Reply

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I welcome the comments by Kopp who brings out two important points concerning our recent paper on energy diagnosis during condensation (Tag, 1980; hereafter Tag). Kopp's first point concerns the constancy of L' when $(C_w - C_{pv})$ is variable (as shown in Fig. 1 of Tag). Kopp's second point concerns the correct temperature to use in defining L for computing the change in temperature resulting from condensation. Both of Kopp's points are valid. The first strengthens the definition of L' as proposed earlier.

1. Determining a value for L'

Fig. 1 from Tag illustrated the variation of L and L' with temperature. The theoretical development leading to L' demanded that L' be constant. However, the use of a variable $(C_w - C_{pv})$ resulted in an 11% variation of L' when computed from specific heats valid between −50 to +60°C. Such a large variation was somewhat disturbing in view of the constancy required by the theory.

The computation of L' was performed using Eq. (9) of Tag, i.e.,

$$L' = L + (C_w - C_{pv})T, \qquad (1)$$

which, as noted in footnote 2, is based on the assumption that $(C_w - C_{pv})$ is constant during an integration from 0 K to T. We noted that a more rigorous definition would take into consideration the fact that $(C_w - C_{pv})$ is unknown [at least in the Smith-

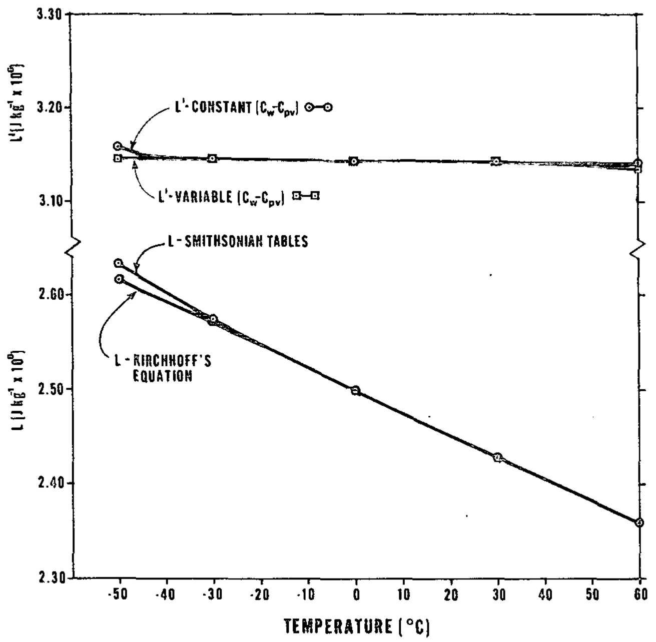

FIG. 1. Variation of L with temperature, and variation of L' when L' is defined according to Eq. (1) at 0°C. "L—Kirchhoff's Equation" uses Kirchhoff's equation ($dL/dT = C_{pv} - C_w$) and shows the variation of L based on L, C_{pv} and C_w defined at 0°C. "L'—Constant ($C_w - C_{pv}$)" is based on C_w and C_{pv} defined at 0°C. "L'—Variable ($C_w - C_{pv}$)" is based on the form of Eq. (3) when L' is adjusted to reproduce Eq. (1) at 0°C. Values of L, C_{pv} and C_w are taken from the *Smithsonian Meteorological Tables* (List, 1958).

sonian Meteorological Tables (List, 1958)] below −50°C. A two-part integration acknowledges this point:

$$L' = L(T) + \int_{0\,\mathrm{K}}^{223.16} (C_w - C_{pv})dT + \int_{223.16}^{T} (C_w - C_{pv})dT. \quad (2)$$

Since the first integral is unknown, L' is only known to within an arbitrary constant. Consequently, we concluded that 1) the computation of total energy (Tag's Eq. 8) and 2) the definition of L' can be simplified by assuming that the specific heats are constant from 0 K to T. This assumption leads to (1) above, with which we chose to define a numerical value for L' based on 0°C (a temperature more representative of the atmosphere). As noted by Kopp, however, Eq. (1) cannot be used, with variable specific heats, to determine the degree of variation of L' when calculated over the range −50 to +60°C. Kopp correctly points out that an integral form such as (2) must be used to define this variation.

To be consistent with Fig. 1 of Tag, we adjust the arbitrary constant [represented in the first integral of Eq. (2)] to duplicate L' at 0°C when using Eq. (1). And, since specific heats are available at 5° increments above 0°C (averages can be used below), it is convenient to integrate at 10° increments over the entire range of temperature. As an example, computing L' at −30°C would involve two integrations:

$$L'_{243.16} = L(243.16) + \text{constant} + \int_{223.16}^{233.16} (C_w - C_{pv})dT + \int_{233.16}^{243.16} (C_w - C_{pv})dT, \quad (3)$$

where the two integrals would be evaluated using specific heats valid at −45 and −35°C, respectively. Computing L' at −40°C would involve a single integration while −20°C would take three, etc. By using the form of (3) to determine the variation in L' we have eliminated the cumulative error generated by using Eq. (1) by itself.

Using the above, Fig. 1 of Tag has been redrawn and is reproduced here as Fig. 1. The only difference lies in the line labeled "L'—Variable ($C_w - C_{pv}$)." Based on this recomputation L' varies by only 0.19% over the entire 110° range. This minimal variation is now consistent with the theory developed in Tag.

2. Computation of ΔT resulting from condensation

By equating the total energy before and after condensation, the resulting change in temperature can be computed [see Eq. (13) in Tag]:

$$\Delta T = \frac{LM_c}{M_d C_{pd} + (M_v - M_c)C_{pv} + M_c C_w}, \quad (4)$$

where M_d, M_v and M_c are the respective masses of dry air, vapor, and cloud water. Tag made the statement that L should be defined at a temperature midway during the change ($T + \Delta T/2$). Kopp correctly points out that L must be defined at T. This requirement becomes obvious if one examines the energy equation leading to (4). Although ΔT will normally be small and any difference in L usually ignored, a prescription of L at T does simplify computations when an accurate change in temperature is required.

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REFERENCES

List, R. J., 1958: *Smithsonian Meteorological Tables*. Smithsonian Institution, Washington, DC, 527 pp.

Tag, P. M., 1980: On the diagnosis and conservation of energy during condensation. *J. Atmos. Sci.*, 37, 2347–2351.