Stationary Rossby Wave Propagation through Easterly Layers

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ABSTRACT

The linearized shallow water equations on a sphere are solved numerically to examine the sensitivity of the steady response to midlatitude mountain forcing to the zonal mean basic state. The zonal mean basic state consists of meridionally varying zonal winds $\bar{u}(y)$ and meridional winds $\bar{v}(y)$. Cases are considered where $\bar{u}$ is westerly everywhere, outside a tropical region where it is easterly. A zonal wavenumber three mountain confined to the Northern Hemisphere midlatitudes, where $\bar{u} > 0$, provides the forcing.

When $\bar{v} \equiv 0$ the usual result of negligible Southern Hemisphere response to the mountain forcing is found. However, a modest mean meridional velocity [O(3 m s^{-1})] that is directed from north to south through the easterly layer leads to significant Southern Hemisphere response. An argument based on the local dispersion relation is offered to explain this effect. It is concluded that critical latitude effects on wave propagation are sensitive to the structure of the mean meridional circulation in the critical latitude region of the model. The result of the simplified model suggests that a more relevant model with a zonally symmetric basic state consisting of zonal winds and meridional circulation varying with height as well as latitude should be investigated.

1. Introduction

A standard approach to studying the stationary wave response of the atmosphere to orographic or imposed thermal forcing is to linearize the equations of motion about a longitudinally independent basic state consisting of geostrophically balanced zonal mean zonal winds $\bar{u}$, and heights, $\bar{\phi}$, while the mean meridional circulation of the basic state is taken to be zero. The linearized wave equations are derived by representing the dynamical variables as a zonal mean plus a derivation from the zonal mean (i.e., $u = \bar{u} + u'$, where overbar denotes zonal mean and prime denotes deviation from the zonal mean), and assuming that the deviations from the zonal mean (the "basic state") are infinitesimal. Unless further approximations are made, the atmospheric wave equations contain terms in which the mean meridional and vertical velocities $\bar{v}$ and $\bar{w}$ appear. Presumably, the solutions to the wave models that are relevant to the understanding of the atmosphere are those in which the basic state is sufficiently realistic. It is the purpose of this article to show that the sensitivity of the response to the inclusion of a nonzero basic state mean meridional circulation is not negligible under some circumstances. Thus it may be necessary to include a sufficiently realistic nonzero meridional circulation in the basic state to understand certain features of the general circulation.

Analyses of tropospheric observations (e.g. Oort and Rasmussen, 1971) have produced a qualitative picture of the zonally averaged atmospheric dynamical variables which is probably correct. Some of the features of the observations relevant to this article are summarized below. The zonal wind $\bar{u}$ reaches magnitudes O(30 m s^{-1}) in subtropical westerly jet streams, and is generally westerly except for somewhat weaker easterlies in the equatorial zone. The zonal winds and heights $\bar{u}$ and $\bar{\phi}$ are approximately in geostrophic balance. The largest $\bar{v}$ is to be found in the tropics and subtropics and is about 3 m s^{-1}, an order of magnitude smaller than the jet stream $\bar{u}$. Midlatitude and upper latitude $\bar{v}$'s are difficult to observe but appear to be of order 1 m s^{-1}. Thus it can be said with some confidence that by and large $|\bar{v}| \ll |\bar{u}|$. This feature of the observed atmosphere and convenience of model interpretation have apparently been the motivation for the choice of basic states in wave models with zonally symmetric basic states in which $\bar{v}$ and $\bar{w}$ are taken to be zero. Obviously, a basic state containing a strong enough mean meridional circulation of some structure could be expected to significantly effect the properties of a wave model.

In this paper we shall examine the sensitivity of stationary wave solutions to various basic state meridional circulations when the basic state zonal wind is westerly except for tropical easterlies. The influence of the mean meridional circulation on the propagation of stationary Rossby waves has been considered by Opsteegh (1982) in a two-level model forced by a local heat source. He concluded that the presence of a 1 m s^{-1} cross-equatorial Hadley circulation should allow interhemispheric propagation of long waves across the easterlies in the direction of the meridional winds. Nigam (1983) has also examined the influence of the mean meridional circulation on stationary waves in a linearized version of the GFDL general circulation model. He found that inclusion of the basic state meridional circulation for Northern Hemisphere winter reduced the Southern Hemisphere response to Northern Hemisphere topographic and diabatic forcing.

Most theoretical research on atmospheric stationary waves since Rossby's time has been concerned with quantifying the properties which are contained qualitatively in the $\bar{v} \equiv 0$ Rossby wave dispersion formula. The dispersion relation and its application to stationary waves were first discussed by Rossby and collaborators (1939). In the case where the analysis is carried out on an infinite barotropic β plane with a basic state $\bar{u}$ = constant, $\bar{v} \equiv 0$, the dispersion relation is (e.g. Holton, 1979, p. 167)

$$c = \bar{u} - \frac{\beta}{k^2 + l^2},$$

where the disturbance has been taken to be proportional to $\exp[i(kx + ly - kct)]$. Here, k is the (real) zonal wavenumber, l the meridional wavenumber, and c the zonal phase velocity. Taking $\beta > 0$, meridionally propagating solutions (l real) imply $\bar{u} - c > 0$, or that the phase speed of the Rossby wave is westward with respect to $\bar{u}$.

For stationary waves, $c = 0$, and

$$l^2 = \frac{\beta}{\bar{u}} - k^2$$

determines the meridional wavenumber. Then stationary waves are non-propagating ($l^2 < 0$) when $\bar{u} < 0$. Also, only long waves can propagate in regions of strong westerlies. These considerations also apply when $\bar{u} = \bar{u}(y)$ and $\beta - d^2\bar{u}/dy^2 > 0$. Near a critical latitude, where $\bar{u} = 0$, a crude approximation to the wave behavior is given by the local dispersion relation: $l^2 \rightarrow \infty$ as $\bar{u} \rightarrow 0^+$ and $l^2 \rightarrow -\infty$ as $u \rightarrow 0^-$. The meridional wavelength $(l/2\pi)^{-1}$ approaches zero as the critical latitude is approached from the side where $\bar{u} > 0$, and the wave is damped with an e-folding distance which approaches zero as the critical latitude is approached from the side where $\bar{u} < 0$. The crude description then predicts that the critical latitude acts as an impermeable barrier. Forcing confined to the easterlies produces no response in the westerlies, and forcing in the westeries produces no response in the easterlies. More careful treatments of the linear wave problem near the critical latitudes (Dickinson, 1968; Tung, 1979) show that the linear critical latitude is not an impermeable barrier, as the effective e-folding distance in the easterlies is finite near the critical latitude. Those studies indicate, however, that the description of the critical latitude as a barrier to meridional propagation is correct, and that the correction to the communication between the easterlies and westerlies is exponentially small.

The calculated response to stationary forcing in the westerlies depends strongly on the details of what happens near the $\bar{u} = 0$ critical latitude. The linear theory with $\beta - d^2\bar{u}/dy^2 > 0$ everywhere gives total critical latitude absorption of a wave incident from the westerlies (Dickinson, 1968; Nigam and Held, 1983) or partial absorption there (Tung, 1979). However, the linear inviscid theory breaks down near the critical latitude as singularities in the vorticity occur, unless $\beta - d^2\bar{u}/dy^2 = 0$ where $\bar{u} = 0$. This breakdown does not occur when the effect of nonlinearity of the waves is included (e.g. Benney and Bergeron, 1969, for gravity waves), or the waves are allowed to interact with the mean flow in an initial value approach (Geisler and Dickinson, 1974; Nigam and Held, 1983). In the quasilinear Rossby wave case $\beta - d^2\bar{u}/dy^2 \rightarrow 0$ at the critical latitude. In the nonlinear wave or quasi-linear Rossby wave calculation, the steady state inviscid critical latitude acts as a perfectly reflecting barrier. A reflecting critical latitude allows for the possibility of a resonant response to stationary Rossby wave forcing, while an absorbing critical latitude does not. The phase distribution of the inviscid response is affected everywhere by the character of the critical latitude behavior.

The behavior near the critical latitude is also of importance in the understanding of the midlatitude response to stationary forcing in the zonal mean easterlies. There is evidence (Horel and Wallace, 1981) that interannual variability in the Northern Hemisphere winter midlatitude stationary wave pattern is related to interannual variability in tropical sea surface temperatures, and that the structures of the interannually varying stationary waves have some of the characteristics of northward propagating Rossby waves forced at low latitudes (Hoskins and Karoly, 1981). The tropical sea surface temperature variations are presumably translated into variations in the thermal forcing for the atmosphere via cumulus convection. The perturbations in the thermal forcing directly connected with the tropical sea surface temperature variations occur primarily at latitudes where the zonal mean zonal winds are easterly (Liebmann and Hartmann, 1982), and linear stationary wave theory suggests that the response to these perturbations should then be confined to the tropical easterlies. The theoretical problem is to understand the mechanisms, if any, by which significant interannual variability in the midlatitude stationary waves can be connected to interannual variability in forcing that occurs in the zonal mean easterlies.

One such mechanism was suggested by Hoskins *et al.* (1977) and has been examined by Simmons

(1982), Webster and Holton (1982), Branstator (1983), and Karoly (1983). The idea is that while the zonal mean winds in the tropics are easterlies, the time mean zonal winds are also longitudinally varying, and at some longitudes in the tropical belt, the zonal winds are westerly. In these westerly ducts, meridional propagation of stationary Rossby waves is allowed and tropical–extratropical interactions may occur. Zonal wavelengths for which this effect is likely to be important are presumably restricted to be of the order of (or smaller than) the width of the westerly duct. Longer zonal wavelengths "see" the longitudinally averaged easterly zonal winds and the critical latitude barrier. Modeling of this effect requires the introduction of forcing to maintain the now zonally varying basic state. The above studies of zonally varying basic states, however, chose to use models in which the zonal mean of the basic state meridional wind is zero, a constraint which is unnecessary when the restriction that the basic state be a solution to the unforced inviscid equations is relaxed. Additionally, it is not clear that the problem of the influence of zonally varying tropical forcing on midlatitudes can be considered to be distinct from the problem of how the zonally varying basic state in the tropics is maintained. The distribution of "basic state" easterlies and westerlies in the tropics and the extratropical stationary wave response may possibly be viewed as responses to the same tropical forcing.

Critical latitudes formally are those locations where the linearized wave equations are singular. For a basic state with $\bar{u} = \bar{u}(y)$, $\bar{v} = 0$, these singularities occur where the phase speed of the waves is zero relative to the basic state winds. When the basic state $\bar{v}$ is not zero, the governing equations are singular at those latitudes where $\bar{v} = 0$, as the most highly differentiated term contains $\bar{v}$ as a coefficient. Our use of the term "critical latitude" in the stationary wave case to refer to locations where $\bar{u} = 0$ is not formally correct when the basic state meridional circulation is nonzero. Additionally, the tropical easterly region in the zonal mean troposphere is the region of the most intense meridional circulation. These considerations motivate the present numerical calculations, which were designed primarily to examine the potential sensitivity of critical latitude effects to a basic state meridional circulation.

It will be shown here that including a "small" mean meridional circulation in the specification of the basic state can significantly alter the dispersion properties of quasi-barotropic stationary waves, to the extent that the waves can readily propagate across critical latitudes and through easterly layers. This propagation, however, is a one-way effect; meridional propagation occurs only in the direction of the mean meridional velocity if the basic state is such that there would be no propagation with the basic state meridional circulation neglected. As long as there is a meridional circulation at the critical latitude, the governing equations there are non-singular; the question of absorbing versus reflecting critical latitudes for stationary planetary waves may be of only academic interest, due to unrealistic assumptions concerning the basic state.

These results will be shown to occur in numerical calculations, using the linearized shallow water equations. A diagnosis of the balances in the governing equation indicates that the wave solutions are approximately nondivergent, so that the linearized barotropic vorticity equation, derived including a basic state $\bar{v}$, should produce the same qualitative behavior. It is shown that the dispersion relation resulting from the linearized barotropic vorticity equation applied locally, which is the result obtained using WKB analysis for slowly varying $\bar{u}(y)$ and $\bar{v}(y)$ to lowest order, yields the qualitative behavior found in the numerical calculations.

2. Model

In order to examine in a preliminary manner the potential effects of a basic state meridional circulation on stationary waves, we have chosen the framework of the linearized shallow water equations on a sphere. The basic state zonal winds are taken to be westerly, except for a belt of tropical easterlies. The stationary wave forcing is provided by the mean zonal winds blowing over midlatitude orography confined to the northern hemisphere. The basic state meridional circulation is implicitly forced by a specified longitudinally independent distribution of mass sources and sinks. The shallow water model, of course, cannot represent the vertical structure of the basic state or the waves. The source/sink driven mean meridional velocities are meant to be analogous to the atmospheric mean meridional circulation above the boundary layer, with return flow occurring in the boundary layer. While this representation is necessarily crude, it is justified for these preliminary calculations because of computational convenience and the fact that we are interested in qualitative rather than quantitative effects. Certainly, if the inclusion of an earthlike basic state meridional circulation in this model did not affect the stationary wave behavior significantly, further study of the problem with more realistic models would not be indicated.

The shallow water equations on a sphere (e.g., Gill, 1982) are:

$$\frac{Du}{Dt} - \left(2\Omega y + \frac{y}{\sqrt{1-y^2}}\frac{u}{a}\right)v = -\frac{g}{a\sqrt{1-y^2}}\frac{\partial h}{\partial \lambda} + F_\lambda, \quad (1)$$

$$\frac{Dv}{Dt} + \left(2\Omega y + \frac{y}{\sqrt{1-y^2}}\frac{u}{a}\right)u = -g\frac{\sqrt{1-y^2}}{a}\frac{\partial h}{\partial y} + F_y, \quad (2)$$

$$\frac{\partial h}{\partial t} + \frac{1}{a\sqrt{1-y^2}}\frac{\partial}{\partial \lambda}[(h-h_b)u] + \frac{1}{a}\frac{\partial}{\partial y}[(h-h_b)v\sqrt{1-y^2}] = M, \quad (3)$$

where

$$\frac{D}{Dt} = \frac{\partial}{\partial t} + \frac{u}{a\sqrt{1-y^2}}\frac{\partial}{\partial \lambda} + v\frac{\sqrt{1-y^2}}{a}\frac{\partial}{\partial y}.$$

Longitude λ is the zonal coordinate and $y = \sin\theta$, where θ is latitude, is the meridional coordinate. The eastward and northward components of the velocity are u and v, h is the height of the free surface, and h_b is the height of orography. The gravitational acceleration is g, a the radius of the sphere, and Ω the rotational frequency with the axis of rotation passing through $y = \pm 1$. The zonal and meridional momentum sources (e.g. frictional damping) are represented by F_λ and F_y, respectively, and M represents mass sources.

The equations governing small amplitude, zero zonal mean perturbations, u', v', and h', about a zonally independent basic state, $\bar{u}(y)$, $\bar{v}(y)$, and $\bar{h}(y)$ are straightforward to derive. A Raleigh friction representation of F_λ and F_y is assumed in the perturbation equations:

$$(F_\lambda, F_y) = (-cu', -cv'). \quad (4)$$

The equations governing the small amplitude steady ($\partial/\partial t \equiv 0$) perturbation forced by orography ($h_b = \bar{h}_b + h'_b$) and mass sources may then be written:

$$\frac{\bar{u}}{a\sqrt{1-y^2}}\frac{\partial u'}{\partial \lambda} + \frac{\bar{v}}{a}\frac{\partial}{\partial y}(u'\sqrt{1-y^2}) + \left[\frac{1}{a}\frac{\partial}{\partial y}(\bar{u}\sqrt{1-y^2}) - 2\Omega y\right]v' = -\frac{g}{a\sqrt{1-y^2}}\frac{\partial h'}{\partial \lambda} - cu', \quad (5)$$

$$\frac{\bar{u}}{a\sqrt{1-y^2}}\frac{\partial v'}{\partial \lambda} + \frac{\sqrt{1-y^2}}{a}\frac{\partial}{\partial y}(\bar{v}v') + 2\Omega y\left(1 + \frac{\bar{u}}{\Omega a\sqrt{1-y^2}}\right)u' = -\frac{g\sqrt{1-y^2}}{a}\frac{\partial h'}{\partial y} - cv', \quad (6)$$

$$\frac{(\bar{h}-\bar{h}_b)}{a\sqrt{1-y^2}}\frac{\partial u'}{\partial \lambda} + \frac{\bar{u}}{a\sqrt{1-y^2}}\frac{\partial h'}{\partial \lambda} + \frac{1}{a}\frac{\partial}{\partial y}[(\bar{h}-\bar{h}_b)\sqrt{1-y^2}v'] + \frac{1}{a}\frac{\partial}{\partial y}(\bar{v}\sqrt{1-y^2}h') = \frac{\bar{u}}{a\sqrt{1-y^2}}\frac{\partial h'_b}{\partial \lambda} + \frac{1}{a}\frac{\partial}{\partial y}(\bar{v}\sqrt{1-y^2}h'_b) + M'. \quad (7)$$

The basic state heights and zonal winds are assumed to be geostrophically balanced. The basic state zonal winds and global average height H_0 are specified, as are the basic state meridional velocities, and the zonal mean heights are calculated from

$$2\Omega y\left(1 + \frac{\bar{u}}{2\Omega a\sqrt{1-y^2}}\right)\bar{u} = \frac{-g}{a}\sqrt{1-y^2}\frac{\partial \bar{h}}{\partial y} \quad (8)$$

such that

$$\frac{1}{2}\int_{-1}^{1}\bar{h}dy = H_0 \quad (9)$$

We do not concern ourselves with the maintenance of the basic state in this article, other than remarking that those zonal mean momentum and mass sources which are necessary to produce the assumed steady zonally averaged basic state are implicit in the model.

Zonal mean mass sources and sinks, $\bar{M}$, are analogous to the zonal mean apparent diabatic heat sources in a stratified atmosphere (i.e. those heat sources that are balanced by the heat flux divergences of the mean meridional circulation). The mass source/heat source analogy has been locally exploited by Gill (1980) and Webster and Holton (1982). The zonally averaged zonal momentum sources, $\bar{F}_\lambda$, can be thought of as representing the divergence of the horizontal eddy momentum fluxes in an atmospheric model. It is not known how these forcings should be parameterized in terms of the zonal mean variables. The approach of assuming those forcings necessary to maintain the basic state has been taken by Simmons (1982) and Branstator (1983).

The perturbation zonal velocities are assumed to be of the form

$$u' = \text{Re}\{e^{im\lambda}[u_r(y) + iu_i(y)]\} = u_r\cos m\lambda - u_i\sin m\lambda,$$

with similar representations of v', h', and h'_b, where m is the zonal wavenumber. Only a single zonal wavenumber forcing, $m = 3$, is considered in the numerical calculations. The six ordinary differential equations in y resulting from this representation are finite differenced, using second-order centered differences, with grid points equally spaced in y. Solutions are obtained by the Gaussian elimination method of Lindzen and Kuo (1969). An independently coded time dependent model was used to verify the matrix inversion solution. A grid spacing $\Delta y = 2/400$, chosen for comparison with the model of Nigam and Held (1983), was used in the calculations.

3. Numerical calculations

The model equations were solved for the wavenumber 3 response to forcing provided by Northern

Hemisphere midlatitude orography, with various choices for the basic state $\bar{u}$ and $\bar{v}$. The orography and $\bar{u}$ were chosen to be those used by Nigam and Held (1983) in their barotropic model, as those calculations were sufficiently well documented for use in the verification of our model.

The zonal wind was represented by the symmetric about the equator form

$$\bar{u}(y) = E \sin\left[\frac{3\pi}{2}(1 + y)\right] + F(1 - y^2) \qquad (10)$$

The profile u_1 obtained by using E = 18 m s^{-1} and F = 14 m s^{-1} was used in the analogous calculations of Nigam and Held (1983) and was adopted here for most of the experiments. Profile u_2 was obtained by setting E = 15 m s^{-1} and F = 18 m s^{-1}. An experiment was also performed with profile u_3 (E = 22 m s^{-1}, F = 10 m s^{-1}) which had intensified and more extensive tropical easterlies.

Figure 1 shows u_1. Westerlies, with midlatitude maxima of about 27 m s^{-1}, occur everywhere, except for a belt of tropical easterlies with maximum speed of 4 m s^{-1} at the equator. The stationary wave critical latitudes are located near 9°N and 9°S. Also shown in Fig. 1 are u_2 and u_3.

We used the wavenumber 3 orography employed by Nigam and Held (1983):

$$h_b = \hat{h}(y) \cos 3\lambda, \qquad (11)$$

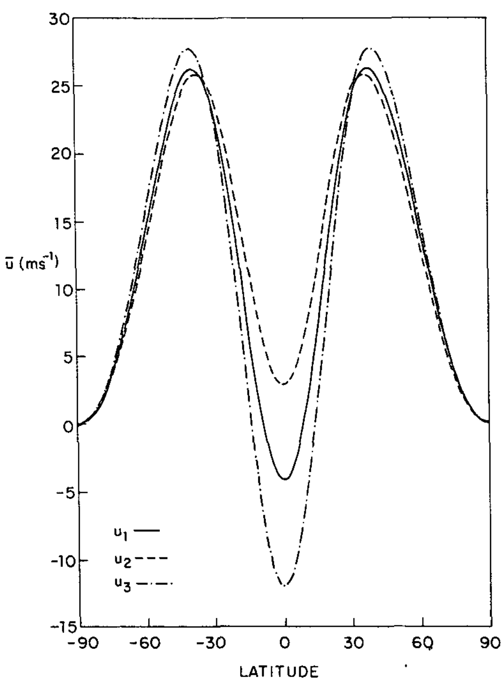

FIG. 1. Meridional structure of basic state zonal velocities u_1, u_2 and u_3.

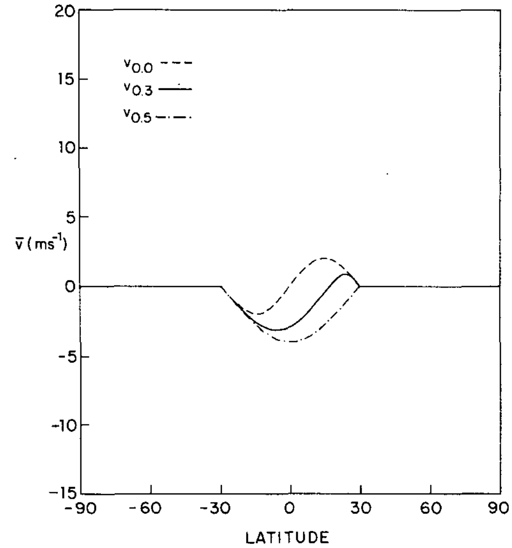

FIG. 2. Meridional structure of basic state meridional velocities $v_{0.0}$, $v_{0.3}$ and $v_{0.5}$.

with

$$\hat{h}(y) = \begin{cases} h_0 \sin\left[\dfrac{(\theta - 34.65°)\pi}{20.25°}\right], & 34.65° < \theta < 54.9° \\ 0, & \text{otherwise,} \end{cases} \qquad (12)$$

where θ is latitude in degrees, and h_0 = 300 m. The zonal mean height of this orography, $\bar{h}_b$ in (7) is zero. The drag coefficient used by Nigam and Held (1983) and also here was $c = (13.5 \text{ days})^{-1} = (1.1664 \times 10^6)^{-1}$ s^{-1} and H_0 was taken as 8000 m. Additional constants appearing in the equation were taken to be g = 9.8 m s^{-2}, a = 6.371 × 10^6 m, and Ω = 7.292 × 10^{-5} s^{-1}.

The basic state $\bar{v}$'s that we chose were either $\bar{v} \equiv 0$ or are given by the one parameter family (variable parameter y_0)

$$v_{y_0}(y) = \begin{cases} V_s \sin\left(\pi \dfrac{y - y_0}{y_0 + 0.5}\right), & -0.5 \leqslant y \leqslant y_0 \\ V_n \sin\left(\pi \dfrac{y - y_0}{0.5 - y_0}\right), & y_0 \leqslant y \leqslant 0.5 \\ 0, & |y| > 0.5 \end{cases} \qquad (13a)$$

such that V_s and V_n are positive,

$$V_s + V_n = V_0 \qquad (13b)$$

and

$$\frac{|V_s|}{|V_n|} = \frac{|y_0 + 0.5|}{|0.5 - y_0|}. \qquad (13c)$$

The value V_0 was taken to be 4 m s^{-1} and $|y_0|$

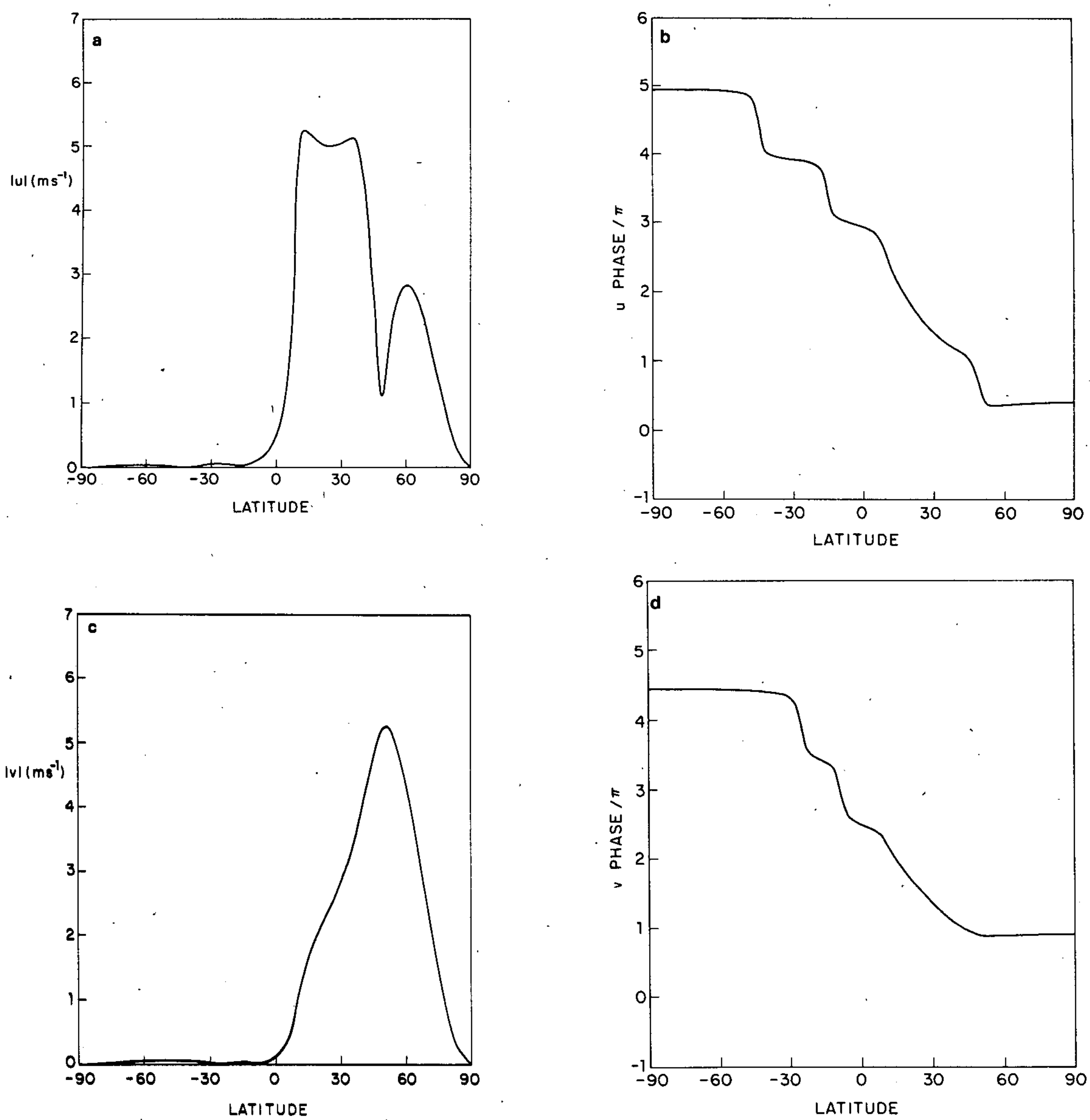

FIG. 3. Meridional structure of perturbation solution for basic state $\bar{v} \equiv 0$, u_1 (a) amplitude ($|u_r + iu_i|$) and (b) phase [$\tan^{-1}(u_i/u_r)$] of u', (c) amplitude and (d) phase of v', (e) amplitude and (f) phase of h'.

< .5. This form of $\bar{v}$ is meant to crudely represent the seasonal behavior of the upper tropospheric tropical Hadley circulation (e.g. Newell *et al.*, 1972), with y_0 being the sine of the latitude separating upper tropospheric southerlies from northerlies. For $y_0 = 0$, $v_{0.0}(y)$ is antisymmetric about the equator with maximum meridional wind speeds of 2 m s^{-1}. As y_0 is moved from the equator into the "summer" hemisphere, the "summer" hemisphere Hadley cell contracts and becomes weaker, while the "winter" hemisphere Hadley cell expands and strengthens. The analysis of Newell *et al.* (1972) indicates that values of y_0 between about ± 0.3 produce meridional circulations that may be analogous to those which occur in the atmosphere, although we shall not limit ourselves to this range. Figure 2 shows $v_{0.0}$, $v_{0.3}$ and $v_{0.5}$.

The first case presented is the one analogous to a case considered by Nigam and Held (1983). The basic state zonal winds were u_1 and the meridional circulation was identically zero. The amplitudes and phases of the u', v', and h' results are shown in Fig. 3. The phase P is defined by

$$x = |x| \cos(m\lambda + P(y)).$$

The sign convention was chosen for comparison with the results of Nigam and Held (1983). Note that with this definition of phase, P increasing northwards implies that trough and ridge lines slope in the

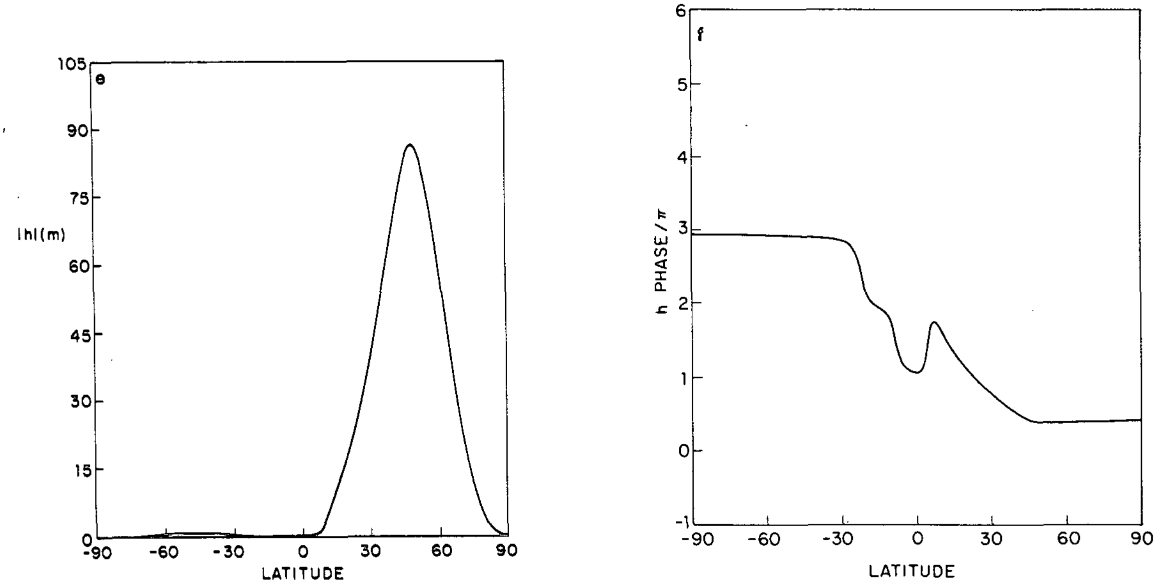

FIG. 3. (*Continued*)

opposite direction, or southeast to northwest. The amplitude of the streamfunction $2\psi'$, defined by

$$v' = \frac{1}{a\cos\theta}\frac{\partial}{\partial\lambda}(2\psi'),$$

is close to but not exactly the same as that shown by Nigam and Held (1983), with relative differences <5%. The differences were greatly reduced if $\bar{h}$ was held constant in our model as it is in that of Nigam and Held. The perturbation mass flux was found to be very close to nondivergent for this case. An outstanding feature of this case is the negligible amplitude of u', v', and h' in the Southern Hemisphere. As predicted by the Rossby wave dispersion relation, the mountain waves do not propagate through the easterlies.

Using the same $\bar{u}$ as the previous case, v_{y_0} was included in the basic state for various values of y_0 between ±0.5. The major result is summarized in Fig. 4, which shows the maximum value of the

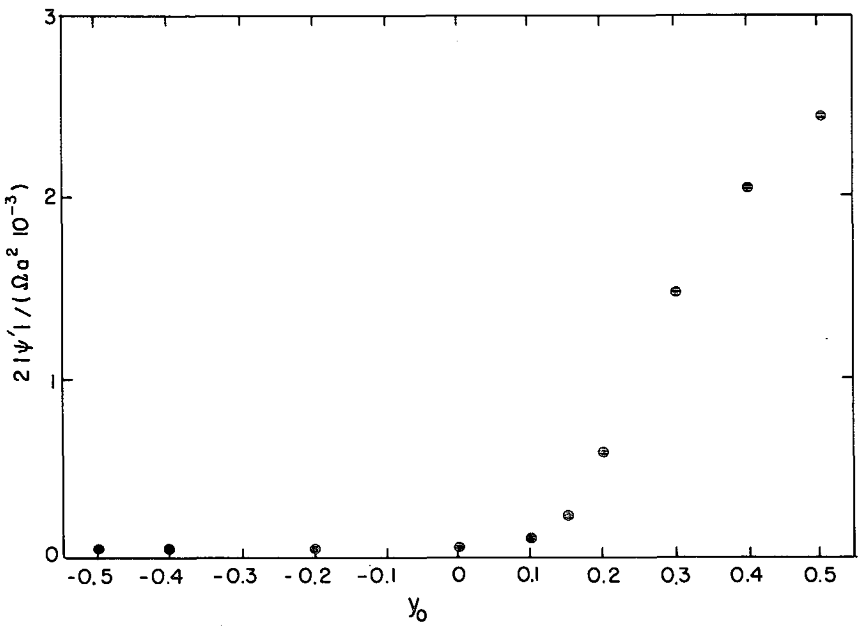

FIG. 4. Maximum Southern Hemisphere streamfunction amplitude for perturbations to the basic state u_1, v_{y_0} as y_0 is varied between −0.5 and +0.5.

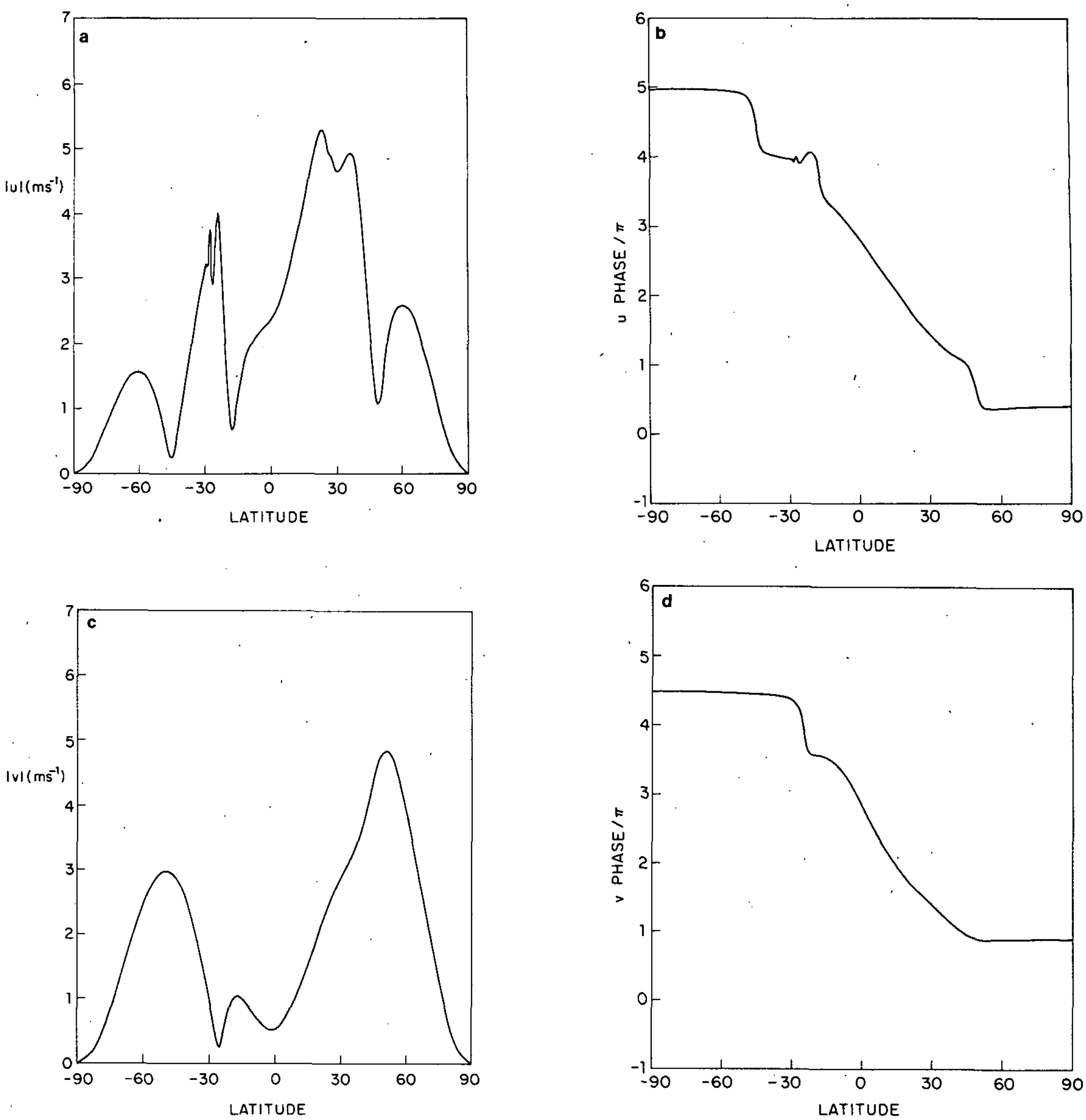

FIG. 5. As in Figs. 3a–f, respectively, but for basic state u_1, $v_{0.3}$.

summer hemisphere streamfunction as a function of y_0. For comparison the value for the $\bar{v} \equiv 0$ case is 0.02. The Southern Hemisphere response is very sensitive to the structure of the basic state meridional circulation. For $y_0 < 0.1$ the Southern Hemisphere response is negligible, and the solutions are essentially those which are found with $\bar{v} \equiv 0$. As y_0 increases from .1 the Southern Hemisphere response increases dramatically, until for $y_0 = 0.5$ the Southern Hemisphere response is almost as large as the Northern Hemisphere $\bar{v} \equiv 0$ response. Wave transmission through the easterly layer is significant when the basic state meridional wind is directed away from the wave generation region throughout the easterlies. Basic state meridional winds towards the forcing region in the easterlies do not allow transmission of the forced waves into the Southern Hemisphere.

The details of the response for $y_0 = 0.3$ are shown in Fig. 5. For comparison the response for $\bar{v} \equiv 0$ and a $\bar{u}$ with no critical latitudes, u_2, is shown in Fig. 6. The responses for these two cases are very similar in both amplitude and phase. This basic state meridional circulation with $\bar{v}$ of order 3 m s^{-1} causes the model to respond almost as if there were no critical latitudes or easterlies. It turned out for all the cases considered that the wave velocities were approximately nondivergent everywhere. Figures 5a and 5b show small scale structure near 30°S, a latitude where $\bar{v} \rightarrow 0$.

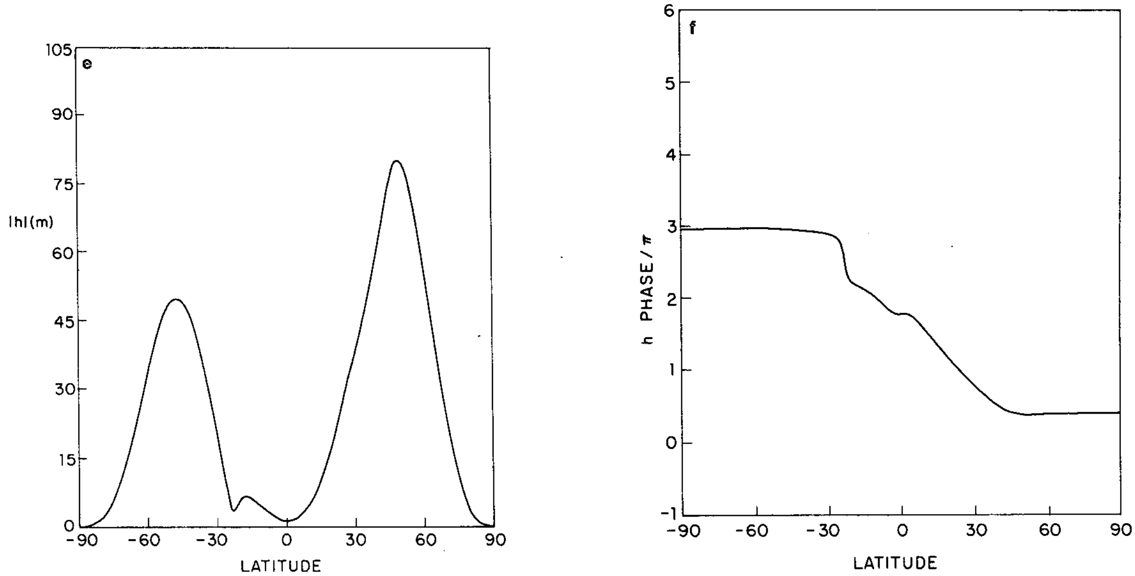

FIG. 5. (*Continued*)

This behavior is also seen in the dispersion relation discussed in the Section 4 and results from a singularity in the equation. Introduction of a small higher order damping (e.g., viscosity) greatly reduces these oscillations with negligible change to the rest of the solution.

A case was performed to test how stronger and more intense tropical easterlies affect the meridional circulation induced wave transmission. The basic state zonal wind was taken to be u_3 (shown in Fig. 1), with equatorial easterlies of -12 m s^{-1} and critical latitudes $\pm 14°$. The basic state $\bar{v} \equiv v_{0.3}$, as in the case shown in Fig. 5, was specified. The results are shown in Fig. 7. The amplitude of the Southern Hemisphere response is reduced by the stronger and wider easterlies, being about 40% of those found with u_1.

4. Discussion

The numerical solutions to the linearized shallow water equations show that stationary waves can be transmitted across critical latitudes where $\bar{u} = 0$ and through easterly layers when the basic state mean meridional velocity is favorable. Apparently, the direction of transmission is related to the sign of $\bar{v}$, with easterly layer transmission occurring only in the downstream (in terms of $\bar{v}$) direction. When $\bar{v} \sim 1$ m s^{-1} at critical latitudes, the wave transmitted downstream has a meridional wavelength near the critical latitude that is apparently comparable to the meridional wavelength in the region of midlatitude westerlies, whereas the $\bar{v} \equiv 0$ analysis gives meridional wavelength approaching zero near critical latitudes.

We have also noted that the wave solutions of the shallow water equation model were quasi-nondivergent. Thus, the behavior found numerically should be, and is, contained in the "barotropic" wave vorticity equation, obtained by linearizing the shallow water equations about a basic state $u(\bar{y})$, $v(\bar{y})$ and assuming that the divergence of the wave velocities is negligible.

For simplicity, a Cartesian β-plane geometry will be considered. The equation governing zero divergence waves $(u'_x + v'_y = 0)$ is then

$$\frac{\partial \xi'}{\partial t} + \bar{u}\frac{\partial \xi'}{\partial x} + \frac{\partial}{\partial y}(\bar{v}\xi') + (\beta - \bar{u}_{yy})v' = 0, \quad (14)$$

where

$$\xi' = \frac{\partial^2 \psi'}{\partial x^2} + \frac{\partial^2 \psi'}{\partial y^2}$$

and

$$u' = -\frac{\partial \psi'}{\partial y}, \quad v' = \frac{\partial \psi'}{\partial x}.$$

ξ' is the perturbation vorticity, ψ' the perturbation streamfunction, x and y are in units of distance, and β is the meridional gradient of f.

A local dispersion relation may be derived by taking $\psi \alpha \exp[ik(x - ct + \gamma y)]$ and substituting in (14). Assuming that the basic state meridional length scale is large compared to the wave meridional length scale, so that a WKB type analysis is appropriate (and $\bar{v}_y\xi' \ll \bar{v}\xi'_y$), gives the local dispersion relation

$$f(\gamma) = \bar{v}\gamma^3 + (\bar{u} - c)\gamma^2 + \bar{v}\gamma + (\bar{u} - c) - \frac{\beta^*}{k^2} = 0, \quad (15)$$

where $\beta^* = \beta - \bar{u}_{yy}$. Stationary waves with $c = 0$

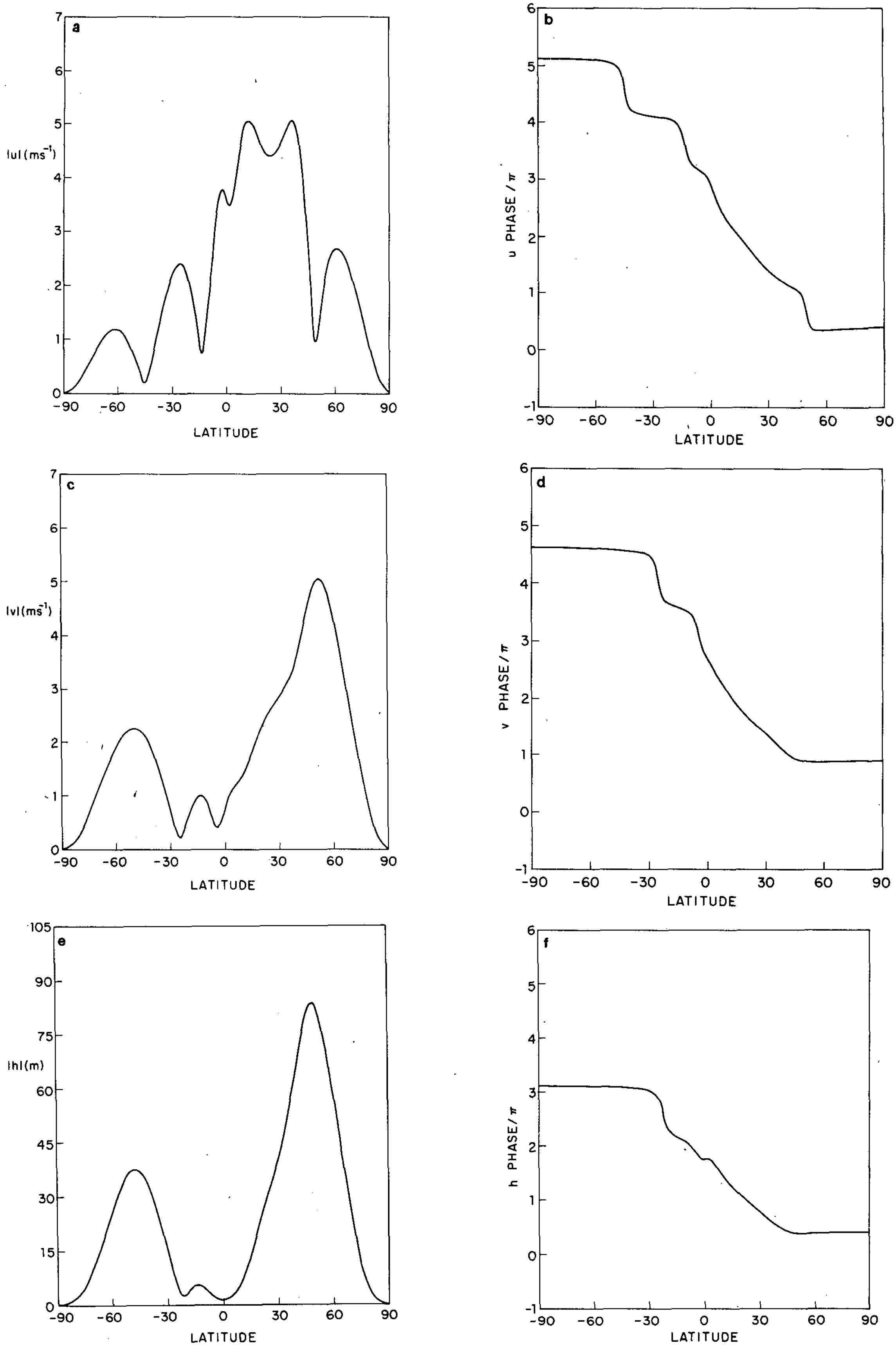

FIG. 6. As in Figs. 3a–f, respectively, but for basic state $\bar{v} \equiv 0$, u_2.

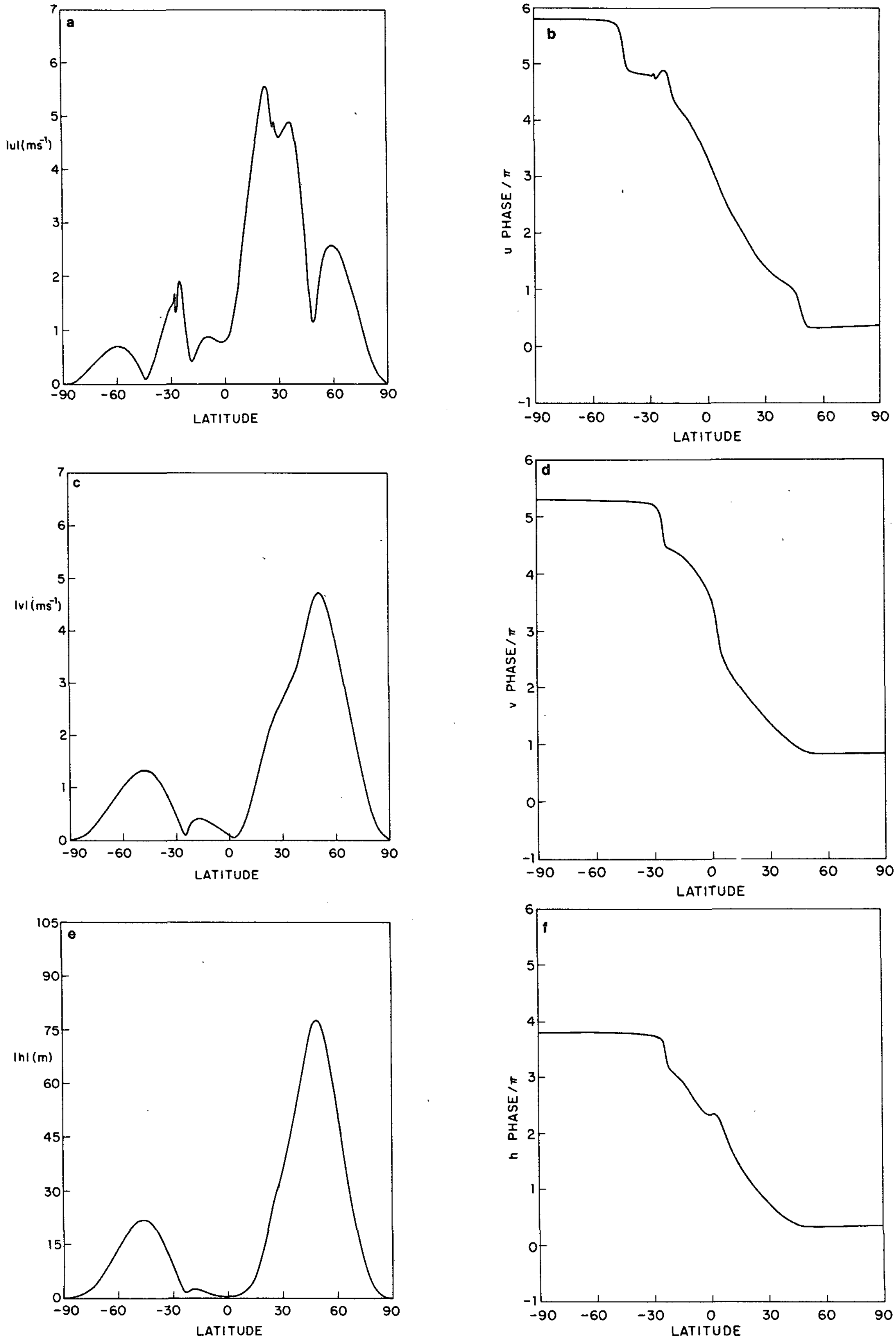

FIG. 7. As in Figs. 3a–f, respectively, but for basic state $v_{0.3}$, u_3.

and $\beta^* > 0$ will be assumed in the following. The results may be extended to $c \neq 0$ by substituting $\bar{u} - c$ for $\bar{u}$. A version of (15) has been discussed by Opsteegh (1982).

When $\bar{v} \equiv 0$, (15) reduces to the usual Rossby wave dispersion relation,

$$\gamma^2 = \frac{\beta^*}{k^2\bar{u}} - 1. \tag{16}$$

Referring to solutions with real γ as (meridionally) propagating, and solutions with imaginary γ as evanescent, (16) allows propagating solutions only when $0 < \bar{u} < \beta^*/k^2$. Additionally, (16) gives $|\gamma| \to \infty$ and the meridional wavelength $\to 0$ when $\bar{u} \to 0_+$.

The dispersion relation (15) with $v \neq 0$ has rather different behavior. This equation is cubic in γ with real coefficients, and hence has either one or three real (propagating) roots. In the case of only one real root, the other two roots are complex conjugates, and the solution may be mixed propagating/evanescent. Thus it immediately follows in this approximation that if $\bar{v} \neq 0$, there is always at least one propagating solution, whatever the value of $\bar{u}$.

In order for three distinct propagating solutions to (15) to exist, $f(\gamma)$ must have two extrema for real γ, γ_1 and γ_2, and $f(\gamma_1)$ and $f(\gamma_2)$ must have opposite signs. The extrema of $f(\gamma)$ are the solutions to

$$\frac{\partial f}{\partial \gamma} = 3\bar{v}\gamma^2 + 2\bar{u}\gamma + \bar{v} = 0,$$

so that $\gamma_{1,2}$ are

$$\gamma_{1,2} = -\frac{\bar{u}}{3\bar{v}}\left[1 \pm \left(1 - 3\frac{\bar{v}}{\bar{u}^2}\right)^{1/2}\right].$$

Therefore, for three distinct propagating solutions to exist, the necessary condition

$$\bar{v}^2 < \frac{\bar{u}^2}{3} \tag{17}$$

must be satisfied. Assuming (17) is satisfied, so that real γ_1 and γ_2 exist, it is sufficient for the existence of three propagating solutions that $f(\gamma_1)$ and $f(\gamma_2)$ have opposite signs.

At a critical latitude, where $\bar{u} = 0$, (17) is violated if $\bar{v} \neq 0$. Then only one propagating solution exists there. The meridional wavelength at a critical latitude is found from the real solution to

$$\bar{v}\gamma^3 + \bar{v}\gamma - \beta_*/k^2 = 0. \tag{18}$$

For the zonal wind profile and zonal wavelength that were used in the numerical experiments and using $\beta^* = \beta$, $\beta^*/k^2 \sim 100$ m s^{-1}, and $\beta^*/(k^2\bar{v}) \gg 1$. Then at the critical latitude

$$\gamma = \lambda/k \sim \left(\frac{\beta^*}{k^2\bar{v}}\right)^{1/3},$$

where λ is the meridional wavenumber, $2\pi/L_y$, and the meridional wavelength L_y is

$$L_y \sim 2\pi\left(\frac{\bar{v}}{\beta^* k}\right)^{1/3}. \tag{19}$$

For zonal wavenumber 3 and $\bar{v} = 1$ m s^{-1}, (19) gives $L_y \sim 2900$ km. The component which propagates across the critical latitude has γ the same sign as $\bar{v}$ (taking β^* as positive), so that for $\bar{v} > 0$, the lines of constant phase of this component of the solution are oriented SE to NW, while for $\bar{v} < 0$ the lines of constant phase run SW to NE across the critical latitudes.

Singularities of (15) occur at the latitudes where $\bar{v} = 0$. The solutions to (15) approach the two roots of (16) as $\bar{v} \to 0$, and the third (real) root has $|\gamma| \to \infty$. The short wavelength γ is given approximately by

$$\gamma \sim \frac{-\bar{u}}{\bar{v}} \tag{20}$$

as $\bar{v} \to 0$.

The dispersion relation corresponding to (15) for spherical geometry was used to find the local meridional wavenumber corresponding to the wavenumber 3 numerical experiments, with $\bar{u} = u_1$ and $\bar{v} \equiv 0$ (Fig. 8) and $\bar{v} = v_{0.3}$ (Fig. 9). The real parts of the roots are represented as meridional wavelength in degrees of latitude, and the imaginary parts as the relative growth of amplitude in a 5° north latitude distance. Amplification as well as decay, with amplitude amplification the inverse of the decay, are shown in Figs. 8b and 9b as the roots appear as complex conjugate pairs. The appropriate root representing the dominant part of the solution to a forced, damped problem is presumably that which decays as distance from the forcing increases.

When $\bar{v} \equiv 0$ the two solutions are propagating outside the easterly layer, except for evanescence in the polar regions. As the critical latitudes are approached from the propagating regions, the meridional wavelength approaches zero. The waves cannot tunnel through the easterlies, as they are attenuated to zero amplitude in an infinitesimal distance in the easterlies as the critical latitudes are passed. Thus in the WKB approximation, midlatitude stationary wave forcing in the propagating region of one hemisphere cannot produce a response in the tropical easterlies or in the other hemisphere if there is no damping. Similarly, there can be no response outside the easterlies to stationary wave forcing inside the easterlies. As mentioned in the introduction the correction of the breakdown of the WKB approach near the critical latitudes does not change the qualitative notion of the critical latitude as a barrier.

The dispersion relation with $\bar{v} = v_{0.3}$ behaves somewhat differently from the $\bar{v} \equiv 0$ case in the vicinity of the easterlies. The roots are the same for

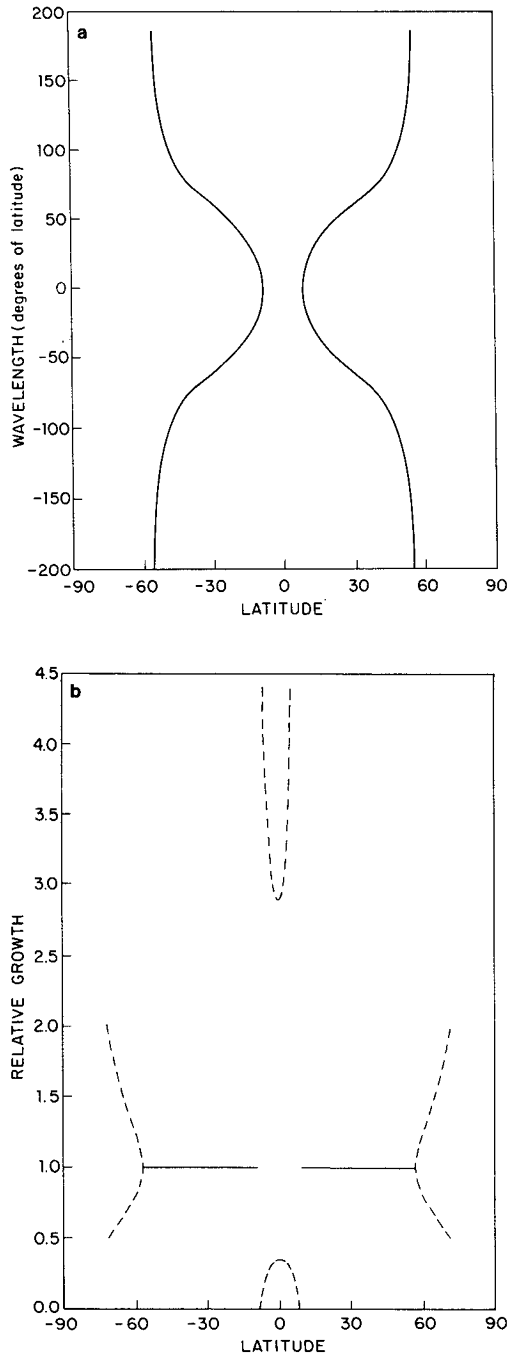

FIG. 8. (a) Local wavelength in degrees of latitude for $\bar{v} \equiv 0$, u_1. (b) Relative growth of amplitude in a 5° north latitude distance. Growing or decaying waves are represented by dashed lines. There are two complex conjugate roots at each latitude.

|latitude| > 30°, as $\bar{v} = 0$ in those regions. The Rossby wave root with negative meridional wavelength (SW to NE tilt of lines of constant phase) is propagating (imaginary part = 0) throughout the region occupied by the meridional circulation. The meridional wavelength of this branch is finite everywhere, with a minimum magnitude of about 30° latitude near 10°N. The average wavelength through 25°N to 15°S is very close to the 40° wavelength found in that region in Fig. 5. Two other propagating roots, one with short and the other with longer meridional wavelengths, are found between 30–21°S and also between 14–30°N. The short waves have zero meridional wavelength where $\bar{v} = 0$, while the longer wave is the continuation of the positive meridional wave-

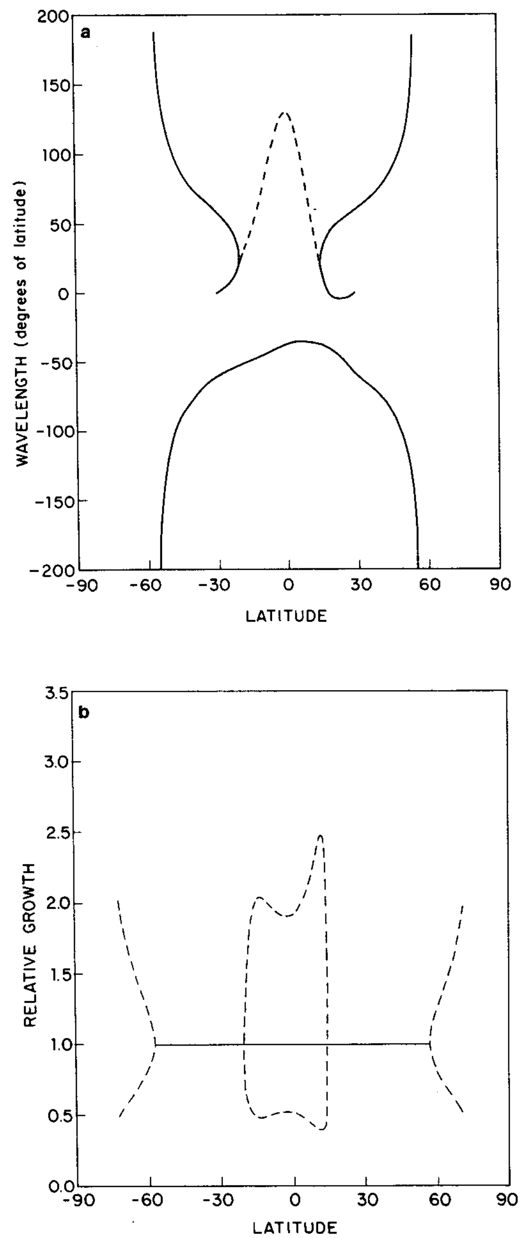

FIG. 9. As in Figs. 8a,b, respectively, but for basic state $v_{0.3}$, u_1. In the equatorial region (21°S to 14°N) there are two complex conjugate roots with the same northward wavelength and one southward wave of constant amplitude. Elsewhere in the $\bar{v} \neq 0$ region there are three real roots.

length Rossby wave. The two roots merge into a single root near 21°S and 14°N and become mixed propagating/evanescent complex conjugates between these latitudes. The meridional wavelength of the mixed waves is largest in the deep tropics. The amplitude decay is largest near 12°N, and is finite everywhere. The propagating/evanescent waves can tunnel through the easterlies to some extent; the propagating wave is unaffected by them. The inclusion of the $\bar{v}_y$ term in the dispersion relation leads to three complex roots in the $\bar{v} \neq 0$ region. The long meridional wavelength propagating root becomes propagating/evanescent with a decay factor of about 0.75.

The one-way wave transmission across the tropical easterly layer affect conditioned by $\bar{v}$ found in the numerical results is contained in this barotropic wave dispersion relation. If $\bar{v}$ is such that stationary wave forcing in the westerlies of the Northern Hemisphere produces a significant Southern Hemisphere response, then stationary wave forcing in the westerlies of the Southern Hemisphere would produce a negligible Northern Hemisphere response. Stationary wave forcing in the easterlies should produce a significant but asymmetric global response so long as the easterly layer $\bar{v}$ is large enough. The tropics and midlatitudes cannot be considered as being dynamically independent in this model.

The effect of $\bar{v}$ on the group velocity may also be found from (15). Using $\sigma = kc$ and $\gamma = \lambda/k$, $C_{gx} = \partial\sigma/\partial k$ and $C_{gy} = \partial\sigma/\partial\lambda$ give

$$C_{gx} = \bar{u} + \beta^* \frac{k^2 - \lambda^2}{(k^2 + \lambda^2)^2} = C_{grx}, \qquad (21)$$

$$C_{gy} = \bar{v} + \frac{2k\lambda\beta^*}{(k^2 + \lambda^2)^2} = \bar{v} + C_{gry}. \qquad (22)$$

These group velocity formulae are given by Karoly (1983) in the context of a zonally varying $\bar{v} = 0$ basic state. The result (21) for C_{gx} is formally identical to that for a barotropic Rossby wave with $\bar{v} \equiv 0$, denoted as C_{grx}, while the result (22) for C_{gy} is the y group velocity expression for a $\bar{v} \equiv 0$ barotropic Rossby wave, C_{gry}, plus $\bar{v}$. However, the numerical evaluation of C_{gx} and C_{gy} with $\bar{v} \neq 0$ produces results that differ significantly from the $\bar{v} \equiv 0$ barotropic Rossby wave case for stationary waves, since the dispersion relation (15) produces real or complex λ solutions in the easterlies and finite real or complex λ's at critical latitudes.

Figure 10 shows a ray path calculated from the spherical analogues of (15), (21), and (22) for u_1 and $v_{0.3}$ for the southward propagating zonal wave number 3 stationary wave root. The group velocity for this root is tangential to the path at each latitude. The packet is started at 45°N and 10°E, propagates eastward for three days, reaching 15°N, 34°E, then propagates westward for six days crossing the easterly layer until it reaches 20°S, 7°E, and then propagates eastward again, reaching 45°S, 24°E in two more days. The wave packet starting at 45°N reaches 45°S in about 11 days, with a net eastward displacement of about 14° longitude in this case. Therefore the significant amplitude reached in the Southern Hemisphere in Fig. 5 is consistent with the 13.5 day damping in the model having limited effect over this time.

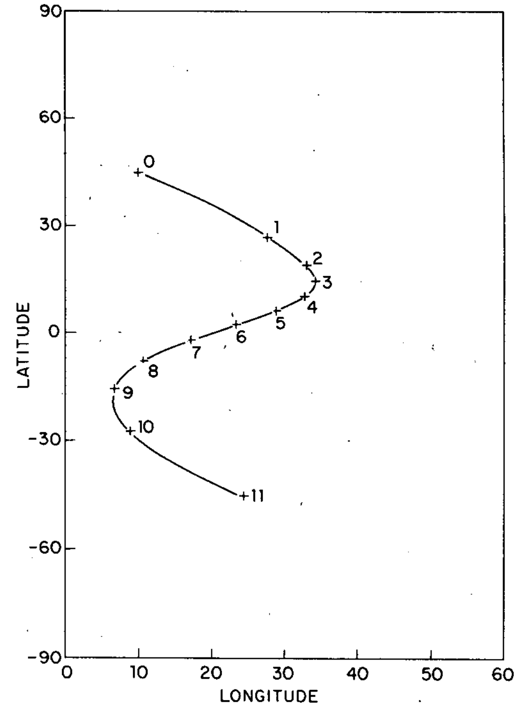

FIG. 10. Ray path for the southward propagating wave of Fig. 9 starting at 45°N, 10°E. Elapsed time in days is shown at points along the path.

5. Conclusion

We have shown that a basic state mean meridional circulation can significantly affect barotropic Rossby wave propagation in the vicinity of critical latitudes and easterly layers. This result has been shown by numerical example where the numerical values chosen for the basic state are not outside the bounds established by observations for the zonally averaged atmospheric winds. These results *do not* show that the atmospheric mean meridional circulation would have similar significant effects on three-dimensional atmospheric waves. This study does indicate, however, that problems of three-dimensional atmospheric waves on a zonally averaged basic state including reasonable meridional circulations and zonal winds should be done, and that the effort expended might not be unrewarding. We have also indicated the circumstances in which a mean meridional circulation effect might or might not be important and discussed the properties of such effects.

The examples chosen for illustration of the potential effect of inclusion of a basic state mean meridional circulation on critical latitude behavior were those of interhemispheric transmission of orographically forced planetary waves. These examples were constructed to attempt to show these effects in the clearest possible way. It was thought that this aim could be best accomplished by separating the wave forcing region from the region in which the effects of the modified basic state on wave propagation were to be examined. The shallow water model produced responses in the unforced hemisphere that were comparable in amplitude to the responses in the forced hemisphere when the basic state meridional winds were favorable, despite the presence of tropical easterlies. However, these examples were not constructed to explain an effect that is observed to occur in the atmosphere, nor are we suggesting that midlatitude orography in one hemisphere may have a substantial direct impact on midlatitudes in the other hemisphere that may have been missed due to insufficient observational data. Indeed, preliminary experiments with a more realistic multilevel model indicate that the wave response in the midlatitudes of the unforced hemisphere is much reduced from the analogous experiments with the shallow water model.

However, we believe the meridional circulation effect may be significant in the forcing of midlatitude stationary waves by tropical cumulus heating, and in the influence of midlatitude orography on the location of the tropical convective heat sources. In the case of tropical heat source forcing, experiments with both the linearized shallow water model and a multilevel linearized primative equation model, which will be reported in due course, are in agreement. Stationary tropical heating confined to the easterlies influences the winter hemisphere more strongly than the summer hemisphere due to the one way propagation effect discussed here, even when the heating is confined to the summer hemisphere. The midlatitude response is of sufficient amplitude to cause observable deviation from climatology, and these results indicate the possibility that the climatological midlatitude winter hemisphere stationary waves may contain a significant direct contribution from zonally asymmetric tropical convective forcing.

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