

Radiative Damping and Amplitude of Long-Wavelength Modes in the Stratosphere

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ABSTRACT

The dependence of radiative damping rates in the stratosphere is investigated as a function of the distribution of active radiative gases and the wavelength of the temperature perturbation.

Damping rates are found to depend considerably on the scale of the temperature perturbation. The same radiative model used to calculate damping rates is employed to study the effects of more accurate radiative calculations on the amplitude of long-wavelength Green modes in the stratosphere. A quasi-geostrophic β -plane model is used for this purpose and the results show a quite large reduction of the amplitudes of the modes up to wavenumber 3. These results are qualitatively discussed by taking into account a nonlinear damping mechanism and the dependence of the Newtonian cooling coefficient on the scale of the temperature perturbation.

1. Introduction

In the stratosphere a strong coupling exists between photochemistry and radiation. Radiative heating (or cooling) plays a major role in the general circulation and in transport processes of minor species in this region. In order to simplify the treatment of the radiative terms, parameterization schemes have been used in most of the proposed transport mechanism. Photochemical-radiative relaxation rates were studied by Lindzen and Goody (1965) and Blake and Lindzen (1973). More recently Strobel (1977, 1978) and Hartmann (1978) have shown how large perturbations in the ozone mixing ratio could affect considerably the relaxation rates. Similarly the infrared radiative terms have been parameterized following the Newtonian cooling approximation. In this respect the most widely used coefficients are those of Dickinson (1973). Previous studies by Dickinson (1969) had shown how the introduction of parameterized radiative effects could have a major role in the propagation of planetary waves in the stratosphere. The Newtonian cooling approximation also has considerable importance in scale analysis (Geirasch *et al.*, 1970) and atmospheric models (Stone, 1972; Cunnold *et al.*, 1975). Harshvardan and Cess (1976) used the Newtonian cooling approximation to assess the climatic effects of stratospheric aerosols.

Considerable insight in the ozone transport processes mechanism in the stratosphere has been provided by the work of Geisler and Garcia (1977) and Hartmann and Garcia (1979). In the same direction but in the framework of Lagrangian formulation the

eddy diffusion concept has been given a physical basis (Holton, 1981; Strobel, 1981). In all these schemes the Newtonian cooling approximation is also adopted.

However since the evaluation of the Newtonian cooling coefficient in the stratosphere by Dickinson (1973) not too much attention has been paid to the dependence of the damping rates on the scale of temperature perturbation and on the distribution of active gases. Pioneering work in this area is due to Spiegel (1957) and Sasamori and London (1966). They showed how the damping rates changed considerably taking into account the scale of temperature perturbation. Spiegel introduced an analytic method to treat the problem in an isothermal atmosphere and did not take into account the distribution of active gases. A realistic distribution of gases was used by Sasamori and London who calculated damping due to H₂O, O₃, and CO₂ below 30 km. A major step in including the scale of atmospheric absorber was made by Prinn (1977). He also considered an isothermal atmosphere but showed that a critical role in determining the damping rate is the ratio between the scale of the perturbation to the scale of the atmospheric absorber. Also Prinn's analysis showed that the nature of the ground of the underlying surface is important. The entity of the ground response to the atmospheric temperature perturbation would depend on the conduction properties of the surface.

An indirect effect of the dependence of the Newtonian cooling on the temperature has been explored by Ramanathan and Groose (1977, 1978). They compared the results of two spectral 3-dimensional quasi-geostrophic circulation models of the stratosphere.

One of these models used a detailed radiative transfer code as compared to the other for which a Newtonian cooling approximation was used. The authors claim that a detailed treatment of the radiative effects could strongly affect the propagation of planetary waves and the latitudinal gradients of temperature.

An important contribution to the development of a simple method to account for these scale dependent rates has been given by Fels (1982). He has used a detailed radiative transfer code to show the dependence of the damping rates on the wavelength of a simple sinusoidal temperature perturbation. These results are used as a basis to develop an analytical parameterization which takes into account the perturbation scale.

At this point a number of studies seems to point out the importance of the perturbation scale for the radiative damping rates. The question remains whether this dependence could affect wave propagation in the stratosphere especially the long wave modes studied by Geisler and Garcia.

In this paper, in order to address this problem, independent calculations are presented on the dependence of the Newtonian cooling coefficients on the distribution of active gases and on the wavelength of the temperature perturbation. The radiative code takes into account the contribution of O₃, CO₂ and H₂O while some of the temperature profiles to be perturbed are calculated at radiative equilibrium.

As a practical approach to study the effect of the Newtonian cooling parameterization on the propagation of long wavelength modes we have run the quasi-geostrophic model of Geisler and Garcia (1977) using radiative damping rates calculated for the appropriate temperature perturbation.

The results obtained are discussed in view of both a local nonlinearity of the damping rate and its dependence on the perturbation scale.

No attempt has been made to develop a new parameterization to the cooling which should include nonlinearities and the scale of the perturbation.

2. Radiative transfer

Radiative long-wave fluxes have been evaluated with the method described by Ramanathan (1976). Absorption of solar radiation by O₃ and H₂O is evaluated following Lacis and Hansen (1974).

In the stratosphere we have taken into account the contribution from the 15 μ band of CO₂ and 9.6 μ band of ozone. The same code used for the stratosphere adopts the radiative transfer formulation for water vapor given by Sasamori (1968).

The model considers 51 levels with the ratio $p_i/p_{i+1} = 0.816$ where p_{i+1} is the level below the i th. This criterion would give a height resolution of roughly 1.5 km. For this reason we assume that perturbation cases

with wavelength less than 15 km do not give reliable results for the stratosphere. Also in the troposphere the water vapor scale height is less than 2 km requiring a very high resolution for the vertical grid.

For this reason we used a code developed from the method outlined by Cess (1974) for the troposphere.

There are slightly significant modifications in our numerical scheme with respect to those of Ramanathan and Cess.

These are extensively reported in the paper by Alimandi and Visconti (1979). A short summary of the modifications, is also given here.

a. Carbon dioxide

The net flux at height z (positive downward) is given by

$$F(z) = -B(z_{\text{top}})A(z_{\text{top}} - z) - \int_{z_{\text{top}}}^0 A(|z - z'|)dB, \quad (1)$$

where B is the Planck function at 667 cm⁻¹ (15 μ CO₂ band), $A(|z - z'|)$ is the absorptance between heights z and z' . For CO₂ we take into account seven bands of the fundamental isotope.

A major simplification is introduced in order to cut computational time in Eq. (1). For each level i the integral is numerically reduced to a sum of the form:

$$F_i = \sum_j A_{ij}(B_j - B_{j+1}), \quad (2)$$

where A_{ij} is the absorptance between level i and j . This absorptance can be scaled appropriately either for the Doppler or Lorentz formulation. The main assumption is that the pathlength does not depend strongly on temperature. As a result it can be easily shown that A_{ij} will scale approximately as the temperature in case of Doppler broadening while it will scale as the square root of temperature for Lorentz broadening, that is:

$$\left. \begin{aligned} A_{ij} &= A_{ij}^{\text{ref}} \left(\frac{T_i}{T_{i,\text{ref}}} \right), & \text{Doppler} \\ A_{ij} &= A_{ij}^{\text{ref}} \left(\frac{T_i}{T_{i,\text{ref}}} \right)^{1/2}, & \text{Lorentz} \end{aligned} \right\} \quad (3)$$

With this argument computational time is considerably reduced. It is sufficient to store $i \times j$ values of A , scale them according to (3) for each temperature profile and perform the summation (2). This procedure has been tested by using a seasonal and latitudinal range of temperature profiles and found to give a maximum 10% deviation with respect the exact computation.

Another minor difference with the Ramanathan code is the criteria for matching the Doppler and Lorentz region. Following Goody and Belton (1967) in the integral (1) we adopt the larger value for the

absorptance between two levels. The original matching condition was to impose the same derivative.

b. Ozone

We found that the strong line approximation is never applicable to ozone so that for Lorentz lines the full expression for the absorptance has to be used. Also in this case the matching between the Doppler and Lorentz region was obtained as described for CO₂.

3. Damping rates

At each altitude z , the temperature perturbation $T'(z)$ is given by

$$T'(z) = T_p \cos[2\pi(z - z_0)\lambda^{-1}] \quad (4)$$

so that the new temperature profile results as

$$T(z) = T_0(z) + T'(z). \quad (5)$$

In Eqs. (4) and (5), T_p is the amplitude of the perturbation (typically 1°K), λ the wavelength of the perturbation and z_0 the altitude at which the damping rate has to be evaluated.

If $Q_0(z_0)$ is the cooling rate at altitude z_0 for the unperturbed temperature profile $T_0(z)$ and $Q(z_0)$ is the cooling rate for the temperature profile $T(z)$, the damping rate is defined as

$$\alpha(z_0) = [Q_0(z_0) - Q(z_0)]T_p^{-1}. \quad (6)$$

In the stratosphere we allow for different ozone distributions. These in turn determine the temperature structure and eventually cooling and damping rates. We proceed in choosing a number of ozone distributions which are reported in Table 1.

We define an analytical distribution for ozone following Green (1964). (This is not related at all to the Green modes we refer to later in the paper.)

This analytical distribution gives the amount of ozone (cm) above altitude z :

$$u(z) = \frac{a[1 + \exp(-b/c)]}{1 + \exp[(z - b)/c]}, \quad (7)$$

where a , b and c are parameters defining the total amount of ozone above the ground, the altitude of maximum concentration and the width of the layer.

TABLE 1. Ozone distributions used to evaluated damping rates.

O ₃ distribution	a (cm)	b (km)	c (km)	Temperature profile
Green	0.25	25	4	Radiation equilibrium
Green	0.4	25	4	Radiation equilibrium
Green	0.5	18	4	Radiation equilibrium
Observed		40°N January		Observed
Observed		0°N January		Observed
Observed		80°N January		Observed

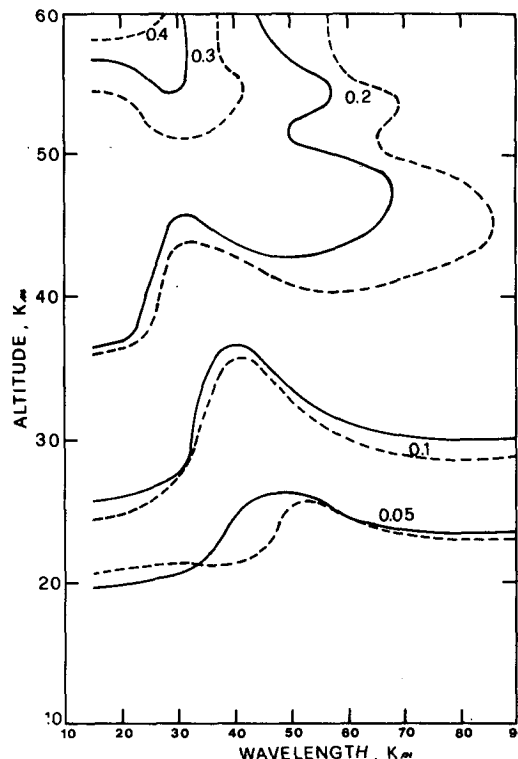


FIG. 1. Damping rates (days⁻¹) as a function of altitude and wavelength of the temperature perturbation. Solid line refers to a Green distribution with $a = 0.25$ cm, $b = 25$ km and $c = 4$ km and a radiative equilibrium profile for the temperature. Dashed line refers to observed temperature and ozone distribution at 0° latitude in January.

As indicated in Table 1 we also adopted different temperature profiles. For the observed ozone distributions the corresponding observed temperature profile was adopted. For each of the Green distributions we have calculated a radiative equilibrium profile. This was obtained by applying a time marching technique to the equation

$$\frac{\partial T}{\partial t} = \frac{1}{\rho c_p} \frac{\partial F}{\partial z}, \quad (8)$$

where ρ is the atmospheric density, c_p is the specific heat and F is the net flux. We follow closely the method outlined by Manabe and Wetherald (1967) for the convective adjustment in the troposphere. No clouds were present in our scheme. The CO₂ mixing ratio was taken at 320 ppm and the water vapor mixing ratio 3 ppm in the stratosphere. We assume that Green distributions approximate typical observed distributions. Consequently the 0°N January distribution is similar to the first distribution in Table 1; the 40°N is approximated by the second Green distribution and the 80°N is approximated by the Green distribution with $b = 18$ km and $a = 0.5$ cm.

Results for the damping rates as a function of wavelength and height are given in Figs. 1, 2 and 3. As

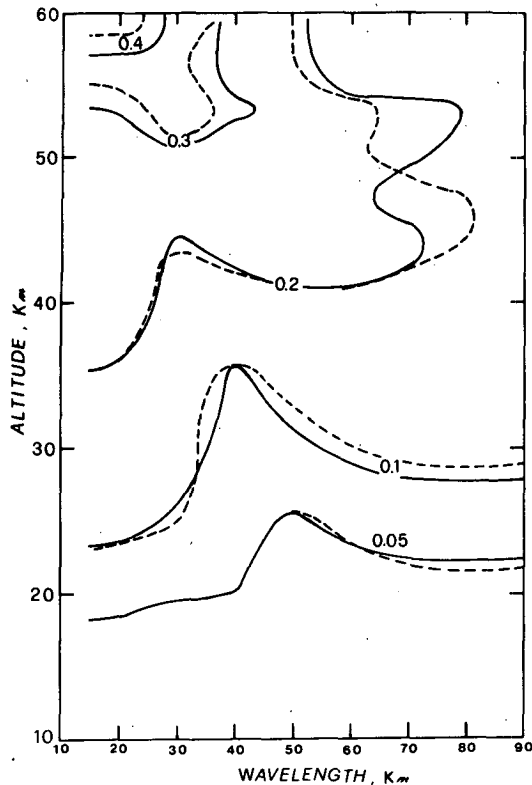


FIG. 2. As in Fig. 1 for a Green distribution with $a = 0.4$ cm, $b = 25$ km, $c = 4$ km and temperature and ozone distribution observed at 40°N , January.

already noted due to the poor resolution results for wavelength less than 15 km cannot be trusted entirely.

As expected, the higher the stratospheric temperature the higher is the damping rate. For the equatorial case typical rates of 0.2 day^{-1} are obtained at 50 km for perturbation wavelength greater than 50 km. This result reproduces values given by Dickinson (1973). For the lower stratospheric temperature the damping rate decreases. Notice also the similarities between the damping rates for the observed and calculated profiles of ozone and temperature.

It is interesting to note that the behavior of the damping rates as a function of wavelength is quite similar in the stratosphere to the one reported by Fels (1982). This means relaxation times at a given altitude decrease with increasing wavelength. With our resolution the maximum damping rates are observed above 60 km to be 0.4 day^{-1} . Our results show considerably more structure. It is to be noted, however, that they are not directly comparable with Fels because he reports details mainly between 30 and 4 km wavelength, so that the overlap with the present results is limited.

We also ran some tropospheric cases (not reported in this paper) which reproduce the same behavior found by Prinn (1977).

A similar trend in damping rate is also reported by

Sasamori and London (1966) for ozone. This dependence on the wavelength in the stratosphere could be attributed to the different structure in temperature and distribution of active gases, with respect to the troposphere.

In the troposphere both temperature and water vapor mixing ratio decreases with altitude. In the stratosphere the temperature increases up to 50 km and then decreases with a much lower gradient than in the troposphere. Also, although most of the damping effect could be attributed to carbon dioxide, the ozone distribution is critical in determining the temperature structure and thus the behavior of the damping rate with altitude.

4. Effects of damping rates on Green Modes propagation

Geisler and Garcia (1977) studied the propagation of long wavelength slowly growing modes. They found that these modes could give considerable amplitude in the stratosphere. The same physical process could be responsible for the ozone transport in the stratosphere as was shown in subsequent papers (Garcia and Hartmann, 1979; Hartmann and Garcia, 1980).

Mode growth was studied by solving the quasi-geostrophic potential vorticity equation in the β -plane approximation:

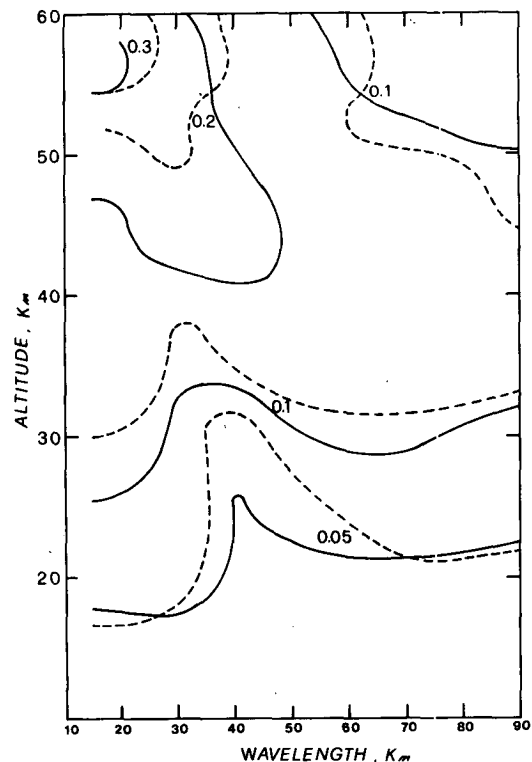


FIG. 3. As in Fig. 1 for a Green distribution with $a = 0.5$ cm, $b = 18$ km and $c = 4$ km. Dashed line refers to the observed temperature and ozone profile at 80°N January.

$$\left(\frac{\partial}{\partial t} + \bar{u} \frac{\partial}{\partial x}\right) \left(\nabla^2 \Psi + \frac{1}{\rho_s} \frac{\partial}{\partial z} \left(\rho_s \frac{f_0^2}{N^2} \frac{\partial \Psi}{\partial z}\right)\right) + \beta_e \frac{\partial \Psi}{\partial z} = -\frac{1}{\rho_s} \frac{\partial}{\partial z} \left(\rho_s \frac{f_0^2}{N^2} \alpha \frac{\partial \Psi}{\partial z}\right). \quad (9)$$

In Eq. (9) notations used are the same as in Geisler and Garcia (1977). In the same equation the Newtonian cooling approximation is represented by the third term with α being the appropriate damping rate. In the framework of quasi-geostrophic theory the perturbation temperature T' is related to the perturbation streamfunction ψ by

$$T' = \frac{f_0 H}{R} \frac{\partial \Psi}{\partial z} \quad (10)$$

with f_0 the Coriolis parameter, H the atmosphere scale height and R the gas constant. Using the same model provided by Garcia we have calculated the Newtonian cooling coefficient in two different ways to assess the dependence on a temperature perturbation obtained from a perturbation streamfunction. In the first case starting from the non-isothermal atmosphere as given in Geisler and Garcia (1977) we have evaluated $\alpha(z)$ as in Dickinson (1973) by perturbing the basic temperature profile by a small

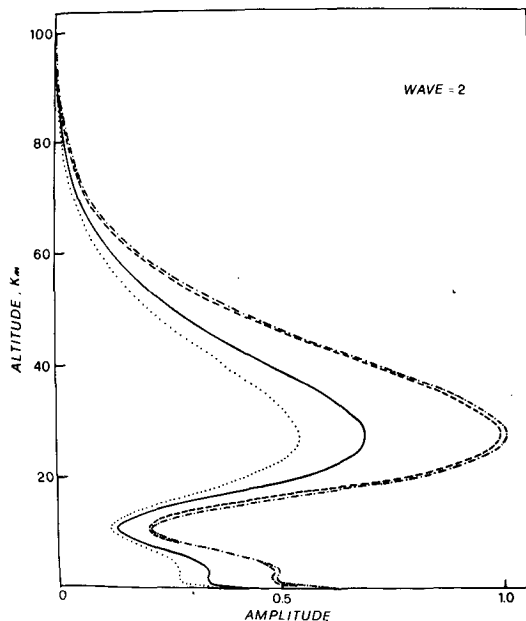


FIG. 4a. Normalized mode amplitude as a function of altitude for wavenumber 2. Dashed: results obtained with the constant Newtonian cooling coefficient as given in Dickinson (1973). Dashed-dotted: results with constant Newtonian cooling coefficient calculated for the basic state temperature. Solid: results obtained with damping rates calculated according to the perturbation temperature obtained from the derivative of the streamfunction (see text). Dotted: results obtained with a damping rate relative to a perturbation wavelength of 30 km.

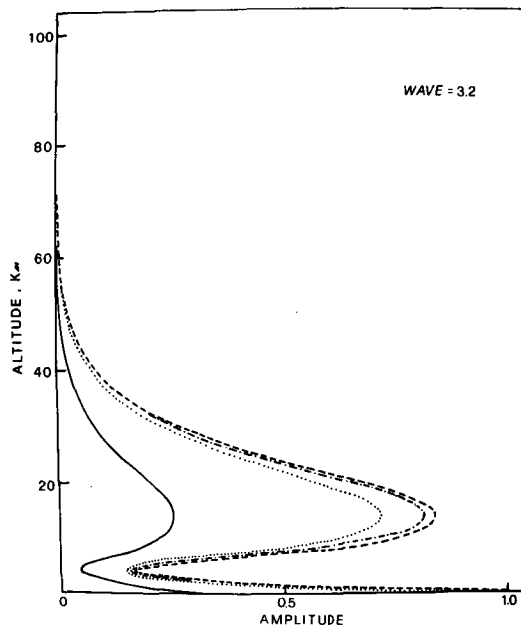


FIG. 4b. As in Fig. 4a for wavenumber 3.2.

amount only at the altitude z , where α is being calculated. In the second case at each integration time step for Eq. (9) we have evaluated the perturbation temperature at altitude z as proportional to $\partial\psi/\partial z$ according to the relation (10). In order to obtain reasonable values for T' the lower boundary condition was changed in such a way to have typical geopotential amplitude for the stratosphere at the end of the run. As we will see later this is not a critical point because the Newtonian cooling coefficient shows a very small nonlinearity, locally, so that its computation is quite independent of the amplitude of T' .

Results of the computations performed by using the two approximations are shown in Fig. 4, where results are compared with a standard case obtained by using α as given in Dickinson (1973). The amplitudes reported in Fig. 4 refer to the same time elapsed since switching on of the forcing, typically 50 days. The reduction of amplitude, very much evident when T' is obtained from Eq. (10), is actually due to a decrease in the imaginary part of the phase speed. This in turn implies a slower growing rate for the mode. It is to note that after 50 days the amplitude has a constant growth rate so that our conclusions are not affected by the choice of the time interval elapsed as long as the comparison between different cases is made at the same time.

The decrease in amplitude depends on the wavenumber of the mode with a greatest effect for wavenumber around 3.2. This effect could be explained on the basis of a more realistic approximation to the cooling calculations. The usual Newtonian approximation assumes that cooling at altitude z is propor-

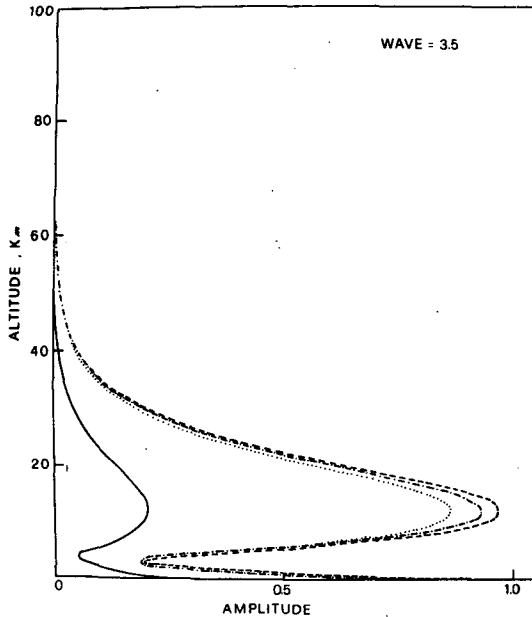


FIG. 4c. As in Fig. 4a for wavenumber 3.5.

tional to the local temperature perturbation T' . This assumption could be unrealistic because it does not consider higher order terms in T' which could not be negligible for large perturbations. Also results of the previous section show how cooling at some altitude z , depends on the scale and distribution of the perturbation simply because the divergence of the radiative flux takes into account coupling between different levels (Fels, 1982).

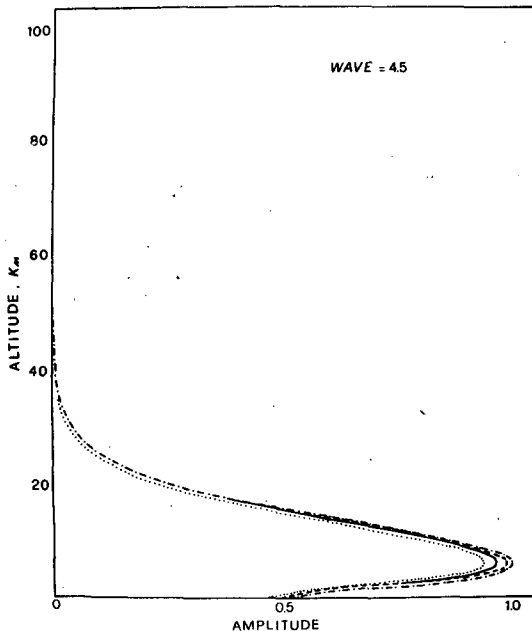


FIG. 4d. As in Fig. 4a for wavenumber 4.5.

The damping rates widely used and calculated by Dickinson refer to a local perturbation in temperature. To understand qualitatively the effect of local, nonlinear damping, we have used a simple model similar to the one reported by Dickinson (1969) for the study of Newtonian cooling on vertical wave propagation. In this case the assumptions are a zonal wind speed and a damping rate independent of altitude. For stationary waves Eq. (9) could be reduced to the simple form

$$\frac{\partial \Psi}{\partial z} + n^2 \Psi = 0, \tag{11}$$

where $\Psi(x, z, t)$ is the vertical amplitude of the perturbation stream function which is of the form

$$\Psi(x, z, t) = \psi(z) \exp[(ik(x - ct) + z/2H)]. \tag{12}$$

In Eq. (12) k is the wavenumber, c the complex phase speed. The equivalent index of refraction n in Eq. (11) is given by

$$n^2 = \left(\frac{N^2}{f_0}\right) ik \frac{(\beta - \bar{u}k)}{\alpha + iku} - \frac{1}{4H^2}. \tag{13}$$

Solution to Eq. (11) is then of the form $\psi(z) = Ae^{(i\gamma - \Gamma)z}$ with γ and Γ being related to n by $2(\gamma^2 + \Gamma^2) = |n|^2$. Within our assumption the calculation of a damping rate as a function of the perturbation temperature is equivalent to introducing a nonlinear damping term of the form $\delta \partial \Psi / \partial z | \partial \Psi / \partial z |$, so that the dissipative term on the rhs of Eq. (9) is given by

$$\text{rhs} = -\frac{1}{\rho_s} \frac{\partial}{\partial z} \left[\rho_s \left(\frac{f_0^2}{N}\right) \left(\alpha \frac{\partial \Psi}{\partial z} + \delta \frac{\partial \Psi}{\partial z} \left| \frac{\partial \Psi}{\partial z} \right| \right) \right]. \tag{14}$$

A few arguments can be used to justify this form of nonlinear damping. First of all it would simply correspond to include a quadratic term in the expansion of the perturbation heating rate. The absolute value of $\partial \Psi / \partial z$ ensures that damping is working properly, decreasing the perturbation when T' is positive or increasing it when T' is negative. The coefficient δ in Eq. (14) has dimension of an inverse length and can be easily obtained from the cooling as we will see later. If a solution for $\psi(z)$ is of the form given before as a first approximation we can neglect all terms except

$$\frac{\delta}{H} \frac{\partial \psi}{\partial z} \left| \frac{\partial \psi}{\partial z} \right|.$$

This is an oversimplification for the sake of discussing qualitatively the effect of nonlinear damping and would probably result in an underestimation of the effects. In this case, however, Eq. (9) reduces to

$$\frac{\partial^2 \Psi}{\partial z^2} + \frac{\delta}{H(\alpha + iku)} \frac{\partial \Psi}{\partial z} \left| \frac{\partial \Psi}{\partial z} \right| + n^2 \Psi = 0. \tag{15}$$

Solutions to similar equations are discussed in a number of books on nonlinear oscillations (Minorsky, 1974; Nayfeh and Mook 1979). It is shown that if A_0 is the amplitude of the solution to the linear equation (11), solution to the equation including the nonlinear damping term results in a decrease in amplitude by the amount

$$A = \frac{A_0}{1 + 4\epsilon n A_0 z / 3\pi}, \quad (16)$$

with A the amplitude of damped solution and $\epsilon = \delta / [H(\alpha + iku)]$. In Eq. (15) δ is obtained by perturbing the basic temperature state by an amount T' only at the altitude z . The amplitude for T' is changed and at the same altitude z the cooling rate perturbation is fitted with a second degree polynomial in T' , that is,

$$Q(T + T') - Q(T) = \alpha_0 T' + \alpha_1 T'^2. \quad (17)$$

The coefficients α_0 and α_1 can then be obtained as a function of altitude. Typical values for α_1 are $1.4 \times 10^{-3} \text{ K}^{-1} \text{ day}^{-1}$. With this value for α_1 , δ can be evaluated to be of the order of $4 \times 10^{-13} \text{ cm}^{-1}$. Assuming typical geopotential amplitude at 10^3 m (Hartmann and Garcia, 1979) the amplitude at 30 km is reduced by a factor 1.3 for wavenumber 3, constant zonal wind of 10 m s^{-1} and $\alpha = 0.05 \text{ day}^{-1}$. This rather qualitative argument would indicate that introduction of local nonlinear damping would influence marginally the amplitude of the modes.

To show the effect of the scale of the disturbance on the damping rates we have studied the growth of the modes with the cooling coefficient obtained for a sinusoidal wave distribution with 30 km wavelength. Results are given in Fig. 4, and also, in this case, a considerable reduction is observed.

5. Conclusions

A detailed radiative transfer scheme has been used to calculate radiative damping rates in the stratosphere as a function of altitude, wavelength and amplitude of the temperature perturbation. These calculations have been performed for a number of ozone and temperature distributions. They show that the well-known Newtonian cooling coefficient is a strong function of the wavelength of the temperature perturbation while the nonlinear behavior is of minor importance. For a given altitude, especially around the temperature maximum, the Newtonian cooling coefficient increases with decreasing wavelength. On the other hand the nonlinearity can be expressed with a second degree polynomial giving values of $1.4 \times 10^{-3} \text{ K}^{-1} \text{ day}^{-1}$ for the T'^2 coefficient.

The effects of this dependence of the damping rates on the propagation of planetary wave disturbances have been studied with a quasi-geostrophic model. Within this approximation the perturbation temper-

ature is related to the perturbation streamfunction so that the Newtonian cooling coefficients can be evaluated at each time step during the growth of the mode. It is found that the amplitudes of the modes are strongly reduced especially for small wavenumber (long waves). To explain this effect it is assumed that introduction of temperature dependent damping rates is equivalent to a nonlinear damping. A simple model for stationary waves and quadratic damping with a constant damping rate and zonal wind is used to discuss qualitatively the results obtained with the quasi-geostrophic theory. With a damping rate of 0.05 day^{-1} and $u = 10 \text{ m s}^{-1}$ for typical geopotential amplitude of 10^3 m this simple model shows a 30% reduction in amplitude. Growth of the same mode has been also studied with a Newtonian cooling coefficient corresponding to sinusoidal temperature distribution with 30 km wavelength and also in this case the amplitude reduction is considerable. In conclusion, the local nonlinearity of the damping rate seems to have a minor effect on wave amplitude compared with the dependence on the scale of temperature perturbation.

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