NOTES AND CORRESPONDENCE

Wave–Mean Flow Statistics

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10 December 1981 and 15 June 1982

ABSTRACT

A relation between the statistics of large-scale waves and the mean flow is derived from the potential enstrophy equations integrated over an isobaric surface. The difference between time-averaged zonal-mean state and the radiative–dynamical equilibrium state due to the symmetric circulation is determined by three components: the steady wave enstrophy, the variance in the wave enstrophy and the variance mean flow enstrophy. With some simplifications, the relationship between these components can be used to estimate the maximum amplitude for Rossby waves derived from a statistical data set. We obtain an upper limit of ~1200 gpm for a wave disturbance with a meridional scale of ~1800 km. If the Rossby wave amplitudes are observed near that upper limit, then the wave energy spectrum should exhibit a −5 power law.

The three enstrophy components are estimated for a parameterized model of wave–mean flow interaction at a single level. We find that the steady wave enstrophy, the wave enstrophy variance and the mean enstrophy variance all are within a factor of 2 of each other with the wave variance being the largest. These results suggest that attempts to model the time-mean stratospheric structure in winter, using only the time-mean stationary wave forcing of the mean flow, may not be successful.

1. Introduction

The wave–mean flow non-acceleration theorem derived by Eliassen and Palm (1960) and generalized most completely by Andrews and McIntyre (1978) provides a fundamental relationship between wave activity and changes in the zonal mean flow. In essence, waves tend not to accelerate the mean flow unless special (but not uncommon) conditions prevail. Those conditions require the presence of wave "transience" or "dissipation." Transience is defined as the time rate of change of the wave amplitude (squared) and dissipation is any diabatic or frictional process acting on the wave. Thus, mean-flow acceleration may occur wherever there are growing or decaying, damped or forced waves. The growth or decay of the wave may be due to specific events, such as a wave switch-on, critical level interaction, baroclinic or barotropic instability, etc.

The non-acceleration theorem provides a powerful constraint on the dynamical interaction of the waves and the mean flow. It also seems plausible that the non-acceleration theorem might provide a constraint on the statistics of the flow field. The generation of such statistical constraints is the subject of this paper. We will focus on the statistics associated with wave–mean flow interaction which occurs in the winter stratosphere, associated with sudden stratospheric warming. A variety of models have been able to reproduce sudden warming phenomena in the stratosphere with some degree of success (Matsuno, 1971; and others). One of the simplest models yet devised is the vacillation model of Holton and Mass (1976) (hereafter HM), which produces cyclic sudden warmings in a β-channel. We shall use a vacillation model like that of HM, but more highly parameterized, to generate a sample statistical data set for the stratosphere. Using a model to provide the statistics avoids many of the pitfalls of observational data sets and guarantees consistency with the dynamical assumptions used in the theory. However, models often tend to oversimplify the dynamics and the non-dynamical physical processes. Applications to actual observational data sets and more sophisticated models will be the topic of later papers.

In the next section we derive a statistical relation for the various components of the enstrophy. Two kinds of terms appear in the statistical equation: time-mean components and variances. Each has a role in determining the total enstrophy budget. However, we may look on this equation with another perspective. If the eddy enstrophy means and variances are known, as well as the zonal mean flow variance, then the difference between the radiative–dynamic zonal mean enstrophy and the time-averaged zonal mean enstrophy is determined. This viewpoint is important for understanding how eddies can perturb the time-average, zonal-mean state away from radiative–dynamic equilibrium.

In Section 3 we use the enstrophy equations to estimate the upper limit on Rossby wave amplitude in the stratosphere. Interestingly, this procedure gives rise to an expression for the energy spectrum of Rossby waves near saturation. The results of a statistical analysis of a parameterized vacillation model is presented in Section 4.

2. The potential enstrophy equations

To a good approximation, the motion field of the extratropical stratosphere may be described by the quasi-geostrophic potential vorticity equation

$$\frac{dq}{dt} = -D(q - \bar{q}_0), \tag{1}$$

where $q = f + \tilde{\nabla}\psi^2$ is the potential vorticity and $\bar{q}_0 = f + \tilde{\nabla}^2\bar{\psi}_0$, where $\bar{\psi}_0$ represents some basic-state flow streamfunction which is x (zonally) independent, and f is the earth's vorticity. d/dt represents the substantial derivative associated with the geostrophic component of the flow on an isobaric surface,

$$\frac{d}{dt} = \frac{\partial}{\partial t} + u_g \frac{\partial}{\partial x} + v_g \frac{\partial}{\partial y}.$$

D is the dissipation rate, i.e., Newtonian cooling and Rayleigh friction. We will take D to be constant which is an oversimplification of the real physical processes which restore the perturbed potential vorticity to the radiative–dynamical equilibrium. The consequences of this assumption will be discussed later. The operator $\tilde{\nabla}^2$ is

$$\tilde{\nabla}^2 = \frac{\partial^2}{\partial y^2} + \frac{1}{\rho}\frac{\partial}{\partial z}\rho\frac{f_0^2}{N^2}\frac{\partial}{\partial z} + \frac{\partial^2}{\partial x^2},$$

where $\rho = \rho_0 e^{-z/H}$, ρ_0 is the reference density, H is a scale height, z is the upward coordinate, $z = H \times \ln(p_0/p)$, where p is pressure, p_0 is a reference pressure. The Coriolis frequency f is given as $f_0 + \beta y$ for a β-channel where y is the northward direction. The channel we shall consider later is 60° wide centered at 60° latitude. N is the buoyancy frequency ($\sim 2 \times 10^{-2}$ s^{-1}).

Multiplying (1) by q and using the geostrophic continuity equation

$$\frac{\partial u_g}{\partial x} + \frac{\partial v_g}{\partial y} = 0,$$

we have the potential enstrophy equation

$$\frac{1}{2}\left[\frac{\partial q^2}{\partial t} + \frac{\partial}{\partial x}(u_g q^2) + \frac{\partial}{\partial y}(v_g q^2)\right] = -D(q^2 - q\bar{q}_0). \tag{2}$$

We now define zonal mean and eddy components and integrate (2) over the horizontal dimension of the flow, along the isobaric surface which encompasses non-zonal perturbations. We obtain

$$\left\langle \frac{1}{2}\frac{\partial}{\partial t}(\overline{q'^2}) + \frac{1}{2}\frac{\partial}{\partial t}(\bar{q}^2) \right\rangle = -D\langle(\bar{q}^2 + \overline{q'^2} - \bar{q}\bar{q}_0)\rangle, \tag{3}$$

where $q' = q - \bar{q}$, $(\bar{\ }) = \frac{1}{X}\int_0^X (\ )dx$ and $\langle\ \rangle = \int_0^L (\ )dy$. The assumption that our integration encompasses all non-zonal perturbation regions implies that the eddies (primed quantities), and therefore the eddy fluxes, vanish along the y boundaries. Note that (3) is valid for flows involving wave–wave interaction as well as wave–mean flow interaction.

To obtain statistics on the wave fields and the mean fields, we define the mean and variance as

$$q_m = \frac{1}{\tau}\int_0^\tau q dt \quad \text{(mean)},$$

$$q_v^2 = \frac{1}{\tau}\int_0^\tau (q - q_m)^2 dt \quad \text{(variance)},$$

where τ represents the averaging time interval. We can divide q into two components, the mean and a time-dependent portion whose mean is zero, *viz.*,

$$q = q_m + q_t,$$

such that

$$\int_0^\tau q_t dt = 0.$$

Therefore

$$q_v^2 = \frac{1}{\tau}\int_0^\tau q_t^2 dt.$$

Integrating (3) over the interval τ, we obtain

$$\frac{1}{2\tau}\langle(\overline{q'^2} + \bar{q}^2)\rangle\Big|_0^\tau = -D\langle[\bar{q}_m^2 + \bar{q}_v^2 + (\overline{q'^2_m}) + (\overline{q'^2_v}) - \bar{q}_m\bar{q}_0]\rangle. \tag{4}$$

For the moment we may neglect the left-hand side (lhs) of (4) since the contribution due to these terms goes to zero as τ increases.

Defining $\bar{Q} = \bar{q} - \bar{q}_0$, we simplify (4) to

$$\langle \bar{Q}_m^2 + \bar{Q}_m\bar{q}_0 + \bar{Q}_v^2 + \bar{q}'^2_m + \bar{q}'^2_v \rangle = 0, \tag{5a}$$

where $\bar{Q}_v^2 = \bar{q}_v^2$. If n waves are present then the general result is

$$\langle \bar{Q}_m^2 + \bar{Q}_m\bar{q}_0 + \bar{Q}_v^2 + \sum_{s=1}^{n}\overline{q'^2_{m,s}} + \sum_{s=1}^{n}\overline{q'^2_{v,s}} \rangle = 0, \tag{5b}$$

where s is the zonal index.

Eqs. (5a, b) are valid at every z level in the atmosphere, if the integral $\langle\ \rangle$ encompasses all non-zonal quasi-geostrophic perturbations. In the stratosphere, the integration interval could be from the equator to the pole for winter conditions, since planetary-wave disturbances are absent in the summer hemisphere or, during equinox, the domain may extend from pole to pole if cross-equatorial propagation of Rossby waves is evident.

The interrelationships between the mean quantities and the variances expressed by (5) shows which processes contribute to the time-averaged mean flow state. They are the horizontally-averaged mean flow and wave variances $\overline{\bar{Q}_v^2}$, $\overline{q_v'^2}$ and the stationary-wave amplitude $\overline{q_m'^2}$. Schoeberl (1982) considered the role of q'_m in altering the mean flow through induced Lagrangian mean motions. But it is apparent from (5) that $\overline{\bar{Q}_v^2}$ and $\overline{q_v'^2}$ may also play an important role. The physical reason for the appearance of the variance terms is related to the "enhancement" of dissipation by time fluctuations. Consider, for example, a wave which is steady and dissipating. The dissipation process forces the mean flow by creating an eddy potential vorticity flux. If the wave field fluctuates in time, then the quadrature ($\overline{\bar{q}^2}$, $\overline{q'^2}$) terms will be larger than the same quantity estimated using only time-mean quantities. The increase is due to the temporal correlations of the fluctuations. An exact analogy to this process occurs in spatial averaging, where the partitioning of eddies and zonal means gives rise to eddy flux terms (spatial correlations), so it is no surprise that dividing the time history into mean and fluctuation quantities gives rise to variance terms. It is also apparent that variance terms must be included in any equations which attempt to determine a time-averaged balance between dynamic quantities.

Given an observational data set at a given level z over an observing period τ, we can compute how the channel-averaged function of $\bar{Q}_m$ is determined, because (5a) can be written as

$$\langle F(Q_m, q_0)\rangle = \langle \overline{\bar{Q}_v^2}\rangle + \langle \overline{q_v'^2}\rangle + \langle \overline{q_m'^2}\rangle. \quad (5c)$$

$\langle F\rangle$, which is only a function of the time-mean, zonal-mean state, is entirely determined by the wave enstrophy and the variance in the mean enstrophy. Since the quantities on the right-hand side (rhs) of (5c) are, in principle, measurable, we may determine how wave activity determines the horizontally-averaged zonal mean climatology.

3. Limiting cases

In order to develop a relationship between (5a–c) and the results derived by Schoeberl (1982), we will consider some special situations starting with the most general problem of determining Q_m and working toward the restrictions implied by separable flows. First, we define some set of orthonormal functions $g_j(y)$ such that

$$\left.\begin{aligned} \langle g_j(y)g_k(y)\rangle &= \delta_{jk} \\ \gamma(y) &= \sum_j A_j g_j(y) \end{aligned}\right\}, \quad (6)$$

where

$$A_j = \langle \gamma(y)g_j(y)\rangle.$$

Now $\bar{q}_0$ consists of the earth's vorticity f and the additional vorticity due to the background flow $\bar{\eta}$:

$$q_0 = f(y) + \bar{\eta}(y, z),$$

where η may be as large as f in the upper stratosphere. Using (6),

$$\left.\begin{aligned} \bar{Q}_m &= \sum Z_j(z)g_j(y) \\ \bar{q}_0 &= \sum [Z_{0j}(z) + B_{0j}]g_j(y) \end{aligned}\right\},$$

where $B_{0j} = \langle f(y)g_j(y)\rangle$ and $Z_{0j}(z) = \langle \bar{\eta}g_j(y)\rangle$. Applying the orthogonality condition,

$$\left.\begin{aligned} \langle \bar{Q}_m{}^2\rangle &= \sum_j Z_j^2(z) \\ \langle \bar{Q}_m\bar{q}_0\rangle &= \sum_j Z_j(z)[Z_{0j}(z) + B_{0j}] \end{aligned}\right\},$$

Eq. (5b) can then be written

$$\sum Z_j[Z_j + (Z_{0j}(z) + B_{0j})]$$
$$= -\langle \bar{Q}_v{}^2 + \sum_{s=1}^{N} \overline{q_{m,s}'^2} + \sum_{s=1}^{N} \overline{q_{v,s}'^2}\rangle = \xi(z). \quad (7)$$

We may also write

$$Z_j(z) = Z_1(z)[C_j + \epsilon_j(z)],$$

so if $\epsilon_j(z) = 0$, $\bar{Q}_m$ is separable, and if $\epsilon_j(z) \ll C_j$, $\bar{Q}_m$ is nearly separable.

a. Separable case

If $\epsilon_j(z) = 0$, then (7) becomes

$$Z_1(z)[Z_1(z)I_1 + I_2(z)] = \xi(z),$$

where

$$I_1 = \sum_j C_j^2,$$

$$I_2(z) = \sum [Z_{0j}(z) + B_{0j}].$$

This is the case treated by Schoeberl (1982). Solving for Z_1 we have

$$Z_1 = \frac{-I_2(z)}{2I_1} - \left\{\left[\frac{I_2(z)}{2I_1}\right]^2 - \frac{\xi}{I_1}\right\}^{1/2}, \quad (8)$$

where the positive root has been chosen for the following reason: consider a flow where $|f| > |\bar{\eta}|$ (e.g., the lower stratosphere). If an upward-propagating wave disturbance $\sim \sin y/L$ is present, then the wave transports $\bar{q}$ southward so $\bar{Q}_m \sim \sin\pi y/L \cos\pi y/L$, which is positive at low latitudes and negative at higher ones. $\sum_j Z_j(z)B_{0j}$ represents the weighting of the earth's vorticity by $\bar{Q}_m$ and is therefore negative if q_0 increases poleward. As a result, I_2 is negative, so the positive root is chosen in (8) such that $Z_1 = 0$ if there are no waves present.

From (8) two conditions must prevail. Since Z_1 is real,

$$\frac{I_2{}^2}{4I_1} \geqslant \langle \bar{Q}_v{}^2 + \overline{q_m'^2} + \overline{q_v'^2}\rangle, \quad (9a)$$

and it also follows that

$$|Z_1| \leqslant |I_2/2I_1|. \tag{9b}$$

Eqs. (9a, b) are similar to the results derived by Schoeberl (1982), and Eq. (9a) gives an upper limit on the time-average wave potential vorticity (p.v.) (squared) plus the wave p.v. variance and the mean flow p.v. variance. In Schoeberl (1982), the equivalent equation gives the maximum wave amplitude allowed for a steady mean flow forced by dissipating (or transient) planetary waves. Since, in the statistical model, time-dependent wave fields do not explicitly exist, it is intuitively reasonable that an equation giving a steadiness criterion will be transformed to an upper limit criterion for the statistical mean.

We can use (9a) to make an estimate on the bounds of planetary waves which might be derived from a data set. Upper limit arguments have been presented by Schoeberl (1982) and Lindzen and Schoeberl (1982) but those arguments apply only to the instantaneous planetary wave field. The arguments below apply to the statistics. From the data set analyzed by van Loon *et al.* (1973), we can crudely estimate the scale of the planetary wave disturbances in the lower stratosphere. We take the meridional scale for wavenumber one to be $\sim$1800 km. If the vertical wavelength is greater than $\sim 14\ \mathrm{km}^{-1}$, then the meridional scale is the smallest scale. We can therefore approximate

$$q'_m \approx \left|\frac{k^2hg}{f_0}\right| \sin[ka(\pi/2-\theta)]\cos(mx),$$

where h is the wave geopotential height, k is the meridional wave number, m is the zonal wavenumber and g is the gravitational acceleration. The transport of potential vorticity by the wave q'_m will produce a disturbance in $\bar{q}$ of the form

$$\bar{Q}_m \approx \sin[ka(\pi/2-\theta)]\cos[ka(\pi/2-\theta)].$$

The basic state vorticity in the lower stratosphere can be crudely approximated by

$$q_0 \approx 4\Omega\cos(60°)(\theta - 60°) + 4\Omega\sin(60°) = 2\beta y + 2f_0\,,$$

based on zonal mean circulation models (Schoeberl and Strobel, 1978). Performing the horizontal integrals in (9a) with the above approximations and neglecting the variances we obtain

$$q' \leqslant \frac{k^2h_mg}{f_0} \approx \sqrt{2}\beta/k \quad \text{or} \quad h \leqslant 1200\ \text{gpm}. \tag{10}$$

The data set presented by van Loon *et al.* (1973) shows Z'_m for wavenumber one to be $\sim$650 gpm at 30 km. A recent analysis of NOAA satellite data by M. A. Geller (1982, personal communications) shows $h \approx 1200$ gpm at $\sim$1 mb. Since $h \sim k^{-3}$, the upper limits given by (10) should not be taken as too rigorous.

Eq. (10) generalizes into a power law behavior for the saturated spectrum of Rossby waves near their maximum amplitudes. If

$$|q'_m|^2 \approx \beta^2/k^2, \quad \text{then} \quad E(k) \approx \beta^2 k^{-5}, \tag{11}$$

where $E(k)$ is the energy spectrum defined such that the total energy E_t is given by $\int E(k)dk$.

Even though (11) was obtained using a rather empirical argument and is valid only for separable systems, there appears to be theoretical support for the -5 power law from Rhine's (1975) study of β-plane turbulence. The -5 power law holds as long as the wavenumber of the disturbance is larger than the critical wavenumber $k_\beta = \beta/2U$ where U is the horizontal velocity scale of the wave. For disturbances with $k > k_\beta$, Rossby waves form and propagate out of the turbulent region thereby spatially smoothing the energy of the disturbance. For conditions given by (10) it is easily shown that $k > k_\beta$ for all wavenumbers.

b. *Non-separable case*

For the nonseparable case, it may not be possible to obtain the simple limiting conditions expressed by (10). But we may be able to minimize the function ϵ_j/C_j by a judicious choice of expansion functions. For example, Schoeberl and Geller (1977) used a Fourier–Hough expansion, whose eigenmodes vary with altitude, to examine planetary wave propagation. The variation in the zonal wind with altitude causes a large change in the structure of the Hough mode and this is the fundamental reason for the sensitivity of planetary wave structure to the mean zonal winds in steady state models. From Schoeberl and Geller's results it would appear that ϵ_j/C_j could be minimized for wave fields and perturbed mean-flow fields using a Fourier–Hough series.

To summarize, if $\bar{Q}_m$, q'_m and $\bar{q}_0$ are separable, and the variances are zero, certain upper limit constraints on the amplitude of the wave and perturbed mean flow field can be generated. Under these conditions, the time-mean wave amplitude cannot exceed $\sim$1200 gpm in the stratosphere. Unfortunately, separability is a powerful assumption, which would normally restrict the applicability of this type of analysis to separable models such as those by Holton and Mass (1976) and Davies (1981). Nevertheless, it may be possible to minimize the meridional mode coupling which precludes separability by a judicious choice of expansion functions such as a Fourier–Hough series.

4. The statistics for a parameterized model of stratospheric vacillation

a. *Model*

While it may be possible to set an upper limit on the stationary wave amplitude only for a limited number of cases, a good deal of knowledge can still be extracted concerning wave influence on the zonal mean flow using (5). Separability or modal decomposition assumptions are not required to evaluate the integrals on the rhs of (5c) so this equation may be used to make a climatological assessment of the role of waves in determining the time-averaged zonal-mean state.

To provide a crude statistical data set to test (5), we use the parameterized model of vacillation presented by Schoeberl (1982). The model assumes

$$\bar{q} = f_0 + \beta y + Z_1(z)\sin(ky)\cos(ky),$$

$$q' = Z'(z)\sin(ky)e^{imx}, \quad \bar{q}_0 = f_0 + \beta y$$

($\bar{\psi}_0$ is assumed to be zero, which is the case for a constant zonal wind). The maximum amplitude for Z_1 is restricted to its maximum time mean value given by (10). The other parameters are given in Schoeberl (1982). The results of the model are shown in Fig. 1, which gives Z_1 and $|Z'|$ versus t. Three regions are marked on the figure, which indicate the important physical process. The deceleration of the mean flow in the channel by wave transience, as the wave amplitude increases, is labeled as "switch-on." Once Z_1 reaches the maximum value allowed, the wave is turned off, which produces a transient reversal and rapid reacceleration of the mean flow. Note that the flow does not return to the initial state because of irreversible deceleration during the cycle due to the damping of the waves. The time constant for damping is (5 days)$^{-1}$. The waves are shut off for 30 days before being switched on again so the mean flow relaxes almost to its initial value. This is the "decay" phase indicated in the figure. The maximum wave amplitude achieved during this cycle is 215 gpm.

b. Statistics

Using the vacillation model described in the previous section, we may now obtain the statistics re-

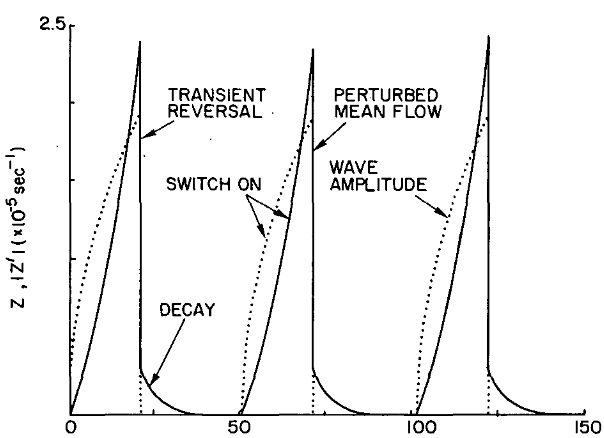

FIG. 1. Mean flow and wave potential vorticity perturbation as a function of time for the vacillation model.

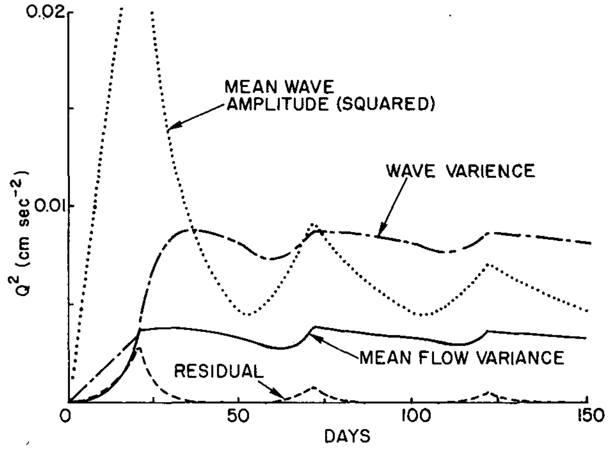

FIG. 2. Planetary wave–mean flow statistics for the vacillation cycles shown in Fig. 1, as a function of the measurement interval τ, in days.

quired for (5). We begin by comparing the three components of (5) which determine the time-averaged mean-flow state when channel averaged: $\overline{\bar{Q}_v^2}$, $\overline{q_v'^2}$, $\overline{q_m'^2}$. These variables and the absolute value of the residual of Eq. (5) are plotted as a function of τ, the measurement interval, in Fig. 2. The residual is simply the sum of all the terms in (5) and tells us the magnitude of the lhs of (4).

The statistical quantities shown in Fig. 2 converge slowly, since the 150 day interval encompasses only three vacillation cycles. It is clear that forcing of the time-averaged mean p.v. is principally due to the wave p.v. variance (~45%) followed by the forcing due to the mean wave p.v. The mean flow p.v. is the smallest, but still it is not negligible. We conclude that the variance of the wave p.v. and the mean flow p.v. can play a very important role in the determination of the time-averaged zonal-mean p.v. Note that the residual slowly approaches zero as expected.

In a preliminary analysis of a β-plane model of the stratosphere developed at the Naval Research Laboratory, we have found that the largest statistical component which determines the time-average zonal-mean p.v. below ~40 km is the time-mean wave component. In this model, the stationary wave saturates near 40 km so that above 40 km the mean flow should show a high degree of variability. In fact, the wave p.v. variance and zonal mean p.v. variance are found to be most important above 50 km. This is not unexpected, since the wave variance combined with the steady wave component will produce a mean flow variance.

It has been suggested that planetary waves might substantially heat the lower polar stratosphere during winter, by producing a poleward downward residual circulation as they dissipate. These results indicate that a computation of this heating, using only stationary wave amplitudes, might underestimate this effect, unless the winter stratosphere is very quiet. The variance of the waves and the mean field further increase the dissipation of the flow and therefore enhance the poleward downward residual circulations. Consequently they are also important in determining the net adiabatic heating of the lower stratosphere by wave-induced vertical motions.

5. Summary and concluding remarks

We have investigated the statistics of wave–mean flow interaction and have derived the statistical equation (5) which relates the horizontal average zonal mean and wave enstrophy variances and time averages for a quasi-geostrophic flow. This relation shows that the difference between the radiative–dynamic equilibrium state and the actual time-averaged mean enstrophy may be thought of as being determined by the variance in the zonal mean enstrophy, the variance in the wave enstrophy, the mean wave enstrophy (squared). The statistical equation which determines these relations is independent of the dissipation rate D, although D cannot be identically zero. For a more complex function of the damping, the relationship (5) will still be approximately valid as indicated by Schoeberl (1982, Appendix A) although the form of the equations will be much more complex.

The relation formed from Eq. (9a) which is valid for separable flows, can be used to estimate the upper limit on the amplitude of Rossby waves in the stratosphere. Based on the observed length scales from van Loon *et al.* (1973), we find that the square root of the sum of the wave amplitudes squared cannot exceed ~1200 gpm. This upper limit occurs when Rossby waves have saturated the mean flow, or, in other words, the potential vorticity of the wave field is of the same magnitude as the basic state.

Using a simple empirical argument, it can be shown that saturated Rossby waves should exhibit a −5 power law for the energy spectrum. This result is in agreement with Rhines (1975) theoretical prediction for turbulence on a β-plane.

With a simple vacillation model we have examined the three statistical contributions to the time-averaged mean flow. The wave variance was found to be the most important component in determining the time-averaged zonal-mean flow. For the real stratosphere, the variances and the means probably have approximately equal importance in determining the time-averaged zonal-mean state above the mid stratosphere (~40 km), although the time-mean wave potential vorticity is probably most important at lower altitudes.

Of fundamental importance is the magnitude of the (downward) residual circulation flow in winter at high latitudes. This flow is believed to advect ozone toward the pole and may provide substantial dynamic heating in the polar regions. Schoeberl (1982) examined the Lagrangian mean flow induced by stationary dissipating planetary waves. But it is clear that the eddy variance and zonal mean variance may play equally important roles in inducing a residual circulation. The contribution of these terms should not be neglected by modelers.

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