The Fluxes of Specific Enthalpy, Sensible Heat and Latent Heat near the Earth's Surface[1]

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ABSTRACT

Corrections to the sensible heat flux due to fluctuations in the specific humidity recently proposed by Brook (1978) have been shown to be incorrect by Frank and Emmitt (1981). However, it is easy to misinterpret Frank and Emmitt's paper. Here an effort is made to clarify the issue and to sketch what its importance is to the energy balance of the hydrological cycle. By expanding the specific enthalpy flux into the fluxes of sensible heat and latent heat, it is found that a good first-order approximation shows that no corrections are necessary, which is in agreement with Frank and Emmitt's result.

A recent paper by Frank and Emmitt (1981) shows that the correction to the sensible heat flux due to the moisture flux as suggested by Brook (1978) is incorrect. The argument by Frank and Emmitt is somewhat confusing, and, in fact, misled the author to believe that it was wrong. Therefore there is some need to clarify the issue and this is attempted here.

In order to keep the argument simple we shall assume that no liquid water is present in the air near the surface and that the geopotential is constant.

Following Frank and Emmitt, the specific enthalpy h_m of moist air may be expressed as the weighted sum of the enthalpies of dry air, h_d, and water vapor, h_v; i.e.,

$$h_m = (1-q)h_d + qh_v\,, \tag{1}$$

where q is the specific humidity.

The specific enthalpies may be expressed by (see Iribarne and Godson, 1973)

$$h_d = c_pT + b_1\,, \tag{2}$$

$$h_v = c_{pv}T + b_2\,, \tag{3}$$

$$h_l = c_lT + b_3\,, \tag{4}$$

where c_p is the specific heat at constant pressure for dry air, c_{pv} the specific heat at constant pressure for water vapor, c_l the specific heat of liquid water, and b_1, b_2 and b_3 are constants reflecting energies present in the respective substances. Substitution of (2) and (3) into (1) yields

$$h_m = (1-q)c_pT + qc_{pv}T + b_4\,, \tag{5}$$

where

$$b_4 = (1-q)b_1 + qb_2\,.$$

The flux of specific enthalpy may now be expressed by

$$\overline{\rho wh_m} = \overline{(1-q)\rho wc_pT} + \overline{\rho wqc_{pv}T} + \overline{\rho wb_4}\,, \tag{6}$$

where the overbar indicates an average over time assuming a stationary time series and horizontal uniformity, ρ is the density of the moist air, and w the vertical velocity.

At this point it is necessary to consider the mass flux $\overline{\rho w}$ more closely; by recognizing that the density is composed of a dry air component ρ_d and a water vapor component ρ_v, we may write

$$\overline{\rho w} = \overline{\rho_d w} + \overline{\rho_v w} = \overline{\rho(1-q)w} + \overline{\rho qw}$$

and because we assume no contributions from horizontal advection

$$\overline{\rho_d w} = \overline{\rho(1-q)w} = 0, \tag{7}$$

$$\overline{\rho w} = \overline{\rho qw} \simeq \bar{\rho}\,\overline{q'w'}. \tag{8}$$

Using (7) and (8) in the expansion of (6) yields after neglecting small terms (see Appendix)

$$\overline{\rho wh_m} \approx \bar{\rho}\,\overline{w'T'}[c_p + \bar{q}(c_{pv}-c_p)] + \bar{\rho}\,\overline{w'q'}(c_{pv}\bar{T} + b_2). \tag{9}$$

If we had used $\overline{\rho w} = 0$ and neglected b_1 and b_2, we would have obtained Brook's (1978) result in the form

$$\overline{\rho wh_m} \approx \bar{\rho}\,\overline{w'T'}\,[c_p + \bar{q}(c_{pv}-c_p)] + \bar{\rho}\,\overline{w'q'}[(c_{pv}-c_p)\bar{T}]. \tag{10}$$

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Because the specific enthalpy of water vapor contains the latent heat of vaporization ($L = h_v - h_l$), the specific enthalpy may also be written in the form

$$h_m = (1 - q)c_pT + qL + qc_lT + b_5\,, \quad (11)$$

where $b_5 = (1 - q)b_1 + qb_3$.

The flux of specific enthalpy may now be written as

$$\overline{\rho wh_m} = \overline{(1 - q)\rho wc_pT} + \overline{\rho wqL} + \overline{\rho wqc_lT} + \overline{\rho wb_5}\,. \quad (12)$$

Again using (7) and (8) the expansion in this case yields

$$\overline{\rho wh_m} = \bar\rho\,\overline{w'T'}[c_p + \bar q(c_{pv} - c_p)] + \bar\rho\,\overline{w'q'}(c_l\bar T + \bar L + b_3), \quad (13)$$

because $L' = (c_{pv} - c_l)T'$.

It is easy to verify that the right-hand sides of (9) and (13) are the same by using (3) and (4): $b_3 = b_2 + T(c_{pv} - c_l) - L$. Substituting this in (13) yields (9).[3]

Although it is satisfying that (9) and (13) agree with each other, the problem is not closed because we do not know what the constants b_2 or b_3 are. This is because water vapor is allowed to pass through the reference surface and we are therefore dealing with an open system. In order to focus on this problem it is useful to consider the sensible and latent heat fluxes at the surface. It is quite clear how much energy was transformed at the surface in order to provide these fluxes.

The sensible heat flux E_H is given by heat conduction at the surface to the atmosphere:

$$E_H = -k_0(\partial T/\partial z)_0\,, \quad (14)$$

where k_0 is the thermal conductivity of air at the surface. The latent heat flux can be given similarly by considering the vapor flux at the surface due to molecular diffusion and by assuming that this flux is entirely due to evaporation at the surface. Therefore

$$E_E = -L_0\rho_0D_0(\partial q/\partial z)_0\,, \quad (15)$$

where D is the molecular diffusivity of water vapor in air. All we need to consider then is the difference in specific enthalpy between the liquid and the vapor at the surface.

The fluxes of sensible and latent heat expressed by (14) and (15) must be equal to the enthalpy flux expressed by (13). This requires that we choose

$$b_3 = -c_lT_0\,, \quad (16)$$

where T_0 is the average surface temperature. This choice of b_3 is equivalent to taking the difference between the two enthalpies at the interface. The energy required to provide for the enthalpy of the liquid water at the surface is not considered. We will come back to this point below.

Substitution of (16) in (13) yields

$$\overline{\rho wh_m} = \bar\rho\,\overline{w'T'}[c_p + \bar q(c_{pv} - c_p)] + \bar\rho\,\overline{w'q'}[L_0 + c_l(\bar T - T_0)],$$

but because

$$L_0 = \bar L + (c_{pv} - c_l)(\bar T - T_0),$$

this may be written

$$\overline{\rho wh_m} = \bar\rho\,\overline{w'T'}[c_p + \bar q(c_{pv} - c_p)] + \bar\rho\,\overline{w'q'}[\bar L + c_{pv}(\bar T - T_0)]. \quad (17)$$

The last term on the right-hand side represents the specific heat released to the surface layer by the water vapor passing through and is therefore usually a negative term in the total specific enthalpy flux. This term as well as $\bar q(c_{pv} - c_p)$ are small terms, usually less than 2% of the major terms. By neglecting the small terms we obtain

$$\overline{\rho wh_m} = \bar\rho\,\overline{w'T'}c_p + \bar\rho\,\overline{w'q'}\,\bar L. \quad (18)$$

The terms on the right-hand side express the sensible and latent heat flux, respectively, and correspond to the surface fluxes as expressed by (14) and (15). We see, therefore, that no corrections of the type Brook has suggested are needed.

Although this result is pleasing to the micrometeorologist who can continue to work along well-established procedures, the arbitrary closure given by (16) requires further discussion. When we considered the transfer of energy at the interface we could restrict ourselves to the difference in specific enthalpy on both sides of the interface. In the larger context we have to consider the fact that for the water vapor flux we are dealing with an open system. Consequently, careful bookkeeping is required when we are dealing with the energy related to the hydrological cycle. For example, the water in the ground before it evaporated had to be warmed up from a temperature T_p which was the temperature of the precipitation that brought the water to the surface to the

[3] If we set $b_3 = 0$ and use $\overline{\rho w} = 0$, we obtain

$$\overline{\rho wh_m} = \bar\rho\,\overline{w'T'}[c_p + \bar q(c_{pv} - c_p)] + \bar\rho\,\overline{w'q'}[(c_l - c_p)\bar T + \bar L],$$

which is the same as Eq. (21) in Frank and Emmitt. They used this equation to compare with Brook's equation after adding the latent heat flux in the form $\bar\rho\,\overline{w'q'}L$. This illustrates that even for the rather special case of $\overline{\rho w} = 0$ Brook's equation is erroneous.

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temperature at which evaporation took place. This amount of energy is neglected over land because it is assumed that the heat flux into the ground vanishes in the long-term average. A small but systematic error is introduced in this way.

In order to facilitate careful bookkeeping of the energy, it is convenient to set T_0 in (16) equal to 0°C. With this convention, we must keep in mind that with precipitation a negative enthalpy flux is associated, which may be expressed by

$$-Ph_l = -Pc_l(T_p - T_0), \qquad (19)$$

where P is the rate of precipitation (kg m^{-2} s^{-1}).

Averaged over the world and over a sufficiently long time interval the flux of water vapor and the rate of precipitation must be equal and the net specific enthalpy flux may be given in a schematic way by

$$\overline{\rho w h_m} = \bar{\rho}\,\overline{w'T'}[c_p + \bar{q}(c_{pv} - c_p)] + \bar{\rho}\,\overline{w'q'}[c_l(T - T_p) + L_0], \qquad (20)$$

which again reduces to (18) by neglecting the small terms. In this case neglecting the small terms is not advisable, because these terms tend to be systematic and may cause a cumulative error in a climatological sense. Also an amount of energy should be added to (20) in order to account for the precipitation that falls in the form of snow. This involves the specific enthalpy of ice, h_i, which, after using the 0°C reference temperature and recognizing that the latent heat of fusion, $L_{il} = (h_l - h_i)$, may be written

$$h_i = c_i(T - T_0) - L_{il0}, \qquad (21)$$

where c_i is the specific heat of ice and L_{il0} is the latent heat of fusion at the reference temperature T_0.

It is not our intent to give a complete account of the energy involved in the hydrological cycle, but only to sketch how the present analysis of the specific enthalpy flux fits into this larger context.

Instead of using T_0 = 0°C in (17), Frank and Emmitt prefer to set T_0 = 0 K. Eq. (18) should then be written as

$$\overline{\rho w h_m} = \bar{\rho}\,\overline{w'T'}c_p + \overline{\rho w'q'}(\bar{L} + c_l\bar{T}). \qquad (18a)$$

This is possibly a better convention because the term which includes $c_l\bar{T}$ will remind us of the necessity of careful bookkeeping. On the other hand, using 0 K as a reference temperature, i.e., setting $b_3 = 0$, easily leads to the oversight of latent heat of fusion. Also in reality it is not possible to integrate the enthalpy from 0 K to whatever atmospheric temperature is desired with constant specific heat. Thus, although 0 K seems like an attractive absolute reference point, it has an illusive character.

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APPENDIX

Expansion of Eq. (6)

In order to obtain (9) we have to expand the individual terms on the right-hand side of (6). The first term is

$$\left.\begin{aligned} &\overline{(1-q)\rho w c_p T} = c_p\overline{\rho w T} - c_p\overline{q\rho w T} \\ &\overline{\rho w T} = \underset{\text{I}}{\bar{\rho}\,\bar{w}\,\bar{T}} + \underset{\text{II}}{\bar{\rho}\,\overline{w'T'}} + \underset{\text{III}}{\bar{w}\,\overline{\rho'T'}} + \underset{\text{IV}}{\bar{T}\,\overline{\rho'w'}} + \text{TC} \end{aligned}\right\} . \qquad (A1)$$

Here TC stands for triple correlation, which we will neglect from hereon as well as higher correlations. Terms I and IV may be combined with (8) to give $\rho T w'q'$; since term III is negligible, we have

$$\left.\begin{aligned} &\overline{\rho w T} = \bar{\rho}\,\overline{w'T'} + \bar{\rho}\,\bar{T}\,\overline{w'q'} \\ &\overline{q\rho w T} = \underset{\text{I}}{\bar{q}\,\bar{\rho}\,\overline{w'T'}} + \underset{\text{II}}{\bar{q}\,\bar{\rho}\,\bar{T}\,\overline{w'q'}} + \underset{\text{III}}{\bar{\rho}\,\bar{w}\,\overline{q'T'}} + \underset{\text{IV}}{\bar{T}\,\bar{w}\,\overline{q'\rho'}} + \underset{\text{V}}{\bar{T}\,\bar{\rho}\,\overline{w'q'}} + \text{TC} \end{aligned}\right\} . \qquad (A2)$$

Since terms II, III and IV are at least an order of magnitude smaller than I and V, we have

$$\overline{q\rho w T} \simeq \bar{q}\,\bar{\rho}\,\overline{w'T'} + \bar{T}\,\bar{\rho}\,\overline{w'q'}. \qquad (A3)$$

Substituting (A2) and (A3) into (A1) yields

$$\overline{(1-q)\rho w c_p T} \approx \bar{\rho}\,\overline{w'T'}c_p(1 - \bar{q}). \qquad (A4)$$

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The second term on the right-hand side of (6) is analogous to (A3), i.e.,

$$\overline{\rho w q c_{pv} T} \approx \bar{\rho}\, \overline{w'T'} c_{pv} \bar{q} + \bar{\rho}\, \overline{w'q'} c_{pv} \bar{T}, \quad \text{(A5)}$$

while the third term is

$$\overline{\rho w b_4} = \overline{\rho w (1-q)} b_1 + \overline{\rho w q} b_2 .$$

Using (7) and (8) this becomes

$$\overline{\rho w b_4} = \bar{\rho}\, \overline{w'q'} b_2 .$$

Combining (A4), (A5) and (A6) results in (9). Eq. (13) is derived in much the same fashion.

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