Shear Zone Asymmetry in the Observed and Simulated Quasi-Biennial Oscillations[1]

TIMOTHY J. DUNKERTON[2]

Department of Atmospheric Sciences, University of Washington, Seattle 98195

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ABSTRACT

Reasons underlying the asymmetry in shear-zone intensity in the observed and simulated quasi-biennial oscillations are investigated. It is shown that much of the incorrect model asymmetry originates in the differing equipartition laws of the Kelvin and Rossby gravity waves. The observed asymmetry cannot entirely be explained by vertical advection due to the residual mean meridional circulation. It is suggested that latitudinal shear plays a role in the observed shear zone asymmetry by reducing the degree of inflection in the dependence of Rossby-gravity wave vertical group velocity on intrinsic frequency via a curvature-induced change in the effective planetary vorticity gradient. The experiments are suggestive of a possible mechanical dissipation of the Rossby-gravity wave.

1. Introduction

In recent years it has been shown (Holton and Lindzen, 1972; Plumb, 1977; Plumb and Bell, 1982b) that the upward-propagating equatorial waves discovered by Yanai and Maruyama (1966) and Wallace and Kousky (1968) can successfully account for the observed tropical zonal wind cycle known as the "quasi-biennial oscillation" (Reed *et al.*, 1961; Veryard and Ebdon, 1961; Reed, 1965; Wallace, 1973; Coy, 1979). Holton and Lindzen (1972, hereafter referred to as HL) found that an effective wave stress

$$\langle S_{(xz)}\rangle \equiv \rho_0\langle \overline{u'w'} - \frac{f}{N^2}\overline{v'\theta'}\rangle, \qquad (1.1)$$

would produce, in the presence of thermal damping and a small mean flow viscosity, a long-period oscillation in the latitudinally-averaged zonal mean wind

$$\langle \bar{u}\rangle \equiv \int_{-\infty}^{+\infty} \bar{u}dy, \qquad (1.2)$$

the overall period and structure of which agreed reasonably well with the observed oscillation. A major simplification in HL's method was the use of a two-scaling approximation for the waves (Lindzen, 1971; Andrews and McIntyre, 1976a,b; Boyd, 1978a,b; Dunkerton, 1981a,b) to determine their vertical propagation. The accuracy of this approximation is evident in their simulation, and also receives recent support from Plumb and Bell (1982a). Finally, it should be mentioned that the laboratory analog (Plumb and McEwan, 1978) has gone a long way toward providing credibility for the HL theory.

At this point it might seem that the remaining details of the theory merely involve tying up loose ends, so to speak. However, as will be evident below, some of these loose ends actually form very interesting and challenging questions. To name a few, there is a new interest in wave transience in the oscillation (Dunkerton, 1981a,b); there is the very difficult question as to how latitudinal shear affects the oscillation (Boyd, 1978a,b; Holton, 1979; Dunkerton, 1982; Plumb and Bell, 1982a,b); there are some questions concerning the vertical extent of the oscillation and the role of wavelength-dependent radiative relaxation (Hamilton, 1981); and, there is the realization that eventually the model simulation must extend to the sphere, something which also touches upon the ozone oscillation question (Plumb and Bell, 1982b).

In this paper we wish to explore what is on the surface perhaps the most obvious question raised by the HL theory. This concerns a discrepancy between simulated and observed shear-zone strengths. Holton and Lindzen recognized this discrepancy in their paper; in their simulation the easterly shear zones appeared too sharp in height and time[3], compared to the westerly shear zones. This asymmetry is just the opposite of that observed (Wallace, 1973). The observed asymmetry is clearly evident in each and

[1] Department of Atmospheric Sciences Contribution 603.

[2] Current address: National Center for Atmospheric Research, P.O. Box 3000, Boulder, CO 80307.

[3] Shear zone strength may be evaluated with two independent quantities, either $\bar{u}_t$ and $\bar{u}_z$, or $|\nabla\bar{u}|$ and $(dz/dt)_{\bar{u}}$. Because observations suggest that the asymmetry is primarily in $|\nabla\bar{u}|$, and *not* in the descent rate, we shall use $|\nabla\bar{u}|$ as a measure of shear zone strength, or "sharpness."

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every documented phase, and is characterized by a slightly sharper westerly shear than that of HL, together with a much weaker easterly shear-zone sharpness.

Holton and Lindzen attributed this discrepancy to the lack of a residual vertical advection in their one-dimensional model. This advection may be approximately described by the thermodynamic relationship

$$\bar{w}^* N^2 \doteq -\bar{\alpha}_T \bar{\phi}_z , \quad (1.3)$$

implying that warm (cold) anomalies (positive and negative $\bar{\phi}_z$) are maintained by downward (upward) advection (negative and positive w^*) against the thermal relaxation $\bar{\alpha}_T$ and stable stratification N^2. By the thermal wind law at the equator

$$\beta \bar{u}_z = -\bar{\phi}_{zyy} \doteq \frac{\bar{\phi}_z}{L^2} . \quad (1.4)$$

This implies that regions of westerly (easterly) shear descend faster (slower). As originally recognized by Reed (1964), for example, the estimated magnitude of $\bar{w}^*$ is perhaps a significant fraction of the actual shear zone descent rate (Plumb and Bell, 1982b). Hence it seems likely that the residual advection effect could prove important in the atmosphere. Interestingly the recent two-dimensional model of Plumb and Bell (1982b) has correctly simulated the observed asymmetry, a fact those authors partly attribute to the residual advection.

The purpose of the present paper is to investigate in some detail the simulated shear zone asymmetries. This question divides itself into two parts. First, what is the origin of the incorrect asymmetry in the HL model? Second, what is the correct explanation of the observed asymmetry? The first question is actually an important one, despite the fact that Holton (1975) attributed their incorrect asymmetry to the more inflected dependence of Rossby-gravity wave vertical group velocity on intrinsic frequency. In fact, as will become clear in the next section, the reason for the incorrect model asymmetry involves more than the vertical group velocity.

The second question is not rhetorical either, despite the answer to it proposed by HL, and reiterated by Plumb and Bell (1982b) concerning the residual advection. The reason for this is that latitudinal shear is a contributing factor in the observed oscillation also. This fact will become evident below when the "gamma-plane" approximation is invoked to deal with the latitudinal shear (Boyd, 1978a,b) as it affects the Rossby-gravity wave.

In fact, it is possible to crudely compare the relative importance of residual advection and latitudinal shear in the context of the classical HL prototype model. This is done in Section 3, except that the HL model is modified by neglecting O(k) terms where k is zonal wavenumber (Boyd, 1978a, Section 6). First, the latitudinal shear is incorporated by assuming that the wind shear exists in the form of a symmetric parabola having some time-dependent curvature which depends only upon intrinsic frequency. For this shear in the gamma-plane approximation the net effect is a replacement of β, the planetary vorticity gradient, with the geometric mean value $[\beta(\beta - \delta_0)]^{1/2}$, where δ_0 is the curvature $\bar{u}_{yy}$. Because the easterly Rossby-gravity waves induce easterly curvature, the effect of the curvature is to lessen the degree of inflection in the dependence of vertical group velocity on intrinsic frequency.

Secondly, the residual advection is incorporated using (1.3) and (1.4). This provides some justification for the heuristic formula

$$\bar{u}_t = \frac{\bar{\alpha}_T L^2 \beta}{N^2} \bar{u}_z^2 + \text{Eliassen-Palm flux convergences}, \quad (1.5)$$

to be used below.

An important conclusion of this work, coming out of both Sections 2 and 3, is that the effectively reduced thermal damping rate of the Rossby-gravity wave is crucial to the incorrect asymmetry. Because the equipartition of wave action in the Rossby-gravity wave differs from that of the Kelvin wave, the thermal (mechanical) damping of this wave is relatively smaller (larger). This fact actually explains much of the incorrect asymmetry in our simulations. It is found, for example, that a comparable (though small) reduction in the incorrect asymmetry occurs when equal mechanical and thermal damping is used, as when latitudinal shear is employed (with the gamma-plane approximation) to alter the vertical group velocity. The implication is that latitudinal shear cannot overcome this difference in damping rates, and cannot, by itself, explain the observed asymmetry. It could, however, suffice in a model with equal thermal-mechanical damping, but such a model is speculative at best. Hence the residual advection, or some other factor, seems necessary.

Consistent with the results of Plumb and Bell (1982b), it is demonstrated that the residual advection by itself does indeed help to explain the observed asymmetry. This effect, however, does not seem to adequately reduce the *sharpness* of the easterly shear zones while creating an asymmetric descent rate. Therefore, another important conclusion to be drawn from this study is that the residual advection also cannot entirely explain, by itself, the observed asymmetry, since the latter involves not the shear-zone descent rate, but primarily the overall sharpness.

Combining the effects of latitudinal shear and residual advection further improves the simulated asymmetry, provided that mechanical dissipation is incorporated. The latter remains speculative, although Andrews and McIntyre (1976a) suggested possible sources of mechanical damping. In view of the uncertainty in wave parameters, part of the observed asymmetry may be due to asymmetric wave forcing, although a recent simulation by Plumb and Bell (1982a) gave a correct asymmetry without this effect.

2. The Holton-Lindzen simulation

The first question to be addressed in this paper concerns the reason for the shear-zone asymmetry in the original Holton-Lindzen simulation. The Holton-Lindzen model consists of three coupled equations for the (latitudinally-integrated) mean zonal wind, Kelvin-wave action density, and Rossby-gravity wave action density. These equations are listed in Dunkerton (1981b) as Eqs. (14), (15a) and (15b), respectively, with the exception being that the wave transience terms were omitted in the HL model. In the interest of computational expediency we shall follow the latter authors in neglecting these terms, although Dunkerton demonstrated that there are times in each cycle when the transience terms contribute. Some justification for our ignoring these terms is given in the beginning of the next section; suffice it to say here that it is our belief that wave transience does not alter the conclusions of any of the following discussion.

In the interests of self-consistency in this paper we shall use the gamma-plane version of the HL equations. This approximation is discussed further in Section 3; essentially it neglects $O(k)$ terms in the Rossby-gravity wave action equation. There exist in our model, therefore, two modifications of the HL equations; first, the Rossby-gravity vertical group velocity becomes

$$W = -\frac{k^3(c-\bar{u})^3}{2N\beta}, \tag{2.1}$$

while the Rossby-gravity thermal-dissipation rate becomes

$$F = \tfrac{1}{2}\alpha_T A. \tag{2.2}$$

The implications of neglecting the $O(k)$ terms are discussed below. Our model equations are as follows:

$$\frac{\partial \bar{u}}{\partial t} = \nu \frac{\partial^2 \bar{u}}{\partial z^2} - \frac{1}{\rho_0}\frac{\partial}{\partial z}\rho_0(B_K + B), \tag{2.3}$$

$$\frac{1}{\rho_0}\frac{\partial}{\partial z}\rho_0 B_K = -F_K, \tag{2.4a}$$

$$\frac{1}{\rho_0}\frac{\partial}{\partial z}\rho_0 B = -F, \tag{2.4b}$$

where B_K and B are the wave-action flux for the Kelvin and Rossby gravity waves, respectively, given by the product of the respective vertical group velocities and wave action densities A_K and A. The

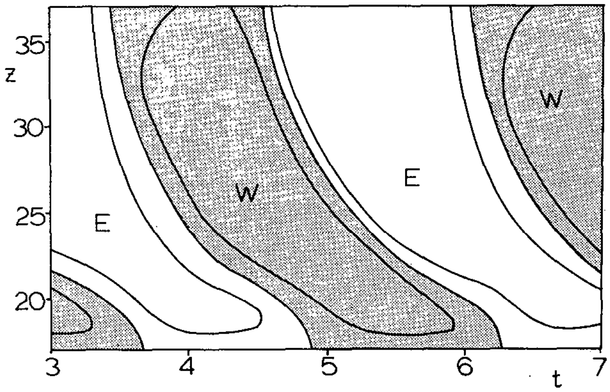

FIG. 1. Time-height cross section of latitudinally-averaged zonal mean wind for the "control" experiment (see text for details). Contours: 10 m s^{-1}.

Kelvin vertical group velocity and thermal dissipation rate are respectively

$$W_K = \frac{k_K(c_K - \bar{u})^2}{N}, \tag{2.5}$$

$$F_K = \alpha_T A_K. \tag{2.6}$$

In (2.3) $\bar{u}$ condenses the notation of (1.2) and is now the latitudinal average; the term ρ_0 is the basic state density having a scale height H; our convention is to take the Kelvin and Rossby-gravity wave action quantities as positive and negative, respectively. Our model parameters are $H = 6$ km, $\nu = 0.3$ m^2 s^{-1}, $\beta = 2.29 \times 10^{-11}$ m^{-1} s^{-1}, $N = 2 \times 10^{-2}$ s^{-1}, $k_K = 1$ a^{-1}, $k = 4$ a^{-1}, $a = 6.37 \times 10^6$ m, $c_K = 25$ m s^{-1}, $c = -c_K$, and the lower boundary fluxes of wave action imposed at $z = 17$ km are $\pm 4 \times 10^{-3}$ m^2 s^{-2}. The zonal wind is set to zero at 17 km and the model grid extends upward to $z = 37$ km at which point the vertical derivative is set to zero; the grid spacing is $\Delta z = 250$ m and $\Delta t = 1$ day, as in HL. The imposed semi-annual oscillation of HL is omitted here. Finally, a profile of damping similar to that of HL is adopted, i.e.,

$$\alpha(z) = 0.4 \times 10^{-6}\left(1 + 3\frac{z-17}{20}\right)\mathrm{s}^{-1} \tag{2.7}$$

and we shall explore variations in the ratio of mechanical to thermal damping

$$\lambda = \alpha_M/\alpha_T, \tag{2.8a}$$

$$\alpha(z) = \alpha_M + \alpha_T. \tag{2.8b}$$

Of particular interest are the two cases $\lambda = 0$ and $\lambda = 1$. The latter case alters (2.2) and (2.6) so as to make both dissipation rates equal to the product of α and wave-action density.

The $\lambda = 0$ case will be referred to as the control experiment. It is analogous to the HL simulation apart from the $O(k)$ terms. The corresponding oscillation is shown in Fig. 1. It is in all respects very similar to the HL simulation, having a period of ~930 days. The feature to which we call attention is the notable asymmetry in the strength of the easterly and westerly shear zones, with the easterly phase experiencing a much larger gradient. (In this and the following figures, a small segment of each integration is presented. However, the excellent repeatability of the oscillation indicates that these segments are almost exactly representative of the whole.) The question is immediately raised as to the reason for this asymmetry.

Holton (1975) attributed the asymmetry to the differing *frequency dependences* in the governing equations. The more inflected dependence for the Rossby gravity wave appeared responsible for the more intense easterly shear zone.

Now while his conclusion might appear superficially correct, it is, we believe, not the only contributing factor. Instead, it appears that much of the blame lies in a comparison of (2.2) and (2.6). Due to the equipartition of energy (or, more precisely, the equipartition of wave action) in the Rossby gravity wave, there is relatively less thermal damping of this mode, compared to the Kelvin wave. In the gamma-plane approximation, when there is either no shear or parabolic shear, it can be shown that the ratio of "kinetic" to "thermal" wave action is in fact 3:1. Hence when mechanical dissipation is entirely absent as in the above formulas, there is relatively less damping of the Rossby-gravity wave for a given amount of wave action. This fact directly leads to a considerably sharper easterly shear zone, apart from any frequency-dependence differences [Dunkerton, 1981a, Eq. (42)].

The neglect of the $O(k)$ terms does not alter this conclusion. Their only effect seems to be that of further reducing the Rossby gravity wave dissipation rate in strong westerlies; however, the only example quoted in the literature in which these terms had any dynamical importance is the anomalous oscillation in Fig. 8c of Plumb (1977), in which the westerlies were observed to continue for almost three times their normal length, due to a vanishing in the Rossby-gravity wave dissipation rate at its "westerly cutoff" point. This anomaly, however, has not been observed in any other simulation, and is therefore considered unimportant (see footnote 5).

Further evidence supporting our assertion comes from two sources. First, it is possible to consider equal mechanical and thermal damping (it is important to note that the *total* damping rate remains constant throughout our simulations). Secondly, it is possible to alter the frequency dependence in the vertical group velocity by incorporating latitudinal shear. This will be discussed further below, but what will be demonstrated in Figs. 2 and 3 is that a similar, though small, reduction in the incorrect asymmetry occurs when either of these changes are incorporated *separately*.

Fig. 2 indicates that indeed the differing frequency dependences in the vertical group velocities contribute to the incorrect asymmetry observed by HL. However, Fig. 3 (to be discussed below) indicates that even when these frequency dependences are made exactly equal (with the help of a curvature parameterization), a similar incorrect asymmetry remains, now due entirely to the reduced Rossby gravity wave thermal dissipation.

3. Reasons for the observed asymmetry

Interestingly, the observed shear-zone asymmetry is just the opposite of that simulated by HL. There seems to be no doubt about the significance of the observed asymmetry since it clearly is evident in each and every oscillation cycle. In comparing the observed and simulated asymmetries, the most notable difference is the much weaker gradient of the easterly shear zone in the observed oscillation. There is, however, a slightly stronger westerly gradient also. This latter fact may be significant in explaining the overall discrepancy, as will become clear below.

Before concentrating on what we believe are the most important factors regulating the observed asymmetry, it is necessary to set aside some other factors, which despite their likely importance in the actual oscillation have little to do with the asymmetry question. These are as follows:

1) Much of the documented quasi-biennial oscillation originates in data obtained well off the equator. It sometimes goes unrecognized that two of the more important reporting stations (Balboa, Kwajalein) are at 9°N and hence are relatively far from the equator. This fact is important because when mechanical dissipation or transience are present, the Rossby-gravity wave acceleration is actually *westerly* at some latitude away from the equator. In Eulerian terms this arises from the equatorward flux of easterly momentum set up by the growing or frictionally dissipated disturbance. Unfortunately the net importance of this effect is difficult to assess since the location of zero acceleration depends on the relative importance of thermal damping and the latitudinal scale, both of which vary markedly throughout each cycle. In the westerly phase the wave scale is large, implying easterly accelerations even at these outer stations. However, at some point in time the easterly acceleration here is much reduced or even reversed (before completion of the easterly acceleration phase as a whole).

All other factors being the same, the question then becomes whether or not the easterly accelerations are any more intense at the equator. There is little evidence for this in the Singapore record at 1°22'N. Furthermore, the recent model simulation of Plumb and Bell (1982b) gives the weaker easterly phase on the equator (their Fig. 2). Therefore the shear zone asymmetry would seem to remain.

2) Wave transience is a significant contributing factor in the oscillation. Despite the chronological importance of wave transience demonstrated by Dunkerton (1981b), there is little reason to believe that transience would alter the vertical distribution of the acceleration so as to reverse the asymmetry (although transience would greatly alter the latitudinal structure of the acceleration as discussed in item 1). Evidence for this is provided by the fact that Dunkerton's transient wave simulations were characterized by the incorrect shear zone asymmetry.

3) A spectrum of forced waves is involved. In view of the possible role of the frequency dependence, it is conceivable that a spectrum of forced waves could result in an overall change in the asymmetry under the right circumstances. Whether or not this is significant, however, will need to be decided later; at this point it seems unlikely since Plumb and Bell's (1982b) oscillation, which involved only two discrete wave frequencies, gave the correct asymmetry.

4) There may be an accelerated thermal damping rate at small vertical wavelengths. This is likely to be an important factor in the actual oscillation insofar as Newtonian cooling is itself important. However, it does nothing to explain the observed asymmetry! An accelerated damping rate would in fact enhance the existing simulated asymmetry in view of the more inflected dependence of vertical wavelength on intrinsic frequency for the Rossby gravity wave (Hamilton, 1981).

Investigation of the relative importance of these effects in the overall oscillation remains a problem for the future. Discussion here will now center on two effects believed to be important in the observed shear-zone asymmetry.

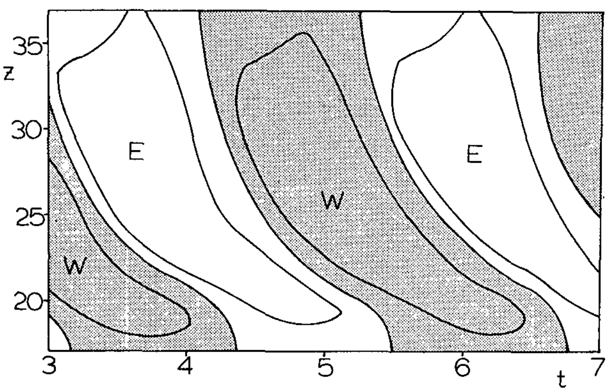

FIG. 2. As in Fig. 1 except with equal mechanical-thermal damping.

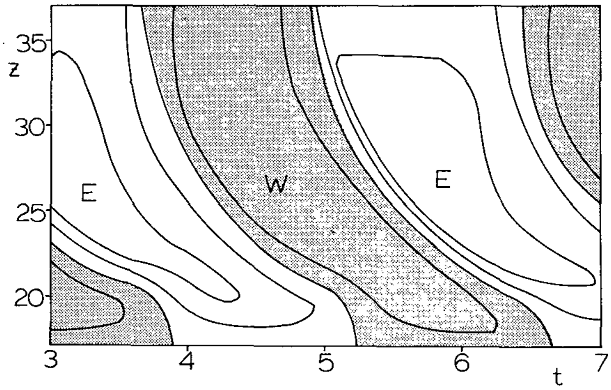

FIG. 3. As in Fig. 1 except with the latitudinal-curvature parameterization.

a. Latitudinal shear

While a detailed discussion of latitudinal shear lies outside the scope of this paper, it is possible to make what we believe is a very accurate qualitative assessment of the impact of latitudinal shear on the oscillation by invoking the gamma-plane approximation (Boyd, 1978a,b). This is a perturbation method designed for ultralong Rossby-gravity and gravity waves *in horizontal* shear, that is to say, waves whose zonal extent is much greater than the latitudinal extent. For these waves the dominant influence of the shear is felt in the north–south advection of the effective planetary vorticity gradient.[4] The otherwise difficult variation in intrinsic frequency is thereby ignored and is small due to the smallness of zonal wave number. Hence it can be said that the zonal wind is "held constant except where differentiated" (as $\gamma = \bar{u}_y$)—thus the "gamma-plane" terminology. The most notable achievement of the gamma-plane approximation is the existence of exact solutions at lowest order in any linear or parabolic horizontal wind profile. These involve the familiar Hermite polynomials of the original no-shear beta-plane problem, except that they are modified by the shear. These modifications alter the scale of the eigenfunctions and the location of the effective equator.

It can also be shown that within the context of this approximation the net effect of a symmetric, para-

[4] Hence Kelvin wave latitudinal shear effects, though nonzero (Boyd, 1978a,b) are nevertheless small, and are ignored here.

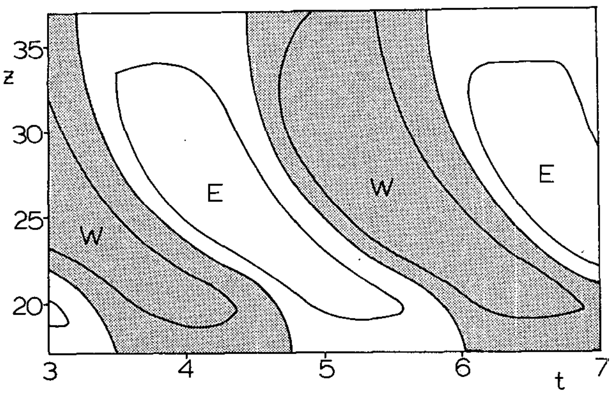

FIG. 4. As in Fig. 1 except with the latitudinal-curvature parameterization and equal mechanical-thermal damping.

bolic shear is to alter the dispersion relation of the Rossby gravity wave by replacing the planetary vorticity gradient with the geometric mean value $[\beta(\beta - \delta_0)]^{1/2}$, where δ_0 is the curvature (Boyd, 1978a). Furthermore, this symmetric shear does not alter the equipartition theorem at all. For the purposes of our model, then, the only change manifests itself in the vertical group velocity equation (2.1) which becomes

$$W = -\frac{k^3(c - \bar{u})^3}{2N[\beta(\beta - \delta_0)]^{1/2}}. \qquad (3.1)$$

(The quantity $\bar{u}$ retains its latitudinally-integrated sense although Boyd used the equatorial value; the former is more suitable for our purposes and is perfectly acceptable in view of the arbitrariness in the definition of this quantity at lowest order.) Evident in (3.1) is a reduction in the degree of inflection in the dependence of W on intrinsic frequency; this follows from the fact that easterly curvature ($\delta_0 > 0$) is associated with easterlies, and *vice versa*.

To illustrate the importance of this curvature in the quasi-biennial oscillation, consider a simple monotonic functional parameterization of the form

$$\delta_0 = \delta_0(\omega_0), \qquad (3.2)$$

where $\omega_0 = k(c - \bar{u})$. (In reality the curvature will be determined by the wave structure and other factors, possibly giving rise to some intercyclical variability.) The form of (3.2) must satisfy the following constraints:

1) The curvature at the equator must not exceed the planetary vorticity gradient as $\omega_0 \to 0$;

2) the net acceleration in westerly curvature should be perhaps twice as strong as in the absence of shear.

Item (1) is consistent with the curvature diminution theorem of Dunkerton (1982). On the other hand, item (2) follows from the numerical calculation of Boyd (1978b) who found the net acceleration due to the Rossby gravity wave in the observed westerly curvature to be 270% of its no-shear value (his Fig. 8). We note that while the gamma plane approximation becomes invalid in strong westerlies (the parameter $k\omega_0/\beta$ approaches unity), it is yet possible to *mimic* the exact numerical result of Boyd (1978b) by using (3.2). The following dependence is examined here

$$\delta_0 = \beta\left[1 - \left(\frac{\omega_0}{\omega}\right)^2\right], \qquad (3.3)$$

where $\omega = kc$. This expression satisfies item (1) and also agrees with our expectation of a relatively rapid development of easterly curvature (Andrews and McIntyre, 1976a). Also, in the westerly jet $\omega_0 \doteq 2\omega$ and the acceleration is twice its no-shear value.[5] Perhaps of most interest in (3.3) is the now equal dependence of Kelvin and Rossby-gravity wave vertical group velocity on intrinsic frequency. (Hence our choice of this functional form has an ulterior motive, besides being based on a crude representation of a real shear effect, thereby exhibiting the importance of the reduced Rossby-gravity thermal dissipation unambiguously.)

Fig. 3 shows the resulting oscillation in the $\lambda = 0$ case employing (3.3). Inspection of the numerical output revealed a slight reduction in the strength of the easterly shear zone in this case. However, the strong remaining (and incorrect) asymmetry supports the claim of Section 2 regarding the cause of the HL model asymmetry.[6] The weaker thermal damping of the Rossby gravity wave is crucial to our simulated asymmetry. Furthermore, the latitudinal shear as parameterized here cannot overcome this.

The corresponding oscillation with $\lambda = 1$ is shown in Fig. 4. Unlike the preceeding figure this oscillation is characterized by roughly symmetric shear zones, with the westerlies actually a bit sharper (due to unequal wave parameters). This integration is in part

[5] That is to say, the acceleration is twice its no-shear *gamma-plane* value. The acceleration is actually more than twice the no-shear beta-plane value since we have neglected the $O(k)$ term in the dissipation rate. Exactly *how much* more depends on what the beta-plane value is, and that depends on the value of λ. When $\lambda = 0$, the gamma-plane dissipation rate is a poor approximation because of the vanishing of the no-shear beta-plane dissipation rate in strong westerlies (as mentioned above), but when $\lambda = 1$, the inaccuracy is much less. The inclusion of the $O(k)$ terms would then seem to reinforce one major assertion of our paper; i.e., that the reduced Rossby gravity thermal dissipation rate is crucial to the incorrect asymmetry! For reference purposes, the no-shear beta-plane value contains the factor $(1 + \chi)$ when $\lambda = 0$ and $(1 + \frac{1}{2}\chi)$ where $\lambda = 1$, where $\chi \equiv k\omega_0/\beta$. In the basic state $\chi \approx -\frac{1}{2}$. Boyd's calculation assumed $\lambda = 1$, so doubling the gamma-plane acceleration actually slightly exceeds his observed increase as $\omega_0 \to 2\omega$. This fact remains true despite Boyd's Fig. 8 being plotted assuming constant maximum ϕ (instead of constant B).

[6] A reduction in dissipation for one wave is isomorphic to an increase in the zonal wave number; hence our Fig. 3 finds it equivalent in Fig. 6b of Plumb (1977) who considered the latter case.

a fulfillment of Boyd's words (1978b, p. 2265) to the effect that the dramatic changes accompanying the latitudinal shear are extremely important. We note here a fact previously overlooked; namely, that Boyd's numerical prediction suggests a tendency for latitudinal shear to produce the correct shear zone asymmetry.

By way of caution it is not being suggested that the Rossby-gravity wave vertical group velocity (3.1), with (3.3) incorporated, is necessarily precisely what will be the case in reality—obviously the actual vertical group velocity will involve intrinsic frequency and curvature separately—nevertheless, it can be reasonably expected that the formula we have used here is qualitatively representative of the exact expression.

What *is* being said, however, is that the changes due to the shear would need to be much larger than that which we have employed here in order to overcome the reduced Rossby-gravity wave thermal damping. (This remark assumes that the shear does not significantly alter the equipartition law.) In this regard we find it quite interesting that Plumb and Bell's (1982b) simulation, which gave the correct asymmetry, employed a $\lambda(z)$ profile ranging from one-half over most of the model to one in an upper sponge layer.[7]

b. Residual advection

For many years it has been felt that the observed asymmetry owes its origin to the presence of a mean vertical advection due to the thermal dissipation of the mean temperature anomalies in the respective shear zones. We refer to this as an advection by the "residual" mean meridional circulation following Andrews and McIntyre (1976a). Plumb and Bell (1981b) discuss this at some length (e.g. their Fig. 1), although those authors speak of it as the Lagrangian mean meridional circulation. In any case, there is considerable reason to justify interest in this circulation, not only because the nature of the advection would help give the observed asymmetry, but also because a rough estimate of its magnitude is on the same order as the shear-zone descent rate itself.

Eq. (1.5) enables us to test the residual advection effect in a crude way in terms of the HL one-dimensional model (despite the two-dimensional character of such a circulation). It should be remembered that a very important aspect of the mean advection that will not be treated here concerns the existence of such a circulation outside the region of wave driving (see Plumb and Bell, 1981b).

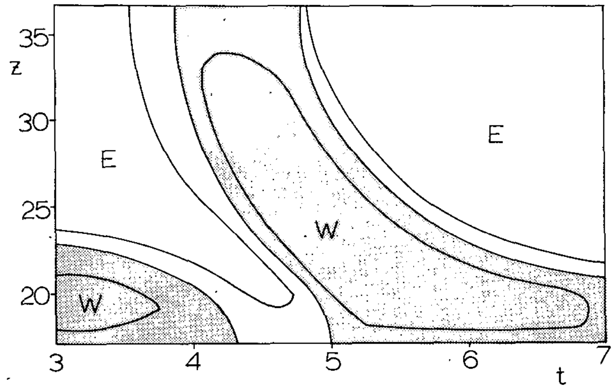

FIG. 5. As in Fig. 1 except with the residual advection included.

The control experiment was repeated with curvature neglected, but with (1.5) incorporated ($\bar{\alpha}_T \equiv \alpha$), the value of L was allowed to vary. It was found that variation of L in the range 10^5–10^6 m was critical to the existence and appearance of the oscillation. For example, the value $L = 10^5$ m resulted in only a very slight change from the control experiment. On the other hand, the value 10^6 m suppressed the oscillation altogether. Instead, a steady state resulted, having a westerly jet at low levels, overlain by an easterly jet. The latter was prevented from descending by the residual advection.

A value of $L = 5 \times 10^5$ m generated an oscillation having a more correct asymmetry, but with a somewhat longer period than the control experiment. This oscillation is shown in Fig. 5. The period is now 1170 days, the lengthening obviously due to the longer descent of the easterlies.[8] Interestingly the enhancement of the westerly descent was insufficient to prevent the overall lengthening of period.

In agreement with Plumb and Bell's (1982b) simulation, Fig. 5 displays asymmetry in the respective length of the easterly and westerly phases at different levels, with the westerlies predominant at lower levels and *vice versa*. This is simply a consequence of the differing descent rates.

In a crude calculation of the kind presented here it is not possible to deduce very much from the precise value of L; however, it is pleasing to note that the value used in Fig. 5 agrees both with the e-folding scale of the oscillation, and with the general magnitude of the associated temperature anomaly (1–2 K).

In our opinion the integration displayed in Fig. 5 suffers from an overly strong easterly shear-zone sharpness. The weaker shear zone of the observed easterly phase seems to be a significant feature of

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[7] Those authors employed a wavenumber 2 Kelvin wave, and although this yields less Kelvin wave attenuation and a relatively sharper westerly phase onset, it does nothing to reduce easterly shear zone sharpness according to the HL theory.

[8] Because (1.3) is not valid near horizontal boundaries, the elongation of the easterly descent near the tropopause might be an artifact of our model insofar as the tropopause may behave like a rigid boundary.

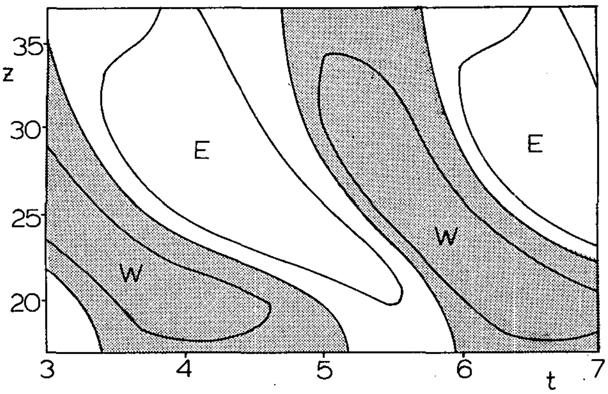

FIG. 6. As in Fig. 1 except with the latitudinal-curvature parameterization, equal mechanical-thermal damping, and the residual advection included.

the oscillation, and in particular it had been hoped that at least one of the simulations undertaken here could have given this result. In view of the importance of the reduced Rossby-gravity thermal damping, it has already been shown that latitudinal shear, at least within the context of the model used here, cannot account by itself for the observed asymmetry, although it does definitely help to reduce the sharpness of the easterly shear zones. Therefore, the residual advection seemed necessary.

Unfortunately, however, there does not seem to be much reason to think that the advection effect can lessen the easterly shear zone sharpness; indeed, it does very little of this in Fig. 5.

c. *Other experiments*

The inadequacy of either the advection or curvature effect taken separately raises the possibility that perhaps both are of importance in the observed oscillation. Also of possible importance might be some other factor, such as the magnitude of the wave forcings. For example, Plumb (1977, Fig. 6a) and Dunkerton (1981b, Fig. 5) give examples of how a correct asymmetry may be aided by increasing one wave forcing with respect to the other. In the second paper it was suggested that a reduction in the Rossby gravity wave forcing may have been responsible for the lengthening of the observed oscillation period after 1962. Unfortunately, it is difficult to specify precisely the value of the wave forcings, since both are quadratic wave quantities. However, Plumb and Bell (1982b) obtained the correct asymmetry without invoking an asymmetry in the wave forcing. Therefore it seems doubtful that the wave forcing alone should explain the observed asymmetry, although it may be of importance on certain occasions.

In any case for the purposes of this paper it was felt that an investigation of the combined advection-curvature effect should be performed unambiguously without invoking an asymmetric wave forcing. The experiments of this type seemed to reiterate a point made in the above discussion, namely, that the damping of the Rossby-gravity wave is simply inadequate if mechanical dissipation be ignored. A more realistic simulation involved a variation of the experiment shown in Fig. 4 in which the curvature parameterization was employed with equal mechanical-thermal damping, but with the residual advection effect included. The resulting oscillation is shown in Fig. 6.[9]

At this point the conclusion that would follow from this paper is that either the HL model is inadequate (either in terms of its original conceptual framework or in terms of the representation of the curvature and advection effects employed in this paper) to explain the observed shear zone asymmetry, or that perhaps mechanical damping *does* play some role in the atmospheric oscillation. Because the latter alternative seems somewhat speculative, one should not insist on its being true given the results of this paper; nevertheless, in itself it does seem to be an interesting possibility in need of further study (Andrews and McIntyre, 1976a).

In simplest terms the possible importance of mechanical dissipation of the Rossby-gravity wave would follow from the equipartition theorem as applied to this wave. Because a substantially greater fraction of the total wave action is of the "kinetic" type, there would be more wave action available for dissipation were there to be some form of mechanical damping.

4. Conclusion

It has been shown that the question of the simulated and observed shear-zone asymmetry in the quasi-biennial oscillation is more complex than previously thought, for two reasons. First, the original explanation of the model asymmetry appears inadequate, since in reality what is being seen in the model is not merely the differing frequency dependences in the vertical group velocity, but also the differing partitions of wave action for the two waves. Second, the residual advection seems unable to entirely account for the observed asymmetry, since it would predominantly alter the shear zone descent rate without significantly affecting the overall sharpness of the shear zones. To account for the latter, we have suggested the possible importance of latitudinal shear (Boyd, 1978b). We have also found that a correct asymmetry is greatly aided by an increase in the mechanical damping of the Rossby-gravity wave. Perhaps a further explanation needs to be sought in the magnitudes of the wave forcing on some occasions.

[9] A wavenumber 2 Kelvin wave (Plumb and Bell, 1982b) would further improve this correctly simulated asymmetry in the westerly shear zone; observations sometimes suggest a stronger westerly gradient than shown in Fig. 6.

Note added in proof

Upon submission of this manuscript it was found that the time-mean easterly component would help the observed asymmetry since, according to Wallace (1973), the observed Doppler-shifted Rossby-gravity wave frequency in the westerlies is only about 130% of its basic value instead of almost two times as in HL. For the wavenumber 1 Kelvin wave case the conclusions of this paper stand; however, for the wavenumber 2 case it is conceivable that either mechanical damping or latitudinal shear as parameterized here could *alone* account for the relative observed asymmetry. As to why the observed westerlies are weak, planetary wave critical level interaction remains of possible importance.

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