An Efficient Method for Computing the Absorption of Solar Radiation by Water Vapor

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ABSTRACT

An efficient method has been developed to compute the absorption of solar radiation by water vapor. The method is based on the molecular line parameters compiled by McClatchey et al. (1973) and makes use of the far-wing scaling approximation and k-distribution approach previously applied by Chou and Arking (1980) to the computation of the infrared cooling rates. The entire near-IR spectrum between 0.83 and 4.0 μm is treated as one region with the effect of the variation of the incoming solar flux with wave-number incorporated into precomputed functions. For clear atmospheres, the solar fluxes are computed from a table in which the scaled water vapor amount is the independent variable. For cloudy atmospheres, the k-distribution method is used. Using the line-by-line method as a standard, the maximum error introduced by this method is ~4% of the peak heating rate. The present method has the additional advantage over previous methods in that it can be applied to any portion of the spectral region containing the water vapor bands.

1. Introduction

The increasing sophistication of numerical models of the atmosphere requires more accurate methods of treating radiative energy transport. At the same time, speed is essential, especially for the long-term integration required in climate modeling. In a previous paper, Chou and Arking (1980) developed a fast but accurate method for computing the cooling rate in the infrared due to water vapor. Using a similar approach, this paper presents a method for computing the heating rates due to the absorption of solar radiation by water vapor in the wavelength range between 8.3 and 4.0 μm.

Most of the previous methods of modeling the molecular absorption of solar energy (e.g., McDonald, 1960; Manabe and Möller, 1961; Yamamoto, 1962; Sasamori et al., 1972) were based either on laboratory measurements or on direct solar radiation measurements. The laboratory measurements consist of transmittance through a cell containing a uniformly mixed gas at a fixed temperature and pressure, whereas the direct solar radiation measurements are made through an atmospheric slant path along which pressure, temperature and humidity vary. In either case, the transformation to other paths and the calculation of transmittance are not straightforward. It is complicated by the large number of variables involved, including the high degree of variability in the water vapor distribution. It has been found that solar heating rates computed on the basis of these measurements could differ by nearly a factor of 2 (e.g., Wang, 1976) depending upon the methods used.

A further complication is the need to take into account atmospheric scattering by the molecular atmosphere and by cloud and aerosol particles. Since the molecular absorption measurements are made for wide spectral intervals, they cannot be applied directly to a scattering atmosphere, where the solution to the problem of multiple scattering in the atmosphere requires monochromatic values of the absorption coefficient. In order to apply the radiative transfer scheme, including scattering, the transmittance within each spectral interval is approximated by an exponential series, in which each term represents the transmittance associated with an equivalent absorption coefficient; the series coefficients and the equivalent absorption coefficients are empirically determined (Yamamoto et al., 1970; Lacis and Hansen, 1974; Liou and Sasamori, 1975). Although the representation of transmittance over a spectral interval as an exponential series (the so-called k-distribution method) is justified by theoretical considerations (Arking and Grossman, 1972), nonetheless, the empirical approach leads to a plethora of non-unique fits to the data. There is just not enough information in the wide spectral measurements to uniquely determine the distribution of absorption coefficients.

A far more preferable approach is the direct com-
putation of transmittance from molecular line parameters. This is made possible by the extensive work in recent years to compile molecular line parameters for atmospheric gases (McClatchey et al., 1973). Within the limit of uncertainty in line parameters, a line-by-line computation along the spectrum can be considered as the most accurate method. Because absorption is a rapidly varying function of wavenumber, the size of spectral interval required in a line-by-line computation is of the order of 0.01 cm\(^{-1}\) or less. For repeated computations in an atmospheric model, the required computing time is far beyond what anyone can afford. Chou and Arking (1980) took advantage of the nature of the atmospheric humidity distribution to develop an efficient method for computing transmittances in the thermal infrared water vapor bands. The essence of that method is 1) to scale the absorption coefficient based on the conditions representative of the region in the atmosphere where cooling is most significant, and 2) to assume that the dependence of the absorption coefficient on temperature and pressure corresponds to that in the far wings of absorption lines. It leads to the separation of wavenumber dependence of transmittance from pressure and temperature and results in a one-parameter scaling. The k-distribution method (Arking and Grossman, 1972) can then be applied to the calculations of transmittance and flux in an inhomogeneous atmosphere, thereby greatly reducing computing time, but retaining a high degree of accuracy. In its final form, our solar radiation routine resembles that of Lacis and Hansen (1974), which is based upon the absorption curve of Yamamoto (1962) and is used in the GLAS general circulation model. The essential differences are that 1) we compute the distribution of absorption coefficients from molecular line parameters, instead of an exponential series fit to wide spectral measurements, and 2) the reference pressure for scaling the water vapor amount in our method is optimally selected instead of STP.

In this study, we extend the work of Chou and Arking (1980) to computing the absorption of solar radiation by water vapor. The validity of the method is verified by comparison with line-by-line calculations. The spectral region covered in this study ranges from 0.83 to 4 \(\mu\)m, which includes five water vapor bands. The weak 0.7 and 0.8 \(\mu\)m bands are not included since the molecular line parameters compiled by McClatchey et al. (1973) do not include these two bands. In Section 2, we outline an efficient method for transmittance and flux computations based upon actual line parameters. In Section 3, we compare the computed heating rate in clear atmospheres using the present method with that obtained from direct line-by-line calculations; comparison is also made with the Lacis and Hansen method, which is based on Yamamoto's absorption curve. Sample calculations for cloudy atmospheres are given in Section 4. In Section 5, the sensitivity of the results to scaling parameters are examined. A summary is presented in Section 6.

2. Formulation of the method

The parameters that affect the absorption coefficient are the line position \(\nu_o\), intensity \(S\), width \(\alpha\) and shape. The broadening of absorption lines is mainly due to the Doppler effect in the higher atmosphere and molecular collision in the troposphere. Within the spectral range concerned in this study, the region of equal importance of molecular collision and Doppler effect is between 60 and 300 mb. For combined Doppler and collision broadening, the line shape can be represented by the Voigt function in which the far wings follow closely the Lorentz function due to collision effect. In the stratosphere where molecular lines are very narrow (\(\alpha\) is of the order of 0.01 cm\(^{-1}\) depending on pressure and wavenumber, which is much smaller than the mean line spacing of 0.3 cm\(^{-1}\)), the absorption coefficient is usually very large near line centers and falls off rapidly away from line centers. As the solar beam traverses the upper atmosphere into the stratosphere, most of the incoming energy near line centers is quickly absorbed, and the line wings are important to the radiative heating in the troposphere and lower stratosphere. The importance of the line wings increases with increasing path length.

For a Lorentz function, the absorption coefficient at wavenumber \(\nu\), pressure \(p\) and temperature \(T\) is given by

\[
k_s(p, T) = \sum_i \frac{S_i(T)}{\pi} \frac{\alpha_i(p, T)}{(\nu - \nu_i)^2 + \alpha_i^2(p, T)},
\]  

(1)

where the summation is over all absorption lines, denoted by the subscript \(i\), the half-width is given by \(\alpha(p, T) = \alpha(p_r, T_r)(p/p_r)(T/T_r)^{1/2}\), and the subscript \(r\) denotes reference values. In the far wings of a line where \(|\nu - \nu_o| \gg \alpha\), the absorption coefficient can be written as

\[
k_s(p, T) = k_s(p_r, T_r) \frac{p}{p_r} R_s(T, T_r),
\]  

(2)

where

\[
R_s(T, T_r) = \left(\frac{T}{T_r}\right)^{1/2} \sum_i \frac{S_i(T)\alpha_i(p_r, T_r)}{(\nu - \nu_i)^2} \frac{\alpha_i(p_r, T_r)}{(\nu - \nu_i)^2}.
\]  

(3)

The effect of temperature on \(R_s\) is mainly through its effect on line intensity. However, the dependence of line intensity on temperature is not large except for transitions involving high rotational quantum num-
bers, which occur in the outer portions of the absorption bands. From line-by-line calculations, it has been found that for atmospheric conditions the absorption of solar radiation is significant only near the centers of absorption bands. The effect of temperature on the absorption coefficient can therefore be neglected especially if we choose \( T_r \) to be in the middle range of atmospheric temperatures. Furthermore, the water vapor absorption lines in the solar spectral region are several orders of magnitude weaker than that in the infrared; thus, the far-wing scaling approximation of (2), which is used by Chou and Arking (1980) to compute radiative transfer in the infrared with wavelength longer than 4 \( \mu \text{m} \), is modified to

\[
k_s(p) = k_s(p_r, T_r) \left( \frac{p}{p_r} \right)^m,
\]

where the scaling parameter \( m \) should be somewhat less than 1 to account for the non-saturation situation near line centers. The value of \( m \) is to be determined empirically.

Since our main concern is to compute the transmittance between the top of the atmosphere and pressure levels within the atmosphere, the reference pressure \( p_r \) should be chosen somewhere in the upper troposphere for the following reasons. Although the absorption in the stratosphere, where line center regions are important, will be underestimated using the wing-scaling approximation, the absorption is weak due to the small amount of water vapor and the very narrow absorption lines above the tropopause. Its effect on the absorption of solar energy is therefore small. In the lower troposphere, the optical path is so long that a large portion of the spectrum near the line centers is saturated, and the absorption of solar energy is largely determined by the line wings. Hence, the wing-scaling approximation can be used in the lower troposphere and the results would not be sensitive to the choice of \( p_r \).

In the upper troposphere where absorption is neither weak nor strong, the absorption of solar energy is in the line center regions as important as in the line wings, but the ratio \( p/p_r \) is close to 1 and \( k_s(p, T) \) is not much different from \( k_s(p_r, T) \) if we choose \( p_r \) to be in the upper troposphere. The use of (4) is therefore suitable for computing the absorption of solar energy:

The reference temperature, being least important, was chosen to be 240 K and the reference pressure was chosen to be 300 mb. The parameter \( m \), which should be close to zero for weak absorption and one for strong absorption, was chosen to be 0.8. For this particular choice of reference pressure, the parameter \( m \) has the effect of reducing errors in the stratosphere and lower troposphere. The sensitivity of the results to the choice of parameters is examined in Section 5.

To compute the absorption of solar energy, we start with spectral intervals \( \Delta \nu_i, i = 1, \ldots, n \), such that the solar energy incident at the top of the atmosphere can be considered constant within each spectral interval. The downward solar flux at pressure level \( p \) is

\[
\mu F(p) = \mu \sum_{i=1}^{n} F_i(0) \tau_i(p) \Delta \nu_i,
\]

where \( \mu F_i(0) \Delta \nu_i \) is the incoming solar flux at the top of the atmosphere, \( \tau_i(p) \) is the mean transmittance between the top of the atmosphere and pressure level \( p \), given by

\[
\tau_i(p) = \frac{1}{\Delta \nu_i} \int_{\Delta \nu_i} \exp \left[ -\frac{1}{\mu g} \int_{0}^{p} k_s(p', T) q(p') dp' \right] d\nu,
\]

where \( q \) is specific humidity, \( g \) the acceleration of gravity, and \( \mu \) the cosine of the solar zenith angle.

With the use of the wing-scaling approximation (4), Eq. (6) is reduced to

\[
\tau_i(p) = \frac{1}{\Delta \nu_i} \int_{\Delta \nu_i} \exp \left[ - k_s(p_r, T_r) w(p) / \mu \right] d\nu,
\]

where \( w(p) \) is the scaled water vapor amount,

\[
w(p) = \frac{1}{g} \int_{0}^{p} \left( \frac{p'}{p_r} \right)^m q(p') dp'.
\]

The advantage of using the wing-scaling approximation is that the wavenumber dependence of \( k_s(p_r, T) \) is separated from pressure and temperature, and wavenumbers with the same value of \( k_s(p_r, T) \) can be treated as one identity. As a result, the transmittance averaged over a small spectral interval can be computed by using the \( k \)-distribution method in which the integration over wavenumber is replaced by the integration over the absorption coefficient, and Eq. (7) becomes

\[
\tau_i(p) = \int_{\log k_{\text{min}}}^{\log k_{\text{max}}} \exp \left[ - k w(p) / \mu \right] h_i(k) d \log k,
\]

where \( h_i(k) \) is the distribution of the magnitude of the absorption coefficient \( k_s(p_r, T_r) \) in the spectral interval \( \Delta \nu_i \), with the normalization

\[
\int_{\log k_{\text{min}}}^{\log k_{\text{max}}} h_i(k) d \log k = 1,
\]

such that the fraction of the spectral interval with \( \log k \) contained in the range \( \log k \pm 0.5 d \log k \) is \( h_i(k) \times d \log k \), and \( k_{\text{min}} \) and \( k_{\text{max}} \) are the minimum and maximum values of \( k_s(p_r, T_r) \) within the spectral interval.

In practice, the integration in (7) and (9) is approximated by a summation to compute mean transmittances. Since the absorption coefficient is a
rapidly varying function of wavenumber, thousands of points are needed along the spectrum to compute the mean transmittance over, for example, a 40 cm\(^{-1}\) spectral interval if (7) is used. On the other hand, transmittance is a rather smooth function of the absorption coefficient, and the number of \(k\) needed for computing the mean transmittance using (9) is only of the order of 10. Compared to the use of (7), the computing time can be greatly reduced by using (9).

With the substitution of (9), Eq. (5) becomes

\[
\mu F(p) = \mu \int \exp[-k w(p)/\mu] h(k) d \log k, \tag{10}
\]

where \(h(k)\) is the flux weighted \(k\)-distribution function, defined by

\[
\text{Table 2. Weighted \(k\)-distribution function} \ h \ (\text{mW cm}^{-2}) \ \text{with} \ p_r = 300 \ \text{mb,} \ T_r = 240 \ \text{K for individual absorption bands.} \ k \ (\text{g}^{-1} \ \text{cm}^{-3}) \ \text{is the absorption coefficient.}
\]

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<th>1.87 (\mu m)</th>
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\[
h(k) = \sum_{i=1}^{n} F_i(0) A_i \nu_i h_i(k). \tag{11}
\]

In computing the \(k\)-distribution function we first divide the spectrum between 2600–12040 cm\(^{-1}\) into 236 spectral intervals of 40 cm\(^{-1}\) each. They include the five water vapor absorption bands listed in Table 1. Since water vapor lines with wavenumber > 12040 cm\(^{-1}\) were not available, the weak 0.7 and 0.8 \(\mu m\) bands are not included in this study. Each 40 cm\(^{-1}\) interval is further divided into 2000 points, 0.02 cm\(^{-1}\) apart between neighboring points. The absorption coefficient \(k(p_r, T_r)\), with \(p_r = 300 \ \text{mb and} \ T_r = 240 \ \text{K},\) is then computed at each point using a Voigt profile and the line parameters compiled by McClatchey et al. (1973).\(^1\) Finally, a histogram of \(k(p_r, T_r)\) for each 40 cm\(^{-1}\) interval is created, which is the function \(h(k)\). In computing \(h(k)\) from (11), the solar flux outside the atmosphere \(F(0)\) is taken from Labs and Neckel (1968).

The flux weighted \(k\)-distribution function \(h(k)\) computed separately for each of the water vapor absorption bands and for the total spectrum is shown in Table 2. The total solar flux \(\mu F\) at any point in the atmosphere, integrated over the entire spectrum, is a function of the solar zenith angle and the scaled water vapor amount above that point. Values of \(F(w/\mu)\) are shown in Table 3. For a clear atmosphere,
Fig. 1. Solar heating rates in the 0.94 μm band (a), 1.14 μm band (b), 1.38 μm band (c), 1.87 μm band (d), 2.7 μm band (e) and 2600–12040 cm⁻¹ spectral region (f) for a solar zenith angle of 60° and zero ground albedo. The solid curve is computed by taking the pressure and temperature effects on absorption exactly into account according to the Voigt function. The dashed curve is computed using the wing-scaling approximation of (4).
values of $w(p)$ can be easily computed from (8) and the solar flux from Table 3.

The solar heating rate for any layer is related to flux convergence as

$$\frac{\Delta T}{\Delta t} = -\frac{\mu g}{c_p} \frac{\Delta F(p)}{\Delta p}, \quad (12)$$

where $c_p$ is the specific heat of air at constant pressure and $t$ is time.

3. Heating rate in a clear atmosphere

a. Test of the wing-scaling approximation

To test the accuracy of the wing-scaling approximation, two sets of heating rates have been computed, based on line-by-line calculations of transmittance and flux every 0.02 cm$^{-1}$. In one set, the absorption coefficient is computed from the line parameters for a Voigt line shape which varies with height according to the temperature and pressure profiles of the model atmosphere; this represents the "exact" calculation. In the second set, we proceed as in the first set but substitute the wing-scaling approximation given by (4) for the variation of the absorption coefficient with height (with $p_r = 300$ mb, $T_r = 240$ K and $m = 0.8$). In each set, the heating rate for any atmospheric layer is proportional to the difference in fluxes at the top and bottom of the layer [Eq. (12)] with flux computed using the transmittance given by (6) and (7). The results are shown in Figs. 1a–1f for a clear, mid-latitude winter atmosphere, with a solar zenith angle of 60° and zero surface albedo. It can be seen that in each band the error due to the wing-scaling approximation is $<0.015$°C day$^{-1}$ below 50 mb; in the worst case, the 0.94 μm band, the error is $<7$% of the peak heating rate due to that band. For the total heating rate due to the combined spectrum (Fig. 1f), the error below 50 mb is $<0.025$°C day$^{-1}$, which is $<2.5$% of the peak heating rate.

The errors can be understood as follows. In the stratosphere, most absorption occurs near line centers where the absorption coefficient is either independent of pressure (Doppler effect) or inversely proportional to pressure (effect of molecular collision). By using the wing-scaling approximation, absorption is underestimated since $(p/p_r)^m$ is much smaller than 1 in the stratosphere. As a result, more energy is available in the upper troposphere and the heating rate there is overestimated. In the lower troposphere, because of the long optical path, the
band wings become more important. Since the molecular line intensity in the band wings increases with increasing temperature due to a large energy of the lower state of the transition, the heating rates are underestimated wherever the temperature is higher than the reference temperature of 240 K.

b. The fast method

In order to achieve the high speed necessary for repeated computations in numerical models of the atmosphere, the scaled water vapor is computed using (8) and the flux computed from (10). In practice $F(w/u)$ is stored and a table lookup procedure is used (Table 3).

Examples showing the accuracy of the fast method appear in Figs. 2–4. "Exact" line-by-line calculations of the solar heating rate are shown in Fig. 2 for two model atmospheres of McClatchey et al. (1972): midlatitude winter and tropical. The differences in calculated heating rates between the fast and "exact" line-by-line methods are shown in Figs. 3 and 4. Also shown, for comparison, are the differences from the line-by-line methods when using the formulas of Lacis and Hansen (1974) [their Eqs. (21) and (24) but with the effect of temperature on the scaled water vapor amount neglected], which are based on Yamamoto’s (1962) absorption curve.

It can be seen from Figs. 3 and 4 that differences between the heating rates using the present method and the line-by-line method are small, with a maximum of 0.06°C day⁻¹. This good agreement shows

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Footnote:


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the validity of the wing-scaling approximation. The maximum difference between the heating rates using Yamamoto's absorption curve and the line-by-line method is about 0.24°C day⁻¹. This large discrepancy arises primarily from the use of different sets of absorption data. The line-by-line calculations are based on detailed line parameters, while Yamamoto's absorption curve is derived from the laboratory measurements of Howard et al. (1956) and the solar radiation measurements of Fowle (1915). The choice of the standard pressure as the reference pressure for water vapor scaling is also believed to account for part of the discrepancy. It is clear that line center portions of the spectrum are important to the absorption of solar energy in the stratosphere and upper troposphere because of the small optical depth there. When water vapor is scaled linearly with pressure based on the standard pressure, the absorption in line center portions is greatly underestimated. As shown in Figs. 3 and 4, Yamamoto's heating rates above the 300 mb level are much smaller than the line-by-line calculations. It should be noted that Yamamoto's heating rates include absorption in the 0.7 and 0.8 μm bands, which we have neglected. However, the effect of these bands on solar heating is believed to be sufficiently small, especially for the drier atmosphere (cf. Fig. 1 of Yamamoto, 1962).

Table 4 shows the absorption of solar energy in the 2600–12040 cm⁻¹ spectral region by the entire air column computed from the different methods. The present method is in fairly good agreement (<2%) with the line-by-line calculations for both the midlatitude winter and tropical atmospheres. The calculations based upon Yamamoto's absorption data are within 6% of the line-by-line calculations.

<table>
<thead>
<tr>
<th></th>
<th>Line-by-line method</th>
<th>Present method</th>
<th>Yamamoto (1962)</th>
</tr>
</thead>
<tbody>
<tr>
<td></td>
<td>m = 0.5</td>
<td>m = 1.0</td>
<td></td>
</tr>
<tr>
<td>Tropical</td>
<td>10.56 (0.892)</td>
<td>10.47 (0.884)</td>
<td>11.43 (0.965)</td>
</tr>
<tr>
<td>Midlatitude</td>
<td>7.12 (0.601)</td>
<td>7.27 (0.615)</td>
<td>7.23 (0.612)</td>
</tr>
</tbody>
</table>

Table 4. Absorption of solar energy (mW cm⁻²) by water vapor in clear atmospheres with a solar zenith angle equal to 60° and surface albedo equal to zero. Numbers in parentheses are the heating rate (°C day⁻¹) of the entire air column.

value for all clouds and wavenumbers (Hansen and Pollack, 1970). The relative humidity within the cloud is assumed to be 100%, and the surface albedo and solar zenith angle are set to be 0.07 and 60°, respectively. Since our purpose is to test the validity of the wing-scale approximation but not the absorption by cloud droplets which is not well known, the absorption of solar radiation by cloud droplets is not included.

The computed heating rate profiles using the present method are compared with those using the “exact” line-by-line method and Yamamoto's absorption curve. For the heating rate computations using the present method, we first compute fluxes for each k using the delta-Eddington approximation with w computed from (8) and the incoming solar energy normalized to 1. The total flux is then the sum of these fluxes multiplied by μh(k) d log k as indicated in (10). To apply Yamamoto's absorption curve to the scattering scheme which is derived for a monochromatic case, the mean absorption function is reduced to a form equivalent to (9) (e.g., Yamamoto et al., 1970; Lacis and Hansen, 1974; Liou and Sasamori, 1975) as

\[ \tau_i(w) = \sum_{n=1}^{N} a_n \exp(-b_n w), \]

where \( a_n \) and \( b_n \) are obtained using a least-square fit. \( w \) is given by (8) with \( r = 1013 \) mb, and \( N \) is the number of terms used to fit \( \tau_i \). Total fluxes are computed similarly to the procedures just described for the case using the present method.

Comparisons of the present method (using Table 2) with the “exact” line-by-line method are made in Fig. 5 for the 0.94 and 1.38 μm bands. It can be seen that the results of the present method are in good agreement with the line-by-line calculations. The difference is less than 0.015°C day⁻¹ except at the top layer where the difference is 0.03°C day⁻¹ for the 1.38 μm band.

Two heating rate profiles computed from the present method and using Yamamoto's absorption curve are compared in Fig. 6. The weighted k-distribution function listed in the last column of Table 2 is used.

for the present method, and the coefficients $a_n$ and $b_n$ obtained by Lacis and Hansen (1974) are used for the case using Yamamoto's absorption curve. It can be seen that these two heating rate profiles agree well except in the region above the 300 mb level where the maximum difference is 0.13°C day$^{-1}$. Compared to the present method, Yamamoto's absorption curve predicts smaller heating above the 300 mb level and larger heating in the lower troposphere, which agree with the results shown in Fig. 4 for the clear case. However, the mean column heating rate using Yamamoto's absorption curve (0.39°C day$^{-1}$) is slightly less than that using the present method (0.41°C day$^{-1}$), which is contrary to the results for the clear cases (see Table 4). This is an indication of the uncertainty arising from the use of exponential series fit to mean transmission functions.

Table 5. The maximum error in the solar heating rate (°C day$^{-1}$) and the percentage error in the solar flux at the earth's surface (in parentheses) for a standard midlatitude winter atmosphere with a solar zenith angle of 60°.

<table>
<thead>
<tr>
<th>$T_r$ = 240 K</th>
<th>$T_r$ = 260 K</th>
</tr>
</thead>
<tbody>
<tr>
<td>$m$</td>
<td>200</td>
</tr>
<tr>
<td>$p_r$ (mb)</td>
<td></td>
</tr>
<tr>
<td>0.5</td>
<td>-0.128</td>
</tr>
<tr>
<td>0.6</td>
<td>-0.114</td>
</tr>
<tr>
<td>0.7</td>
<td>-0.081</td>
</tr>
<tr>
<td>0.8</td>
<td>-0.053</td>
</tr>
<tr>
<td>0.9</td>
<td>-0.076</td>
</tr>
<tr>
<td>1.0</td>
<td>0.100</td>
</tr>
</tbody>
</table>

5. Sensitivity to scaling parameters

Although the proper ranges for the choice of the scaling parameters $m$, $p_r$, and $T_r$ can be determined from the physics of radiative transfer in the atmosphere as discussed in Section 2, the specific set of scaling parameters used in this study is determined empirically. It is therefore important to know the sensitivity of the results to different choices of these parameters.

Table 5 lists the maximum error in the atmospheric heating rate and the percentage error in the solar radiation at the earth's surface for the clear midlatitude winter atmosphere when different values of the scaling parameters are used. It can be seen that the optimum choice of the parameters is $m = 0.8$, $T_r = 240$ K and $p_r = 300$ mb (or 400 mb). However, the results are not sensitive to the scaling parameters. For example, the heating rate errors are all smaller than 0.006°C day$^{-1}$ within the ranges of 0.7 ≤ $m$ ≤ 0.9 ($p_r = 300$ mb, $T_r = 240$ K), 200 mb ≤ $p_r$ ≤ 400 mb ($m = 0.8$, $T_r = 240$ K) and 240 K ≤ $T_r$ ≤ 260 K ($m = 0.8$, $p_r = 300$ mb). Compared to the peak heating rate of ~1°C day$^{-1}$, the error can be considered as small.

A surprising result as shown in Table 5 is the small error induced in the computed heating rate within a wide range of the parameter $m$ when the reference pressure is chosen at 1000 mb. If only the error in heating rate is considered, one might like to choose $p_r$ at 1000 mb. However, this choice of $p_r$ systematically underestimates the absorption of solar radiation in the atmosphere. It results in a much larger error in the computed solar radiation at
the earth’s surface as compared to the use of other reference pressures. Since the absorption of solar radiation at the earth’s surface is important in climate studies, this factor should also be considered in choosing the reference pressure.

In utilizing the $k$-distribution method for computing solar radiation, fluxes are first computed for each value of $k$. The total flux is then the sum of these fluxes [see Eq. (10)]. Since the computing time is proportional to the number of $k$ intervals, it is important to know the sensitivity of the results to the number of $k$ intervals used. It is found from sample calculations that increasing the number of these intervals from 10 to 100 changes the heating rate by a maximum of 0.024°C day$^{-1}$. Using the line-by-line calculation as a standard, the maximum heating rate error decreases incidentally from 0.035 to 0.024°C day$^{-1}$ when the number of $k$ intervals decreases from 100 to 10.

6. Summary

A method has been developed for computing the absorption of solar radiation by atmospheric water vapor. It is based on line parameter data with accuracy comparable to that of line-by-line calculations. High speed is achieved by using a one-parameter scaling approximation to convert an inhomogeneous path into an equivalent homogeneous path at suitably chosen reference conditions ($p_r = 300$ mb and $T_r = 240$ K). Precomputed tables (based on line-by-line calculations) of either the absorption coefficient distribution (i.e., $k$-distribution) or the transmittance of solar flux versus equivalent water vapor amount, both of which are tabulated, provide the user with a fast method suitable for climate models or general circulation models. The variation of incident solar flux with wavenumber within the water vapor absorbing region (2600–12040 cm$^{-1}$) has been folded into the tables. The present method is similar in form and speed to that of Lacis and Hansen (1974). It differs primarily in that 1) their transmittances are based upon the empirically derived absorption curve of Yamamoto (1962), whereas we use the line parameter data of McClatchey et al. (1973), and 2) their water vapor amount is scaled to STP whereas we use upper tropospheric values (300 mb and 240 K).

The accuracy of the present method is compared to line-by-line calculations for two model atmospheres: midlatitude winter and tropical. The present method has a maximum error of $\sim 4\%$ of the peak heating rate for both clear and cloudy atmospheres. It has been shown from sample calculations that the results are not very sensitive to the choice of scaling parameters. The present method has the additional advantage over previous methods in that it can be applied to any portion of the spectral region containing the water vapor bands. It thus permits one to take into account the combined effects of aerosols or trace gases that may alter the spectral distribution of solar energy reaching the water vapor layer.

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REFERENCES


