A Preliminary Numerical Simulation of a Shower

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ABSTRACT

A quasi-one-dimensional, time-dependent and precipitating cumulus cloud model incorporated with a time-dependent PBL model has been used to simulate the precipitation record of a local summer afternoon shower induced by sea breeze. The system is so designed that it is subjected only to the variations of the parameters pertaining to the PBL. Through the use of this system, we have obtained relatively good agreement between the observed and model-produced rainfall pattern, peak rainfall intensity and total rainfall; we are also confident of reproducing the time of onset of the shower. It has also been found that the PBL influences the precipitation characteristic of a shower in a complicated way. For a given water vapor content, greater thickness of the PBL, which implies a greater heat supply to the cloud activity above, will delay the onset of a shower, reduce its total rainfall and produce multi-peaked intensities. When the thickness is increased to a critical value, no shower can be produced. If the thickness is increased further, a shower can be produced again. For a given thickness, increasing water vapor will greatly expedite the onset of a shower which also has greater total rainfall and multi-peaked rainfall intensities. Only when the heat content is accompanied by a proper water vapor content can a shower of single-peaked rainfall intensity be produced.

In this model, we have introduced an immediate environmental (IE) region which allows the cloud to generate its own inner or immediate environment, enriched by water mass in both liquid and vapor form. Our work has shown that the IE region plays an important role in enhancing cloud development, delaying the showers to afternoon hours and in providing a recycling process of water mass so that heavy rainfall can be produced by this simple model.

II. Introduction

It is now well known that the conditions in the lower atmosphere or the PBL are vital for the initiation, development and maintenance of the overlying cloud activity (Matsumoto et al., 1967; Warner and Telford, 1967; Betts, 1976; Wang, 1979). However, in the last decade or so, many studies (e.g., Takahashi, 1973, 1974, 1975, 1981; Soong, 1974; Cotton et al., 1976 and Wang, 1979) on the formation and development of warm rain are concentrated on the influences of the microphysics of the raindrops while very few are concerned on the effects of the meso-scale systems in the lowest atmosphere. Acknowledging the importance of the effects of the meso-scale systems such as sea breeze, Cotton et al. have tried to make joint use of a sea breeze model (Pielke, 1974) and a cloud model (Cotton, 1975) to reproduce some observed showers in Florida. In their work, they computed the change of the morning sounding due to the sea breeze up to the time around noon when the cloud activity was most intensive. These theoretically predicted soundings were then used as the reference soundings to simulate the observed clouds and rainfall. From the predicted soundings, we see that the most obvious changes are at the lowest part of the sounding from 1000 to about 900 hPa, and the curve configuration of the changed part on the sounding is quite similar to that of the planetary boundary layer (PBL). This has led us to think about the replacement of the sea breeze model by a simple PBL model, since the computational work is quite cumbersome when a sea breeze model is used.

Therefore, Wang (1979) has proposed another scheme. In addition to the incorporation of the precipitation mechanism, he assumed that the surface layer in his cloud model (Wang, 1983) possesses the characteristics of a PBL which has constant depth. Making use of this model, Wang (1979) tried to simulate an observed shower in Taiwan. The reproduced rainfall pattern, the total rainfall and the peak intensity, were quite close to the observation; however, the time of onset of the shower could not be predicted—a result of the assumption of constancy of the thickness of the PBL. Because of this assumption, the zero time of the experiments could not be defined. Consequently, the time of onset of a model-produced shower could not be determined. Furthermore, this assumption does not appear realistic over land.

As an improvement, in this study we shall formulate and incorporate a very simple one-dimensional, time-dependent PBL model in our cloud model (Wang, 1983) to simulate the observed precipitation curve and to predict the time of onset of a summer shower. Also, we wish to find out some of the influences of the PBL upon the production of a shower and the characteristics of the precipitation. Furthermore, we hope to find out...
the capability of the present simple model in producing a shower with high rainfall rate and total rainfall, which, a pure one-dimensional model cannot (Takahashi, 1974).

2. The models

a. The cloud model

1) THE CONCEPT

The cloud model used in this study is similar to the one proposed by Wang (1983), but the precipitation mechanism is now incorporated. Furthermore, in addition to the original three regions (the core region and the immediate environment (IE) and the far environment (FE) regions) a PBL region has been added below. The configuration of the model together with the sequence of events are shown in Fig. 1. In Wang's (1983) paper, the description of the model was deleted and the reader was referred to Wang's (1979) paper. Since the circulation of Wang (1979) is limited, we give a description of our model in this paper. Furthermore one of the unique things in our model is the

Fig. 1. Geometry of a model cloud. A, B and C, corresponding to time $t_1$, $t_2$, $t_3$ respectively, illustrate the sequence of events. Core is the region housing the updraft of constant radius $R_c$; IE, immediate environment of variable radius $R_f$; FE, far environment region; PBL, planetary boundary layer; $R_f$, radius of a point far away from the axis; $Z_c$, height of surface layer; $Z_p$, height of top of PBL; $Z_b$, height of cloud base; $Z_x$, a reference level; $Z_r$, maximum mass flux $M_f$; $Z_t$, cloud top; $Z_c$, height at which the vertical velocity in the core first vanishes and $Z_{top}$, height of the upper boundary. In A, B and C the configuration of the cloud at the given $t$ is shown on the right and $M_f$ profile, on the left. In A, the vertical mass convergence in the core causes the detrainment of the moist air to form the initial IE region. In B, the drawing is three-dimensional, showing that the IE region surrounds the core. A maximum on the $M_f$ profile appears at $Z_r$. Accordingly, the air in the IE region below $Z_r$ is now entrained back into the core region. In C, it is shown that the rain drops in the IE region, detrained from aloft, represented by the dots, and the evaporationally chilled IE air are moving downward. The four small up-and-down arrows indicate possible gravity oscillations. The broad arrows in the core region indicate the main updraft, while those in the PBL indicate that warm and moist air from far away is supplied to the updraft via the PBL. The other thin arrows indicate schematically the directions of the flow. $W_f$ (dashed line) indicates the vertical velocity in the IE region. For details, see text.
introduction of an immediate environment (IE) region (which is not the same as, but somewhat similar to, that proposed by Lopez, 1973; cf. Wang, 1983). As we shall show, the IE region allows the cloud to generate its own inner environment, enriched by water mass in both liquid and vapor form. Thus, it plays an important role in enhancing the cloud development, delaying the showers to afternoon hours and in providing a recycling process of water mass, so that heavy rainfall can be produced by this simple model. A more detailed description of the IE region is given below.

Let us assume a cold cloud is formed on top of a plume (Morton, 1958; Wang, 1983) which is rooted in the PBL as shown on the right side of Fig. 1a. At the early stage \( t = t_1 \), the vertical velocity of the updraft in the cloud is characterized by the monotonic decrease, as shown by the mass flux profile on the left side of Fig. 1a. This velocity distribution results in vertical mass convergence, and thus horizontal mass divergence in the upper part of the updraft; producing dynamic detrainment of the cloudy air out from the core region, as indicated by the outward-pointing arrow at the level \( Z_R \) in Fig. 1a. This gives rise to the initial IE region. As the cloud continues developing, the IE region continues building upward, following the higher reaches of the updraft. If the development persists, continuous release of latent heat may alter the profile of the updraft so as to produce a maximum aloft, say, at the level \( Z_{fe} \) as shown in Fig. 1b. While the dynamic detrainment process produces or expands the existing IE region aloft, the dynamic entrainment process below \( Z_R \) entrains air from the IE region, which is moist because it was originally detrained from the core region. This entrainment process then causes the outer boundary of the IE region to move inward as indicated by the arrow D in Fig. 1b, which was outward-pointing at \( t = t_1 \) in Fig. 1a. One may think at this moment that if this process persists, the IE region at \( Z_R \) shown in Fig. 1b should vanish. This may not be the case, however. Once the air is detrained out of the core region into the IE region, it is no longer supported by the updraft from below, but it may keep rising for a period at the expense of its residual kinetic energy. When the drier air of the FE region is brought in by the eddy exchange of mass across the outer boundary of the IE region, some of the liquid water droplets will evaporate. Sooner or later, the evaporatively chilled IE air will start to move downward together with its liquid water, as indicated by the small arrow--dots in the IE region in Fig. 1c. Hence, if the replenishment of the liquid water mass from aloft is adequate, the lower part of the IE region will not vanish.

Now, we see that the position of the outer boundary, and thus the horizontal dimension of our IE region, changes with time and height; controlled by the dynamics in both the IE and the core region. Also, we see that the functions of the IE region are: i) to act as a storage for the moist air detrained from the core region for later demand; ii) to provide a passage through which the water drops detrained from aloft can fall to lower levels where they may be entrained back into the core without excessive depletion in mass; and iii) to protect the core region.

As shown in Fig. 1c, the core region houses the updraft, and the FE region represents the vast far environment in which the cloud activity is embedded. Together with the IE and PBL region, these four regions constitute a convective cell. In Fig. 1c of the previous works of this author (Wang, 1979, 1983), a lateral boundary has been drawn at \( r = R_F \), which means that the convective cell is closed. The downward compensating motion in the FE region is deduced from the equation of continuity. After several test runs, it has been found that if the radius \( R_F \) is much larger (say, thirty times) than the radius \( R_C \) of the core region, the development of the clouds is not affected appreciably by the weak, compensating downward motion. Therefore, in the experiments the downward motion in the FE region is ignored, and the stratification in the FE region stays invariant and serves as reference only. This makes the existence of a lateral boundary at \( R_F \) insignificant and the FE region or the convective cell can be considered open in the lateral direction. In the present study, therefore, we simply assume that the convective cell is open and the downward motion in the FE region is negligible. Thus, no boundary is drawn at \( R_F \) as depicted in Fig. 1c. The dynamic consequence is that there is no force depressing and no air mass penetrating the top of the PBL region if we further ignore the mixing process on top of the PBL. These assumptions greatly simplify the formulation of the PBL model.

Figure 1c shows that the PBL region is under the vast FE region and the core region. The initial stratification is the same in all regions. After the convection is initiated, the change of the parameters in the PBL region below the FE region is governed by a PBL model, while the change in the core region below the cloud base (and IE region if it extends to below \( Z_R \)) is governed by the dynamics of both the cloud from above and the PBL from all sides, as shown by the heavy arrows in Fig. 1c. This is why the short horizontal lines at \( Z_p \) and \( Z_i \) in the core region and the outer boundary of the core region below \( Z_R \) are drawn as dash lines. Furthermore, we also see that the PBL functions as a duct for the moist surface air supplied from far away to the cloud above, as shown by the horizontal heavy arrows in the mixed layer shown in Fig. 1c.

We follow Kessler's (1969) concept in dividing the liquid water content into cloud water and rain water, the latter produced through cloud conversion and accretion processes. No icing process is considered, however. In an unsaturated environment, cloud water evaporates instantly, while the evaporation rates of rain drops vary. Furthermore, we assume that cloud water moves with the air in any direction, while rainwater moves at terminal velocity.

Finally, we should notice that due to the simplicity
of the cloud model, many important factors such as wind shear cannot be taken into consideration. The application of the cloud model is then somewhat limited.

2) GOVERNING EQUATIONS

Basic equations. Since the model has circular geometry, cylindrical coordinates are used. The independent variables are \( r, \theta \) and \( z \), where \( r \) is the radial distance, \( \theta \) the azimuthal angle and \( z \) is pointing upward. The basic equations are:

\[
\frac{\partial \omega}{\partial t} = -u \frac{\partial \omega}{\partial r} - w \frac{\partial \omega}{\partial z} + g \left[ \frac{T_v - T_{v'}}{T_{v'}} - q_c - q_R \right],
\]

\( (2.1) \)

\[
\frac{\partial \theta}{\partial t} = -u \frac{\partial \theta}{\partial r} - w \frac{\partial \theta}{\partial z} + \pi \left[ \Delta G - (1 - \Delta)(E_C + E_R) \right],
\]

\( (2.2) \)

\[
\frac{\partial q_v}{\partial t} = -u \frac{\partial q_v}{\partial r} - w \frac{\partial q_v}{\partial z} - \left[ \Delta G - (1 - \Delta)(E_C + E_R) \right],
\]

\( (2.3) \)

\[
\frac{\partial q_c}{\partial t} = -u \frac{\partial q_c}{\partial r} - w \frac{\partial q_c}{\partial z} + \Delta G - P_R - P_C - (1 - \Delta)E_C,
\]

\( (2.4) \)

\[
\frac{\partial q_R}{\partial t} = -u \frac{\partial q_R}{\partial z} - w \frac{\partial q_R}{\partial z} + \frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho \bar{v} q_R \right) + P_R + P_C - (1 - \Delta)E_R,
\]

\( (2.5) \)

\[
\frac{\partial}{\partial r} (\rho w r) + \frac{\partial}{\partial z} (\rho w z) = 0,
\]

\( (2.6) \)

where

\( u, w \) Radial and vertical velocity, respectively

\( g \) Gravity acceleration

\( T_v \) Virtual temperature and \( T_{v'} \), that of FE region

\( \theta \) Potential temperature

\( q_v \) Water vapor mixing ratio

\( q_c \) Cloud water mixing ratio

\( q_R \) Rain water mixing ratio

\( \frac{\pi}{P} = \left( \frac{L}{C_p} \right) \left( \frac{1000}{P} \right) \)

\( \Delta \) An identifier to be defined later

\( G = -W \frac{\partial q_v}{\partial z} \)

\( E_C \) Rate of evaporation of cloud water

\( E_R \) Rate of evaporation of rain water

\( P_C \) Rate of production of rain water due to cloud conversion process

\( P_R \) Rate of production of rain water due to accretion process

\( \rho, \rho(z) \), density of the moist air

\( \bar{v} \) Terminal velocity of the rain drops;

while \( E_C, E_R, P_C, P_R \) take the forms suggested by Kessler (1969).

In arriving at the above system from the basic hydrodynamic equations, the following conditions or assumptions have been considered:

1) no mean motion,
2) Boussinesq's approximation,
3) no pressure perturbation,
4) no molecular viscosity and diffusion,
5) vertical expansion is allowed.

In this study, the radial equation of motion is neglected but the radial motion in other equations is retained. The radial velocity will be computed from the equation of continuity. In order to integrate (2.1)–(2.6) for the \( i \)th region from its inner boundary at \( r_1 \) to the outer boundary at \( r_2 \) to obtain the mean of the variables, we let \( \bar{A} \) denote any variable in a given region and

\[
\bar{A} = \bar{A} + \bar{A}' = \bar{A} + \bar{A}''
\]

where \( \bar{A} \) denotes the areal mean at any given level, \( \bar{A}' \) its departure; \( \bar{A} \) the azimuthal mean at \( r_1 \) or \( r_2 \), \( \bar{A}'' \), its departure, respectively; \( r_1 \) is the radius of the inner boundary and \( r_2 \) is the radius of the outer boundary of the given region.

\[
\bar{A} = \frac{1}{2\pi} \int_0^{2\pi} A dx \] at \( r_1 \) or \( r_2 \)

\[
\bar{A} = \frac{2}{r_2^2 - r_1^2} \int_{r_1}^{r_2} A r dr
\]

\( r_1 \) or \( r_2 \)

\[ = \frac{1}{\pi (r_2^2 - r_1^2)} \int_0^{2\pi} \int_{r_1}^{r_2} A r dr dx \]

Integrating (2.1)–(2.6) with the aid of (2.7), we obtain

\[
2\rho(r_2 \bar{u}_2 - r_1 \bar{u}_1) + \frac{\partial}{\partial z} [\rho \bar{w}(r_2^2 - r_1^2)] - F_i
\]

\( (2.8) \)

\[
\frac{\partial A_i}{\partial t} + \frac{2}{r_2^2 - r_1^2} [(r_2 \bar{u}_2 \bar{A}_n - r_1 \bar{u}_1 \bar{A}_n) + \frac{1}{\rho(r_2^2 - r_1^2)} \frac{\partial}{\partial z} \times [(\rho \bar{w}_i \bar{A}_n + \rho \bar{w}_i A_i)(r_2^2 - r_1^2))] = F_i.
\]

\( (2.9) \)

Here, (2.8) is the equation of continuity and (2.9) is the general form of the prognostic equation for any one of the five variables \( w, \theta, q_v, q_c \) and \( q_R \) in the \( i \)th region. The right-hand term \( F_i \) represents the areal average of the sources and sinks for the corresponding variable. On the left side of (2.9), \( \bar{u}, \bar{A} \) are the dynamic entrainment of \( A \) due to \( \bar{u} \), the axisymmetric, organized
radial flow deduced from (2.8). The \( \vec{u}' \vec{A}' \) is the unorganized radial exchange due to eddy mixing across a boundary, which can only be approximated. To convert (2.8) and (2.9) into two sets of equations for the core and the IE region, we introduce the boundary conditions:

\begin{align*}
\text{(2.10a) } & \quad w = 0 \text{ at } z = z_{\text{top}}, \\
\text{(2.10b) } & \quad u = 0 \text{ at } r = r_F \text{ (very far away)},
\end{align*}

and let the eddy terms take the following forms:

\[ u' \vec{A}' = -(\nu_r + \beta|w_2 - w_1|)(A_2 - A_I), \]

\[ -\frac{1}{\rho} \frac{\partial}{\partial z} \left( \rho w' \vec{A}' \right) = \nu_z \frac{\partial^2 A}{\partial z^2}, \]

where \( \nu_r \) is the eddy mixing coefficient due to the pre-existing atmospheric turbulence, \( \beta \) is that due to the turbulence created by the cloud itself, and \( \nu_z \) the conventional vertical eddy mixing coefficient.

Using the subscripts \( c, l \) and \( f \) to denote the variables in the core, IE and FE regions, and \( c, l \) and \( f \) to denote the inter-regional boundaries between the core and IE regions and between IE and FE regions, respectively, we get the following equations.

i) For the core region, the mass continuity and the prognostic equations are:

\[ \vec{u}_f = -\frac{r_c}{2\rho_c} \frac{\partial}{\partial z} (\rho_c w_c), \]

\[ \frac{\partial A_c}{\partial t} = -\frac{r_c}{\rho_c} \frac{\partial A_c}{\partial z} - \frac{2}{r_c} \vec{u}_c (A_{cl} - A_c) + F_c \]

\[ + \nu_z \frac{\partial^2 A_c}{\partial z^2} + \frac{2}{r_c} (\nu_r + \beta|w_1 - w_c|(A_I - A_c), \]

where \( F_c \) is given by

\[ F_c = \begin{cases} 
T_e - T_{ef} - q_e - q_R & \text{if } A_c = W_c, \\
\pi[A \Lambda - (1 - \Lambda)(E_c + E_R)c], & \text{if } A_c = \Theta_c, \\
\Lambda[A \Lambda - (1 - \Lambda)(E_c + E_R)c], & \text{if } A_c = q_c, \\
1 - \rho \frac{\partial}{\partial z} (\rho V q_R) + P_R + P_c - (1 - \Lambda)E_R c, & \text{if } A_c = q_R.
\end{cases} \]

ii) For the IE region, the continuity equation has been integrated from \( r = 0 \) to \( r = r_f \) to take the following form for a given \( z \),

\[ \frac{\partial}{\partial t} (r_f^2 - r_c^2) = -\frac{1}{\rho_c} \frac{\partial}{\partial z} (\rho_c w_c r_c^2) \]

\[ -\frac{1}{\rho_f} \frac{\partial}{\partial z} \left( \rho w'(r_f^2 - r_c^2) \right). \]

This integration is done because \( (r_f^2 - r_c^2) \) is needed in the following prognostic equation. In (2.16), the zero quantity \( \partial r_{cl}/\partial t \) is added. Also, \( w' = w_1 - \vec{V}_I \), and \( \vec{V}_I \) is the mean terminal velocity (positive downward) of the rain drops in the IE region. The prognostic equation for \( A_I \) is

\[ \frac{\partial A_I}{\partial t} = -\frac{r_f}{r_f^2 - r_c^2} \frac{\partial}{\partial z} (A_{cl} - A_I) + \nu_z \frac{\partial^2 A_I}{\partial z^2} + \frac{2}{r_f^2 - r_c^2} [r_c(\nu_r + \beta|w_c - w_1|(A_I - A_c) \]

\[ - r_f(\nu_r - \beta w_1)(A_F - A_I)), \]

where \( F_I \) are the same as (2.15) if the subscript \( c \) in (2.15) is replaced by \( I \). In (2.14) and (2.17), \( A_{cl} \) takes the upstream value. That is,

\[ A_{cl} = A_c, \quad \text{if } \vec{u}_c > 0 \]

\[ A_{cl} = A_I, \quad \text{if } \vec{u}_c < 0. \]

Cotton (1975) notes that (2.18) will cause over-dilution and hence under-predict cloud height even when the eddy mixing process is completely deleted from the model. Since there is an IE region in our model, (2.18) seems to be a good assumption.

In solving (2.14), we should also be cautious about the availability of the IE air. When dynamic entrainment prevails at some level in the core region, the entrained air will come from the IE region; however, if the IE region does not exist at that time and level, or there is not enough stored-air in the IE to meet the demand, the entrained air must be totally or partially supplied by the FE region. Therefore, some care must be taken in computation. In case of stored-air insufficiency, we compute the deficit and extract it from the FE region. In this case, the IE region vanishes, and we set \( r_f = r_c \).

**Thermodynamic adjustment.** In the prognostic equations for thermal energy and water mass, there are terms containing an identifier \( \Lambda; \Lambda = 1 \) indicates condensation and \( \Lambda = 0 \), evaporation. In the course of computations, we assign

\[ \Lambda = 1, \quad \text{if } w > 0 \text{ and } q_e > q_{ew}, \]

\[ \Lambda = 0, \quad \text{if } w \leq 0 \text{ or } q_e \leq q_{ew}. \]

This means that the changes due to condensation are computed with the other terms in the prognostic equations, while evaporation changes are delayed to the final stage of each iteration. We call this stage the thermodynamic adjustment stage.
When the cloud water content alone is not sufficient to saturate the parcel, the relevant variables are given as
\[ q' = q_c + q_e + E_R \delta t \]
\[ q'_e = 0, \text{ (all cloud droplets are evaporated)} \]
\[ T' = T - \frac{L}{C_p} (q_c + E_R \delta t) \]
\[ \theta' = T'(1000/p)^k, \quad (k = R/C_p) \]
\[ q'_R = q_R - E_R \delta t \]
\[ E_R = -1.93 \times 10^{-6} N_0^{2/3} q_R (q_c + q_e - q_{ua}) \]
where the primes indicate the adjusted quantities and \( T' \) is the temperature (greater than wet-bulb temperature \( T_w \)) of a chilled unsaturated parcel containing no cloud droplets but where rain drops may exist.

Otherwise,
\[ T_w = T + \Delta T \]
\[ q'_c = q_c + \Delta q_c \]
\[ q'_e = q_e - \Delta q_e > 0 \]
\[ \theta' = T' \left( \frac{1000}{P_0} \right)^k, \quad k = R/C_p \]
where
\[ \Delta T = C \left[ \frac{C_p}{L} + \frac{0.622 f(T)}{P_0 - e(T)} \right]^{-1} \]
\[ \Delta q_c = - \frac{C_p}{L} \Delta T, \]
\[ C = \frac{q_c - q_{ua}(T)}{< 0, \text{ condensation}} \]
\[ f(T) = \frac{5.31}{T^2} (1296 - T)e(T), \]
\[ e(T) = 6.11 \times \exp \left[ 25.22 \left( 1 - \frac{273}{T} \right) \left( \frac{273}{T} \right)^{5.31} \right] . \]
The derivation of these expressions is given in the Appendix.

b. The PBL model

1) THE CONCEPT

As mentioned before, we wish to make use of a PBL model in lieu of the sea breeze model (Cotton et al., 1976) to simulate the changes in the lowest part of the reference atmosphere in the simulation of a shower. The PBL model that we shall formulate will be a very simple one. This is because the development of a PBL under various conditions is still a complicated problem which is currently under intensive study (e.g., Ogura and Cho, 1974; Deardorff, 1975; Betts, 1976; Mahrt and Lenschow, 1976; Gutman and Melgarejo, 1981; McNider and Pielke, 1981; Johnson, 1981; Kaimal et al., 1982; Han et al., 1982). In the present study, we intend to study only the response of the clouds to the PBL, not the influences of the overlying clouds or rough terrain, upon the PBL. Furthermore our cloud model is a simple one which is certainly not compatible with a complicated PBL model. Hence, a simple PBL model will be formulated and incorporated in our cloud model. Since the shower that we shall simulate is under the influence of rough terrain, the use of a simple PBL model is only a first approximation to reality.

As shown in Fig. 1c, the PBL region is open in the lateral direction and is divided into two layers: a superadiabatic, thin surface layer of depth \( \delta \) and a well mixed isentropic layer of thickness \( h \). As we have mentioned above, there is no downward motion in the FE region. This implies that there is neither force to depress nor organized, downward moving air to penetrate the top of the PBL. Ignoring further the mixing process at the top and assuming horizontal homogeneity, the depth and the heat content of the PBL are then controlled solely by the heat flux through the top of the superadiabatic surface layer into the mixed layer.

When the PBL grows to some height, as determined by a governing equation, the potential temperature in the FE region at this height will then be assigned to all levels below except the surface, in accordance with our later assumption of isentropy in the PBL. The FE region and the PBL region are thus coupled in this manner insofar as the thermodynamics is concerned. When an updraft is initiated in the core region, it will draw up the warm and moist source air from the PBL and produce horizontal convergence in the PBL, as indicated by the arrows in Fig. 1. Since there is no lateral boundary, the homogeneous source air may come from far away via the PBL. If downdraft in either core or IE region extends down to the levels close to the ground, horizontal divergence is produced. In either case, only horizontal motion exists in the PBL region. The FE region and PBL are then not coupled dynamically. Another point worth mentioning is that the cloud base may not coincide with the top boundary of the PBL. In this respect, our concept is best illustrated by Fig. 1 of Deardorff's (1975) comments on the work of Ogura and Cho (1974). This figure is reproduced as our Fig. 2, in which some minor changes have been made. They are: i) the dimension of the updrafts under the clouds, as we think, should be smaller than that of the visible cloud bases; ii) the surface layer is added. In Fig. 2, we see that the cloud bases are above the "mean height" \( h \) of the PBL, as shown by the long dash line. This mean height is then the height of our PBL region in our Fig. 1. This is why the cloud base in Fig. 1 is drawn above the top of the PBL. It may not always be so, however, since the cloud base height depends upon the properties of the air in the PBL.
2) THE GOVERNING EQUATION

To derive the equation which governs the thickness of the isentropic layer, we follow Ball's (1960) two assumptions: i) the mean potential temperature gradient in the stable layer is not altered by the convection; ii) the total mean upward heat transport $H_f$ decreases linearly with height in the mixed layer. Together with the configuration shown in Fig. 3 and following Kuo and Sun's (1976) work, we obtain the following two equations

$$\frac{d\theta_m}{dt} = \beta_0 \frac{dh}{dt},$$  
(2.21)

$$\frac{d\theta_m}{dt} = \frac{1}{\rho C_p h}(H_b - H_s),$$  
(2.22)

where $\theta_m$ is the mean potential temperature in the isentropic layer, $\beta_0$ the lapse rate of the undisturbed potential temperature above the mixed layer, $h$ the thickness of the isentropic layer and $H_b$ and $H_s$ are the values of the total heat transport at the base and the top of the isentropic layer. Equating (2.21) and (2.22), we get

$$\frac{d^2h}{dt^2} = \frac{2k}{\beta_0 \delta} (\theta_s - \theta_m).$$  
(2.24)

From Fig. 3,

$$\theta_m = \theta_b + \beta_0 h,$$

where $\theta_b$ is the potential temperature at the base of the isentropic layer extrapolated from the basic state as shown in Fig. 3. Substituting into (2.24), we get

$$\frac{d^2h}{dt^2} = \frac{2k}{\beta_0 \delta} (\theta_s - \theta_b) - \frac{2k}{\delta} h.$$  
(2.25)

This equation can be written as

$$\frac{d^2h}{dt^2} + A h = B (\theta_s - \theta_b),$$

where

$$A = \frac{2k}{\delta},$$

$$B = \frac{2k}{\beta_0 \delta} = \frac{A}{\beta_0},$$

$$\theta_b = \theta_0 - \beta_0 \delta$$

and $\theta_0$ is the surface potential temperature extrapolated from the basic state as shown in Fig. 3. Note, while $\theta_s$ is time-dependent, $\theta_b$ and $\theta_0$ are not.

In the above expressions, large $A$ may be due to small $\delta$ or large $k$. In the case of small $\delta$, it means that the surface layer is thin and the super-adiabatic lapse rate is large. In the case of large $k$, it means that the eddy conductive process is very different. In either case, the heat input into the isentropic layer from the super-adiabatic layer is large. This is also true for $B$, since $\beta_0$ is taken to be a constant. From Fig. 3, we see

FIG. 3. The structure of the planetary boundary layer: $Z_s$ and $Z_p$ are the height of the top of the surface and mixed layers; $h$ the thickness of the isentropic layer; $\delta$ that of the surface layer; $\theta$ the horizontally averaged basic potential temperature; $\theta_\circ$ the extrapolated surface basic potential temperature; $\theta_b$ the extrapolated basic potential temperature at the base of the isentropic layer; $\theta_m$ the mean potential temperature in the isentropic layer; $\theta_\circ$ the observed surface potential temperature; $\beta_0$ the lapse rate of the basic potential temperature; $H_f$ upward heat transfer; and $H_b$ and $H_s$, that at the base and the top of the isentropic layer. The dot–dash lines are the mixing ratio profiles and $y_s$, the lapse rate.
that higher $\theta_i$ means more heat input into the isentropic layer when $\theta_i$ is kept constant. When $\theta_i$ does not vary, higher $\theta_i$ means less heat input into the isentropic layer from the surface layer. Hence, large $A$, $B$ and $\theta_s$, or small $\delta$ and $\theta_m$, imply a greater growth rate for $h$, and vice versa. On the other hand, (2.24) can be written as

$$\frac{dh}{dt} = \frac{B}{2h}(\theta_s - \theta_m).$$

This equation indicates that the initial growth rate is large because of small $h$ and the rapid rising of the surface temperature in the early morning. Finally, we note that (2.25) becomes invalid when $\theta_i$ becomes smaller than $\theta_m$ in the late afternoon, since the configuration in Fig. 3 is destroyed.

3) THE SENSITIVITY TEST

In finite difference form, (2.25) can be written as

$$h_{i+\Delta t} = [h_i^{\Delta t} - 2Ah_i\Delta t + 2B(\theta_s - \theta_m)\Delta t]^{1/2}. \quad (2.26)$$

This equation is then to be incorporated in the model and solved for $h$ at each iteration in the numerical experiments. The potential temperature of the lowest isentropic section of depth $h$ of the FE region then varies in accordance with the computed value of $h$ and the configuration shown in Fig. 3.

Before we put (2.26) into our program, several sensitivity test runs have been made with several different values of coefficients $A$, $B$ and $\delta$. For these test runs, we let the initial values of $h_0$ and $h_{i-\Delta t}$ be $500 \text{ m}$, $\beta_0 = 0.009 \text{ K m}^{-1}$, $\theta_0 = 294.5 \text{ K}$ and $\theta_h = (294.5 + 0.009\delta) \text{ K}$. In (2.26) $\theta_s$ takes the values converted from some observed surface temperature. The assignment of these values will be explained later. In Kuo and Sun (1976), they assumed that the eddy conduction coefficient $k$ has the values of 1 and 2.5 $\text{m}^2 \text{s}^{-1}$. They found that their results were insensitive to these changes. In order to enlarge the difference and perhaps better approximate the case of a moderate wind blow-

![Figure 4. Results of the test runs of the PBL model.](image)

<table>
<thead>
<tr>
<th>$k$</th>
<th>$\beta_0$</th>
<th>$\delta$</th>
<th>$\theta_s$</th>
<th>$\theta_m$</th>
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</thead>
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<td>10</td>
<td>0.2</td>
<td>22.2</td>
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<tr>
<td>2.5</td>
<td>0.009</td>
<td>10</td>
<td>0.5</td>
<td>55.6</td>
</tr>
<tr>
<td>5</td>
<td>0.009</td>
<td>10</td>
<td>1</td>
<td>111.1</td>
</tr>
<tr>
<td>10</td>
<td>0.009</td>
<td>10</td>
<td>2</td>
<td>222.2</td>
</tr>
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</table>

The atmosphere between $z = 0$ and $z_{top}$ is divided into $N$ layers of equal thickness $\Delta z (=100 \text{ m})$, so that there are $N + 1$ grid points in the vertical in each region. Initially, the atmosphere is assumed to be strat-
ified without mean motion. After the convection is initiated in the core region, the finite difference analogue of the system of equations is then applied to the grid points in the core region and to those grid points in the IE region if the IE region exists at those levels. The variables above the PBL in the FE region stay invariant and serve as the reference. After the thickness \( h \) of the isentropic, mixed layer is determined by (2.26) at each iteration, the depth of the PBL is then \( (h + \delta) \). The potential temperature at \( (h + \delta) \) in the FE region is then assigned to all the levels in the PBL except the surface to simulate the isentropy. The other variables at these low levels are then computed accordingly. The forward time difference and upstream space difference scheme are used. The time increment \( \delta t \) is not fixed. Initially, it is 8 seconds. Afterward, it is governed by the condition

\[
\delta t < \frac{\Delta x}{3w_{\text{max}}}
\]

where \( w_{\text{max}} \) is the maximum vertical velocity in the core region. Whenever \( \delta t \) does not fulfill the above condition as \( w_{\text{max}} \) increases, it is reduced by half.

3. Case study

The convective shower chosen for this study occurred on 4 August 1973, in Taoyan Shihmen region in the northern part of Taiwan Province of the Republic of China. The map of this region is shown in Fig. 5. This shower, which occurred at site A in Fig. 5, is chosen because of its simplicity, in the sense that the recorded precipitation curve exhibits only one peak rainfall rate of moderate intensity which follows two minor, very small forerunners (relative maxima). In addition, the shower area is small, since there is no rainfall recorded at either sites B and C (Fig. 5) at the shower time. As shown in Fig. 5, the coastal area is relatively flat and is susceptible to the flow from the sea on the north and west. To the south of site A, there are high mountains which indicate that the vicinity of site A is a sea breeze convergence zone. Also, the steep slopes of the mountains may act as a source of high level heating and act as a barrier. Both may trigger convective clouds. All these features make the whole region an excellent area for cloud activity, especially in the proximity of site A. In fact, on summer days, almost all the local afternoon shower clouds first form and develop over the mountains at site C and then move over site A. In the experiments, a velocity impulse is used to trigger off the convection. The impulse then simulates implicitly the above physical processes.

a. Observations

The relevant synoptic charts of 3 and 4 August 1973 are shown in Fig. 6, where we see that the air mass is maritime and the flow is confluent northward. Furthermore, there was no synoptic scale disturbance nearby. This suggests that the shower is of local origin. The lower part of the 0800 CST (China Standard Time) and 2000 CST soundings taken at Makung, a city on the Penghu Islands (also called Pescadores, Fig. 5) is shown in Fig. 7; those taken at site B (Fig. 5), in Fig. 8, where the thick solid lines (temperature) and dashed lines (mixing ratio) are of 0800 CST and the thin lines, of 2000 CST. The 0800 CST (0000 GMT) upper air wind above Makung (dashed lines) and site B (solid lines) are shown in Fig. 9. From Figs. 7 and 8, we see that the water vapor content above both places has increased during the day, due to the deep southerly flow (dashed lines in Fig. 9) over the Taiwan Strait. Also in Figs. 7 and 8, the four temperature profiles do not differ much, except for the appearance of a warm layer between 700 and 850 hPa on the 0800 (hereafter, all times local) sounding taken at site B (thick solid line in Fig. 8). Referring to Fig. 9, we see that above 850 hPa, an easterly wind prevails in a deep layer. Evidently, the warm layer is a result of the downslope wind riding over the very high mountains on the east (outside of the map).

On the other hand, we see from Fig. 5 that Penghu is a group of islands of low elevation and is in the middle of the Taiwan Strait. Therefore, the soundings taken at Makung, which show a very high water vapor
The observed surface temperature (°C), at both sites A and B are shown by the curves in Fig. 10. The relative humidity (%) is shown in parentheses and the mixing ratio (g kg⁻¹) is shown by the numbers attached to the curves shown in Fig. 10. The temperature and mixing ratio observed at site A were lower than those at site B. This is perhaps due to the cloud shadow in the early morning and evaporational cooling after 1100. Also, the fact that site A is about 200 m higher than site B may partly account for the difference. The rapid warming between 0800 and 1100 may be a result of insolation. The observed surface wind vectors (m s⁻¹) are shown in the upper part of Fig. 10, where we see that the sea breeze of increasing strength started at sometime between 0900 and 1000 and ceased at about 1500. From these vectors and also the observed mixing ratio and temperature at site B between 1000 and 1300, we see that the sea breeze steadily transported air of remarkably uniform properties from the sea toward land and converged in the vicinity of site A. The convergence zone surrounding A, perhaps, is the major factor in deciding the location of the shower. Fur-

FIG. 7. Sounding taken at Makung 4 August 1973. The thick lines are the 0800 CST sounding and the thin lines, 2000 CST. Heights of the standard pressure surfaces at 0800 CST are shown on the left and those at 2000 CST, on the right.

FIG. 8. Sounding taken at site B on 4 August 1973. Otherwise as in Fig. 7.
thermore, the uniformity in air properties of the sea breeze may justify our assumption that the properties of the air in the PBL are horizontally homogeneous. The original precipitation record is shown on the left side of Fig. 11. The rainfall rate is displayed on the right, which is the slope of the precipitation curve. The peak intensity is about 45 mm h⁻¹ and the total rainfall, 17.5 mm. The starting time of the shower was at 1220 and that of the two small forerunners was about 1050. Our task is then to reproduce not only this precipitation curve as closely as possible, but also the time of onset of the shower. In this study, a great flaw is the lack of information about the cloud. The only available data is that the observed height of the cloud base was 915 m (3000 ft) at 0800 and 662 m (2500 ft) at 0900 and 1000. Finally, we should mention that the initial values used in the sensitivity test of (2.26) are taken from these observations.

b. Initial conditions and numerical experiments

As shown in Fig. 5, site B is about 25 km from site A. The 0800 sounding taken at site B is then considered to be representative of the environment during the entire period of the moderate cloud activity over site A. This is permitted because, first, the synoptic situation did not change much during the cloud activity, and second, the environment is of sea breeze scale which is much larger than the shower area (Wang, 1979, 1983). Finally, not much change is shown on the temperature profiles as shown in Figs. 7 and 8. The increase of the moisture content shown by the dashed-lines in Figs. 7 and 8 and the disappearance of the warm layer on the 2000 sounding (thin solid line, Fig. 8) are ignored, because we do not know how the change take place. Hence, the initial stratification in all regions is computed according to the sounding in Fig. 8 and stays invariant in the FE region except for those levels within the PBL. In the core regions, the stratification will be disturbed after the convection is triggered.

In reality, the conditions in the lowest atmosphere will change with time after sunrise. Accordingly, the lowest part of the reference stratification should also change. In Cotton et al. (1976), this modification is computed through the use of their sea breeze model (Picike, 1974) until the time when clouds are most active and then used as an invariant reference. This is also true in Wang (1979), except that a PBL of constant thickness is used. In this work, the modification is made at every time step according to the time-dependent PBL model formulated in Section 2. As shown in Fig. 8, the layer below 950 hPa (about 548 m MSL) is isentropic. This indicates that by 0800, the top of the PBL has risen to about 500 m at site B (48 m MSL). Therefore, we take 0800 as the initial time and 500 m as the initial thickness of the mixed layer in all our experiments. Afterward, the variation of the thickness of the mixed layer is governed by (2.26) with prescribed A, B and δ. The simulation of the isentropy is then made in the manner described in Section 2.

On the other hand, the heavy dash line below 950 hPa shown in Fig. 8 provides the initial profile for the mixing ratio in the PBL. How this profile changes afterward is not well known. Since both Figs. 7 and 8 show that all of the mixing ratio profiles in the PBL are linear, we assume that the mixing ratio profile in

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![Fig. 9. The 0800 CST upper air winds observed at Makung (dashed line) and at site B (solid line). The pressure heights shown on the right are taken from the sounding of site B.](image)

![Fig. 10. Surface observations taken at sites A and B as indicated: (lower part) temperature is indicated by the solid lines; relative humidity (%) by parenthesized numbers; mixing ratio (g kg⁻¹) by unparenthesized. Upper part: wind vectors (speed m s⁻¹ indicated by numbers). Time is indicated on the abscissa.](image)
our PBL region below the FE region always remains linear. The change of the whole profile then follows the surface mixing ratio observed at site B, as shown schematically in Fig. 3 and discussed at the end of Section 2b. The slope of the profile, namely the lapse rate $\gamma_v$, is one of the parameters pertaining to the PBL and will assume different values from one case to another. These specifications of the temperature and mixing ratio in the PBL then simulate the transport of low level air inland from the sea with increasing moisture content by the mesoscale sea breeze circulation after 0900, and maintain uniformity of the maritime air properties.

On the surfaceboundary, the temperature and the mixing ratio are functions of time. Since we intend to isolate our system from all variables but the PBL, we simply assign the observed values of temperature and mixing ratio shown for site B in Fig. 10 to the surface boundary. For the core region, those observed at site A are assigned and for the FE region, those observed at site B. The assigned surface mixing ratio in the FE region then leads the movement of the mixing ratio profiles shown in Fig. 3 as we have mentioned earlier. Moreover, the assigned surface temperature will bring forth the changes of $h$ through the term $(\theta_s - \theta_v)$ in Eq. (2.26). If the IE region reaches the ground because of the falling rain or downdraft, the surface temperature and mixing ratio of the IE region then take the values observed at site A.

The other parameters take the following values in all experiments:

- $r_c = 1$ km (Radius of the core region),
- $z_0 = 48$ m (Elevation of site B above MSL),
- $p_0 = 1005.5$ hPa (Observed surface pressure at site B),
- $v_r = 0.1$ m s$^{-1}$ (Corresponding to $\nu = 10^3$ cm$^2$ s$^{-1}$ with mixing length $l$ of 100 m ($=\Delta z$), and is very close to that computed by Richardson’s law $= 0.2l^{1/3}$),
- $\beta = 0.1$ m$^{-1}$ s (Derived from the entrainment constant),
- $\nu_c = 1.0$ m$^2$ s$^{-1}$ ($= 10^4$ cm$^4$ s$^{-1}$),
- $c^2 = 2 \times 10^4$ (Cloud conversion coefficient),
- $q_c = 0.5$ g kg$^{-1}$ (Threshold).

Finally, we should mention that the difference in height above sea level between sites A and B is ignored for the purpose of simplicity.

In this work, the parameters taking different values for different cases of the numerical experiments are $A$, $B$, $\delta$ and $\gamma_v$. Our cloud system is then isolated from all the other factors and responds to the variations of the variables, such as temperature, mixing ratio, etc., of the PBL only; all these variables vary with the thickness of the PBL. To initiate the convection, a velocity impulse of 1 m s$^{-1}$ is added only at the first time step at the first grid point in the core region above the lower boundary (earth surface). The convection so initiated, if any, is then free convection and is rooted in the PBL. It induces horizontal convergence and draws warm and moist air in the PBL for further development as shown in Fig. 1.

c. Results and discussion

In case 1, $A = 0.1$, $B = 11.1$, $\delta = 10$ m, and $\gamma_v = -0.0006$ (100 m)$^{-1}$. This $\gamma_v$ is chosen from Fig. 7, which shows that the $\gamma_v$ is between $-0.0005$ (100 m)$^{-1}$ to $-0.0006$ (100 m)$^{-1}$ at Makung. The result is shown in Fig. 12 which includes the observed rainfall rate and total rainfall from Fig. 11. In case 1, a very light rain starts at 1040 and then increases suddenly in the form of a shower at about 1340. The maximum rainfall intensity, reached at about 1350, is 73 mm h$^{-1}$, about 28 mm higher than observed. The total rainfall is a little more than 20 mm, about 2.5 mm too high. Comparing case 1 with the observation, we see that the time of the beginning of the small rain is 1040, while
the observed time is 1050. This can be considered in good agreement with the observation, because the rain gauges may not be able to indicate the exact starting time of such a small rainfall. However, the starting time of the model-produced shower is about 1340 while the observed is 1220, showing a large discrepancy. In case 2, A and B have been increased to 0.3 and 33.3, respectively. Here, no small rainfall similar to case 1 is produced and the precipitation curves move even more to the right as shown in Fig. 10. The rainfall rate reaches a maximum of 130 mm h⁻¹ which is much higher than observed but the total rainfall (12.4 mm) is less. Besides these two cases, we have also run another case with A = 1 and B = 111.1 and no rain was produced at all, which is quite surprising.

From Fig. 4, we see that at a given time, the PBL for greater values of A and B is deeper than that for smaller values of A and B. From our PBL model shown in Fig. 3, we notice that an increase of the depth of the PBL will bring forth an increase in the mean potential temperature in the whole layer. When an increase in heat flux from the surface layer (A and B large) is strong enough, it may offset the increase in potential temperature gradient aloft. Therefore, the boundary layer grows to a greater depth before the shower and has a higher temperature. However, the mixing ratio is not altered much. It increases only at the very top of the PBL with a small amount. Thus, when the magnitudes of A and B are increased from small to large values, the PBL deepens and the model clouds have an increase in heat supply but without a proportional increase in water vapor supply. Therefore, in case 2 which has a higher heat supply, a lower relative humidity and a stronger updraft at low levels than in case 1, no small rainfall similar to case 1 could reach the ground. Furthermore, because the stronger updraft has a greater resistance to falling drops, both rain and cloud drops have been raised to higher levels. This will not only delay the onset of the shower, but will also cause a greater loss of water mass aloft due to stronger nonlinear eddies acting for a longer time than in case 1. As a result, in case 2, not only does the rain pour down as an intense shower at the beginning, but the total rainfall is also less than in case 1. In the case A = 1 and B = 111.1, the updraft is strong enough to prevent all the raindrops from reaching the ground for a time long enough for the eddy mixing processes to erode the whole cloud completely. Takahashi (1974) has mentioned that when a one-dimensional cloud model is used, a much steeper environmental lapse rate will result in weaker rainfall intensity. This situation is rather similar to, but not the same, as ours, because when other conditions are the same, a steeper environmental lapse rate will also produce stronger updraft and thus will cause stronger nonlinear loss of water mass.

Due to curiosity, we have also increased A and B to 2 and 222, respectively, in case 3. Surprisingly enough, a shower is produced. The onset of the shower is even earlier than in case 2 as shown in Fig. 10, because the updraft is strong enough to bring forth earlier formation and development of the cloud, which then reduces the time for the eddies to erode the cloud, resulting in an earlier initiation of a shower.

In case 4, we model the PBL as in case 2 but increase γe in the PBL to a higher-than-observed value of −0.0004 (100 m)⁻¹, with A = 0.3, B = 33.3 and δ = 10. The result, plotted in Fig. 13 depicts that the onset of the shower is much earlier. This shows that a higher water vapor supply from the PBL will greatly expedite the production of a shower. Comparing cases 2 and 4, it appears that a good prediction of the starting time of a shower is surely possible if the chosen γe is truly representative of the moisture content of the PBL or the sea breeze. Now if we compare all the rainfall rate curves of case 1 to case 4 as shown in Figs. 12 and 13, we see that the higher the values of A, B and γe, the more the notches on the precipitation curves; therefore, we made another run with A = 0.06, B = 6.7 and δ = 30, but reduced γe to −0.0006, which is case 5. As illustrated in Fig. 14, it appears that the patterns of the rainfall intensity and total rainfall curves of case 5 are much closer to the observation. Comparing Figs. 13 and 14, it seems that even better agreement between the observation and prediction can be obtained, if we let γe take a value of nearly −0.0005, and reduce A and B further.

Besides the above discussions, there are other points worth mentioning. First, case 5 shows the best agree-
ment in rainfall pattern. In case 5, the thickness of the PBL increases from 500 to 700 m. Coincidently, in a previous study (Wang, 1979), the case which produced the best result had a PBL depth of 500 m, which seems to be too shallow for a hot summer day in Taiwan. However, in the two studies of Pennell and LeMone (1974) and Nicholls and LeMone (1980), the observed thicknesses of the PBL over the tropical Atlantic Ocean were about 600 m. This observation also holds true over the tropical Pacific Ocean, as shown in the works of Reed and Recker (~700 m, 1971), Soong and Tao (top of the isentropic layer at ~940 mb, 1980), and Soong and Ogura (~500 m, 1980). In our case, the depth of the PBL over the Taiwan Strait of the case day is about 500 m as shown in Fig. 7. After the sea breeze starts, part or the whole of the PBL over the sea will be transported over the hot land surface. The depth of the PBL will then increase due to surface heat flux. However, the time interval during which the PBL undergoes heating from below before it is entrained into the cloud at site A is only about two hours, since it takes from 93 to 139 min for the sea breeze air to move from the sea to site A (corresponding to a velocity of 4.5 and 3 m s⁻¹, respectively, over a distance of about 25 km at a direction 320° from site A to the coast). Because of the short time interval, the depth of the PBL cannot increase too much, perhaps only 100 or 200 m. Hence, the depth of the PBL on the case day would be much shallower than on other days without any sea breeze or showers.

Second, in this study, the initial time of all experiments is 0800, but all of the model showers occur in afternoon hours, except the extremely wet case 4. Besides other factors such as the sea breeze, the surface temperature and the change in depth of the PBL, this achievement is a result of our IE region. In order to reveal the physical processes involved, the vertical velocity of the updraft and the thickness of the IE region of case 3 are plotted in Figs. 15 and 16, respectively. From Fig. 15, we see that the low level updraft stays weak for six hours from 0800 to 1400 although water vapor content in the PBL has increased from 0900 to 1000, the depth of the PBL has increased to its peak value, and the surface temperature under the cloud has passed its peak value at 1100. During this time interval, the weak updraft keeps pumping warm and moist air upward and detrains it to build up the IE region in the lower part of and below the inversion. When the updraft has gained some strength and starts to develop higher, it entrains moist air into its lower part from the IE region, which will then reduce its thickness. As its thickness decreases, its moisture content also decreases, for the loss of water mass due to eddy transfer across the outer boundary of the IE region is inversely proportional to its dimension (Eq. 2.17). As a result, the air in the IE region becomes progressively drier. The updraft, now entraining this drier air into its lower part, will suffer increasingly greater dilution and will eventually stop from further development and then weaken. As this happens, the updraft will return to its original work, pumping moist air upward and re-building the IE region. This process will repeat, as depicted by the wave-like and closed contours before the development in Figs. 15 and 16. Of course, the variations in the updraft strength may also be partly due to the super-imposed gravity oscillations of Brunt–Väisälä frequency, which also enhances the pumping process and the re-building of the IE region. In Fig. 16, we see that only when the IE region becomes thick enough, exceeding 800 m at about 1420, the updraft will not be over-diluted and becomes able to penetrate through the inversion. Once the inversion is penetrated, the updraft becomes buoyant and will develop quickly to a great height on its own. To prove that the IE region does have the above advantages, we have experimented without an IE region by setting the \( r_1 \) equal to zero at every iteration. The result is a low, steady plume cloud without development at all. Hence, it is the dynamic IE region in our model which is responsible for delaying the showers until afternoon hours, well in agreement with the hypothesis that the buildup of a near-environment is necessary before a cloud can precipitate. Furthermore, as mentioned previously, the IE region also functions as a store of a large amount of moist air and as a passage for the rain drops detrained aloft to fall through it to low levels where they will be entrained back into the cloud without much mass depletion. These last two functions then provide a recycling process of water substance, which a purely one-dimensional cloud model lacks (Takahashi, 1974). Consequently, high rainfall intensity and total rainfall can be produced. Viewing these facts, our model should not be visualized as a purely one-dimensional model. Rather, it should be envisaged as a simplified version of a two-dimensional, axisymmetric model.

Third, the only available data about the cloud is the heights of the cloud base observed at site A. They are 915 m (3000 ft) at 0800 and 662 m (2200 ft) at 0900 and 1000. The computed cloud base heights are in good agreement with the observations. The heights on the printed list are 600 m for cases 1, 4 and 5. Since the cloud water mixing ratios at the base are different, the cloud base heights of these model clouds are also different (the difference is less than 100 m). For cases
2 and 3, the computed cloud base height is 700 m, also with a difference in cloud water mixing ratio. These clouds are then above the PBL in the early stage. Toward noon the depth of the PBL increases. Some of the model cloud bases are then lower than the top of the PBL, particularly when the showers have started. From these results, we see that taking the cloud base height as an estimate of the depth of the PBL is good only when the cloud is shallow, as those in fair weather conditions observed by Nicholls and LeMone (1980).

4. Conclusion

In this work, we have incorporated the precipitation mechanism and a simple PBL model in the cloud model presented in Wang (1983). The model is employed in lieu of the sea breeze model in Cotton et al. (1976). By isolating this system from all influential factors but the PBL, a series of model clouds have been produced for the simulation of a recorded precipitation curve of a local, summer afternoon shower under the influence of a sea breeze. The results strongly suggest that under simple synoptic situations and given surface temperature and mixing ratio, the rainfall pattern, peak intensity, total rainfall and the time of onset of the shower can be reproduced quite closely from the morning sounding.

Only some tentative conclusions can be drawn from such a limited number of cases about the effect of the PBL upon the production of a shower and its precip-
itation characteristics. From the resulting precipitation curves, it appears that the PBL influences the rainfall characteristics in a complicated way. Under the given conditions, it seems that only when the heat and water vapor supply from the PBL have a well-proportioned ratio, namely, a proper relative humidity, the shower will have only one peak intensity. The precipitation curve thus appears smooth. When the thickness of the PBL is increased, which, in our model, implies an increase of heat supply to the cloud without a proportional increase of moisture supply, the onset of the shower is delayed. The total rainfall is reduced and the intensity curve is multi-peaked. When it is increased further to a critical thickness, no shower can be produced. If increased beyond this critical thickness, a shower will be produced again. When we increase the water vapor supply alone, the onset of the shower will be greatly expedited. Its total rainfall is increased and the intensity curve is multi-peaked. This shows that in determining the rainfall pattern and time of onset, the rate of supply of heat and water vapor from the PBL is very important. From these results, it seems that a well-designed sensitivity test of the present model is necessary to further clarify how the clouds respond to variations of the parameters pertaining to the PBL. In this work, a great limitation is the lack of data about the cloud (only cloud base height is available). Otherwise, we would be in a much better position to draw more conclusions.

The results from our cloud model show that the IE region has the virtue of building up a good near-environment to protect its core, thus enhancing the cloud development in a rather unfavorable environment. Since it takes time to build up a good near-environment, local summer showers then usually occur in afternoon hours. Also, the results show that this model, due to its IE region, can produce showers of high intensity and total rainfall, which a pure one-dimensional model cannot (Takahashi, 1974).

To produce a summer afternoon shower, many hours of continuous heat and water vapor supply via the PBL from far away are necessary. Therefore, cloud models with closed lateral boundaries cannot achieve this purpose. Recently, Soong and Ogura (1980) have proposed a model on which the large scale forcing can be imposed. It has been shown by Soong and Tao (1980) that the amount or intensity of the deep or shallow clouds are significantly controlled by the imposed mesoscale advection. Their work and ours, therefore, have provided a very clear indication that in doing numerical simulations of an observed cloud activity spanning several hours, the incorporation of large scale influences in some way is a necessity.

Unfortunately, we are unable to reproduce the forerunners of the shower. This is perhaps due to our neglect of raindrop microphysics, because it is generally experienced that the short rainfalls before onset of the main shower are composed of mostly giant drops. If we want to simulate the forerunners, the consideration of raindrop microphysics seems necessary. On the other hand, since the rainfall of forerunners contributes only a very small fraction to the total rainfall, it seems that Kessler's scheme is still appropriate for use in studies similar to ours.

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APPENDIX

Thermodynamic Adjustment

In the prognostic equations for thermal energy and water substance, there are terms containing an identifier, \( \Lambda \), which we define

\[
\Lambda = \begin{cases} 
1, & \text{if } w > 0 \text{ and } q_v \geq q_v (\text{condensation}) \\
0, & \text{if } w < 0 \text{ or } q_v < q_v (\text{evaporation}) 
\end{cases}
\]

In case \( \Lambda = 1 \), the changes of the variables due to condensation are computed with the other terms. In case \( \Lambda = 0 \), the changes due to downward motion or evaporation are not computed. Therefore, cloud water may exist in an unsaturated air parcel. Even in case \( \Lambda = 1 \), the eddy terms may also lead to this state or even to supersaturation. Hence, at the end of every iteration, adjustment must be made to bring every parcel to a state of thermodynamic equilibrium which is defined as

1) no cloud water exists in an unsaturated parcel, but rain water may,
2) no supersaturation.

This thermodynamic adjustment is simply a wet-bulb process, that is, wet-bulb condensation or evaporation. It is then necessary to find out the wet-bulb temperature \( T_w \). From general textbooks, the equation for \( T_w \) is given by

\[
\frac{T - T_w}{e(T_w) - e} = \frac{0.622L}{pC_p},
\]

where \( e(T_w) \) is the saturation vapor pressure at temperature \( T_w \) and \( e \), the existing vapor pressure. The equation is transcendental and could be solved for \( T_w \) by successive approximations. This is, however, too cumbersome. We will derive another form more suitable for computation. For convenience, we will consider first the nonprecipitating case, namely, \( q_r = 0 \).

Suppose, at time \( t = t \), a parcel has the properties \( T, q_v(T), q_v, \) and \( q_c \), which are not in equilibrium, i.e.,

\[
q_v \neq q_v (T), \quad q_c > 0; \quad \text{or} \quad q_v > q_v (T).
\]

After the adjustment, the following condition should hold,

\[
q_v' = q_v' (T_w), \quad q_c \geq 0,
\]

where \( q_v' \) is the new water vapor content of the parcel.
where the primes indicate the adjusted quantities. The increments of $T$, $q_v$, and $q_v^*(T)$ can be written
\begin{align}
\delta T &= T_w - T, \\
\delta q_v &= q_v' - q_v, \\
\delta q_v^* &= q_v^*(T_w) - q_v^*(T).
\end{align}
Comparing (4) and (5) in light of (2) and rearranging, yields
\begin{equation}
\delta q_v - \delta q_v^* = q_v - q_v^*(T) = C. \tag{6}
\end{equation}
Here, $C$ is a known quantity, since $q_v$ and $q_v^*(T)$ are given at $t$. Thus, (6) indicates that the required adjustment is
\begin{equation}
C > 0, \text{ condensation \quad } C < 0, \text{ evaporation, \quad if } q_c > 0.
\end{equation}
The next step is to derive an expression for computing the equilibrium temperature $T_w$. With
\begin{equation}
q_v^*(T) = \frac{0.622 e(T)}{p - e(T)} \tag{7}
\end{equation}
and, according to Kuo (1965),
\begin{equation}
e(T) = 6.11 \exp[25.22(1 - 273/T)/(273/T)^{5.31}], \tag{8}
\end{equation}
we have the increment of $q_v^*(T)$ given by
\begin{equation}
\delta q_v^*(T) \approx \frac{0.622 \delta e(T)}{p - e(T)}, \tag{9}
\end{equation}
where the increment of $e(T)$ in the denominator is neglected against $p$. Here,
\begin{equation}
\delta e(T) = f(T) \delta T, \tag{10}
\end{equation}
and
\begin{equation}
f(T) = \frac{25.22 \times 273}{T^2} - \frac{5.31}{T}. \tag{11}
\end{equation}
Combining (10) and (9), we find
\begin{equation}
\delta q_v^*(T) = \frac{0.622 f(T) \delta T}{p - e(T)}. \tag{11}
\end{equation}
Substituting (11) into (6), we get
\begin{equation}
\delta q_v = \frac{0.622 f(T) \delta T}{p - e(T)} - C. \tag{12}
\end{equation}
Note,
\begin{equation}
\delta q_v = -\frac{C_p}{L} \delta T. \tag{13}
\end{equation}
Comparing (12) and (13) and rearranging yields
\begin{equation}
\delta T = C \left[ \frac{C_p}{L} + \frac{0.622 f(T)}{p - e(T)} \right]^{-1}. \tag{14}
\end{equation}
Note, in case of condensation, $C > 0$ and $\delta T > 0$, and vice versa. To see whether or not this is true, we factor our $5.31/T^2$ in $f(T)$ and get
\begin{equation}
f(T) = \frac{5.31}{T^2} [1296 - T]e(T). \tag{15}
\end{equation}
From this expression, we see for $f(T) < 0$, $T$ must be greater than 1296 K. This is far beyond the normal range of the temperature in the atmosphere; thus
\begin{equation}
f(T) > 0, \text{ always,}
\end{equation}
and
\begin{equation}
\delta T = (T_w - T) \lesssim 0, \text{ if } C \lesssim 0.
\end{equation}
Further, we should mention that (14) is valid only if the parcel is not too far from its equilibrium state and we expect that this is always the case.
Since every quantity on the right side of (14) is given, $\delta T$ can be calculated. Then, unless $C < 0$ and $q_c < q_v = -(C_p/L) \delta T$, which is the case when $q_c$ is not large enough to saturate the parcel, we have
\begin{equation}
T_w = T + \delta T \\
q_v' = q_v - \frac{C_p}{L} \delta T \\
q_c' = q_c - \delta q_v > 0 \tag{15}
\end{equation}
\begin{equation}
\theta' = T_w \left( \frac{1000}{p} \right)^{R/C_p} \tag{15}
\end{equation}
In case $C < 0$ and $q_c < q_v$, we have
\begin{equation}
q_v' = q_v + q_c \tag{15}
\end{equation}
\begin{equation}
q_c' = 0, \text{ (All droplets are evaporated)} \tag{15}
\end{equation}
\begin{equation}
T' = T - \frac{L}{C_p} q_c \tag{15}
\end{equation}
\begin{equation}
\theta' = T \left( \frac{1000}{p} \right)^{R/C_p} \tag{15}
\end{equation}
where $T'$ is the temperature of a chilled, unsaturated parcel containing no cloud droplets. Eqs. (15) and (16) can be used in nonprecipitating cases.
In precipitating cases, (16) needs to be modified. When all the cloud water is evaporated and the parcel is still unsaturated, rainwater will evaporate. Following Kessler (1969), the rate of evaporation of rainwater can be written
\begin{equation}
E_R = -1.93 \times 10^{-6} N_i^{3/2} q_d (q_v + q_c - q_v'), \tag{16}
\end{equation}
and the amount of rain water evaporated during the time interval $\delta t$ is $E_R \delta t$. Eq. (16) then takes the following forms:
\[ q' = q_v + q_c + E_R \delta T \]
\[ q'_c = 0 \]
\[ T' = T - \frac{L}{C_p} (q_c + E_R \delta T) \]
\[ \theta' = T' \left( \frac{1000}{p} \right) ^ {R/C_p} \]
\[ q'_R = q_R - E_R \delta T \]

(17)

In the present study, (15) and (17) are used. In the computations, the quantities \((\delta q_v - q_c)\) and \(E_R \delta t\) should be compared first. In case \((\delta q_v - q_c) < E_R \delta t\), only that amount of rainwater needed to evaporate to saturate the parcel is extracted from the rainwater content.

REFERENCES


