RADIATIVE COOLING IN THE LOWEST LAYERS OF AN ATMOSPHERE WARMER THAN THE GROUND

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ABSTRACT

Radiative cooling is computed for a warm, isothermal atmosphere streaming out over a cold surface. The study shows that the rate of cooling at heights from a few centimeters up to a few meters varies inversely as the square root of the distance from the ground, and that it varies somewhat more rapidly at larger distances.

The difference in radiative cooling at low levels between an isothermal and an adiabatic atmosphere is computed and found to be negligible in comparison with values derived from the isothermal atmosphere.

Comparison of radiative with observed cooling shows that radiative cooling is relatively small compared with cooling by turbulence in winds around 15 mph.

1. Introduction

When relatively warm air is brought into contact with a cool surface, it emits more radiation than it absorbs. Hence cooling is to be expected.

Cases of this type have been studied by Wexler [6] and Elsasser [3]. Wexler considered the formation of polar continental air over a snow surface, and Elsasser considered the cooling above a nocturnal inversion. Elsasser's results are equally applicable to ground inversions produced by advection of warm air over a cold ground. Both Elsasser and Wexler treated the mean cooling in layers having thicknesses of the order of 100 m.

In this paper, the rate of cooling is computed for specific levels from 100 m down to about 10 cm. The results are applied in a discussion of the conditions under which radiation can play an appreciable part in the cooling of warm air streaming out over cold water.

The general problem of cooling by infrared radiation has been treated by Elsasser [4]. He has constructed a chart from which mean cooling in atmospheric layers can be evaluated, provided that vertical distributions of the temperature and moisture are known.

The emission and absorption by water vapor and carbon dioxide are important for the radiative cooling in the lower layers of the atmosphere. In general, for polyatomic gases, emission and absorption are complicated functions of the thickness of substance traversed. For layers of air a few centimeters thick, emission and absorption vary linearly with the thickness [5]. For the layers of thickness between 10 cm (10^{-4} cm of precipitable water) and about 3 m (3×10^{-3} cm of precipitable water), both theory [4, 5] and observation [1, 2, 4] indicate that absorption and emission vary as the square root of the thickness. For still thicker layers, the relation becomes more complicated [4].

When the radiative properties of the atmosphere are horizontally homogeneous, the temperature variation is given by

$$\frac{dT}{dt} = -\frac{1}{c_p\rho}\frac{\partial F_N}{\partial z} \quad (1)$$

where T is temperature, t is time, c_p is specific heat at constant pressure, ρ is air density, F_N is net flux of energy, and z is the vertical coordinate.

If F_u and F_d are defined as the upward and downward flux, respectively, then $F_N = F_u - F_d$. Thus

$$\frac{dT}{dt} = -\frac{1}{c_p\rho}\left(\frac{\partial F_u}{\partial z} - \frac{\partial F_d}{\partial z}\right). \quad (2)$$

First, an isothermal atmosphere of temperature T_1 and constant specific humidity q superimposed upon a cooler ground of temperature T_0 will be considered. Under these conditions $\partial F_d/\partial z = 0$, since each layer absorbs as much downward radiation as it emits. Thus, equation (2) reduces to

$$\frac{dT}{dt} = -\frac{1}{c_p\rho}\frac{\partial F_u}{\partial z}. \quad (3)$$

2. Effect of water vapor

If the term du denotes the mass of water vapor contained in a vertical column of height dz and of unit area in horizontal cross section, $du = \rho q\, dz$. Thus, for water vapor, equation (3) may be written

$$\frac{dT}{dt} = -\frac{q}{c_p}\frac{\partial F_u}{\partial u}. \quad (4)$$

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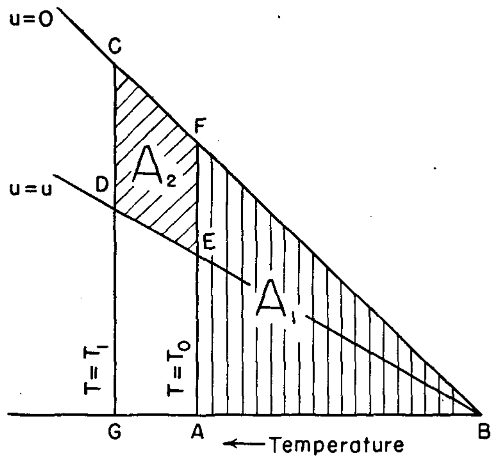

FIG. 1. Sketch based on the Elsasser radiation chart. The sum of the two hatched areas represents the upward flux.

At a level below which the mass of water vapor is u, the upward flux of radiation depends on the black-body flux at temperature T_0 and on the modification of this flux by the warmer intervening layer. On an Elsasser radiation chart, the upward flux is represented by the area ABCDEA (fig. 1). This area consists of two parts, A_1 and A_2. The symbols A_1 and A_2 will be used for the energies represented by the respective areas. Now A_1 is the black-body radiation of the ground and is not a function of u. Hence, from (4),

$$\frac{dT}{dt} = -\frac{q}{c_p}\frac{\partial A_2}{\partial u}. \qquad (5)$$

From tables printed on the radiation chart, A_2 was plotted against $\sqrt{u}$ for $T_1 = 293\text{K}$ and $T_0 = 283\text{K}$. The result is a straight line for small values of u. The equation of this line is

$$A_2 = 3.2\sqrt{u}(T_1 - T_0) \qquad (6)$$

where A_2 is in cal(3 hr)$^{-1}$cm^{-2}, and u is in g cm^{-2}. The equation is applicable at elevations from about 10 cm up to about 2.5 m (with specific humidity of 10 per mille).

For the range of u within which (6) holds, (5) may be written

$$dT/dt = -6.7\sqrt{q/\rho z}\,(T_1 - T_0) \qquad (7)$$

where the unit of t is 3 hours, z is in cm, and ρ is in g cm^{-3}.

For elevations between 2.5 m and 70 m, the variation of A_2 with u is not given by a simple function; hence dT/dt was computed directly from equation (5) written in the form

$$\frac{dT}{dt} = -\frac{q}{2c_p\sqrt{u}}\frac{\partial A_2}{\partial\sqrt{u}}. \qquad (8)$$

The quantity $\partial A_2/\partial\sqrt{u}$ was measured from a graph of A_2 against $\sqrt{u}$.

For elevations greater than 70 m there is again a simple relationship between A_2 and u, namely,

$$A_2 = (0.12 \ln u + 0.80)(T_1 - T_0) \qquad (9)$$

where A_2 is in cal cm^{-2}(3 hr)$^{-1}$, u is in g cm^{-2}, and T is in centigrade. On substitution of equation (9) into equation (5), and application of $u = q\rho z$,

$$\frac{dT}{dt} = -\frac{4.0}{z}(T_1 - T_0) \qquad (10)$$

where z is in meters and the unit of t is 3 hours. It is to be noted that this relation is independent of the value assumed for specific humidity.

The values of dT/dt were computed from equations (7), (8), and (10) with $T_1 = 293\text{K}$, $T_0 = 283\text{K}$, $q = 10$ per mille and 5 per mille, and $\rho = 1.25 \times 10^{-3}$ g cm^{-3}. The cooling due to water vapor is shown as a function of height by the solid lines in fig. 2.

3. Effect of carbon dioxide

The radiation chart cannot be used directly to compute the effect of carbon dioxide, because its construction is based on the assumption that the carbon dioxide in any layer absorbs and emits the same amount of radiation. When a temperature discontinuity exists, this assumption does not hold, and a procedure similar to that used by Elsasser [3] has to be employed. Again, equation (3) is applicable. Let $a(z)$ be the fraction of radiation absorbed in a layer of thickness z. Then the upward flux at an elevation z is given by the black-body radiation from the ground, σT_0^4, plus the energy emitted by the intervening layer, minus the energy absorbed by the intervening layer:

$$F_u = \sigma T_0^4 + a(z)\sigma(T_1^4 - T_0^4).$$

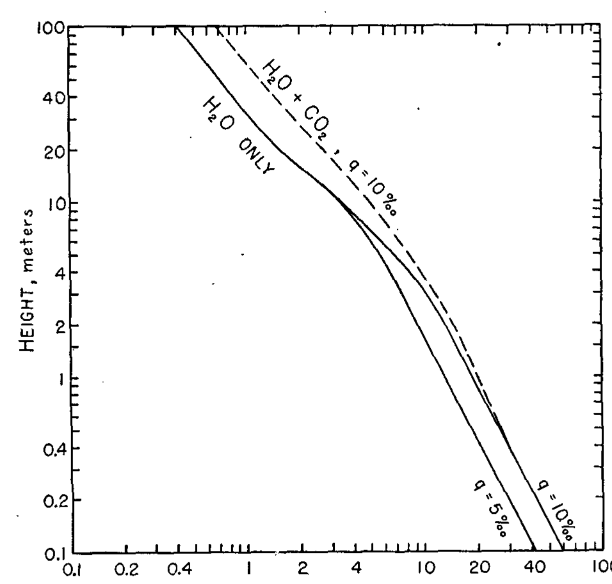

FIG. 2. Rate of radiative temperature change as a function of elevation for a temperature discontinuity of 10C at the ground.

Hence,

$$\frac{dT}{dt} = -\frac{1}{c_p\rho}\frac{\partial a}{\partial z}\sigma(T_1^4 - T_0^4). \qquad (11)$$

The absorption coefficient, a, has been measured by several investigators for unidirectional radiation. Since the radiation in the atmosphere is diffuse, these values cannot be used directly in equation (11). Elsasser [4] found that an absorption tube of length z absorbs the same fraction of unidirectional radiation as a layer of thickness 1.78 z absorbs of diffuse radiation, provided that a varies linearly with $\sqrt{z}$. This condition is satisfied for layers of air with normal carbon dioxide content for values of z between 4 m and 100 m. For thinner layers, the factor 1.78 has to be increased slightly, but for such layers the contribution of carbon dioxide to the total cooling is relatively unimportant. Hence, the factor 1.78 will be assumed to be valid for the whole range of thicknesses under consideration.

Let z' denote the length of an absorption tube having the same absorption coefficient as a layer of thickness z. Then $z' = 1.78\ z$. On substitution into (11),

$$\frac{dT}{dt} = -\frac{1.78}{c_p\rho}\frac{\partial a}{\partial z'}\sigma(T_1^4 - T_0^4).$$

The quantity $\partial a/\partial z'$ was determined from Elsasser's figure [4, fig. 26]. Since this figure is a graph of a against $\sqrt{z'}$, the slope on this graph divided by $2\sqrt{z'}$ yields the required value of $\partial a/\partial z'$.

The total cooling due to water vapor and carbon dioxide, for $T_1 = 293$K, $T_0 = 283$K, and $q = 10$ per mille, is given by a dashed line in fig. 2. It is nearly proportional to $\Delta T = T_1 - T_0$ and increases slowly with increasing mean temperature.

The cooling due to water vapor and, therefore, the total cooling vary only slowly with the specific humidity, as seen from fig. 2.

4. Comparison between an isothermal and a nearly adiabatic atmosphere

For comparison with observations, an atmosphere having a dry-adiabatic lapse rate is more suitable than an isothermal atmosphere. A nearly adiabatic atmosphere of the following specifications is assumed: ground temperature 10C, air temperature 20C, lapse rate dry-adiabatic from 1000 mb (ground) to 910 mb and halfway between dry-adiabatic and saturation-adiabatic from 910 mb up, specific humidity 10 per mille up to 930 mb and one per mille less than the saturation specific humidity at lower pressures.

This atmosphere was analyzed with the aid of Elsasser's radiation chart and compared with the isothermal atmosphere. In all layers analyzed (0–50 m, 0–100 m, 0–300 m), the mean rate of cooling due to water vapor in the adiabatic atmosphere exceeded that in the isothermal atmosphere by 0.2 C(3 hr)$^{-1}$. Since the value is the same for layers of widely varying thickness, it expresses the difference between the cooling in the two atmospheres not only in the mean of these relatively thick layers, but at specific elevations as well.

It is seen from fig. 2 that below about 40 m this difference in cooling between an isothermal and an adiabatic atmosphere of the same surface temperature and specific humidity may be neglected.

5. Comparison with observation

Fig. 2 shows that the total *radiative* cooling at 6 m is 7.2 C(3 hr)$^{-1}$ or 0.4C in 10 min. Since the assumed initial discontinuity is 10C, this amounts to a modification of 4 per cent in 10 min. The computation was made under the assumption that during the 10 min the radiative cooling is constant and equal to its initial value. Actually, the cooling decreases rapidly for two reasons: the temperature of the air at 6 m approaches the ocean temperature, thus decreasing ΔT; and the air above 6 m becomes warm relative to the air at 6 m. Thus the cooling of 4 per cent given above is an upper limit.

Three soundings[2] taken 2–3 miles from shore, in offshore winds with speeds near 15 mph, show that the initial temperature discontinuity between the ground and the air at an elevation of 6 m had been decreased, on the average, by 20 per cent in the first 10 min over the sea. This value is much larger than that to be expected from cooling due to radiation alone. It is probable that most of the cooling was caused by eddy conduction and that radiation played only a minor role. This conclusion is strengthened by the fact that the specific humidity is modified to about the same extent as the temperature.

At lower wind speeds, turbulence is less important. However, no observations are available for a determination of the critical wind speed below which the effect of radiation exceeds that of turbulence.

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[2] These soundings were obtained by the Radiation Laboratory of the Massachusetts Institute of Technology and analyzed by Richard A. Craig at the Woods Hole Oceanographic Institution.