Generalization of the Quasi-Geostrophic Eliassen-Palm Flux to Include Eddy Forcing of Condensation Heating

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ABSTRACT

The Eulerian form of the Eliassen–Palm flux for quasi-geostrophic motion is generalized to include large-scale eddy forcing of condensation heating. Only the vertical component of the flux has to be modified; it is increased. The Eliassen–Palm theorem still holds, but instead of assuming that the diabatic heating is zero, one assumes that the diabatic heating excluding condensation associated with the large-scale motions is zero. The non-acceleration theorem also still holds, provided one adds the assumption that the Eulerian zonal mean moisture field is stationary. The generalization preserves the relationship between stationary wave energy flux and the Eliassen–Palm flux, but now wave condensation effects are automatically included. In addition, the generalization leads to a function describing the total eddy forcing of the moisture field.

The divergence of the generalized Eliassen–Palm flux is calculated from atmospheric observations and compared with the divergence of the standard flux. The eddy forcing of the zonal mean zonal wind and temperature fields is much stronger when condensation effects are included—for example, in the annual mean it is about two and one half times as strong. The eddy forcing of the moisture field is also calculated. It shows the expected tendency to dry out the subtropics and moisturize middle and high latitudes, but the effect of the meridional eddy flux of moisture is greatly enhanced by the effect of the Ferrel cell induced by the eddy heat fluxes.

1. Introduction

The Eliassen–Palm theorem (Eliassen and Palm, 1961) states that for steady, conservative flow, $\nabla \cdot \mathbf{F} = 0$, where $\mathbf{F}$ is a two-dimensional vector flux in the meridional plane which is a function of the zonal mean eddy fluxes. We will follow recent usage and refer to $\mathbf{F}$ as the Eliassen–Palm flux. The conditions under which the Eliassen–Palm theorem can be proved have been generalized considerably since Eliassen and Palm's original paper. For example, we note the work by Holton (1974), Andrews and McIntyre (1976 and 1978), Boyd (1976), and Edmon *et al.* (1980). As a result of these studies we now know that the theorem holds rather generally, given an appropriate definition of $\mathbf{F}$. In particular the Eulerian form holds for quasi-geostrophic motions with arbitrary eddy amplitude (Edmon *et al.*, 1980).

A closely related theorem, the non-acceleration theorem of Charney and Drazin (1961), states in effect that the forcing of the zonal mean zonal wind and temperature fields by eddies is described completely by $\nabla \cdot \mathbf{F}$, apart from boundary effects. As a result of the above cited references we also know that this result holds under rather general conditions, e.g., the Eulerian form again holds for eddies of arbitrary amplitude under quasi-geostrophic conditions (Edmon *et al.*, 1980). Consequently Edmon *et al.* advocated the use of the Eulerian form of $\nabla \cdot \mathbf{F}$ as a diagnostic of forcing by atmospheric eddies. They presented the first calculations of this quantity, and subsequently other authors have also made use of $\nabla \cdot \mathbf{F}$ as a diagnostic (e.g., Dunkerton *et al.*, 1981; Karoly, 1982). There is no doubt that these theorems and the diagnostic calculations have added greatly to our understanding of the role of eddies in the general circulation.

An important point that one must keep in mind in interpreting these results is that $\nabla \cdot \mathbf{F}$ only represents the explicit internal eddy forcing of the zonal mean zonal wind and temperature fields. These fields are also forced by frictional forces and diabatic heating, and these quantities are themselves modified by the action of the eddies. Thus there is an implicit forcing by the eddies in addition to the explicit forcing described by $\nabla \cdot \mathbf{F}$. Two examples come readily to mind: 1) frictional forces act to decelerate the westerly winds near the surface in midlatitudes, but the forces are themselves magnified because the surface westerlies are accelerated by eddy momentum fluxes; and 2) differential diabatic heating tends to generate a latitudinal temperature gradient, but the effect is weakened by the differential condensation associated with the poleward eddy transport of latent heat.

Recently Salustri and Stone (1983) have shown how the effect of eddy forcing on condensation and the associated diabatic heating can be accounted for in calculating the eddy forcing of the Ferrel Cell. It is our purpose in this paper to show how, using their method, one can define a modified Eulerian form of the Eliassen–Palm flux, which also includes the effect of eddy forcing on condensation heating. With **F** replaced by the modified flux the Eliassen–Palm theorem and a modified, but more general form of the non-acceleration theorem for quasi-geostrophic motion still hold. We will also present calculations of the divergence of the modified flux and of the eddy forcing of the moisture field, based on observations.

2. Fundamental equations

We follow Edmon *et al.* (1980) and write the zonal mean quasi-geostrophic equations in pressure coordinates and spherical coordinates (see Appendix for symbols):

$$f\frac{\partial[u]}{\partial p}=\frac{R}{ap}\frac{\partial[\theta]}{\partial\phi}, \tag{1}$$

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi[v])+\frac{\partial[\omega]}{\partial p}=0, \tag{2}$$

$$\frac{\partial[u]}{\partial t}=f[v]+[\mathcal{F}]-\frac{1}{a\cos^2\phi}\frac{\partial}{\partial\phi}(\cos^2\phi[u^*v^*]), \tag{3}$$

$$\frac{\partial[\theta]}{\partial t}+\sigma[\omega]=\frac{[Q]}{C_p}-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi[v^*\theta^*]). \tag{4}$$

We note that the derivation of these equations depends on the assumption that the latitudinal variation of the basic state stratification σ is smaller than the stratification itself by a factor of order the Rossby number. These latitudinal variations must also be assumed to be small in order to derive the Eliassen–Palm flux and theorem for arbitrary eddy amplitudes. This is in contrast to the case when eddy amplitudes are assumed to be small, where it is not necessary to neglect these latitudinal variations. This difference was not mentioned by Edmon *et al.* in their discussion of the finite amplitude case.

We now introduce explicitly the effects of condensation associated with the large-scale motions. First, we let

$$q=q_s+q',$$

where q_s is the time and zonal mean specific humidity (dependent on ϕ and p in general) and q' the fluctuation specific humidity. (We note that $q'^*=q^*$.) Next we wish to write the moisture conservation equation using the quasi-geostrophic approximation. Since this approximation is not valid for smaller-scale motions, we separate the velocity field into two parts, a large-scale part for which the Rossby number is small and a small-scale part which is just the remainder velocity field. We then define S to be the moisture convergence associated with the remainder velocity field. The convergence associated with the larger-scale motions can then be simplified by using the quasi-geostrophic approximation. The derivation, which is analogous to that for the quasi-geostrophic thermodynamic equation, is given by Salustri and Stone (1983). The resulting form of the zonal mean moisture conservation equation is

$$\frac{\partial[q']}{\partial t}+\frac{\partial q_s}{\partial p}[\omega]=-[C-S]-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi[v^*q^*]). \tag{5}$$

We note that in a stationary state, moisture conservation requires that the new source function, $S-C$, be balanced by the convergence of moisture associated with just the larger-scale motions [e.g., see Eq. (5)]. Thus in a stationary state $C-S$ is the condensation rate attributable to just the larger-scale motions. We now define a new thermodynamic source function,

$$Q_m=Q-L(C-S). \tag{6}$$

It follows that, *in a stationary state Q_m is the net diabatic heating excluding the condensation heating attributable to the larger-scale motions.* Substituting Eqs. (6) and (5) into (4), we obtain

$$\frac{\partial}{\partial t}\left[\theta+\frac{L}{C_p}q'\right]+\sigma_m[\omega]=\frac{[Q_m]}{C_p}-\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\left(\cos\phi\left[v^*\theta^*+\frac{L}{C_p}v^*q^*\right]\right), \tag{7}$$

where

$$\sigma_m=\sigma+\frac{L}{C_p}\frac{\partial q_s}{\partial p}. \tag{8}$$

We note that Eq. (7) expresses the conservation of equivalent potential temperature, and could be written more simply in terms of it. However the data available on eddy fluxes is given in terms of fluxes of temperature and moisture, so it is convenient to leave the equation in the above form. Also, the thermal wind relation (1) has a much simpler form when written in terms of temperature rather than equivalent potential temperature.

In order to generalize the usual definitions of the residual mean meridional circulations and of the Eliassen-Palm flux, it is necessary to introduce a latitude-independent static stability parameter σ_0:

$$\sigma_m(\phi,p)=\sigma_0(p)+\sigma'(\phi,p). \tag{9}$$

(One possible choice for σ_0 will be given in Section 4.) Then we can define generalized residual mean circulations

$$\tilde{v}_m = [v] - \frac{\partial}{\partial p} \frac{[v^*(\theta^* + (L/C_p)q^*)]}{\sigma_0}, \quad (10)$$

$$\tilde{\omega}_m = [\omega] + \frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}\frac{\cos\phi[v^*(\theta^* + (L/C_p)q^*)]}{\sigma_0}, \quad (11)$$

and a generalized Eliassen–Palm flux,

$$\mathbf{F}_m = (F_\phi{}^m, F_p{}^m), \quad (12a)$$

$$F_\phi{}^m = -a\cos\phi[u^*v^*], \quad (12b)$$

$$F_p{}^m = \frac{af\cos\phi[v^*(\theta^* + (L/C_p)q^*)]}{\sigma_0}. \quad (12c)$$

Substituting these definitions into Eqs. (2), (3), (5) and (7), we obtain a new form of the transformed Eulerian-mean equations,

$$\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi\tilde{v}_m) + \frac{\partial\tilde{\omega}_m}{\partial p} = 0, \quad (13)$$

$$\frac{\partial[u]}{\partial t} = f\tilde{v}_m + [\mathcal{F}] + \frac{1}{a\cos\phi}\nabla\cdot\mathbf{F}_m, \quad (14)$$

$$\frac{\partial}{\partial t}\left[\theta + \frac{L}{C_p}q'\right] + \sigma_0\tilde{\omega}_m + \sigma'[\omega] = \frac{[Q_m]}{C_p}, \quad (15)$$

$$\frac{\partial[q']}{\partial t} + \frac{\partial q_s}{\partial p}\tilde{\omega}_m = [S - C] + M; \quad (16)$$

where

$$M = -\frac{1}{a\cos\phi}\frac{\partial}{\partial\phi}(\cos\phi[v^*q^*]), + \frac{1}{a\sigma_0\cos\phi}\frac{\partial q_s}{\partial p}\frac{\partial}{\partial\phi}\left\{\cos\phi\left([v^*\theta^*] + \frac{L}{C_p}[v^*q^*]\right)\right\}. \quad (17)$$

The function M describes the eddy forcing of the moisture field. It includes both the effect of the eddy moisture flux and the effect of the zonal mean vertical motions forced by the eddy fluxes of sensible and latent heat. If one neglects moisture, i.e., sets $q' = q_s = 0$, then Eq. (16) decouples from the other equations, $\tilde{v}_m$, $\tilde{\omega}_m$, and $\mathbf{F}_m$ reduce to the corresponding "dry" quantities $\tilde{v}$, $\tilde{\omega}$, and $\mathbf{F}$, and Eqs. (13) to (15) are identical to those of Edmon *et al.* (1980).

We note that only the vertical component of the Eliassen–Palm flux has been modified by the moisture effects. Now the meridional eddy latent-heat flux has been added to the meridional eddy sensible-heat flux, and the static stability has been replaced by a "moist" static stability. Both changes tend to *increase* the vertical component.

3. The Eliassen–Palm and non-acceleration theorems restated

Equations (1) and (13) to (16) constitute a complete set for $[u]$, $[\theta]$, $\tilde{v}_m$, $\tilde{\omega}_m$, and $[q']$, if one is given the eddy fluxes, the basic state structure (T_s and q_s), and the source functions ($[\mathcal{F}]$, $[Q_m]$, and $[S - C]$). To derive the Eliassen–Palm theorem for arbitrary eddy amplitude one must assume that $\sigma' = 0$, regardless of whether one includes condensation effects or not. We will discuss the validity of this assumption for the atmosphere in Section 4 later. In this section we simply assume it; so Eq. (15) becomes

$$\frac{\partial}{\partial t}\left[\theta + \frac{L}{C_p}q'\right] + \sigma_0\tilde{\omega}_m = \frac{[Q_m]}{C_p}. \quad (18)$$

Because $[q']$ is coupled to the other zonal mean fields solely by its time derivative in Eq. (18), the Eliassen–Palm theorem can be restated as follows:

> If $[\mathcal{F}] = [Q_m] = \frac{\partial[\ ]}{\partial t} = 0$, then $\nabla\cdot\mathbf{F}_m = 0$.

The proof requires only the addition of the boundary condition that v is finite at the pole. This automatically ensures that $[v]$, v^*, and $\tilde{v}$ are also finite at the pole (see Eq. (10) in the last case). The theorem then follows immediately from Eqs. (18), (13), and (14). This alternate statement of the theorem has merely shifted the condensation effects associated with the large-scale motions from one forcing function to another, i.e., from the diabatic heating to the Eliassen–Palm flux.

We note that our statement of the theorem, like that of Edmon *et al.* (1980), starts from the assumption that the Eulerian-mean flow is steady and does not require any assumption about the eddy amplitudes. However, they derived the theorem from the quasi-geostrophic potential vorticity equation and the relation between the eddy potential vorticity flux and the Eliassen–Palm flux divergence. The latter relation does not hold in the presence of condensation, because the hydrostatic and thermal wind equations do not generalize in the same way as the thermodynamic equation. However, the above proof is just as simple and illustrates that the potential vorticity eddy flux relation is not necessary to prove the theorem.

We note further that Eliassen and Palm's original proof of the theorem started from the assumption that the eddy fields are stationary, and also assumed that eddy amplitudes were small enough that the eddies can be described by the linearized wave equations. Our generalization can be proved in the same way. In this case it is more illuminating to withhold the quasi-geostrophic approximation until the last step. The basic derivation is given by Eliassen and Palm (1961). It starts from the perturbation equations for stationary waves with no dissipation and no diabatic heating. Our generalization allows for a perturbation diabatic heating Q' associated with the wave, and requires the addition of the linearized moisture conservation equation,

$$\frac{u_s}{a\cos\phi}\frac{\partial q'}{\partial\lambda}+\frac{v}{a}\frac{\partial q_s}{\partial\phi}+\omega\frac{\partial q_s}{\partial p}=-C'. \qquad (19)$$

Now v, ω, q', and C' are perturbation quantities associated with the wave, and u_s and q_s are the basic state zonal wind and moisture fields.

One uses the above equation to substitute in the thermodynamic equation for the perturbation in the diabatic heating, $Q' = -LC'$. The derivation then proceeds exactly as in Eliassen and Palm's paper, but with θ replaced by $\theta + (L/c_p)q'$ and σ replaced by σ_m. The first step in the derivation yields the following relations for the wave energy flux,

$$[v\Phi]=[v^*\Phi^*]=\frac{u_sF_\phi^{m'}}{a\cos\phi}, \qquad (20a)$$

$$[\omega\Phi]=[\omega^*\Phi^*]=\frac{u_sF_p^{m'}}{a\cos\phi}, \qquad (20b)$$

where Φ is the perturbation geopotential, and

$$F_\phi^{m'}=a\cos\phi\left\{\frac{\partial u_s}{\partial p}\left(\frac{[v^*\theta^*+(L/C_p)v^*q^*]}{\sigma_m}\right)-[u^*v^*]\right\}, \qquad (21a)$$

$$F_p^{m'}=a\cos\phi\left\{\left(f-\frac{1}{a}\frac{\partial u_s}{\partial\phi}+\frac{u_s\tan\phi}{a}\right)\times\left(\frac{[v^*\theta^*+(L/C_p)v^*q^*]}{\sigma_m}\right)-[u^*\omega^*]\right\}. \qquad (21b)$$

We see that the vector $\mathbf{F}'_m = (F_\phi^{m'}, F_p^{m'})$ could be regarded as a generalized Eliassen–Palm flux for ageostrophic motion, if $\nabla\cdot\mathbf{F}'_m = 0$.

The next step in the derivation requires that the above relations be substituted into the wave-energy equation. When the wave condensation effects are included, the generalized result for ageostrophic motions is

$$\frac{u_s}{a\cos\phi}\nabla\cdot\mathbf{F}'_m=-\frac{L}{C_p}\left\{[\omega^*q^*]+\frac{1}{a\sigma_m}\frac{\partial q_s}{\partial\phi}\left(\left[v^*\theta^*+\frac{L}{C_p}v^*q^*\right]\right)\right\}. \qquad (22)$$

Thus $\nabla\cdot\mathbf{F}'_m$ is not, in general, zero. Again, this is because the hydrostatic and thermal wind equations do not generalize in the same way as the thermodynamic equation. However, if q_s is taken to be independent of ϕ and if we now invoke the quasi-geostrophic assumption, which allows us to neglect the vertical eddy fluxes, then Eq. (22) does reduce to $\nabla\cdot\mathbf{F}_m = 0$.

This second proof of the quasi-geostrophic Eliassen–Palm theorem is not as general as our first one because of the linearization assumption. However, it does show that two important aspects of the theorem in the "dry" case do carry over to the "moist" case. First, it makes no difference whether one assumes steady conservative waves or steady conservative mean flow: the two cases are equivalent (at least for small eddy amplitudes.) Of course in the moist case "conservative" means $Q_m = 0$ rather than $Q = 0$. Second, the second proof shows that the relationship between stationary wave energy flux and the Eliassen–Palm flux is maintained by our generalization. The second proof also shows that our generalization to the "moist" case only works under quasi-geostrophic conditions.

We now consider what happens to the non-acceleration theorem when condensation effects are included in the Eliassen–Palm flux. We note that even if they are not included, the exact converse of the Eliassen–Palm theorem does not hold, i.e., if $[\mathcal{F}] = [Q] = \nabla\cdot\mathbf{F} = 0$, the motions need not be steady, as the equations do allow nonsteady solutions, depending on the boundary conditions. However, the converse follows in the restricted sense that the conditions $[\mathcal{F}] = [Q] = \nabla\cdot\mathbf{F} = 0$ are *consistent* with the trivial solution $\tilde{v} = \tilde{\omega} = \partial[u]/\partial t = \partial[\theta]/\partial t = 0$. Thus in the "dry" case, if $\nabla\cdot\mathbf{F} = 0$ there is no explicit *internal* eddy forcing of $[u]$ and $[\theta]$. This restricted converse is generally referred to as the non-acceleration theorem of Charney and Drazin (1961).

When the equations are rewritten as in Eqs. (13), (14), (16) and (18), there are two terms which contain explicit internal eddy forcing, namely $\nabla\cdot\mathbf{F}_m$ and M. This might appear to be a complication compared to the conventional formulation where there is only a single such term, $\nabla\cdot\mathbf{F}$. However, what we really desire is a representation of the total eddy forcing—and $\nabla\cdot\mathbf{F}$ does not give us this anyway because some of the eddy forcing is hidden in the $\mathcal{F}$ and Q terms.

The advantage of our formulation is that it allows us to state a modified but more general form of the non-acceleration theorem, namely,

If $[\mathcal{F}] = [Q_m] = \nabla\cdot\mathbf{F}_m = 0$, and *if the moisture field is stationary*, i.e., $\dfrac{\partial[q']}{\partial t} = 0$, then there exists the trivial solution $\tilde{v}_m = \tilde{\omega}_m = \dfrac{\partial[u]}{\partial t} = \dfrac{\partial[\theta]}{\partial t} = 0$.

This means that, in states with stationary moisture fields, $\nabla\cdot\mathbf{F}_m$ does contain all the explicit internal forcing of the $[u]$ and $[\theta]$ fields. Such stationary states are of great interest—for example, they include the annual mean and seasonal extreme states (e.g., see Peixoto, 1972). Thus the function $\nabla \cdot \mathbf{F}_m$ retains the utility of $\nabla \cdot \mathbf{F}$ as a diagnostic of explicit internal eddy forcing for stationary states. Moreover, it has the fundamental advantage that it includes the effect of that portion of the eddy forcing which acts by modifying the large-scale condensation and the associated diabatic heating. This advantage makes $\nabla \cdot \mathbf{F}_m$ a more meaningful representation of the internal eddy forcing of $[u]$ and $[\theta]$ than $\nabla \cdot \mathbf{F}$, if moisture is present.

Under the conditions assumed in either of the above two theorems, the other eddy forcing function M need not be zero. However, both theorems do place a constraint on it. The constraint is the same in both cases. Applying either theorem to the moisture conservation equation, Eq. (16), we obtain

$$\boxed{M = [C - S].}$$

This corollary states that the eddy forcing of the moisture field must just balance the condensation associated with the larger-scale motions. Alternatively, the modified non-acceleration theorem could be stated, as follows:

> If $[\mathcal{F}] = [Q_m] = \nabla \cdot \mathbf{F}_m = 0$ and $M = [C - S]$, then there exists the trivial solution $\tilde{v}_m = \tilde{\omega}_m = \frac{\partial [u]}{\partial t} = \frac{\partial [\theta]}{\partial t} = \frac{\partial [q']}{\partial t} = 0.$

This alternate statement is closer in spirit to the conventional versions of the non-acceleration theorem. However, the utility of the modified form of the theorem is more readily apparent from our first statement of it.

In the more general case when the non-acceleration theorem does not apply but the motions are still stationary, the above corollary is replaced by the more general relation, derived from Eq. (5),

$$[C - S] = -[\omega] \frac{\partial q_s}{\partial p} - \frac{1}{a \cos\phi} \frac{\partial}{\partial \phi} (\cos\phi [v^* q^*]). \quad (23)$$

This equation gives us a way of calculating the condensation associated with the larger-scale motions from the motions themselves. The difference between this $[C - S]$ and M is a measure of the extent to which non-acceleration conditions do not occur in the moisture balance. We note that the derivation of Eq. (23) does not require that q_s be independent of latitude.

Finally we note that a Lagrangian formulation of our generalization of the non-acceleration theorem does not appear to be possible. This is because q is in fact not conserved (except in the trivial case $M = 0$). In particular, the assumption that $[q']$ is stationary and the conclusion that $[\theta]$ is stationary are strictly Eulerian statements with no obvious Lagrangian parallel.

4. Diagnostic calculations

To establish the usefulness of the above formulation, we need to justify the neglect of the latitudinally varying part of the static stability parameter σ' in Eq. (15). In the atmosphere most of the latitudinal variation is associated with q_s rather than T_s, so neglecting σ' is a better approximation when moisture effects are excluded. However, even when they are not, it is a good approximation for midlatitudes. To establish this we chose $\sigma_0 = \sigma_m$ (50°N, p), and then calculated the terms $\sigma_0[\tilde{\omega}_m]$ and $\sigma'[\omega]$ in Eq. (15). The σ_0 and σ' were calculated from Oort and Rasmusson's (1971) analyses, while $[\tilde{\omega}_m]$ and $[\omega]$ were calculated from the streamfunctions described in Section 4b.

We found that in both January and July, from 35 to 75°N, the second term was an order of magnitude smaller than the first. However, in low latitudes they were comparable in magnitude. Thus, diagnostic calculations based on $\mathbf{F}_m$ are most useful for the midlatitude region 35–75°N. Of course the quasi-geostrophic formulation will break down in low latitudes anyway.

For all of the results presented, unless otherwise stated, we used the definition of σ_0 given above. Latitude 50°N is a convenient choice for defining σ_0 because many important eddy statistics, such as $[v^*\theta^*]$ and $\partial/\partial\phi(\cos^2\phi[u^*v^*])$ peak at or near 50°N in both January and July.

All of the differences between the "moist" and "dry" versions of the Eliassen–Palm flux involve only the vertical component of the flux (see Eq. 12). Furthermore, these differences are significant only in the middle and lower troposphere because that is where the moisture is concentrated. In those layers the dry flux vector is already predominantly vertical (see Edmon *et al.*, 1980). Thus the pattern of the moist vector field is the same as that of the dry vector field and has been illustrated by Edmon *et al.* (1980). Of course the magnitude of the vertical component of the flux is changed, particularly in the lower troposphere. The changes in the magnitude of this component are directly reflected by the changes in the divergence. Therefore, we only need to illustrate the changes in the divergence of the flux. Also we note that our calculations are based on analyses of atmospheric eddies for which one cannot assume small eddy amplitudes. The significance of the Eulerian vector flux itself for such eddies is not clear; only its divergence has a clear-cut relation to the mean flow-eddy interaction problem when the eddy amplitudes are not small.

a. *Divergence of the modified Eliassen–Palm flux*

Using the definition of $\mathbf{F}_m$ in Eq. (12), we calculated $\nabla \cdot \mathbf{F}_m$ from Oort and Rasmusson's (1971) analyses. Their results are tabulated at eleven pressure levels, 50—100—200—300—400—500—700—850—900—

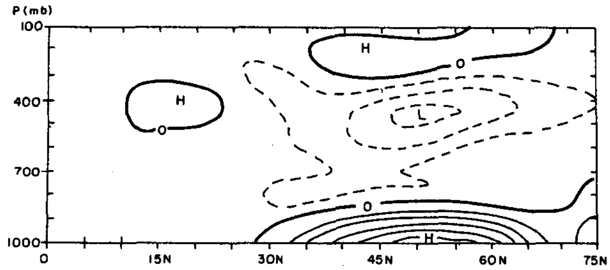

FIG. 1. Meridional cross section of the quasi-geostrophic Eliassen–Palm flux divergence for January, with eddy forcing of the condensation omitted. Solid curves are positive contours and dashed curves are negative contours. The zero contours, maxima (H), and minima (L) are indicated. The contour interval is 200 $m^2 s^{-2}$.

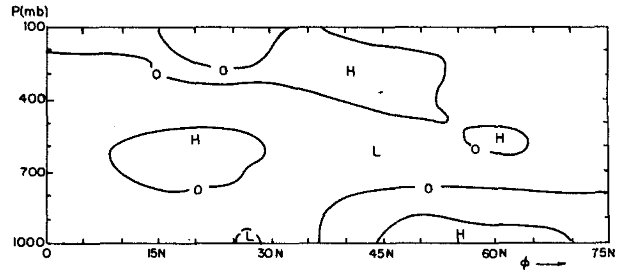

FIG. 3. As in Fig. 1 but for July.

950—1000 mb, and at every 5 degrees of latitude from 10°S to 75°N. We only considered the data from 0 to 75°N and linearly interpolated in order to have data at ten equispaced pressure levels from 100 to 1000 mb, inclusive. All derivatives were computed with a centered finite difference scheme. The eddy fluxes $[u^*v^*]$, $[v^*\theta^*]$, and $[v^*q^*]$ were computed by adding the separate contributions due to the stationary and transient eddies. For comparison, we also computed $\nabla\cdot\mathbf{F}$, i.e., the "dry" divergence, corresponding to $q' = q_s = 0$.

Figures 1 and 2 show the divergence in January, with the eddy effects on condensation omitted in Fig. 1, but included in Fig. 2. Figure 1 is qualitatively similar to Edmon *et al.*'s (1980) calculation of $\nabla\cdot\mathbf{F}$ for winter. Figure 2 shows that the condensation effects do not greatly modify the divergence maximum at the ground near 55°N. On the other hand, the convergence maximum in the midtroposphere near 45°N is about 50% stronger. (N.B., the contour interval in Fig. 2 is twice that in Fig. 1.) Also a new convergence maximum has appeared, at the ground near 30°N. This latter convergence maximum is more than double that of the now secondary convergence maximum in the upper troposphere. Thus the main effect of eddy forcing of condensation in January is a marked strengthening of the eddies' tendency to decelerate the zonal wind, especially below the mean jet.

Figures 3 and 4 show the divergence in July with the eddy effects on condensation omitted and included, respectively. Figure 3 is again similar to the Edmon *et al.* calculation for summer. Comparing Figs. 3 and 4 we see that the pattern of the divergence field is not greatly modified by the condensation effects. The only significant change in the pattern is that the convergence maximum near 30°N has been raised from the ground level to the 800 mb level. On the other hand the magnitude of the divergence and convergence maxima have been greatly increased. (N.B., the contour interval in Fig. 4 is twice that in Fig. 3.) The maximum divergence is about eight times stronger and the maximum convergence is about three times stronger when condensation effects are included. Thus, the major change when eddy forcing of condensation is included in July is a drastic strengthening of the eddy forcing of the zonal mean zonal wind. In fact, the maximum divergence in July is now about twice that in January.

Figures 5 and 6 show the divergence for the annual mean state, with eddy forcing of the condensation omitted and included, respectively. The changes caused by including the moisture effects are similar to those noted in January and July. A new convergence maximum has appeared near the ground at 30°N, and the convergence and divergence maxima are about two and one half times larger.

Finally we note that the vertical component of the Eliassen–Palm flux dominates the flux divergence even more in the moist case than in the dry case. In the moist case the maximum contributions of the vertical component to the magnitude of the divergence

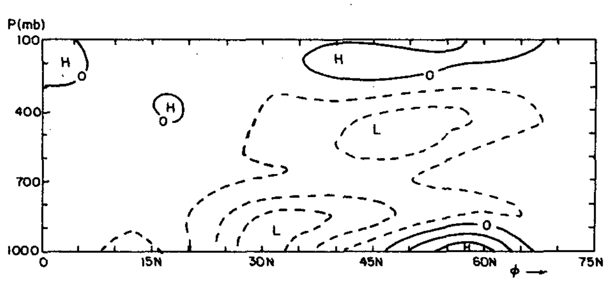

FIG. 2. As in Fig. 1 but with eddy forcing of the condensation included, and the contour interval is 400 $m^2 s^{-2}$.

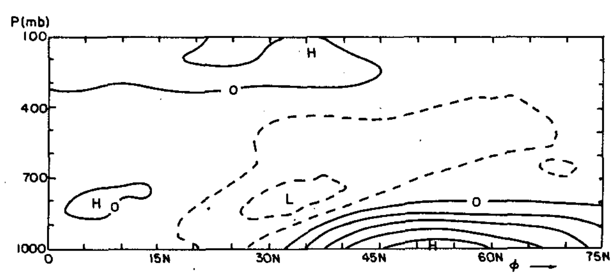

FIG. 4. As in Fig. 3 but with eddy forcing of the condensation included, and the contour interval is 400 $m^2 s^{-2}$.

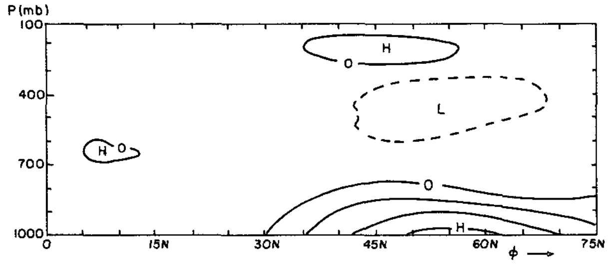

FIG. 5. As in Fig. 1 but for the annual mean.

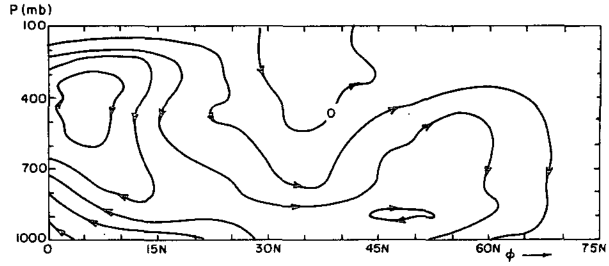

FIG. 7. Quasi-geostrophic residual streamfunction divided by $2\pi a^2/g$ for January, with eddy forcing of the condensation omitted from F. Solid curves are positive contours and dashed curves negative contours. The zero contour is indicated, and the contour interval is 100 m mb s^{-1}.

are 7 and 35 times larger, in January and July, respectively, than the maximum contributions of the horizontal component. Thus, the eddy forcing of the zonal mean wind and temperature fields is dominated by the eddy heat flux rather than by the eddy momentum flux.

b. *Modified residual mean meridional circulation*

We introduce the mass streamfunction ψ for the mean meridional circulation, defined by

$$\frac{\partial\psi}{\partial p} = \frac{2\pi a^2}{g}\cos\phi[v], \tag{24a}$$

$$\frac{1}{a}\frac{\partial\psi}{\partial\phi} = -\frac{2\pi a^2}{g}\cos\phi[\omega]. \tag{24b}$$

If we substitute for $[v]$ and $[\omega]$ from Eqs. (10) and (11), we identify the mass streamfunction for the residual mean meridional circulation

$$\tilde{\psi}_m = \psi - \psi_e, \tag{25}$$

where

$$\psi_e = \frac{2\pi a^2}{g}\cos\phi\,\frac{[v^*(\theta^* + (L/C_p)q^*)]}{\sigma_0}. \tag{26}$$

To compute ψ we used Oort and Rasmusson's (1971) balanced values for $[v]$, and integrated Eq. (24a) downwards. We assumed $[v] = \psi = 0$ at $p = 0$, and used a trapezoidal rule for the integration. The ψ_e was computed from Oort and Rasmusson's values for the eddy fluxes, and the residual streamfunction ψ_m was calculated from Eq. (25).

Figures 7 and 8 show the residual streamfunction, normalized by $2\pi a^2/g$, in January, for the "dry" and "moist" cases, respectively. Figure 7 is very similar to the winter "dry" result of Edmon *et al.*. Figure 8 shows that the direct cell in the residual circulation in mid-latitudes is greatly strengthened by including the eddy forcing of condensation in F, being about three times stronger when they are included. Also, the weak Ferrel cell in the upper troposphere near 35°N is much less prominent when condensation effects are included in F.

Figures 9 and 10 show the normalized residual streamfunction in July, for the "dry" and "moist" cases, respectively. Figure 9 is again very similar to the "dry" result for summer of Edmon *et al.* Figure 10 shows again that the direct cell in the residual circulation in midlatitudes is greatly strengthened by the inclusion of condensation effects in F, this time by a factor of about ten. (N.B., the contour intervals are different in Figs. 9 and 10.)

c. *Eddy forcing of the moisture field*

The eddy forcing function M (Eq. 17) was also calculated from Oort and Rasmusson's (1971) analyses. The results for January and July are shown in Figs. 11 and 12, respectively. The eddies tend to dry out the atmosphere at lower levels in the subtropics in both seasons, with the magnitude of the drying being a little larger in January. In midlatitudes the

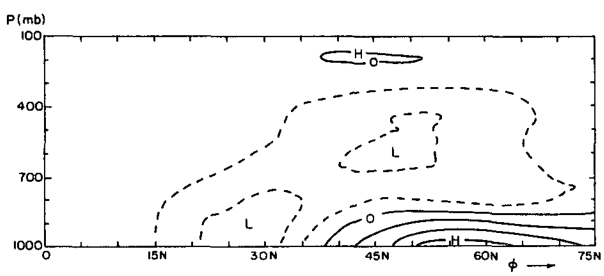

FIG. 6. As in Fig. 5 but with eddy forcing of the condensation included, and the contour interval is 400 m^2 s^{-2}.

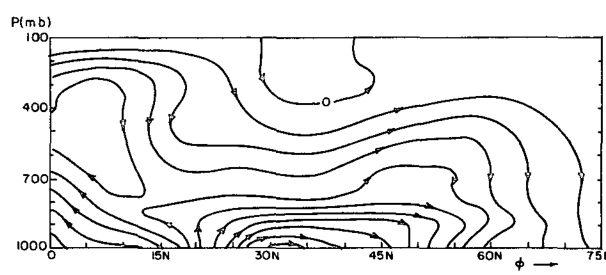

FIG. 8. As in Fig. 7 but with eddy forcing of the condensation included in F.

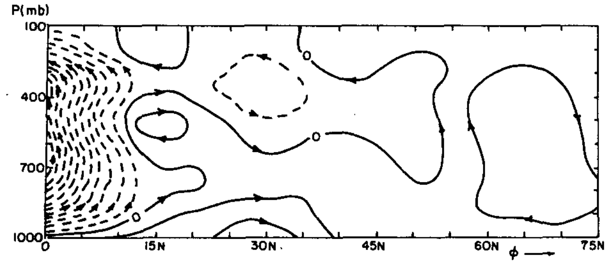

FIG. 9. As in Fig. 7 but for July and the contour interval is 30 m mb s^{-1}.

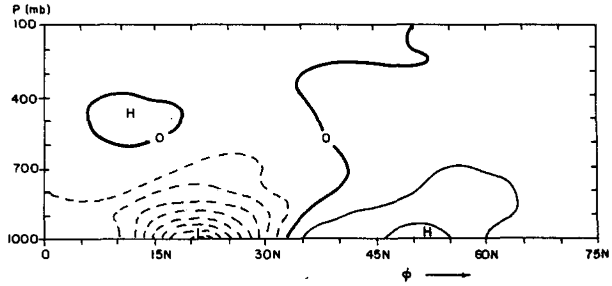

FIG. 11. Meridional cross section of the function describing the eddy forcing of the moisture field in January. Solid curves are positive contours and dashed curves are negative contours. The zero contours, maxima (H), and minima (L) are indicated. The contour interval is 3×10^{-6} g Kg^{-1} s^{-1}.

eddies tend to moisten the atmosphere at lower levels. In January the moistening effect is rather weak, with a peak at 50°N. In July the moistening effect is much stronger and spreads into much higher latitudes, with several peaks spread out over mid and high latitudes.

The forcing in both seasons is considerably more than it would be if just the effect of the eddy meridional flux of moisture were included. This is because the rising branch of the Ferrel cell induced by the eddy heat fluxes coincides with convergence of the eddy flux, and the sinking branch coincides with divergence. For example, this effect increases the maximum in the midlatitude moistening by a factor of 2 in January and by a factor of 4 in July.

d. Condensation associated with the large-scale motions

Another interesting quantity to calculate is $[C - S]$, which measures the condensation associated with just the larger scale motions. The interpretation of $[C - S]$ follows directly from the equation of moisture balance and does not require the concept of the transformed Eulerian-mean equations. Nevertheless it does not appear to have been calculated previously. In addition it is of interest because the difference between $[C - S]$ and M is a measure of the extent to which non-acceleration conditions do not hold in the moisture field.

Using Oort and Rasmusson's (1971) analyses $[C - S]$ was calculated from Eq. (23). Since the derivation of Eq. (23) does not require the assumption that q_s is independent of latitude, we included the latitudinal variations of q_s in these calculations. These variations have very little effect north of 25°N. The results for January and July are shown in Figs. 13 and 14, respectively. (The values near the equator are not accurate because of our use of the quasi-geostrophic approximation.)

Figures 13 and 14 show that the condensation has maxima near the equator and in midlatitudes, and a minimum in the subtropics. The minimum in the subtropics is stronger in January while the maximum in midlatitudes is about equally strong in both seasons. The extreme values all occur in the lower troposphere, and in July a second midlatitude maximum has formed at the ground, displaced towards the equator compared to the other midlatitude maximum.

Comparing $[C - S]$ with M (Figs. 11 and 12; note the different contour intervals), we see considerable differences between the two fields. In particular, the strength of the eddy forcing is considerably stronger than the condensation. Typically, M is two to three times stronger than $[C - S]$ in January and five to six times stronger in July. Of course, the equatorial maximum in the condensation does not appear at all in the eddy forcing field. Clearly non-acceleration conditions are not at all satisfied in the moisture balance equation in the troposphere.

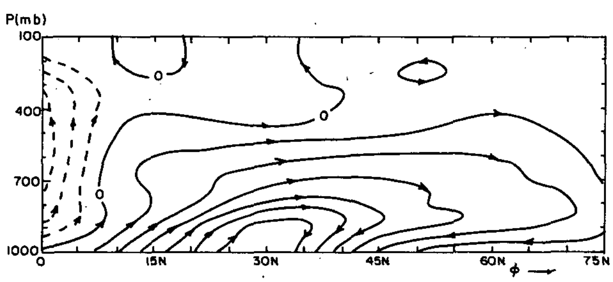

FIG. 10. As in Fig. 9 but with eddy forcing of the condensation included, and the contour interval is 100 m mb s^{-1}.

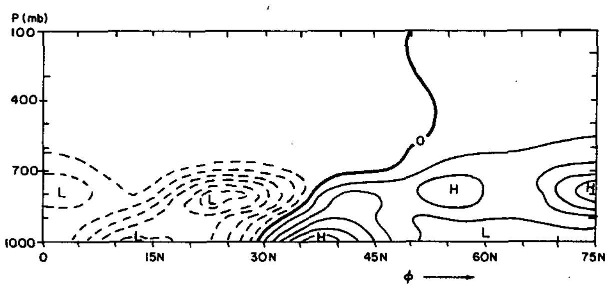

FIG. 12. As in Fig. 12 but for July.

5. Summary

We have defined a more general Eulerian form of the quasi-geostrophic Eliassen–Palm flux, Eq. (12). This form allows us to restate the Eliassen–Palm and non-acceleration theorems in ways that explicitly recognize the effect of eddy forcing on condensation heating. The generalization preserves the form of the relationship between the stationary wave energy flux and the Eliassen–Palm flux, but loses the relationship with potential vorticity.

Our calculations of the modified flux divergence show that the eddy forcing is much stronger than was indicated by calculations using the original form of the flux. The difference provides a good idea of the limitations inherent in using dry models to study the general circulation and the role of eddies in producing the general circulation. We also found that condensation effects lead to stronger eddy forcing in summer than in winter. The changes in the flux divergence are all due to the enhancement of the eddy heat flux component of the Eliassen–Palm flux. In an overall sense this component is much more important than the eddy momentum flux component in forcing the zonal wind and temperature fields.

Our reformulation also leads to a function describing the total eddy forcing of the moisture field, Eq. (17). Calculations of this function show the expected tendency to dry out the subtropical atmosphere and to moisturize middle and high latitudes. However the effect is greatly enhanced because of the effect of the Ferrel cell induced by the eddy heat fluxes. In addition we calculated the condensation forced by the large-scale motions and found that it differs substantially from the eddy forcing. This difference illustrates via the moisture field that non-acceleration conditions do not apply in the troposphere.

One must keep in mind that our modified form of the Eliassen–Palm flux still omits other implicit effects of eddy forcing. In particular, the effect on the frictional forces in midlatitudes near the surface may also be important (e.g., see Crawford and Sasamori, 1981). It would be valuable if generalizations of the eddy forcing functions could be developed which also included such effects.

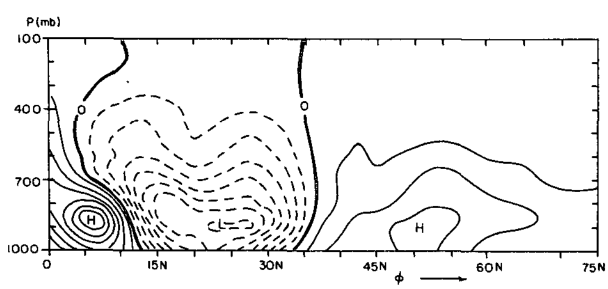

FIG. 13. Meridional cross section of the condensation forced by the larger-scale motions in January. Solid curves are positive contours and dashed curves are negative contours. The zero contours, maxima (H), and minima (L) are indicated. The contour interval is 1×10^{-6} g kg^{-1} s^{-1}.

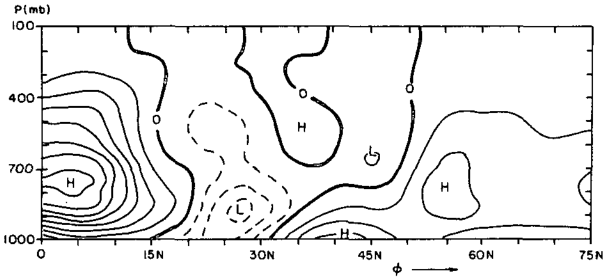

FIG. 14. As in Fig. 13 but for July.

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APPENDIX

Symbols

a	mean radius of the earth
C	total condensation rate
f	Coriolis parameter (latitude dependent)
g	acceleration of gravity
L	latent heat of vaporization
p	pressure
q	specific humidity
Q	diabatic heating rate per unit mass
R	gas constant
t	time
T	temperature
T_s	temperature of the basic state (dependent on pressure and latitude)
u	zonal velocity
v	meridional velocity
c_p	specific heat at constant pressure
ϕ	latitude
λ	longitude
σ	basic state static stability parameter, $dT_s/dp - (R/c_p)T_s/p$
θ	$T - T_s$
ζ	quasi-geostrophic potential vorticity
ω	vertical pressure velocity, dp/dt
$\mathcal{F}$	zonal frictional force per unit mass
$[x]$	zonal mean of x
x^*	deviation from zonal mean, $x - [x]$

REFERENCES

Andrews, D. G., and M. E. McIntyre, 1976: Planetary waves in horizontal and vertical shear: The generalized Eliassen–Palm relation and the mean zonal acceleration. *J. Atmos. Sci.*, **33**, 2031–2048.

——, and ——, 1978: An exact theory of nonlinear waves on a Lagrangian-mean flow. *J. Fluid Mech.*, **89**, 609–646.

Boyd, J., 1976: The noninteraction of waves with the zonally averaged flow on a spherical earth and the interrelationship of eddy fluxes of energy, heat and momentum. *J. Atmos. Sci.*, **33**, 2285–2291.

Charney, J. G., and P. G. Drazin, 1961: Propagation of planetary scale disturbances from the lower into the upper atmosphere. *J. Geophys. Res.*, **66**, 83–109.

Crawford, S. L., and T. Sasamori, 1981: A study of the sensitivity of the winter mean meridional circulation to sources of heat and momentum. *Tellus*, **33**, 340–350.

Dunkerton, T. J., C.-P. Hsu and M. E. McIntyre, 1981: Some Eulerian and Lagrangian diagnostics for a model stratospheric warming. *J. Atmos. Sci.*, **38**, 819–843.

Edmon, Jr., H. J., B. J. Hoskins and M. E. McIntyre, 1980: Eliassen–Palm cross sections for the troposphere. *J. Atmos. Sci.*, **37**, 2600–2616. [See also Corrigendum, *J. Atmos. Sci.*, **38**, 1115, 1981, especially the next to last item.]

Eliassen, A., and E. Palm, 1961: On the transfer of energy in stationary mountain waves. *Geofys. Publ.*, **22**(3), 1–23.

Holton, J. R., 1974: Forcing of mean flows by stationary waves. *J. Atmos. Sci.*, **31**, 942–945.

Karoly, D. J., 1982: Eliassen–Palm cross sections for the Northern and Southern Hemispheres. *J. Atmos. Sci.*, **39**, 178–182.

Oort, A. H., and E. M. Rasmusson, 1971: Atmospheric circulation statistics. NOAA Prof. Pap. No. 5, U.S. Dept. of Commerce, Washington, DC, 323 pp.

Peixoto, J. P., 1972: Pole-to-pole water balance for the IGY from aerological data. *Nordic Hydrol.*, **3**, 22–43.

Salustri, G., and P. H. Stone, 1983: A diagnostic study of the forcing of the Ferrel Cell by eddies with latent heat effects included. *J. Atmos. Sci.*, **40**, 1101–1109.