Annual Cycle and Spatial Spectra of Earth Emitted Radiation at Large Scales

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ABSTRACT

Monthly averaged, resolution enhanced global distributions of the Earth's emitted radiation, as measured by the Nimbus-6 Earth Radiation Budget (ERB) wide field of view radiometers, have been analyzed for 1 year of data from July 1975 to June 1976. These distributions are expressed in terms of spherical harmonic coefficients, and time and space variability of the emitted radiation field is studied in terms of these coefficients. The average annual distribution accounts for 78% of the space-time power, and the annual cycle accounts for 17% of the power. Spatial variations over the globe are described in terms of degree variance, and longitudinal variations are described in terms of spectral power as a function of latitude. The longitudinal spectra were found to vary strongly with time.

1. Introduction

The nonuniform distribution of radiative heating and cooling over the Earth drives the motion of the atmosphere and oceans, and an understanding of the interaction between the radiation and the dynamics of the atmosphere and oceans is required for understanding climate processes. For this reason, several Earth radiation instruments of increasing sophistication have been flown on spacecraft, the most recent of which has been the Earth Radiation Budget (ERB) instrument aboard the Nimbus 6 and Nimbus 7 spacecraft (Smith et al., 1977; Jacobowitz et al., 1979). The ERB instrument includes two Earth-viewing wide field-of-view (WFOV) radiometers, one of which measures total irradiance and the other shortwave (SW) irradiance. The difference between the two irradiances is the longwave (LW) or Earth-emitted irradiance. The ERB instrument has been providing WFOV measurements since July 1975. These WFOV data are suitable for studying large-scale processes over spatial scales of thousands of kilometers. In this paper, a study is presented of the annual cycle of the distribution of the Earth's LW radiation; this study is based on one year of WFOV data from July 1975 through June 1976. By using a single year of data to compute an annual cycle, one neglects long term trends and interannual variability and assumes that the one year is typical.

The WFOV sensors integrate the radiance contributions from all portions of the Earth-atmosphere system within the field of view; i.e., from limb to limb. For the Nimbus 6 spacecraft, this distance is 7000 km, and half of the radiant power impinging on the sensor is from within a circle of >1800 km diameter; thus, the sensor has low spatial resolution. A resolution enhancement technique (Smith and Green, 1981) has been developed for analyzing the measurements whereby the small-scale features which are attenuated by the WFOV sensor may be amplified to produce a radiant exitance field at the "top of the atmosphere" (taken herein to be 30 km). This technique has been applied by Bess et al. (1981) to extract from the data monthly mean LW exitance fields, which are the basis for the present studies of temporal and spatial variability at large scales. This technique involves the computation of the coefficients in the spherical harmonic expansion of the monthly average of the Earth emitted radiant exitance. These coefficients constitute a relatively small data set which represents the Earth's emitted exitance and are tabulated by Bess et al. (1980) for July 1975 through June 1976. These coefficients are included through the 12th order, corresponding to a resolution of 15° of arc on the Earth. In the present paper the temporal and spatial variability of the Earth's emitted radiation is studied by analyzing these coefficients.

The annual cycle of Earth emitted radiation has been studied by Heddinghaus and Krueger (1981) and Stephens et al. (1981). Using the data set developed from the NOAA operational satellites (Gruber and Winston, 1978; Winston et al., 1979) which extends over 45 months, Heddinghaus and Krueger studied the annual and semiannual cycles as well as interannual variability of Earth emitted radiation by using an empirical orthogonal function (EOF) analysis. Stephens et al. (1981) have worked with a composite set of data from six different experiments covering 48 months of measurements from 1964 through 1977. They computed annual and semiannual am-
AMPLITUDES IN THE PHYSICAL (LATITUDE–LONGITUDE) DOMAIN. IN THE PRESENT PAPER, THE ANNUAL CYCLE IS ANALYZED IN THE SPECTRAL DOMAIN BY USE OF THE SPHERICAL HARMONIC COEFFICIENTS.

2. SPACE–TIME SCALES

Before proceeding, it is useful to put the space–time scales of the present study in perspective. The space–time scales of various weather and climate processes are shown in Fig. 1, where the abscissa is given as the (log) wavenumber or as (log) distance and the ordinate is (log) time. In these processes, the radiation field acts as a driver of and as a responder to the other meteorological fields. The annual cycle is indicated here as having a time scale of 1 year, with a minimum time resolution, or averaging time, of 1 month. The spatial scales associated with the annual cycle are from global scale (wavenumber 0 or 1) to a wavenumber O(5). Longer time scales involve interannual variations and more gradual changes in climate. These changes involve interaction with the ocean, solar variations, etc. For time scales of the order of a month or less, there is a spectrum of atmospheric processes. At the large scales, these include planetary waves and the low wavenumber features which drive baroclinic processes. As one considers progressively smaller phenomena from synoptic scale processes to local weather, the corresponding time scales shorten. Most of these processes are of duration less than the limit of predictability, which is of the order of 2 weeks. For times of less than a week, radiation is usually not considered to be important to the dynamics of the atmosphere; however, the various processes which occur in the atmosphere are strong modulators of radiation, especially through the influence of clouds. For times of a month or greater, radiation is important to the dynamics as the heat source and sink which drives the atmosphere and ocean. The determination of the space–time spectrum of radiation at these various scales should contribute substantially to our understanding of the role of radiation in climate processes.

The capability of various types of observing systems aboard spacecraft can be represented similarly, as shown in Fig. 2. The ERB WFOV radiometer aboard the Nimbus 6 spacecraft produces data from which information to wavenumber 6 can be obtained on a daily basis. This limit is due to the sampling limitation of slightly less than 14 orbits per day, so that the Nyquist limit, due to sampling in the longitudinal direction, is 6 on a daily basis. For a 1-month data set from the LW Nimbus 6 WFOV radiometers, it has been shown by Smith and Green (1981) that reasonable results can be obtained to wavenumber 12, corresponding to a horizontal resolution of 1600 km. For studies of smaller space and time scales, a scanning radiometer is required.

The amount of data processing and computation, and thus cost, required to extract a time average of a given wavenumber component of the global distribution is proportional to the product of the time duration and the number of data points required for global coverage. Thus, it is desirable from the cost standpoint to use WFOV measurements for small wavenumber features of the global distribution which are derivable from them. The WFOV measurements have the additional advantage that for large spatial scales (small wavenumbers) the radiation flux information derived from them is less sensitive to radiation directional model errors than is information derived from scanning radiometer measurements.
3. Outline

The temporal and spatial variability of the Earth's emitted radiation will be described in terms of power and spectra. The general definition of (average) power $P$ of a field which will be used in this paper is

$$P = \int_D |f(x_1, x_2, \ldots, x_n)|^2 \, dx_1 \, dx_2 \cdots dx_n$$

$$\int_D dx_1 \, dx_2 \cdots dx_n$$

where the region of integration is the entire domain $D$ in $(x_1, x_2, \ldots, x_n)$ for which $f$ is defined. Thus, power is the square of the Euclidian norm of the function. It follows that if $f$ is expressed in terms of a set of functions which are orthonormal on $D$, resulting in a set of coefficients $(C_i)$, then

$$P = \sum_i |C_i|^2$$

The spectrum is then defined as the quantities $|C_i|^2$ or suitable partial sums of these.

In this paper the Earth-emitted radiation field is considered in several domains. Following a description of the coefficient data set in Section 4, the monthly average distribution of Earth-emitted radiation is considered over the globe; i.e., over the latitude-longitude $(\Theta, \Phi)$ domain. In this case, the basis set used is spherical harmonics, and the spectrum is discussed as degree variance in Section 5. For the second case, a sequence of these monthly averaged fields is formed in order to study their time variations. In Section 6, the axisymmetric time dependent case is considered, so that the domain becomes $(\Theta, \Phi, t)$. A time series analysis is then computed. This leads to the development of an empirical latitude model of the annual cycle of emitted radiation in Section 7. Next, a time series analysis is computed for the full data set, which describes the field in the $(\Theta, \Phi, t)$ domain, in Section 8. For the final case the longitudinal variation is examined in Section 9. Here, the latitude and time are considered as fixed parameters so that the domain reduces to simply longitude. The longitudinal variations are expressed as Fourier series and the power spectra computed. In this manner the time and space variations of the Earth-emitted radiation field are quantitatively studied.

4. Coefficient data set

The computation of the spherical harmonic coefficients from Nimbus 6 ERB WFOV data will now be discussed. The radiant exitance field $M(\Theta, \Phi, t)$ at the top of the atmosphere may be described in the form

$$M(\Theta, \Phi, t) = \sum_{m,n} b_m^* (t) Y_m^n(\Theta, \Phi),$$

where $\Theta$ is the colatitude, $\Phi$ is the longitude of a point, $t$ is time, and $Y_m^n$ is the spherical harmonic of degree $n$ and order $m$. (Notation is also defined in the Appendix.) The $Y_m^n$ are normalized such that

$$\int_{4\pi} (Y_m^n)^2 \, ds = 4\pi.$$

The order $m$ is the longitudinal wavenumber, and $n$ is the two-dimensional "Spherical wavenumber." Also, $n-n$ is the number of nodes from pole to pole, as well as an index of north-south symmetry. The complex coefficients $b_m^*(t)$ serve as a method of describing the time dependent distribution of emitted radiation. The real and imaginary parts are denoted $C_m^n$ and $S_m^n$, respectively.

The WFOV radiometer integrates incoming radiation over all solid angles within its field of view, as shown in Fig. 3. Its directional response $g$ is assumed to be a function of nadir angle, $\alpha$. The measurement at a point $\Theta, \Phi$, is thus

$$W(\Theta, \Phi) = \frac{1}{\pi} \int_{FOV} R(\theta) g(\alpha) \, d\Omega,$$

where $R(\theta)$ is the limb darkening, a function of zenith angle $\theta$ of the exiting ray. This is an integral equation which may be solved for $M(\Theta, \Phi)$ if $W(\Theta, \Phi)$ is given over a sphere at satellite altitude, resulting in Eq. (1) with the coefficients given by

$$b_m^n = \frac{1}{\lambda_n} \int_{\Phi_{m-1}}^{\Phi_m} \int_0^{2\pi} Y_m^n(\Theta, \Phi)$$

$$\times W(\Theta, \Phi) \sin \Theta_d \, d\Theta_d \, d\Phi_d.$$

where $(\cdot)^*$ denotes complex conjugate and the $\lambda_n$ are eigenvalues of the measurement integral and whose computation incorporates $R(\theta)$, $g(\alpha)$ and measurement altitude. The demonstration of Eq. (3) is given by Smith and Green (1981) based on the spherical symmetry of the measurement operator for which spherical harmonics are the eigenfunctions.

In order to apply Eq. (3), the sphere at satellite altitude was partitioned into $5^\circ \times 5^\circ$ GARP type "quasi-square" boxes as a grid system. The monthly mean value of the measurement in each box was computed. For computing this mean, all data points dur-
Fig. 4. Geographical distribution of Earth-emitted radiant exitance for September 1975.

ing a single pass over a box were averaged to produce a single "measurement" in order to account for the high correlation between successive data points. The resulting monthly mean measurement map was then used in Eq. (3) to compute a set of $b_n^m$, which is taken to be the mean $b_n^m$ for the month. Thus, $t$ is regarded as discretized, with 12 values for the year. As an example of the application of the technique, the LW radiant exitance field computed at the "top of the atmosphere" (30 km altitude) for the month of September is shown in Fig. 4. The derived radiation field from which this map was prepared contained terms through degree 12.

5. Degree variance

Because there are so many numbers required to specify $b_n^m$ (for a 12th-degree expansion, 169 numbers are required to specify both real and imaginary parts), it is useful to define degree variance as

$$\sigma_n^2 = \sum_{m=0}^{n} b_n^m b_n^m \ast,$$

where $( )^\ast$ denotes complex conjugate. Defined on a sphere, this quantity is the analog of the power spectral density which one defines on a line; e.g., the time line for a process. Degree variance is a measure of the variability at various scales of a field on a sphere. An important property of degree variance is that it is invariant with arbitrary rotations of the coordinate system. Degree variance as a function of degree is shown in Fig. 5 for the month of September. This figure may be considered to be a description of the spatial variability of a single realization of the radiant exitance field. It is seen that for degrees 2 through 12, the $\sigma_n^2$ lie in a band with a negative slope, decreasing approximately exponentially with increasing degree. Thus, the lower degree terms, primarily the second degree, dominate the field. It should be noted that this result is for data for which the resolution has been enhanced; i.e. the WFOV measure-

ments for each month have been deconvolved to produce a radiant exitance field at the "top of the atmosphere" with a resolution of 15°.

Fig. 6 is a similar plot in which the variation of the degree variance throughout the year is indicated. During the year, there are 12 realizations of the $\sigma_n^2$ for each degree, $n$. The bar for each degree indicates the range of these $\sigma_n^2$; there is one outlying point above and below each bar. For degrees 1 and 3, the bars are quite wide, showing considerable variation during the year. It is seen that for the higher degrees, the bars are short and the degree variance decreases exponentially with increasing degree, as for September. It is thus concluded that the monthly averaged Earth-emitted radiation field is quite smooth and can be economically represented by spherical harmonics, i.e., for a given accuracy, relatively few terms are needed in order to represent the field.

The quantity

$$P(k) = \sum_{n=1}^{k} \sigma_n^2$$

Fig. 5. Degree variance for September 1975.
will be called summed degree variance. If the exponential decrease of $\sigma_n^2$ with $n$ continues for $n > 12$, then $\sigma_n^2$ is negligible for $n > 12$, and $P(12)$ is the total power of the spatial distribution spectrum, in analogy with the total power of a process on a line. The quantity $P(k)/P(12)$ will be called the normalized summed degree variance and is shown plotted in Fig. 7 for 4 months. Again, it is seen that the second degree variance dominates, containing approximately half of the total power. Less than 5% of the power is due to degrees $> 8$.

The total power is plotted for each month, starting with July 1975, in Fig. 8 and ranges from 1000 to 1320 W m$^{-4}$. It is also useful to define zonal power and the power computed using only axisymmetric terms; i.e., $\sum_n (C_n^0)^2$ (note that the axisymmetric terms have no imaginary part), and the nonzonal power as the total power minus the zonal power. Zonal and nonzonal power are also shown in Fig. 8. The zonal power varies from 840 to 1060 W m$^{-2}$ and the nonzonal power clusters around 225 W m$^{-2}$ ranging from 125 to 345 W m$^{-4}$. It is seen that $80 \pm 8\%$ of the power is in the zonal terms. This is a measure of the fact that at large scales Earth-emitted radiation is strongly latitude dependent and weakly longitude dependent.

6. Time series analysis of zonal terms

The individual coefficients will now be considered. Examination of the tables of $C_n^m(t)$ and $S_n^m(t)$ published by Bess et al. (1980) shows that the axisymmetric or zonal terms ($m = 0$) dominate up to $n = 5$, beyond which the pattern appears almost random. Because of their dominance, the zonal terms will be considered first.

Fig. 9 shows the yearly cycle of $C_0^0(t)$, $C_1^0(t)$ and $C_2^0(t)$. The $C_0^0(t)$ is the global average emitted radia-

![Fig. 6. Degree variance ranges for July 1975–June 1976.](image)

![Fig. 8. Time history of power for July 1975–June 1976.](image)
Its annual cycle is seen to have a nearly perfect sine shape, with a total range between its minimum and maximum values of 20 W m$^{-2}$. It does not oscillate about zero, but has a bias of 2.3 W m$^{-2}$. For a linear response, if the Earth were symmetric about the equator, one would expect the $C_n^0$ terms for odd $n$ values to have time histories which are symmetric about the time axis; and neglecting the effect of orbit eccentricity, the $C_n^3$ terms will have no annual variations but may have semiannual variations. Thus, the 2.3 W m$^{-2}$ bias in the annual cycle of $C_0^0$ is presumed to be due to land/ocean distribution differences between the Northern and Southern Hemisphere (or differences in cloud cover, which is in turn due to land/ocean distribution). The $C_n^3(t)$ term may be considered to be a measure of equator-to-pole gradient. It is seen to have an average value of $-25.6$ W m$^{-2}$, but a small variation.

The $C_n^0(t)$ for $n = 3$ through 12 are shown in Fig. 10. The $C_3^0$ is nearly sinusoidal, with a total variation of 15 W m$^{-2}$ and a mean of 4 W m$^{-2}$. As with $C_0^0$, this bias is presumed to be the result of hemispheric differences of land/ocean distribution. The $C_4^0$ term has a mean of $-6.4$ W m$^{-2}$ with variation of 5 W m$^{-2}$. Its shape is not so sinusoidal as the $C_0^0$ or $C_3^0$ histories. The $C_5^0$ through $C_{10}^0$ terms each have a significant annual sine component, and the $C_6^0$ and $C_8^0$ terms have a bias such that they do not change signs. As one attempts to describe increasingly fine detail, the land/ocean effects become increasingly important. The maximum absolute value decreases with increasing $n$ until $n = 11$ and 12. $C_{11}^0$ and $C_{12}^0$ are small and show little discernible pattern. Previous studies by Bess et al. (1981) indicated that 12th-degree is near the limit of deconvolution for the orbit altitude of Nimbus 6. Whether the lack of pattern in the computed values of $C_{11}^0$ and $C_{12}^0$ is due to the nature of the atmosphere or due to the limitations of sampling and analysis is unclear at present.

A time series analysis of the zonal terms was performed in order to determine the frequency content of the $C_n^0$'s. Each $C_n^0$ is expanded in a Fourier series as

$$C_n^0(t) = C_n^0 + \sum_k \tilde{C}_n^0(k) \cos\left(\frac{2\pi kt}{12} - \eta_{kn}\right),$$

where $t$ is measured in months, with mid-July 1975 being $t = 1$. The $\tilde{C}_n^0(k)$ is the amplitude of the wave with $k$ cycles per year for $C_n^0$ and $\eta_{kn}$ is the phase angle. The results are shown in Table 1. It is seen that the dominant terms are the annual average $C_n^0$ and the annual cycle $C_n^0(1)$. The computed terms of higher frequency than annual have a small contribution, i.e. less than 2 W m$^{-2}$, with one exception ($C_4^0$). Because these results indicate that the higher frequency terms are quite small, the summation will be reduced to only the $k = 1$ term.

Because only the mean and annual cycle appear to be significant when dealing with monthly averaged data, one may tentatively conclude that emitted radiation at large time and space scales has only the variation which is forced (Lorenz, 1979) by the variation in solar declination; free variations are much

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**Fig. 9.** The $C_n^0$ histories for $n = 0, 1, 2$ for July 1975–June 1976.

**Fig. 10.** The $C_n^0$ histories for $n = 3$ through 12 for July 1975–June 1976.
smaller. Obviously, to study effects such as interannual variations and quasi-biennial oscillation requires a longer data period than one year.

7. Latitudinal model for annual cycle of emitted radiation

These results permit the development of a simple empirical model of the time and latitude dependent distribution of Earth-emitted radiation. The zonal average of M, denoted by $[M]$, may be written as by use of Eq. (1) as

$$[M](\Theta, t) = \sum_{n=0}^{9} (2n + 1)^{1/2} P_n(\cos \Theta) \times \left[ C_n^0 + C_n^0(1) \cos \left( \frac{2\pi t}{12} - \eta_{1n} \right) \right]$$

with the $C_n^0$, $C_n^0(1)$ and $\eta_{1n}$ given in Table 1, and $P_n(\cdot)$ is the Legendre Polynomial of degree n. Eq. (6) may be expressed as

$$[M](\Theta, t) = C_n^0(t) + f_d(\Theta) + f_s(\Theta)$$

$$\times \cos \left( \frac{2\pi t}{12} \right) + f_f(\Theta) \sin \left( \frac{2\pi t}{12} \right),$$

where

$$f_d(\Theta) = \sum_{n} (2n + 1)^{1/2} C_n^0 P_n \cos \Theta,$$

$$f_s(\Theta) = \sum_{n} (2n + 1)^{1/2} C_n^0(1) P_n \cos \Theta \cos \eta_{1n},$$

$$= \sum_{n} \alpha_n(2n + 1)^{1/2} P_n \cos \Theta,$$

$$f_f(\Theta) = \sum_{n} (2n + 1)^{1/2} C_n^0(1) P_n \cos \Theta \sin \eta_{1n},$$

$$= \sum_{n} \beta_n(2n + 1)^{1/2} P_n \cos \Theta.$$

The function $C_n^0(t)$ is the globally averaged Earth-emitted radiation flux. The $f_d(\Theta)$, $f_s(\Theta)$ and $f_f(\Theta)$ are latitudinal functions describing the annual cycle of emitted radiation and are shown in Fig. 11 as a function of $\cos \Theta$ (which provides an area weighted representation). The $f_d(\Theta)$ is the annual average of the difference of the zonal value of M from the global mean level, and the $f_s(\Theta)$ and the $f_f(\Theta)$ represent the annual cycle. Eq. (6) and Table 1 or Eq. (7) and Fig. 11 constitute an empirical time and latitude dependent model for the Earth’s zonally averaged emitted radiation field at larger scales.

The $f_0$ curve is high in low latitudes and low in high latitudes, primarily due to the large $C_n^0$ factor. The curve dips at the equator, due to high convective clouds, and has peaks at $\pm 20^\circ$ latitude, due to the belts of subsidence regions in the subtropics. These Hadley circulation type features are defined by the fourth, sixth, and eighth degree terms. Asymmetries of $f_0$ about the equator are due to the odd degree terms. The $f_s$ and $f_f$ curves are approximately odd functions, as their dominant contributions are of odd-degree.

Examination of the $f_s$ and $f_f$ curves shows that they are quite similar, suggesting that the annual cycle for zonal means could be represented to a good approximation by a simple variation which has latitudinally dependent amplitude and a constant phase angle, $\phi$. Thus, define $f_M(\Theta)$ by

$$f_M(\Theta) \cos \left( \frac{2\pi t}{12} + \phi \right)$$

$$= f_s(\Theta) \cos \left( \frac{2\pi t}{12} \right) + f_f(\Theta) \sin \left( \frac{2\pi t}{12} \right) - \epsilon(\Theta, t),$$

where $\epsilon(\Theta, t)$ is a residual which is to be minimized. It is required that $f_M(\Theta)$ be a least squares fit over the annual cycle, giving the condition

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Table 2. Legendre polynomial coefficients defining annual cycle of Earth-emitted radiation.

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The $f_M(\Theta)$ curve is thus found to be given by

$$f_M(\Theta) = \sum_n \gamma_n (2n + 1)^{1/2} P_n \cos \Theta,$$

where

$$\gamma_n = \alpha_n \cos \phi - \beta_n \sin \phi$$

and the phase angle $\phi$ is given by

$$\tan 2\phi = \frac{\sum_n \frac{1}{2n+1} \alpha_n \beta_n}{\sum_n \frac{1}{2n+1} (\alpha_n^2 - \beta_n^2)}.$$

The residual term is given by

$$\epsilon(\Theta, t) = \sin \left(\frac{2\pi}{12} (t + \phi)\right) E_M(\Theta),$$

where

$$E_M(\Theta) = \sum_n \delta_n (2n + 1)^{1/2} P_n \cos \Theta,$$

$$\delta_n = \alpha_n \sin \phi + \beta_n \cos \phi.$$

The coefficients $\alpha_n$, $\beta_n$, $\gamma_n$ and $\delta_n$ are listed in Table 2. The power in a curve $f(\Theta, t)$ is defined to be

$$P = (2T)^{-1} \int_0^T dt \int_{-1}^1 d^2 \cos \Theta.$$

The power in the annual cycle is then shown to be

$$P = \frac{1}{2} \sum_n \alpha_n^2 + \frac{1}{2} \sum_n \beta_n^2$$

or 136.0 W$^2$ m$^{-4}$. The power in the $f_M$ curve is 128.3 W$^2$ m$^{-4}$ and the residual term $\epsilon_M$ contains 7.7 W$^2$ m$^{-4}$. Thus, the $f_M$ curve contains 94.3% of the power in the annual cycle. Fig. 12 shows $f_M$ and $E_M$ as functions of $\cos \Theta$. As one would expect from the motivation for defining $f_M$, the $f_M$ curve looks quite similar to the $f_1$ and $f_3$ curves. The phase angle $\phi$ is found from Eq. (15) to be $-39.4^\circ$ or 40.0 days. Because $t = 0$ corresponds approximately to June 15, 6 days before the Northern Hemisphere summer solstice, this result indicates that the response of the emitted radiation lags the distribution of incident solar radiation by 34 days.

8. Time series analysis of full coefficient set

A time series analysis was performed using the 12 monthly values for the full set of $b_n^m$ to determine the annual component. Thus, each term may be expressed as

$$b_n^m(t) = b_n^m + A_n^m \cos \left[ \frac{2\pi(t - t_0)}{12} \right]$$

$$+ B_n^m \sin \left[ \frac{2\pi(t - t_0)}{12} \right]$$

with time $t$ in months; $t_0$ is taken to be June 15, so that $t = 1$ is mid-July. The $b_n^m$ defines the annual average distribution, and the $A_n^m$ and $B_n^m$ define the annual cycle, the $A_n^m$ component being very nearly in phase with the solar declination and the $B_n^m$ component being 90° out of phase. These coefficients have been computed by a time series analysis of the monthly spherical harmonic coefficients tabulated by Bess et al. (1980) and the results are listed in Tables 3, 4 and 5. Eqs. (1) and (21), together with Tables 3, 4 and 5, constitute a latitude–longitude time-dependent model of the Earth’s emitted radiation field. Based on the monthly coefficients of Bess et al. (1980) the annual average of the total power of the spatial distribution is 1160 W$^2$ m$^{-4}$, excluding the global

![Graph showing annual cycle function $f_M(\Theta)$ and out-of-phase (error) function $\epsilon_M(\Theta)$](image-url)
Table 3. Real and imaginary parts of $C_n^m$ (coefficients in annual average distribution of Earth-emitted radiation).\(^1\)

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For the real part $n$ is listed in left hand column and $m$ across the top. For the imaginary part $n$ is listed in the right hand column and $m$ across the bottom.

average value. The power contained by the annual average $b_n^m$ as listed in Table 3 is 902 W$^2$ m$^{-4}$, or 78% of the total power. The power contained by the annual cycle, as represented by $A_n^m$ and $B_n^m$ and listed in Tables 4 and 5, is 201 W$^2$ m$^{-4}$, or 17% of the total power. Thus, Tables 3, 4 and 5 account for 95% of the total spectral power. Because of the small size of the higher temporal frequency terms such as the semiannual cycle and the effects of potential errors in the coefficients, it is not prudent to compute them

Table 4. Real and imaginary parts of $A_n^m$ (coefficients of cos(2π(t - t₀)/12) in annual cycle of Earth-emitted radiation distribution).\(^1\)

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\(^1\) As for Table 3.
on the basis of a single year of data. Such extensions of this work must await availability of additional data.

According to this analysis, the annual cycle accounts for 78% of the time dependent portion (258 W² m⁻⁴) of the power. Heddinghaus and Krueger (1981) find that 72% of the variance is accounted for by the first two eigenvectors, which are loosely analogous to the $A_n$ and $B_n$ herein. In the present paper, power is defined as a global quantity. Also, a single year of data is analyzed, so that interannual variations are partly aliased into the annual cycle result. Heddinghaus and Krueger cover the domain 60°N to 60°S with a grid and define variance as the sum of the squares of the values. Interannual variability and the semiannual cycle are strongest within this region. Because of these effects, the variance accounted for by their first two eigenvectors would be expected to be smaller than that for the $A_n$ and $B_n$. With these differences in techniques of analysis and data type, the present authors consider the agreement of results to be quite good.

The degree of variance of the $B_n^m$ is shown in Fig. 13 as a function of degree. In addition to the degree variance for all terms of the given degree, the contributions due to the axisymmetric and nonaxisymmetric terms are shown. As with the individual monthly values, it is seen that the degree variance for second degree is the dominant effect, in this case by an order of magnitude, due to the $C_2^1$ term (equator-to-pole gradient). It is obvious that terms of even degree have a trend different from those of odd degree, as was pointed out in discussing the zonal terms.

All terms of even degree are seen to exceed all of the terms of odd degree through degree 9. It is seen that the odd degree terms of degree greater than 3 are dominated by the nonaxisymmetric terms, while terms of even degree are dominated by the axisymmetric term up to degree 10. The non-axisymmetric terms are due only to terrain–ocean distribution.

### Table 5. Real and imaginary parts of $B_n^m$ (coefficients of sin(2π(t − $t_0$)/12) in annual cycle of Earth-emitted radiation).¹

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<td>-0.04</td>
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<td>0.04</td>
<td>0.44</td>
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¹ As for Table 3.

![Fig. 13. Degree variance for annual average distribution of emitted radiation.](image-url)
thereby demonstrating that the odd degree terms are due to terrain–ocean distribution.

The degree variances of the $A_n^m$ and $B_n^m$ are shown in Figs. 14 and 15, respectively. As before, for each degree the total degree variance is shown and the contribution of the axisymmetric and non-axisymmetric terms. Again, two trends may be discerned. The odd degree terms are dominated by the axisymmetric contribution and follow one trend. The even degree terms are dominated by the non-axisymmetric contributions and follow another trend. Usually the trend for the odd order is above the trend for the even order, although the lines cross due to a dip at $n = 5$ for the $A_n^m$ and at $n = 3$ for the $B_n^m$. For the annual cycle of radiation emitted from a symmetric planet in a circular orbit with tilt of the equator relative to the ecliptic plane, the terms of even degree would identically vanish. Thus, the terms of even degree are due to terrain–ocean distribution and the eccentricity of the Earth’s orbit. The fact they are dominated by non-axisymmetric terms means that they are not due to any significant degree to orbit eccentricity, but are due solely to land–ocean distribution.

Terrain–ocean distribution can certainly contribute to the even ordered terms in the annual average distribution $b_n^m$ and to the odd ordered terms in the annual cycle distribution as specified by the $A_n^m$ and $B_n^m$. However, these contributions are not easily separated from the total.

An inspection of Table 3 shows that none of the non-axisymmetric terms exceed 3 W m$^{-2}$, but there are four terms with magnitudes between 2 and 3 W m$^{-2}$: $C_1^1$, $C_1^2$, $S_4^1$, and $S_3^3$. Use of Tables 4 and 5 to compute the amplitude of the annual cycle shows that two terms exceed 3 W m$^{-2}$—$C_2^2$ and $C_2^2$—and six terms have amplitudes between 2 and 3 W m$^{-2}$:

$$C_2^2, C_3^2, C_4^2, S_3^3, S_4^3, S_5^3.$$ 

These coefficients are shown in Fig. 16. All of the remaining coefficients have annual averages which are less than 2 W m$^{-2}$ and annual cycles with magnitudes which are less than two W m$^{-2}$. For convenience, the 12 coefficients listed above are denoted the major tesseral coefficients. The powers contained in the various groupings of terms are listed in Table 6. It is seen that of the total power contained by the tessera, the 12 major tessera contribute over a fourth of the power of the annual average distribution, which is due to the full set of tessera, and nearly half of the power of the annual cycle.

A map of the annual average distribution of emitted radiation as defined by $b_n^m$ is shown in Fig. 17. This result is very much like previous maps published by Raschke et al. (1973) and Vonder Haar and Suomi (1971). The distribution is seen to be basically zonal. On the equator, there is a band of regions of low emitted radiation, specifically over the Amazon Basin, the Congo Basin of Africa, and Indonesia. These regions have high level cloudiness caused by intense convective activity; the high, cold cloud tops result in the low emitted radiation. There are bands of regions of high emitted radiation at approximately 15° north and 15° south, corresponding in the Northern Hemisphere to the Sahara and Arabian Deserts and in the Southern Hemisphere to Africa’s Kalahari Desert, Australia’s deserts, and a vast region of the Pacific Ocean west of Peru. These regions are characterized by subsidence of dry air with very little cloud, except for some low stratus clouds over the Pacific region. This permits radiation from the warm surface to escape directly, resulting in high emitted radiation. In the north polar area, the low emitted radiation region is over the icecap of Greenland. The global low is over Antarctica.

Maps describing the annual cycle will now be considered. A map defined by the set of $A_n^m$ is shown in Fig. 18. It is recalled that this cosine component, with its origin at mid-June, is very nearly in phase with the solar declination. The map shows that from 55°S to 45°N there are vast areas of ocean for which the
Fig. 16. Histories for 12 major tesseral terms for July 1975–June 1976.
Table 6. Power of space-time distribution (W^2 m^{-4}).

<table>
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<th>Annual average</th>
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<tr>
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<td>Total tesserals</td>
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<td>Total annual average</td>
<td>902</td>
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<table>
<thead>
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<td>Remaining tesserals</td>
<td>35</td>
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<tr>
<td>Total tesserals</td>
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<td>Remaining time variation</td>
<td>57</td>
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<tr>
<td>Total Power</td>
<td>1160</td>
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</table>

Twelve major tesserals:

C1, C2, C3, S1, S2, C4, S4, C5, S5, C6, S6,

magnitude is less than 10 W m^{-2}. However, over the continents there are large variations in this component of emitted radiation, ranging from 30 to 40 W m^{-2}. There is a band of high magnitude regions of 15°S, associated with the Amazon Basin, the Kalahari Desert, and Australia. There is also a band of low (negative high) magnitude regions at 15°N, in each case to the northwest of a high region. These low regions have large magnitudes, but are opposite in phase to the solar declination. There is also a band of low (negative high) regions at 40°S, in each case south of a high magnitude region.

A map defined by the set of B^n is shown in Fig. 19. This is the sine component, and lags the solar declination by approximately 90 days. Again, it is seen that from 55°S to 30°N for most of the ocean, the magnitude is less than 10 W m^{-2}. The major highs and lows are associated with land. The strongest features are a band of low (out of phase high) magnitude regions at 10°N and a band of high (in phase) magnitude regions at 10°S. There is also a strong high over southwestern United States and one over Iran.

The A^n and B^n may be compared with the results of Heddinghaus and Krueger (1981) and Stephens et al. (1981). Significantly more power is contained in the A^n set than in the B^n set, thus the A^n map (Fig. 18) may be expected to compare with the first eigenvector of Heddinghaus and Krueger. Comparison shows that this is the case. Although their eigenvector maps are normalized, the features of the first eigenvector agree quite well with Fig. 18. Stephens et al. (1981) have computed the amplitude of the annual cycle based on their composite data set. The amplitudes resulting from Figs. 18 and 19 agree fairly well with their results except over equatorial Africa, where Figs. 18 and 19 show a strong annual variation. Additional work is needed to explain this difference.

9. Longitudinal variation of emitted radiation

Although most of the spatial variability of the Earth’s emitted radiation is in the meridional direction, there are, nevertheless, important variations in the zonal direction. These zonal variations are found to be strongest in the tropics. This will be attributed to two major reasons. First, a latitudinal circle in the tropics is much longer than a latitudinal circle in the polar regions, providing a much longer distance over which parameters can vary. Second, the zonal winds are much greater at mid and high latitudes than in the tropics, and the resulting circulation will tend to zonally average all processes, including emitted radiation.

The longitudinal distribution of Earth-emitted radiation along circles of constant latitude is shown for July 1975 in Figs. 20 and 21 for the Northern and Southern Hemispheres, respectively. It is seen that in the summer (northern) hemisphere, the longitudinal

Fig. 17. Map of annual average Earth-emitted radiant exitance.
variation is stronger than the latitudinal variation. This is largely due to the fact that the daily solar insolation at the summer pole exceeds that of any other point, minimizing the latitudinal variation in the summer hemisphere. In the winter (southern) hemisphere the latitudinal variation is much greater than the longitudinal variation. Corresponding plots for January 1976 are shown in Figs. 22 and 23, in which the seasons have reversed. Again, the strong latitudinal variation of emitted radiation in the winter hemisphere and the weak latitudinal variation in the summer hemisphere are noted. However, a comparison of Figs. 20–23 shows that the longitudinal structure in the Northern Hemisphere is more complex than in the Southern Hemisphere in July and is nearly as complex in January.

The longitudinal structure may be described in terms of its spectrum, or spectral power, which will be defined as follows:

\[ S(m, \Theta, t) = (2\pi)^{-1} \left| \int_{-\pi}^{\pi} M(\Theta, \Phi, t) e^{-im\Phi} d\Phi \right|^2. \]  (22)

By use of Eq. (1), it may be shown that

\[
\begin{align*}
S(m, \Theta, t) &= \left[ \sum_{n=0}^{N} b_n^m(t) N_n^m P_n^m \cos\Theta \right]^2, \quad m = 0 \\
&= \frac{1}{2\pi} \sum_{n=m}^{N} b_n^m(t) N_n^m P_n^m \cos\Theta \right]^2, \quad m > 0
\end{align*}
\]  (23)

Thus, the longitudinal spectrum, which is a function of latitude and time in addition to wavenumber \( m \), can be expressed in terms of the spherical harmonic coefficients.

The spectrum is shown in Fig. 24 as a function of wavenumber for each of several latitudes for July 1976. It is seen that for wavenumbers 1–10, the spec-

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**FIG. 18.** Map of cosine (time) component of annual cycle of Earth-emitted radiant exittance.

**FIG. 19.** Map of sine (time) component of annual cycle of Earth-emitted radiant exittance.
Fig. 20. Longitudinal distribution of Earth-emitted radiant exitance over Northern Hemisphere during July 1975.

Fig. 21. Longitudinal distribution of Earth-emitted radiant exitance over Southern Hemisphere during July 1975.

Fig. 22. Longitudinal distribution of Earth-emitted radiant exitance over Northern Hemisphere during January 1976.
trum varies over three orders of magnitude. One may describe an approximate upper bound for the spectrum, as indicated by the dashed line, which decreases by one and a half orders of magnitude over this range. Because of the low spectral power for wavenumbers \( \geq 5 \), the remainder of this discussion will consider only wavenumbers 1–4.

The latitudinal distribution of spectral power for wavenumbers 1–4 is shown in Figs. 25 and 26 for July 1975 and January 1976, respectively. The major feature is a strong wave 1 and 2 peak in the region of 20\(^\circ\)–50\(^\circ\)N. This is due to high levels of emitted radiation in the Sahara, Arabian, and Kara Kum Deserts, and near the western U.S.; low levels near India and the Tibetan Plateau; and variations of exitance over the subtropical North Pacific Ocean. The wavenumber 1 and 2 features are absent in January, but wavenumber 3 is large at 20\(^\circ\)N. This wavenumber 3 is due to variations of exitance over the tropical North Pacific and Atlantic Oceans, and the Sahara and Arabian Deserts. From 20\(^\circ\)N to 20\(^\circ\)S, there is considerable structure, with wavenumbers 1–4 active. Comparison of the July and January spectra indicate that this activity follows the sun. In July, there is a strong wave 4 peak at 20\(^\circ\)N, and in January this peak has shifted to 20\(^\circ\)S. Except for this "subpolar" peak, wave 4 is weak. Although wavenumber 1 somewhat follows the subsolar latitude, it peaks at the equator for both July and January. This is mainly due to the persistent low emitted exitance in the region of Indonesia and the high values of emitted exitance over the eastern equatorial Pacific Ocean. The relationship between this subsolar activity and the ITCZ should be investigated. In July, there is a strong wave 1 peak at 75\(^\circ\)S; in January this peak is present, but much smaller. This peak is associated with Antarctica, the center of area of which is displaced from the South Pole along the 70\(^\circ\)E meridian; also, much of the terrain is at high elevation above 4 km. The effect of this is to induce the strong wave 1 when there is little or no solar insolation during June and July. Likewise during the northern winter, Greenland with

10. Conclusions

Monthly averaged resolution enhanced global distributions of the Earth's emitted radiation as measured by the Nimbus 6 Earth Radiation Budget (ERB) wide-field-of-view radiometers have been analyzed for one year of data from July 1975 to June 1976. These distributions were expressed in terms of spherical harmonic coefficients.

The spatial variations have been studied in the spectral domain in terms of degree variance, which is the measure of spectral power appropriate to a sphere. It was found that 65% of the spatial power is in first and second degree variances due to the pole-to-pole and equator-to-pole variations. The degree variance decreases approximately exponentially with degree, so that the total power from degree 9 through 12 accounts for less than 5% of the total spatial power.

The annual average distribution and annual cycle distribution were computed by a time series analysis.

![Figure 24](image-url)  
**Fig. 24.** Spectral power for longitudinal variation as a function of wavenumber for July 1975.
of the spherical harmonic coefficients. Because only one year of data are presently available, shorter period terms were not considered. It was found that the $C_3^0$ coefficient, which may be considered to be the pole-to-pole gradient, has a bias of 2.3 W m$^{-2}$, indicating the greater emitted radiation from the Northern Hemisphere on the average than from the Southern Hemisphere, and that it varies sinusoidally with an amplitude of 10.4 W m$^{-2}$. The $C_2^0$ coefficient, which may be considered to be the equator-to-pole gradient, maintains an average value of −25.6 W m$^{-2}$, with very little variation. The average annual distribution accounts for 78% of the space–time power, and the annual cycle accounts for 17%. The zonal terms, which describe the latitudinal variation of the zonal average, comprise 80 ± 8% of the spatial power. Twelve tesseral terms of low order and degree were found to account for approximately one-fourth of the remaining power in the annual average and one-third of the remaining power in the annual cycle. Maps of the annual cycle components are presented. These maps show vast areas of ocean from 55°S to 30°N for which the annual cycle component magnitude is less than 10 W m$^{-2}$. Over continents, however, these magnitudes are 30 to 40 W m$^{-2}$.

A simple time dependent model of the meridional variation of the Earth’s emitted radiation was developed, consisting of annual average distribution and an annual cycle consisting of a single latitude function which varies sinusoidally with time, lagging the solar declination by 34 days.

Longitudinal variations of emitted radiation were found to be greater and more complex in the Northern Hemisphere than in the Southern Hemisphere. Surprisingly, during summer the variation of emitted radiation with longitude is greater than with latitude in the Northern Hemisphere. The power of longitudinal spectra decreases approximately exponentially.
with wavenumber. During July there are very strong waves 1 and 2 between 20 and 50°N, and strong waves 1, 2 and 3 near the equator. These are due to deserts and large high cloud regions. Also, there is a strong wave 1 at 75°S due to the Antarctic Plateau. In January, there are strong waves 1-4 from the equator to 20°S, and there is a strong wave 1 at 80°N, the latter being due to Greenland.

Data from the Nimbus 6 and 7 ERB instruments since June 1976 should be analyzed in a similar manner to establish an annual cycle average over a longer period of time, higher frequency harmonics and interannual variations.

Acknowledgments. The authors wish to express their appreciation to an anonymous reviewer for his thoughtful review and constructive criticisms which resulted in considerable improvements to this paper.

APPENDIX

Notation

\[ A_n^m, B_n^m \]

- cosine and sine component respectively in annual cycle of \( b_n^m(t) \) [Eq. (21)] (W m\(^{-2}\))

\[ b_n^m \]

- complex spherical harmonic coefficient for degree \( n \) and order \( m \) for radiative exitance field [Eq. (1)] (W m\(^{-2}\))

\[ C_n^m, S_n^m \]

- real and imaginary parts of \( b_n^m \) (W m\(^{-2}\))

\[ E_M(\Theta) \]

- latitudinal function describing residual, or out of phase, portion of annual cycle which is not included in \( f_M(\Theta) \) [Eq. (17)]

\[ c, \theta \]

- time- and latitude-dependent residual portion of annual cycle which is not included in \( f_M(\Theta) \)

\[ f_c, f_s \]

- cosine and sine components respectively in annual cycle of zonally averaged \( M \) [Eq. (7)]

\[ f_M \]

- latitude function which contains maximum power of the annual cycle at a single phase angle

\[ f_0 \]

- annual average of difference of zonally averaged \( M \) from global mean

\[ g(\alpha) \]

- angular response function of wide field-of-view radiometer

\[ M \]

- emitted radiance at the “top of the atmosphere” (W m\(^{-2}\))

\[ N_n^m \]

- normalizing factor for spherical harmonics of degree \( n \) and order \( m \) (W\(^2\) m\(^{-4}\))

\[ P_n^m \]

- Legendre polynomial of degree \( n \) and order \( m \)

\[ P_n \]

- associated Legendre polynomial of degree \( n \) and order \( m \)

\[ R \]

- limb darkening function

\[ S \]

- spectral power (W\(^2\) m\(^{-4}\))

\[ t \]

- time (months)

\[ W \]

- measurement (W m\(^{-2}\))

\[ Y_n^m \]

- complex spherical harmonic of degree \( n \) and order \( m \)

\[ \alpha \]

- angle at satellite from nadir to surface element at the “top of the atmosphere”

\[ \alpha_n, \beta_n, \gamma_n, \delta_n \]

- coefficients in expansion of zonally averaged annual cycle functions \( f_c, f_s, f_m, E_m \) respectively (W m\(^{-2}\))

\[ \eta_n \]

- phase angle of \( k \)th Fourier time component of \( C_n^0 \)

\[ \Phi, \phi, \omega \]

- longitude, phase angle, solid angle at spacecraft

\[ (\_ \_ \_ ) \]

- annual mean

\[ (\_ \_ \_ ) \]

- zonal mean

\[ \text{Re} (\_ \_ \_ ) \]

- real part

\[ \text{Im} (\_ \_ \_ ) \]

- imaginary part.

REFERENCES


