anomalies in the length of individual phases always appear to occur at least in pairs (i.e., both phases are anomalous, and never one by itself). A slight change in B_0 for a single mode could produce changes in the length of both phases, as is clear from the simulation presented here.

6. Closing remarks

The theory of the quasi-biennial oscillation presented here is offered in hope that the primary cause of the oscillation, i.e., wave transience be understood properly, together with the essential but secondary role played by wave absorption. Future studies of this phenomenon will no doubt attempt to assess the role of latitudinal shear (Holton, 1979) and the residual mean meridional circulation. The mechanisms of wave absorption deserve more rational consideration, also.

While it is beyond our purposes to discuss the latitudinal distribution of mean flow acceleration here, it should be remembered that the transience hypothesis removes one *paradox* from the Holton-Lindzen theory, which concerns the contribution to $\bar{u}_t(y)$ made by the antisymmetric Rossby-gravity wave. When this mode is thermally damped, there is no acceleration on the equator (where the quasi-biennial oscillation is strongest). On the other hand, a transient (or mechanically damped) Rossby-gravity wave generally produces an easterly jet centered on the equator, even when thermal damping is present (demonstrated convincingly by Andrews and McIntyre, 1976a; refer to their Fig. 1). While the presence of higher order symmetric modes cannot be ruled out, the transience hypothesis makes it quite clear that there is no need to invoke these unobserved modes to explain the latitudinal distribution of easterly mean flow acceleration.

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