

On the Annual Variation in Height of the Tropical Tropopause

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(Manuscript received 28 July 1980, in final form 8 May 1981)

ABSTRACT

The existence of an annual variation in height and temperature of the tropopause over tropical regions has long been recognized, but has not been fully explained. In this paper it is proposed that the variation is a fairly direct response to the annual variation in average tropical surface insolation. The variation in insolation causes a corresponding annual cycle in average tropical sea surface temperature with a total range of order 1 K. The consequent variation in absolute humidity in turn produces an annual variation in upper tropospheric potential temperatures, and hence in the height and temperature of the tropopause.

The physical link between the surface and the tropopause is provided by convection in the cores of the giant cumulonimbus clouds (hot towers) of the tropical oceanic regions, in which air parcels can achieve the maximum possible heating by release of latent heat. The process is modeled quantitatively in a simplified way, and excellent agreement is found between the predicted and observed phase and amplitude of the annual variation in tropopause potential temperature.

Since the regular seasonal variation in insolation is relatively small in the tropics, the annual variation in sun-earth distance is an important factor in the variation of surface insolation. The annual cycle in the properties of the tropical tropopause thus provides the first identifiable effect of the earth's orbital eccentricity on climate parameters.

1. Introduction

The chief features that distinguish the tropical tropopause from its mid-latitude and polar counterparts are its increased height (15–18 km), its decreased temperature (often below 190 K), and its enhanced sharpness. The factors that determine the height and physical properties of the tropopause at any latitude are not fully understood, although the twin facts that the troposphere is heated from below by the surface of the earth and that the stratosphere is heated internally by the direct absorption of solar radiation guarantee that a temperature minimum must exist. The observed abruptness of the transition, however, has long been a puzzling feature and has led to several studies of the relative contributions of radiative effects and dynamical heat transfer to the formation of the tropopause (Goody, 1949; Staley, 1957). The ultimate driving force for both radiative and dynamical effects in the atmosphere is the intensity of solar radiation, and it seems appropriate to seek a relationship between the height of the tropopause and the insolation at the surface. In this paper we shall develop the arguments for such a relationship in the case of the tropical tropopause, and we shall argue that the intermediary mechanism connecting tropopause properties and insolation involves small changes in tropical sea surface temperature (SST) and conse-

quent changes in the intensity of cumulus convection and hence of the upward branch of the tropical Hadley cell. Since the seasonal variation of insolation is relatively weak in the tropics, the annual variation due to the ellipticity of the earth's orbit is an important factor.

The annual variation in height of the tropical tropopause has been noted by a few authors in the past (Staley, 1962; Ramanadham *et al.*, 1961; Cole, 1975). The corresponding annual variation in temperature of the tropical lower stratosphere has received more attention, and several authors have discussed its relationship to other features of the atmosphere. This earlier work will be discussed later in the context of the model presented here.

2. Annual variation in tropopause properties

Fig. 1 shows the monthly mean tropopause height at a number of tropical Pacific stations derived from routine rawinsonde data. The height was determined using the conventional definition of the tropopause (Craig, 1965) as the lowest point (above the 500 mb level) for which the temperature lapse rate decreases to 2 K km^{-1} or less, and for which the average lapse rate from this level to any point within the next 2 km does not exceed 2 K km^{-1} . Table 1 lists the stations and the time periods for which data were used in this study. More data are available than

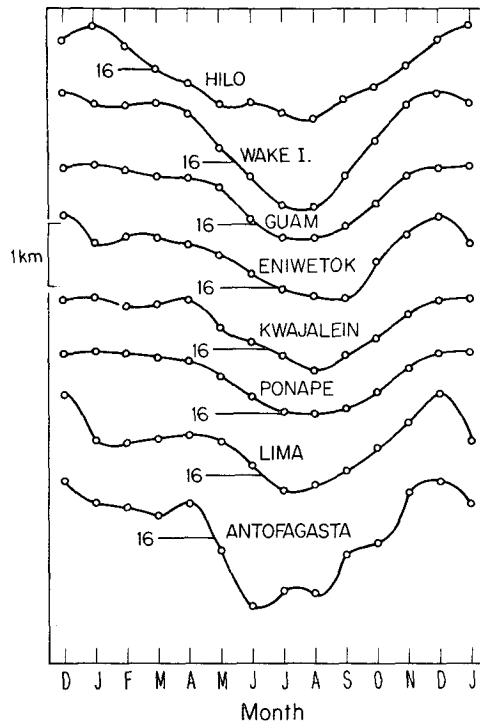


FIG. 1. Monthly mean tropopause height at the stations listed in Table 1. The 16 km height level is indicated on each curve and a 1 km vertical scale is shown at the left.

were actually used, but the lack of dependence of the results on the number of years actually used at any given station is reasonably convincing evidence that the important features would not be changed significantly by using a longer data base.

The principal feature of all of the records in Fig. 1 is a pronounced minimum in tropopause height in the late Northern Hemisphere summer, and a broad maximum in the Northern Hemisphere winter. As might be expected from the rough constancy of tropical lapse rates, there is a corresponding annual variation in tropopause temperature, illustrated in Fig. 2 for Ponape, the station with the longest series of individual records. The tropical Pacific tropopause is ~1–2 km higher and 5 K colder in the Northern Hemisphere winter than in the Northern Hemisphere summer. This forms the observational basis for the arguments that follow.

3. Response of tropical sea surface temperatures to the annual variation of insolation

In the remainder of the paper, we shall develop the theory that the annual variation in tropopause heights represents a response to an annual variation in tropical SST's which is forced by the annual variation in absorbed solar radiation. The diurnal and seasonal variation of solar radiation at the top of the atmosphere can readily be calculated from

TABLE 1. Locations and time periods of data used in the study.

Station	Latitude	Longitude	Years
Hilo	19.7°N	155.1°W	1964–70, 1975–76
Wake I.	19.3°N	166.6°E	1957, 1971–76
Guam	13.5°N	144.8°E	1953–70, 1975–76
Eniwetok	11.3°N	162.4°E	1957, 1968–69
Yap	9.5°N	138.1°E	1953–69
Kwajalein	8.7°N	167.7°E	1968–69
Koror	7.3°N	134.5°E	1953–69
Ponape	7.0°N	158.2°E	1951–73
Lima	12.0°S	77.1°W	1970–76
Antofagasta	23.4°S	70.5°W	1970–72

purely geometrical considerations (e.g., Sellers, 1965). Near the equator two effects control the annual variation: the tilt of the earth's axis tends to produce maxima twice per year at the equinoxes, when the sun is most nearly overhead at noon, while the variation in sun-earth distance caused by the ellipticity of the earth's orbit tends to produce a maximum in January and a minimum in July.

The amplitude and shape of the annual variation change with latitude within the tropics, which we define here as the region within 20° of the equator, but the overall similarity of the curves in Fig. 1 suggests that a substantial amount of spatial averaging is taking place on the time scale of a month. Such averaging is to be expected as a result of horizontal motions in both the ocean and the atmosphere, and suggests that the model calculation should either use the spatially averaged insolation to calculate a single uniform tropical SST or average the temperatures calculated from the actual insolation at suitably chosen latitudes. The latter alternative has been adopted since it seems to be more physically realistic. For illustration, however, Fig. 3 shows the result of averaging the solar radiation absorbed by the ocean over the latitude range +20° to -20°, and over one day in time. A solar

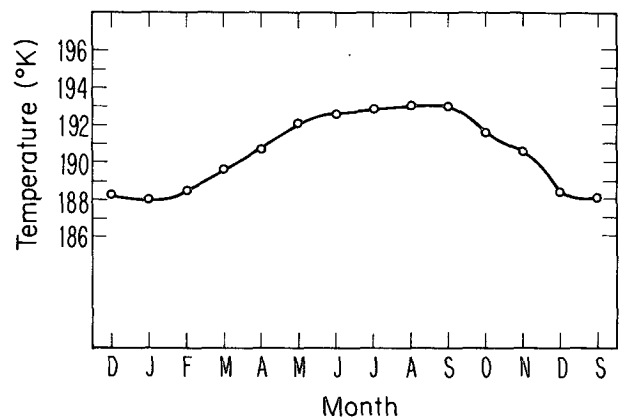


FIG. 2. Monthly mean tropopause temperature at Ponape, 1952–69.

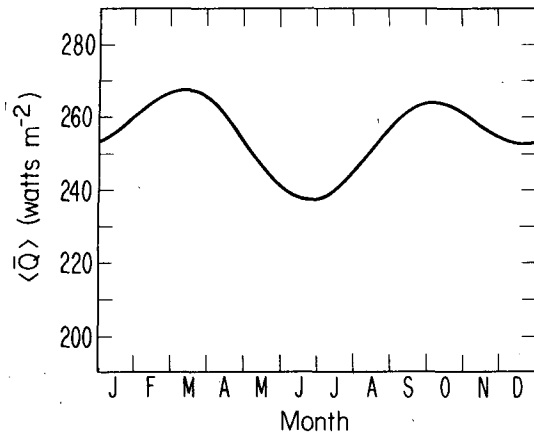


FIG. 3. Annual variation in surface insolation averaged over the latitude belt from +20° to -20°.

constant of 1373 W m⁻² (Fröhlich, 1977) was used, and 61% of the radiation incident at the top of the atmosphere was assumed to be absorbed in the ocean (Vonder Haar and Hanson, 1969). The curve shows two nearly equal maxima at the equinoxes, and two minima of different depths near the solstices. The deeper minimum occurs in July when the earth is farthest from the sun, and the shallower minimum in December, when the earth is closest to the sun.

The model ocean-atmosphere system employed to calculate the SST is described in the Appendix. Briefly, the model consists of a layer of water of depth 20 m that is tightly coupled thermally to the atmosphere. The system is heated by solar radiation and cooled by longwave radiation to space and by horizontal heat transport. A single uniform temperature is calculated for the system, and is referred to as the sea surface temperature.

The average SST over the latitude range ϕ_1 to ϕ_2 is defined as

$$\bar{T} \equiv \int_{\phi_1}^{\phi_2} T d\phi (\phi_2 - \phi_1)^{-1} \quad (1)$$

(the refinement of allowing for the variation in global surface area with latitude is hardly worthwhile for the present purpose). The integration in (1) has been carried out numerically by Gaussian quadrature using the temperatures calculated at the five latitudes 0°, ±10.8° and ±18.1° dictated by the chosen polynomial. The annual variation at each of the five latitudes is shown in Fig. 4, together with the variation of \bar{T} , shown by the broken line. Note that the phases of the variations in the two hemispheres are reversed, as expected, and that the range of the annual variation is appreciably larger in the Southern Hemisphere. This is caused by the reinforcement of the effects of seasonal and orbital variations in the Southern Hemisphere, as compared to their partial cancellation in the Northern Hemi-

sphere. The amplitude and phase of the variation in \bar{T} are similar to those of the variation at the equator, with a maximum temperature in late April and a minimum in August. Comparing with Fig. 3, it is clear that these represent delayed responses to the radiation maximum in March and the minimum in June. The delay is caused by the thermal inertia of the ocean, which also has the effect of reducing the response to the secondary minimum in December to little more than a shoulder in the curve.

Fig. 5 shows the calculated average temperature on an expanded scale, together with the average observed SST's in the latitude range from 7°N to 9°S in the Atlantic Ocean during 1922-38 as reported by Roden (1962). The agreement in phase and amplitude between the observed annual variation and that predicted by the model is excellent, although the temperatures given by the model are 3-4 K higher than those observed. As discussed in the Appendix, the model contains an adjustable parameter γ (the fraction of the solar radiation reaching the surface that is needed to balance the escaping longwave radiation) whose value was chosen to match observed tropopause potential temperatures with values calculated in the manner described in the next section. Clearly, the model requires SST's somewhat higher than those observed in order to match the observed tropopause potential temperatures. The possible reasons for this will be discussed later, but the important point for our present purposes is the excellent agreement in amplitude and phase of the annual variation itself.

Similar agreement can be found between the model results and measurements of the annual variation of the total heat stored in the global oceans as reported by Ellis *et al.* (1978). Fig. 6 shows the

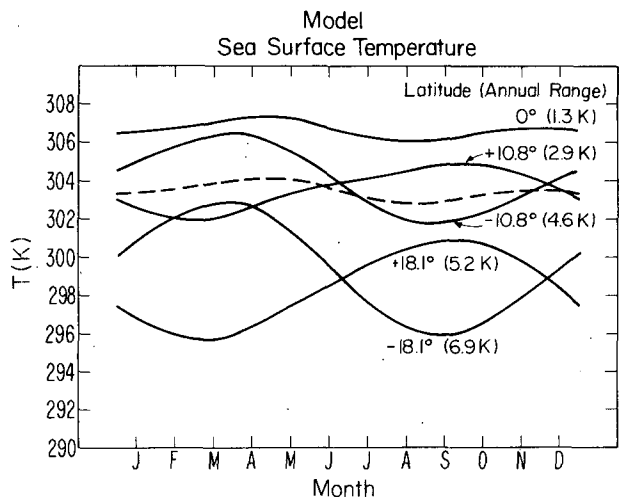


FIG. 4. Calculated annual variation in sea surface temperature at the latitudes indicated (solid lines) and average over the ±20° latitude belt (broken line). Total annual range in temperature is shown in parentheses for each latitude.

comparison: the points are taken from the tabulation by Ellis *et al.*, and the curve is the result of equating the heat stored in any month to ρCh times the increase in SST during the month [see Eq. (A13)]. There is a surprisingly good agreement in phase and amplitude, considering that the model applies only to the tropics while the observations cover the entire earth. The natural conclusion is that the bulk of the heat storage takes place in the tropical oceans, as would be expected.

4. Sea surface temperature and the tropical tropopause

Having arrived at a quantitative estimate of the effect of time-varying insolation on tropical SST's, the final step in the argument is to estimate the influence of these SST variations on the atmosphere.

The long-established view of the upward branch of the tropical Hadley cell circulation as a large-scale gentle uplift of air has been replaced by a view that attributes major importance to strong localized updrafts in large well-developed cumulus clouds—the so-called hot towers (Riehl and Malkus, 1958). In deep cumulus convection the saturated ascending air conserves its equivalent potential temperature, given approximately by

$$\theta_e = \theta \exp\left(\frac{wL}{c_p T}\right), \quad (2)$$

where θ is the ordinary potential temperature, T the absolute temperature, L the corresponding latent heat of condensation, c_p the specific heat of air at constant pressure, and w the humidity mixing ratio, i.e., the mass of water vapor contained in unit mass of moist air. In practice the boundary-layer air over the tropical oceans is not saturated [in which case

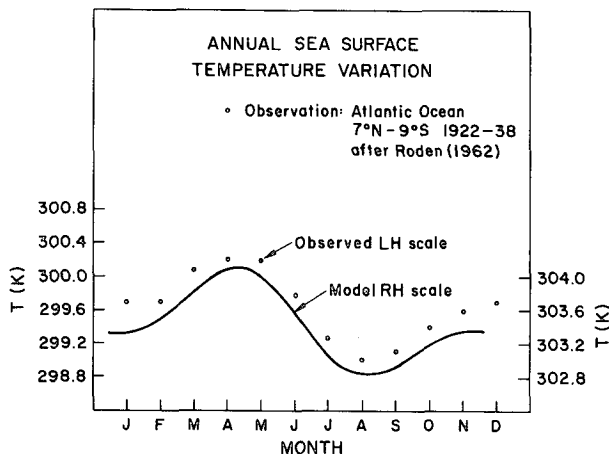


FIG. 5. Annual variation in sea surface temperature averaged over the tropics compared with average observed values over the latitude belt 7°N to 9°S in the Atlantic Ocean.

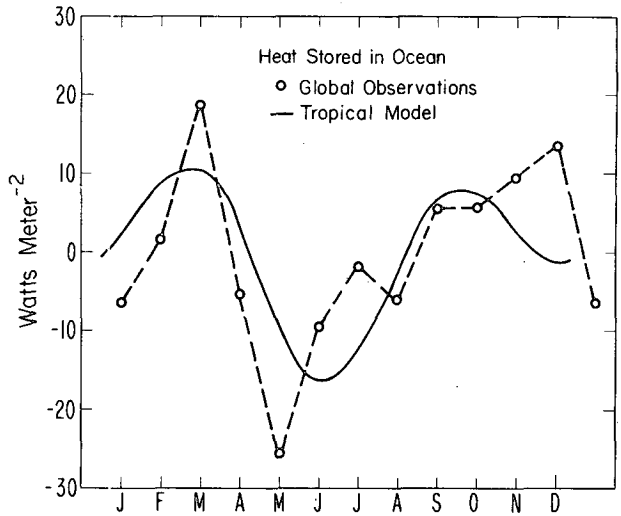


FIG. 6. Annual variation in rate of heat storage in the tropical ocean calculated from the model (solid line), and values computed from observations for the global ocean (from Ellis *et al.*, 1978).

T in (2) should be replaced by the dew-point temperature], and some kind of forced uplift of surface air is needed to reach the point above which the release of latent heat can drive the upward motion. Such forced uplift is thought to be provided by synoptic-scale waves whose energy is in turn derived from the latent heat release, accounting for the preferred growth of hot towers in such waves. Here we shall assume that the mechanisms needed to stimulate the growth of the hot towers exists, and we shall discuss the effects of a variation in SST on their characteristics.

We make the assumption that the relative humidity of the boundary-layer air that is ultimately pumped up in the hot towers has a constant value of 85%, based on an estimate by Kraus (1973) for conditions in the vicinity of tropical storms. The exponential dependence of vapor pressure on temperature then makes the absolute humidity very sensitive to small SST changes. If the temperature is 300 K, for instance, an increase by 1 K will lead to an increase of ~6% in vapor pressure.

Eq. (2) shows that the maximum potential temperature attainable by an ascending air parcel is identical to the equivalent potential temperature of the same parcel at the surface. We can thus use the SST variation given by the model (and implicitly assumed to be identical to the temperature of the surface air) to calculate the annual variation in potential temperature of the air after it has been convected up to the point where practically all of its water-vapor content has been lost, i.e., to the vicinity of the tropopause. In addition to the assumption of constant surface relative humidity, this calculation assumes that there is no mixing with unsaturated

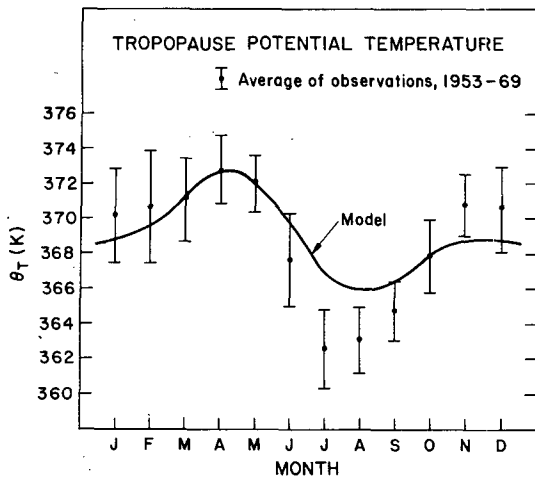


FIG. 7. Calculated annual variation in potential temperature at the tropopause and observed values averaged over the available time period at five tropical Pacific stations.

air during the ascent, a valid assumption for the cores of the hot towers.

The results are shown in Fig. 7, together with the observed monthly mean tropopause potential temperatures averaged over the entire time period 1953-69 and over the five stations Ponape, Guam, Yap, Koror and Eniwetok. The vertical bars indicate standard deviations of the data, but it should be pointed out that the spread is dominated by an apparent systematic variation over the time period that dominates the random deviations. This secular variation appears to have been in phase with the 11-year solar activity cycle, and has been described in an earlier publication (Gage and Reid, 1981). Its magnitude is illustrated in Fig. 8, where the systematic differences between values in the solar-maximum year of 1958 and the solar-minimum year of 1964 are seen to account for most of the scatter.

The agreement in the mean value of potential temperature between theory and observation in Fig. 7 was forced by choice of the parameter γ in the model, but the amplitude and phase of the annual variation are model predictions that appear to be in good agreement with the observations, indicating that the essential features of the model are realistic. In particular, it provides strong evidence that the tropospheric air immediately adjacent to the tropical tropopause has achieved its high potential temperature through the release of latent heat in a saturated environment.

The final step in the argument is the calculation of the annual variation in tropopause height, given the corresponding variation in potential temperature. Since radiative effects play an important role here, this step is not so easy to take, and we have made use of observational data on the temperature gradient in the tropical lower stratosphere instead

of constructing a model from basic principles. Such a model obviously would be a useful tool, but its development is beyond our present scope.

Ascending air parcels in the troposphere are accelerated by buoyancy forces derived from diabatic heating by release of latent heat. The annual variation in latent heat release proposed above must be accompanied by a corresponding variation in the upward acceleration and hence in the upward velocity of the air parcels near the tropopause and in the lower stratosphere. The temperature in the lower stratosphere is determined by a balance between the adiabatic cooling produced by this upward motion and net radiative heating by ozone and carbon dioxide; the annual variation in average upward velocity must therefore be accompanied by an annual temperature variation. The phase of the temperature variation (and of the corresponding potential temperature variation) at a fixed height in the lower stratosphere, however, should be opposite to that of the potential temperature variation at the tropopause itself. The peak SST's occur in March, and the moisture content of the air and thus potential temperatures at the tropopause also reach their maximum then. The consequent maximum in buoyancy forces and in adiabatic cooling, however, produce a minimum in lower stratospheric temperatures. In a similar way, maximum lower stratospheric temperatures should occur in conjunction with minimum tropopause potential temperatures in August. Fig. 9 shows the observed monthly mean potential temperatures at a fixed height of 18 km averaged in the same way as the tropopause observations in Fig. 8. The expected antiphase relationship is clearly present, and again the scatter in the data is nearly all explained by the systematic difference between years

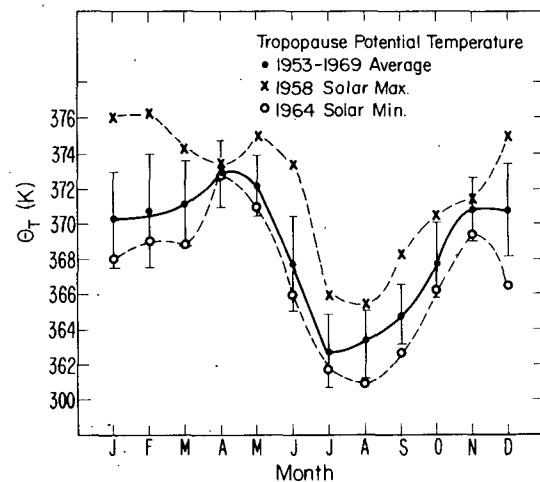


FIG. 8. Observed tropopause potential temperatures averaged over the entire time period (solid line) and for the individual years 1958 and 1964 (broken lines).

of maximum and minimum solar activity. The annual variation shown in Fig. 9 was discussed by Newell *et al.* (1969) and by Reed and Vlcek (1969) as evidence for an annual modulation of the Hadley cell circulation. The explanation we propose is basically the same, but with the addition of the link to tropopause height, and the suggested driving mechanism of deep cumulus convection varying in response to the annual cycles of tropical insolation and sea surface temperature.

The equilibrium level of an ascending air parcel is reached when it encounters stratospheric air of the same potential temperature, and this is the level at which the tropopause should form. In estimating the variation in height, the potential temperature variations in the troposphere and in the lower stratosphere both need to be taken into account. In March the upper tropospheric potential temperature reaches its maximum, thus tending to produce a high tropopause even if the stratospheric temperature were constant. In fact, the associated increase in adiabatic cooling (possibly combined with a variation in stratospheric ozone concentration) causes a minimum in lower stratospheric temperatures at the time, as illustrated in Fig. 9, also causing an increase in tropopause height. These two effects usually reinforce each other, amplifying the change in tropopause height.

To give a numerical example, the average potential temperature gradient in the lower stratosphere

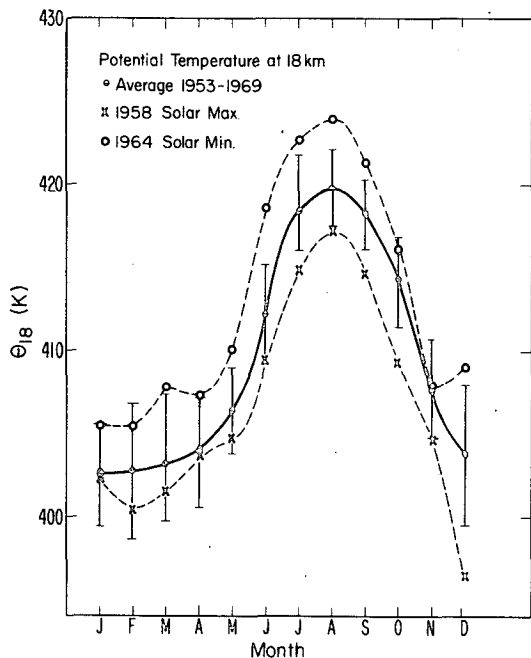


FIG. 9. Observed annual variation in potential temperature at a fixed height of 18 km averaged over the entire time period (solid line) and for the individual years 1958 and 1964 (broken lines).

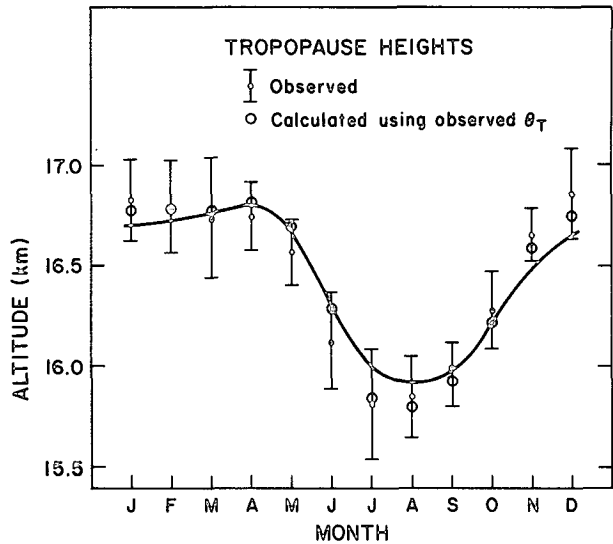


FIG. 10. Calculated annual variation in tropopause height using model potential temperatures (solid line) and observed potential temperatures (open circles), compared with observed tropopause heights.

is $\sim 25 \text{ K km}^{-1}$. The peak-to-peak annual variation in tropopause potential temperature of $\sim 10 \text{ K}$ would thus produce a variation of 0.4 km in tropopause height by itself. The accompanying antiphased peak-to-peak variation of $\sim 16 \text{ K}$ in lower stratosphere potential temperature would produce an additional variation of 0.64 km in the potential temperature contours, giving a total peak-to-peak annual variation in tropopause height of 1.04 km, in good agreement with the observations.

The solid line in Fig. 10 shows the result of calculating the monthly deviation in the height of the tropopause from its average value, using the model tropopause potential temperatures, the observed 18 km potential temperatures illustrated in Fig. 9, and the long-term average, lower stratospheric, potential temperature gradient of 25 K km^{-1} . The open circles show the results of using the observed tropopause potential temperatures instead of the model values, while the dots with vertical bars show the observed tropopause heights with their standard deviations. As in the earlier figures, the data used are averages over the entire time period at the five Pacific stations mentioned earlier. The agreement shown in Fig. 10 is excellent, and provides confidence in the essential correctness of the mechanisms proposed.

5. Discussion

The essential features of the model proposed to explain the annual variation in tropopause height in the tropics are illustrated schematically in Fig. 11. The basic driving force is the annual variation in

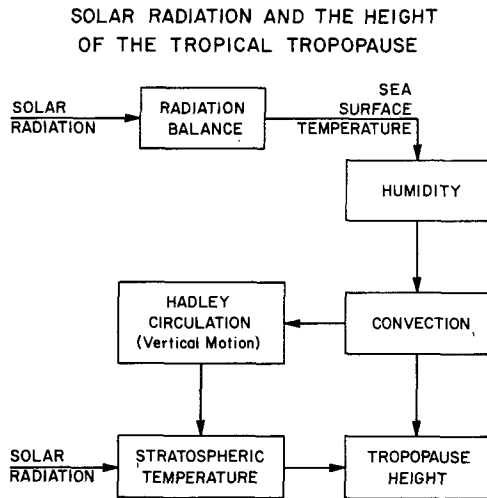


FIG. 11. Schematic diagram of the factors that determine tropical tropopause height.

solar radiation at the ocean surface, which causes a modulation in the SST and hence in the moisture available for heating the troposphere by latent heat release. The consequent variation in tropopause height is amplified by the modulation in adiabatic cooling in the lower stratosphere associated with the general Hadley circulation. The annual variation in solar radiation also may play a role in modulating the temperature of the lower stratosphere directly through variations in the heating due to stratospheric ozone. This effect probably would counteract the observed effects, however, since it would tend to raise the temperature of the stratosphere and lower the tropopause when solar radiation maximized, i.e., when the tropopause is observed to be at its highest. Since the model described here appears to explain the observations at least semi-quantitatively, the effect of a variation in stratospheric radiative heating is probably a minor one.

As mentioned earlier, the model was tuned to match predicted with observed potential temperatures at the tropopause by adjusting the parameter γ . The resulting predicted SST's were ~ 4 K higher than the average observed tropical values (see Fig. 5), although the amplitude and phase of the annual variation showed excellent agreement with observation. There are several possible reasons for the discrepancy. For example, the calculation makes use of the simplifying assumption that ascending air parcels come to rest where the buoyancy forces vanish. In reality, the tropopause will lie above this level, since the inertia of the ascending air causes it to overshoot the equilibrium level. Furthermore, there is a fairly wide range of local sea surface temperatures at any given time, and the hot towers are likely to grow preferentially over the warmer areas. The SST

that is normally measured itself represents some kind of average over the top meter or so of the ocean, and is not necessarily identical to the skin temperature of the water that is in direct contact with the atmosphere. Whatever the reason for the discrepancy, its existence is not relevant to the proposed mechanism for the annual variation, which involves only the magnitude and phasing of the month-by-month departures from the average.

The annual variation in tropical tropopause height has received comparatively little attention in the past, but the related problem of the annual variation in temperature of the tropical lower stratosphere has been discussed by several authors. Newell *et al.* (1969), Reed and Vlcek (1969) and van Loon and Jenne (1970) have found the annual temperature wave to be nearly symmetric about the equator with an amplitude of ~ 4 K and with a phase giving maximum temperature in early August. Newell *et al.* (1969) and Reed and Vlcek (1969) attributed this variation to an annual modulation in the intensity of the Hadley cell. They reasoned that the increased cooling in the lower stratosphere during the Northern Hemisphere winter was primarily a result of adiabatic cooling associated with enhanced upwelling. Reed and Vlcek (1969) argued that the phase of the upwelling should coincide with the variation in intensity of the Northern Hemisphere Hadley cell, suggesting that the seasonal variation of the intensity of the Southern Hemisphere Hadley cell was unimportant.

Several modeling studies have been made that appear to simulate the annual temperature wave in the lower stratosphere. Dickinson (1971) developed a simple analytical model for tropospheric zonal circulation on the equatorial β plane. His model uses an assumed seasonally and latitudinally varying thermal source term that parameterizes the diabatic heating due to latent heat release. In the vicinity of the equator the source term is somewhat stronger in January than in July, and leads to a more intense upwelling.

A more sophisticated numerical general circulation model has been employed by Manabe and Mahlman (1976) to study seasonal and inter-hemispheric variations in the stratosphere. Their model includes the annual variation of insolation associated with the variation in sun-earth distance as well as the seasonal variation. Annually varying SST's are prescribed by interpolation between monthly mean observed values, and the model reproduces the annual variation in temperature in the lower stratosphere very well. In their analysis of the contributing factors, they attribute major importance to the annual variation in the meridional circulation, and also to the dynamical effect of tropical waves. Since the waves are probably driven by latent heat release (Holton, 1972; Beer, 1978),

and are strongly influenced by sea-surface temperatures (Chang and Miller, 1977), they would appear to provide an additional mechanism for influencing the tropical tropopause that acts in the same phase as the one we have described.

As a result of these earlier studies, there seems little doubt that the annual variation of lower stratospheric temperature is associated with an annual variation of the intensity of upwelling associated with the ascending branch of the Hadley cell. The model of Dickinson (1971) showed the importance of the latent heat released by condensation in the tropics, while Bjerknes (1966) suggested that SST variations might play an important role in modulating the intensity of the Hadley cell circulation. The annual SST variation has itself been simulated by Wetherald and Manabe [1972] in a numerical ocean-atmosphere model whose most important feature from the present point of view is that it contains the same land-sea distribution in both hemispheres, thereby eliminating any effects that might arise from the asymmetry that actually exists. The model includes the variations in solar radiation due to the changing seasons and to the variation in sun-earth distance, and predicts an annual variation in such a sense that temperatures are ~ 1 K warmer in January than in July.

The evidence from both observation and theoretical models thus strongly suggests that there is an annual modulation in the intensity of the Hadley cell circulation. We propose that the cause of the modulation lies in the annual variation in average tropical insolation, which leads to a modulation of SST's and hence to a corresponding modulation in the intensity of diabatic heating in the troposphere by release of latent heat in condensation. The variation in insolation is itself strongly influenced by the varying sun-earth distance brought about by the ellipticity of the earth's orbit. The tropical atmosphere thus responds in a uniquely identifiable way to the ellipticity of the earth's orbit. Since the Hadley cell is an important feature of the general circulation, any modulation in its intensity may be reflected in corresponding variations in global climate due to teleconnections of the kind described by Bjerknes (1969). It is conceivable that such relationships could be responsible for the major modulations of global climate that occurred during the Pleistocene, whose dominant period was about 100 000 years (Broecker and van Donk, 1970; Hays *et al.*, 1976), close to the dominant period in the variation of the eccentricity of the earth's orbit. Although speculative, such considerations provide a stimulus for further studies of the connections between tropical and midlatitude climate parameters.

Acknowledgments. We wish to acknowledge the invaluable help of Carl Love, who carried out the programming and computer operations necessary to

extract the data from the original tapes and to compute the tropopause heights. We also wish to acknowledge the important part played by Dr. B. B. Balsley in initiating this study, and to acknowledge helpful discussions with several of our colleagues.

APPENDIX

The Ocean-Atmosphere Model

The interaction between the ocean and the atmosphere is a process of great complexity but of fundamental significance to climatology. Many theoretical models of varying degrees of sophistication have been developed, but all have contained empirical parameterizations that have been designed to avoid the need to treat the poorly understood processes that operate near the interface. Our objective in this study is to demonstrate that the annual variation in height of the tropical tropopause is caused by the annual variation in solar radiation at the surface, acting through the medium of an annual variation in sea-surface temperature. We thus need to calculate the average response of the SST to changing solar radiation, all else remaining constant. Since the details of the air-sea interaction are not of primary concern in this study, the model used is very simple, but appears to lead to results that are in general agreement with observations.

The model ocean consists of a slab of water of depth h , specific heat C and density ρ . The ocean is heated from above by solar radiation and is in good thermal contact with the atmosphere. The ocean-atmosphere system is cooled by infrared radiation escaping from the top of the atmosphere, and is assumed to have a uniform temperature, which we shall call the sea surface temperature. We assume that this model system will reproduce the average phase and amplitude of the response of the real SST to temporal variations in solar radiation.

Adopting the parameterization of Budyko (1969) for escaping longwave radiation in terms of surface temperature T , we can write the equation of heat balance in the form

$$\rho h C \frac{dT}{dt} = (1 - \alpha)S - A - BT - H, \quad (A1)$$

where S is the solar radiation at the top of the atmosphere, α the albedo of the system, and H the rate of horizontal transport of heat to higher latitudes. A and B are the constants in the infrared cooling relation, and were taken directly from Budyko (1969), assuming 50% cloud cover. Changing Budyko's units to watts per square meter, the numerical values adopted for A and B were -193.12 W m^{-2} and 1.44 $\text{W m}^{-2} \text{K}^{-1}$, respectively.

The horizontal heat-flux divergence H was treated by making the heuristic assumption that it is a linear function of the solar radiation received in the tropics,

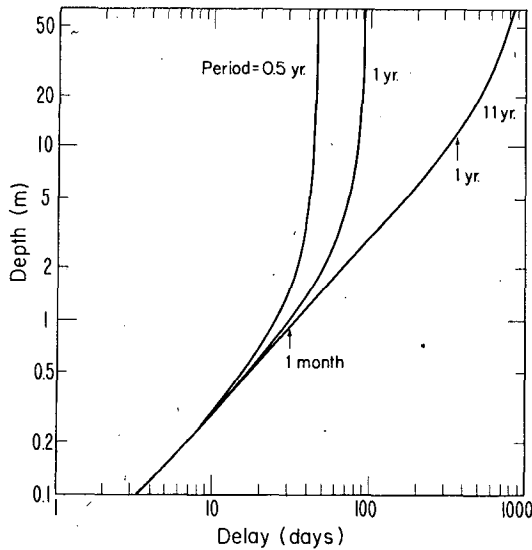


FIG. A1. Delay in time of peak sea surface temperature behind peak in solar radiation, assuming a simple harmonic variation in the intensity of solar radiation with the periods indicated.

so that the quantity $(1 - \alpha)S - H$ can be replaced by $\gamma(1 - \alpha)S$; γ then represents the fraction of the absorbed solar radiation that is used to maintain the temperature of the tropical ocean against long-wave cooling, while $(1 - \gamma)$ is the fraction that is ultimately exported from the tropics. γ was treated as an adjustable parameter whose value was chosen to yield a realistic mean tropopause potential temperature. The value thus determined for $\gamma(1 - \alpha)$ was 0.59, and together with an observed value of 0.24 for the albedo of the tropical oceans (Vonder Haar and Hanson, 1969) implies that ~17% of the incoming solar energy in the tropics is eventually exported to higher latitudes. This agrees well with observationally derived estimates (e.g., Vonder Haar and Suomi, 1971).

In the calculations described in the main text, Eq. (A1) was solved numerically for time-varying S computed for appropriate latitudes. An initial temperature of 280 K was chosen, the time step was 1 day, and the calculation was allowed to run for 20 years, by which time the temperatures were highly reproducible from one year to the next. If the solar radiation is assumed to be a simple harmonic function of time, however, the equation can be solved formally, and the dependence of the sea-surface temperature on various parameters clearly can be seen.

Writing (A1) in the form

$$\rho h C \frac{dT}{dt} = \gamma(1 - \alpha)S(t) - A - BT \quad (A2)$$

and assuming that

$$S(t) = S_0 + S_1 e^{i\omega t}, \quad (A3)$$

we can guess a simple harmonic solution for T with the same angular frequency as that of $S(t)$, but with a phase shift ϕ . Substituting

$$T = T_0 + T_1 e^{i(\omega t + \phi)} \quad (A4)$$

in (A2) together with (A3), and equating the non-time-varying terms, we find

$$T_0 = [\gamma(1 - \alpha)S_0 - A]B^{-1}. \quad (A5)$$

As an example, the calculated annual average value for S_0 at the equator is $\sim 415 \text{ W m}^{-2}$. Taking the values for $\gamma(1 - \alpha)$, A and B quoted above, we find $T_0 \approx 304 \text{ K}$ (31°C), which is encouragingly realistic considering that γ was adjusted to fit observed tropopause potential temperatures rather than observed sea surface temperatures.

Equating the time-dependent terms in (A2) and multiplying by $e^{-i\omega t}$, we find

$$i\omega T_1 \rho h C e^{i\phi} = \gamma(1 - \alpha)S_1 - BT_1 e^{i\phi}. \quad (A6)$$

Equating the real terms in (A6) yields an expression for the phase shift ϕ of the sea surface temperature response given by

$$\tan \phi = -\omega \rho h C B^{-1}. \quad (A7)$$

Since $\tan \phi$ is always < 0 , the SST lags behind the forcing radiation, as expected.

Equating the imaginary terms in (A6) yields a corresponding expression for the amplitude T_1 of the

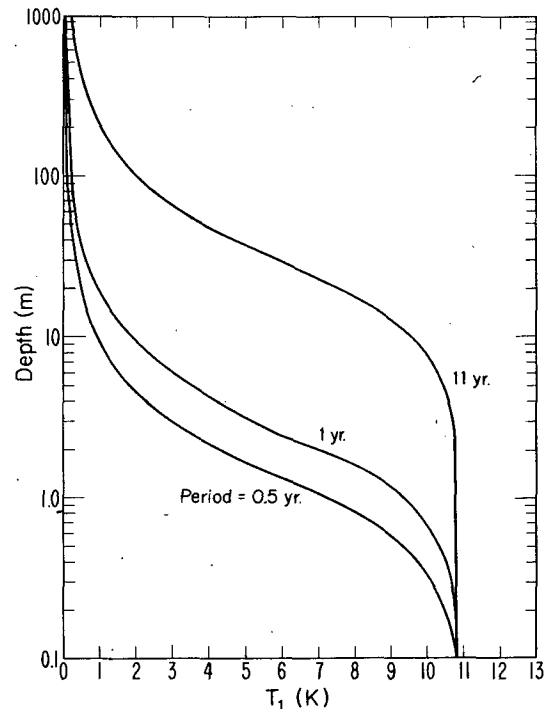


FIG. A2. Amplitude of variation in sea surface temperature for harmonic variation in the intensity of solar radiation with the periods indicated.

temperature variation, given by

$$T_1 = \frac{\gamma(1 - \alpha)S_1}{B \cos\phi - \omega\rho hC \sin\phi} \quad (A8)$$

Fig. A1 shows the time delay ($=\phi/\omega$) between the maximum in the solar radiation and the maximum temperature as a function of ocean surface-layer depth h . The three curves correspond to solar-radiation periods of 0.5, 1 and 11 years. The first two are the dominant periods in the annual variation shown in Fig. 3, while the 11-year period was chosen to illustrate the response to a variation in solar constant with the 11-year solar cycle such as that proposed in an earlier paper (Gage and Reid, 1981).

It is obvious from (A7) that as $\omega \rightarrow 0$ (very long periods), ϕ approaches zero for all depths: the ocean has enough time to remain in phase with the forcing radiation. As $\omega \rightarrow \infty$ (very short periods), $\phi \rightarrow -\pi/2$, and the temperature always lags 90° behind the radiation. These features are clearly seen in Fig. A1: for a depth of 20 m, the 0.5- and 1-year periods are in the $\phi \approx -90^\circ$ region, and the thermal time delays are respectively 45 and 90 days. The temperature is more nearly able to keep up with the 11-year period, and the phase shift is only 47° , corresponding to a time lag of 1.4 years.

Fig. A2 shows the amplitude of the temperature cycle, T_1 , as a function of depth for the same three periods, assuming $S_1 = 26.4 \text{ W m}^{-2}$. This value was derived from the calculated range of values of S at the equator, and is simply intended to illustrate the magnitude of the temperature response. Note the sensitivity of T_1 to the forcing period for depths in the 10–20 m range: for a 20 m depth, T_1 is 0.4 K for a semiannual cycle, 0.9 K for an annual cycle, and 7.5 K for an 11-year cycle. The sea surface temperature is thus much more sensitive to changes in solar radiation that are many years in duration than it is to short-period variations.

Depth of the Ocean Layer

In the model, the ocean is represented by a well-mixed layer of uniform temperature, whereas in reality it is for many purposes infinitely deep, and the annual temperature wave decreases with increasing depth. The appropriate choice of depth for the model ocean can be made by equating the integrated heat stored and released during a full temperature cycle in the model ocean with that in the real ocean.

The latter can be found approximately by assuming that the ocean is heated from the surface and that heat is conducted to lower depths by eddy diffusion (e.g., Neumann and Pierson, 1966). The heat conduction equation is

$$\frac{\partial T}{\partial t} = \frac{\partial}{\partial Z} \left(K \frac{\partial T}{\partial Z} \right), \quad (A9)$$

and if we make the assumption that K is independent of depth and that the surface temperature is given by

$$T = T_0 + T_1 \cos\left(\frac{2\pi t}{\tau}\right), \quad (A10)$$

we find that an appropriate solution for the temperature at depth z is

$$T = T_0 + T_1 e^{-kz} \cos\left(\frac{2\pi t}{\tau} - kz\right), \quad (A11)$$

where

$$k = \left(\frac{\pi}{K\tau}\right)^{1/2}. \quad (A12)$$

Since the heat stored in a thin layer dz in time dt is given by

$$dQ = \rho C \frac{\partial T}{\partial t} dt dz, \quad (A13)$$

we can integrate over depth (from $z = 0$ to ∞) and time (over one-half cycle) to find the total heat stored during one half cycle and released during the other. The result is

$$Q = \sqrt{2}\rho C T_1 k^{-1}. \quad (A14)$$

In the case of the model ocean of finite depth h and uniform temperature, T is given by (A10). Similar arguments show that the heat stored during a half cycle is given by

$$Q = 2\rho Ch. \quad (A15)$$

From (A14) and (A15),

$$h = \frac{\sqrt{2}}{2k}. \quad (A16)$$

Bryan and Cox (1967) found that a value of K of $\sim 1 \text{ cm}^2 \text{ s}^{-1}$ gave realistic poleward heat transport and thermocline depth in their ocean model. Inserting this value in (A12) and (A16), we find that $h \approx 22 \text{ m}$ for $\tau = 1$ year and $h \approx 16 \text{ m}$ for $\tau = 0.5$ year. We have thus adopted $h = 20 \text{ m}$ for the calculations described in the main text.

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