Climatology of Sr-90 in Surface Air: A Simple Model using Diffusion and Scavenging

D. O. STALEY

Department of Atmospheric Sciences, The University of Arizona, Tucson, AZ 85721

(Manuscript received 24 May 1984, in final form 25 September 1984)

ABSTRACT

The evolution of Sr-90 distribution following an instantaneous stratospheric or tropospheric source in an annulus defined by latitude walls is investigated by means of a very simple model that uses 12 stratospheric and 12 tropospheric annular boxes and which assumes diffusional transport between boxes and wet and dry scavenging from tropospheric boxes.

Coefficients for interbox transport are expressed in terms of half-residence times, and these are adjusted to be consistent with reasonable eddy diffusivities and/or variable local stratospheric residence times suggested by previous studies. Precipitation scavenging is proportional to precipitation rate, for which a sinusoidal variation was assumed, with annual mean, amplitude of oscillation and phase all determined by climatology.

The spring peak and fall minimum in tropospheric air are obtained at all latitudes. At middle and high latitudes, these extremes result primarily from annual oscillation of mixing downward from the stratosphere, while at low latitudes they result from the phase of the large annual oscillation of precipitation scavenging. The only reasonable way to obtain, in this model, the progressive delay of the spring maximum with latitude in middle latitudes is by a corresponding delay of the maximum rate of transport across the tropopause, a delay suggested by variations of stratospheric mass.

For reasonable values of vertical and horizontal exchange coefficients and a megaton midlatitude source, half-residence times are obtained for the Northern Hemisphere stratosphere, total stratosphere, and Northern minus Southern Hemisphere burdens that agree with observations. Half-residence times for the northern and total stratosphere increase by about two months over a period of several years, as concentration is depleted in middle and high latitudes where transfer to the troposphere is rapid. Maximum concentration develops over the equator.

Tropospheric debris from a tropospheric source is initially rapidly depleted by precipitation scavenging, and after a few months the tropospheric burden is small compared to the stratospheric debris acquired by initial upward diffusion. Thereafter, the stratosphere becomes the source, and both burdens slowly decrease at the rates given by stratospheric residence times.

1. Introduction

In a previous paper (Staley, 1982), Sr-90 data along the 80th meridian for the period 1963–75 were utilized to identify characteristic seasonal variations of Sr-90 concentration in surface air and to determine various stratospheric half-residence times.

The spring maximum of concentration in surface air was found to extend to the equator and, at low latitudes, to be related to precipitation scavenging associated with seasonal migrations of the intertropical convergence zone (ITCZ). At higher latitudes, seasonal variations of precipitation scavenging were found to be relatively less important while seasonal variations of transport from stratosphere to troposphere became relatively more important.

The seasonal and latitudinal variations of precipitation rate are fairly well known at latitudes such as 80°W, where measurements are available.

Meridional transport of Sr-90 is less well known but, in the troposphere, is vigorous at higher latitudes and sluggish equatorward from about 30° (Staley, 1963a). Meridional transport in the stratosphere is less well documented, but is probably proportional to tropospheric meridional transport.

Vertical transport between stratosphere and troposphere has been investigated by Reed (1955), Staley (1960, 1962, 1963b), Reiter (1963), Danielsen (1964) and Mahlman (1966). It reaches a maximum in spring in middle and high latitudes in association with cyclogenesis and seasonal variation of tropopause height (or stratospheric mass).

Thus enough is known of the relevant scavenging and transport processes on a global and climatological scale to suggest that something may be learned from a simple box model, specifically one that consists of several tropospheric and stratospheric boxes between equator and poles. Ideally, a general circulation model of adequate sophistication should be able to reproduce the climatology of Sr-90 in surface air, but this has not yet been achieved.

The purpose of this paper is to describe such a box model and to simulate Sr-90 evolution, residence times, and tropospheric climatology for arbitrary latitudes and height (stratosphere or troposphere) of Sr-90 injections and for various rates of transport between
boxes. The model is extremely simple. For example, poleward and vertical transport by meridional circulations are not included, and seasonal variation of transport downward to the lower stratosphere may contribute significantly to what is expressed here as a seasonal variation of the coefficients for mixing across the tropopause.

2. The model

Figure 1 depicts the Northern Hemisphere of an atmosphere divided into 12 stratospheric and 12 tropospheric annular boxes. Each hemisphere is divided into six boxes with equal surface area over 360° of longitude. No account, however, is taken of the mass difference between stratosphere and troposphere; half-residence times for transfer between stratospheric and tropospheric boxes and between the whole stratosphere and the troposphere are those commonly discussed in the literature. A finer resolution latitudinally is perhaps justifiable with respect to precipitation scavenging, but probably not for large-scale mixing. Vertical division of stratospheric boxes, in view of the great vertical extent of the stratosphere and seasonal changes of internal vertical transport, is intriguing and possibly justified, but is not done in this simple model. Accordingly, the latitudinal and seasonal variations of the stratospheric burdens obtained here only crudely represent the stratosphere as a whole.

In Fig. 1, the burden in a stratospheric box at any time is $S_i$, and the burden in the underlying tropospheric box is $T_j$. The rate of change of burden in a stratospheric box is assumed to be proportional to the excess burdens in surrounding boxes:

$$\frac{dS_i}{dt} = \lambda_i (T_j - S_i) + \lambda_{i-1,i}(S_{i-1} - S_i) + \lambda_{i+1,i}(S_{i+1} - S_i),$$

where the coefficients are expressed in terms of a half-residence (HR) time $\tau_{1/2}$ in the form

$$\lambda = 0.693/\tau_{1/2}(\phi, t),$$

where $\phi$, $t$ are latitude and time.

In the underlying tropospheric box,

$$\frac{dT_j}{dt} = \lambda_j (S_i - T_j) + \lambda_{j-1,j}(T_{j-1} - T_j) + \lambda_{j+1,j}(T_{j+1} - T_j) - (\lambda_{jg} + \lambda_{jd})T_j,$$

where $\lambda_d$ is a decay rate representing dry removal, and

$$\lambda_{jg} = kP$$

is a decay rate for precipitation removal to the surface. Here $P(t)$ is the precipitation rate, dependent on latitude and time; $k$ is a constant; and the $\lambda$ are defined as in (2). In (2) and (3), $\lambda_{pq} = \lambda_{qp}$ for all $p$ and $q$.

The boundary condition at the bottom of the atmosphere is embodied in the last term of (3). At the top of the atmosphere, vertical transfer is assumed to be zero, as evidenced by the absence of such terms in (1).

3. Values of coefficients

In this section, the general guidelines considered in the selection of coefficients are outlined.

Following the discussion by Staley (1982), $k = 0.1$ cm$^{-1}$ was used in (4) in all cases. The precipitation rate was expressed in the form

$$P(\phi, t) = \bar{P}(\phi) + A_p(\phi) \cos \left[ \frac{2\pi}{12} t - \varepsilon_p(\phi) \right],$$

where $\bar{P}(\phi)$ is the average precipitation rate over a year, $A_p(\phi)$ the amplitude of oscillation about $\bar{P}$, $t$ time in months (mo) and $\varepsilon_p$ is phase angle. In the Northern Hemisphere, maximum precipitation rate was assumed to occur simultaneously at all latitudes, with the maximum in August or September and the minimum in February or March. This simplified variation is based on data presented by Staley (1982) and is most accurate at lower latitudes, where the oscillations of precipitation rate are largest and most crucial, and least accurate in middle and higher latitudes. In the Southern Hemisphere, precipitation rate was assumed to be six months out of phase, with a maximum in February or March. At particular stations, rainfall rates may show extremes at months
other than those chosen, or even double maxima or minima, but the purpose here is to utilize the simplest description of the data.

The average precipitation rate \( \bar{P}(\phi) \) and the amplitude \( A_p(\phi) \) were also taken from data shown by Staley (1982). These, in cm mo\(^{-1}\), together with the month of maximum precipitation rate, are indicated at the bottoms of the tropospheric boxes in Fig. 2. Parenthetical values indicate the month of maximum precipitation in the corresponding box of the Southern Hemisphere. No differences of average or amplitude between hemispheres were assumed, although they clearly exist at latitudes where data are available (e.g., along the 80°W meridian). Only a six-month phase difference was assumed.

Dry removal was assumed to be small compared to wet, and \( \lambda_d = 0.1 \) mo\(^{-1}\), corresponding to \( \tau_{vd} = 6.93 \) mo, was assumed throughout.

The HR time in (2) is expressed, analogously to precipitation rate, as

\[
\tau_{1/2}(\phi, t) = \bar{\tau}_{1/2}(\phi) + A_{\bar{\tau}}(\phi) \cos \left( \frac{2\pi}{12} t - \epsilon(\phi) \right), \tag{6}
\]

where \( \bar{\tau}_{1/2} \) is the average HR time over a year, \( A_{\bar{\tau}}(\phi) \) is the amplitude of oscillation about \( \bar{\tau}_{1/2} \) and \( \epsilon(\phi) \) is phase angle.

The assumed HR times for meridional mixing between boxes and for vertical mixing across the tropopause are shown at the vertical and internal horizontal boundaries, respectively, in Fig. 2. These HR times are much more speculative than those devolving from precipitation rates. When the HR times are the same at both vertical walls of a box, the two horizontal transport terms combine to give a single term \( \{(0.693/\tau_{1/2})(T_{j+1} + T_{j+1} + 2T_{j})\} \) (in the troposphere) implying a horizontal coefficient, \( K_y = 0.693\bar{d}^2/\tau_{1/2} \), where \( \bar{d} \) is the separation of box centers. Table 1 shows HR time \( \tau_{1/2} \) as a function of \( K_y \) and wall latitude.

![Fig. 2. Schematic meridional cross section through annular boxes of the Northern Hemisphere showing parameters used in Cases 1, 2 and 3. Numbers on vertical box walls are values of \( \tau_{1/2} \) in months (assumed independent of time). The first three numbers at the boundaries between stratospheric and tropospheric boxes are \( \bar{\tau}_{1/2}(\phi) \) and \( A_{\bar{\tau}}(\phi) \) (both in months), and the month when the maximum \( \bar{\tau}_{1/2} \) occurs, respectively, for exchange between stratosphere and troposphere. Parenthetical value is month of maximum in Southern Hemisphere. The four numbers at the bottoms of tropospheric boxes are the mean annual precipitation rate (cm mo\(^{-1}\)), amplitude of fluctuation (cm mo\(^{-1}\)), and month of maximum precipitation rate. Parenthetical values are for Southern Hemisphere. \( 10^4 \) and \( 10^5 \) are instantaneous stratospheric and tropospheric annular sources, respectively, on 1 September.](image)

<table>
<thead>
<tr>
<th>Wall latitude (deg)</th>
<th>35.4</th>
<th>41.8</th>
<th>56.4</th>
<th>69.1</th>
</tr>
</thead>
<tbody>
<tr>
<td>( K_y ) (m(^2) s(^{-1}))</td>
<td>10(^{-6} )</td>
<td>3.8</td>
<td>10(^{-6} )</td>
<td>0.77</td>
</tr>
<tr>
<td>5 \times 10(^{-6} )</td>
<td>0.19</td>
<td>0.058</td>
<td>0.041</td>
<td>0.034</td>
</tr>
<tr>
<td>5 \times 10(^{-6} )</td>
<td>1.9</td>
<td>0.58</td>
<td>0.41</td>
<td>0.35</td>
</tr>
<tr>
<td>10(^{-6} )</td>
<td>3.8</td>
<td>1.2</td>
<td>0.82</td>
<td>0.69</td>
</tr>
<tr>
<td>10(^{-6} )</td>
<td>19.2</td>
<td>5.8</td>
<td>4.1</td>
<td>3.4</td>
</tr>
</tbody>
</table>

The meridional spread of radioactivity itself can be used to estimate appropriate values of \( K_y \). Staley (1963a) has inferred that \( K_y \sim 10^5 \) m\(^2\) s\(^{-1}\) in the equatorial troposphere from the spread of gross beta activity introduced over a short period of time by bomb tests at 52 and 75°N in 1961. Phillips (1956) has shown that a diffusion coefficient of \( 10^5 \) m\(^2\) s\(^{-1}\) is appropriate to diffusion by eddies with dimensions less than or equal to 300 km and velocities less than 1 m s\(^{-1}\). He also notes that \( 10^5 \) m\(^2\) s\(^{-1}\) is about 1/40 that necessary to balance the radiative heating loss at higher latitudes. The large-scale disturbances outside the tropics should therefore correspond to a diffusion coefficient some 40 times greater than that appropriate to the tropics. The rate of meridional transport in the troposphere of radioactivity from the bomb tests in 1961 in middle and high latitudes was at least 10 times as fast as in the tropics, suggesting that \( K_y \sim 10^6 \) m\(^2\) s\(^{-1}\) or larger in middle and higher latitudes. Thus, \( K_y \sim 10^8 \) m\(^2\) s\(^{-1}\) at low latitudes and \( K_y \sim 10^6 - 10^7 \) m\(^2\) s\(^{-1}\) at high latitudes are suggested for the troposphere.

In the time–latitude sections of gross beta activity in surface air, there was evidence of more rapid transsequatorial transport by way of the stratosphere than by the troposphere, i.e., radioactivity apparently traveled via the stratosphere and reached the surface at 30°S before air traveling directly through the troposphere. Kida (1983) remarks that two-dimensional transport models for minor constituents in the lower stratosphere have used \( K_y \sim 3 \times 10^6 \) m\(^2\) s\(^{-1}\) or larger, where this \( K_y \) includes both diffusion and advection due to Stokes drift. It is not conclusive that \( K_y \) for the stratosphere, if a single value must be used, should differ from that for the troposphere. No differences are assumed in the numerical examples, although there is no difficulty in doing so.

In any event, from Table 1, appropriate HR times for horizontal transport should probably range from fractions of a month at 56.4 and 41.8° to a few months in the tropics. These properties are incorporated into the cases discussed in Section 4.

In Fig. 2, the assumed HR times \( \tau_{vd}(\phi) \), etc., in months, for vertical transport between the stratosphere and the troposphere, are shown at the boundaries between stratospheric and tropospheric boxes. It has
long been understood that this vertical transport is largest in middle and high latitudes and in spring. The mechanisms of these phenomena have been extensively investigated (e.g., Reed, 1955; Staley, 1960, 1962; Reiter, 1963; Danielsen, 1964; Mahlman, 1966), and need not be reviewed in detail here. Vertical HR times in spring at middle latitudes are of the order of a few months as a result of vertical motion in cyclonic development and upward tropopause migration. In autumn, a year or more is appropriate at middle latitudes in association with a descending tropopause, and at low latitudes as much as two years is suggested by the relative cyclonic inactivity as well as the low radioactivity concentration in surface air, although high precipitation rates can also produce low concentrations.

Half residence times for the stratosphere as a whole are known from many studies (e.g., Staley, 1982), and the average of the yearly average box HR times cannot depart radically from these known times. Along the 80°W meridian, time of the spring maximum of Sr-90 concentration in surface air advances with increasing latitude. Staley (1963b) has related this to tropopause height changes. In the present model, this effect can be simulated by delaying the time of minimum vertical HR time with increasing latitude.

4. Numerical results

In the cases described in this section, instantaneous sources were introduced into the annular boxes at some initial time. This time would correspond to on the order of two weeks after an actual detonation at a particular latitude and longitude or at the end of an initial time interval during which the radioactivity becomes more or less uniformly distributed over longitude and a band of latitude.

a. Case 1

Figure 2 shows schematically the assumed precipitation, local HR times, and instantaneous annular sources in what will be called “Case 1”. Actually this case is the end result of several adjustments of various initially assumed HR times. The adjustments were made for the purpose of improving agreement with observations while maintaining consistency with what is known or suspected about radioactivity transport. The “tuning” necessary to improve the results is not unique: improvement can be achieved in more than one way, by adjusting any number of the many parameters. On the other hand, the time of maximum tropospheric concentration can realistically be advanced with increasing latitude only by advancing the time of minimum local vertical HR time with latitude in a way consistent with the stratospheric mass changes noted by Staley (1963b). The equatorial minimum of Sr-90 activity in surface air cannot reasonably be obtained by adjustment of the transport parameters. The high-latitude January maximum for transport to the troposphere in the model is an attempt to simulate the early spring maximum observed at Thule (76°36’N). Rangarajan and Eapen (1984) point out that this early arrival of the spring maximum does not also occur at Vadso, Eskeblumir and Lerwick (55°-71°N).

The local horizontal HR times, ranging from 3 months at low latitudes to 0.15 and 0.5 months at 41.8 and 56.4°, imply (from Table 1) $K_p$ increasing from about $10^5$ m$^2$ s$^{-1}$ to nearly $5 \times 10^6$ m$^2$ s$^{-1}$ over the same latitude range. No distinction is made in this case between tropospheric and stratospheric values of $K_p$. Hence the numerical values of parameters chosen in Case 1 are fairly consistent with the constraints noted in Section 3. The least questionable HR times are those corresponding to precipitation, at least along the 80th meridian. The most questionable numbers are the means and amplitudes of the vertical HR times and the relative magnitudes of stratospheric and tropospheric horizontal HR times.

To simulate megaton surface detonations, the bulk of the source (10$^9$ arbitrary units) was put into the stratosphere, and a minor part (10$^9$) into the troposphere. Since tropospheric residence times are of the order of two weeks, the tropospheric burden can alternatively be regarded as having been transferred from the stratosphere during the first two weeks after a point source.

The time increment for the forward integration could not exceed the minimum HR time. For convenience 0.1 month was used.

Some comments on mass conservation of the numerical procedure are in order before turning to the specific results. When (1) and (3) are summed, the terms involving interbox transport cancel, and we find

$$\frac{d}{dt} (S + T) = -\sum_{j=1}^{12} (\lambda_{m} + \lambda_{d}) T_j,$$

where the sums on the right represent loss to the surface by wet and dry deposition. In order to assess the extent to which the forward marching scheme conserves mass, we consider first a simple case wherein the Sr-90 tracer mass is the same in all tropospheric boxes and all $\lambda_{d}$ are identical and constant. Additionally, consider the situation at a sufficiently long time after the initial injection, such that the mean annual ratio of stratospheric to tropospheric burdens is a constant, say $\bar{K}$. (The right-hand side of Fig. 7 suggests $\bar{K} \sim 10$.) Now (7) becomes

---

1 Although in the overall spectrum of scientific computing the computations required here are rather modest, the increasingly informal access to computing power is illustrated by the fact that the computations were made on an equally modest Tandy-Radio Shack pocket computer (PC-2).
\[ \frac{dT}{dt} = -\lambda_s T(1 + \tilde{K}), \]  

(8)

where \( \lambda_s = \lambda_g + \lambda_d \) = constant. Integration yields

\[ T = T_0 e^{-0.693/t_1/2}, \]

(9)

where \( t_1/2 = 0.693(1 + \tilde{K})/\lambda_s \) is a half-life of the order of the half-residence time for the total stratosphere, and \( T_0 \) is an initial tropospheric Sr-90 burden at \( t = 0 \).

Although the forward marching scheme also conserves the mass transported between boxes, the transport to the surface goes, from (8), according to

\[ T_n \Delta t = T_0 (1 - 0.693 \Delta t/t_1/2)^n, \]

(10)

where \( n \) is the number of forward steps, \( \Delta t \) the time increment, and \( n \Delta t = t \).

For \( t_1/2 = 11 \) months and \( t = 11 \) months, (9) yields \( T/T_0 = 0.5 \). For \( \Delta t = 0.1 \) month and \( n = 110 \) \((t = 11 \) months), (10) yields \( T/T_0 = 0.499 \). For \( t = 4y \), (9) and (10) yield 0.0486 and 0.0481, respectively. Hence the numerical procedure conserves total mass very well for average burdens and removal rates.

For individual boxes the error can be larger. The minimum HR time for removal from an individual tropospheric box by precipitation and dry scavenging occurs once each year at the lower boundary of the box, next to the equator. For the precipitation rates given in Fig. 2, this HR time is 0.28 months in September. If other transports are neglected and this time is used in (9) and (10) with \( \Delta t = 0.1 \) mo again, the values of \( T/T_0 \) are 0.78 and 0.75, respectively, after 0.1 month; and 0.61 and 0.57, respectively, after 2 months.

The errors are a little larger again in the boxes with vertical walls where the HR time for horizontal transport is 0.15 month. Again, if other transports are ignored, the values of \( T/T_0 \) computed from (9) and (10) with \( \Delta t = 0.1 \) mo are 0.63 and 0.54, respectively; and 0.40 and 0.29, respectively, after 2 month. However, one transport term cannot function independently of others that are present. Thus, for example, if the computed transport out of a box is too large at one step, the gradient between it and the neighboring box will be less at the next step, and the next transport will be reduced. The result probably is that smaller errors spread over several boxes. Some experimentation with smaller time increments indicated insignificant differences in the Sr-90 evolution compared to those that result from different reasonable estimates of the many parameters that appear in the model. In any event, a small decrease in the time increment can substantially reduce any local errors.

Figure 3 shows time–latitude sections of Sr-90 concentration in troposphere and stratosphere for Case 1. In the troposphere (Fig. 3b), the seasonal patterns are established quickly relative to the stratosphere because of the short HR times controlled by precipitation. An autumn minimum occurs in the Northern Hemisphere immediately after the September burst. The spring maximum occurs in middle and high latitudes of the Northern Hemisphere in the following spring. A spring maximum appears in the Southern Hemisphere a little more than a year after the burst. The equatorial minimum establishes itself somewhat more slowly; after about 2.5 years it becomes obvious in the contours. As the years pass, the inventories of the two hemispheres approach each other.

In lower and middle latitudes, the spring maximum is delayed with increasing latitude, in agreement with observations (Staley, 1962, 1982). This feature, and the slightly earlier occurrence at 73.2°, are direct consequences of varying the phase of the vertical HR time.

Figure 3c shows a time–latitude section of observed Sr-90 in surface air along the 80th meridian. The model evolution obviously resembles the observations. However, the latter show more abrupt changes and somewhat larger seasonal variations. To some extent, these features can be mimicked better by tuning the model, but it is probably unrealistic to do so, since the rougher trends of the observations probably result from processes such as advection that are poorly described by diffusion, which tends to smooth. On the other hand, precipitation scavenging may actually change much more abruptly than the assumed sinusoidal variation.

In the stratosphere (Fig. 3a) seasonal patterns are much less obvious, since seasonal variation of precipitation scavenging works only indirectly, and HR times for transport to the troposphere are longer than those for precipitation. A maximum of concentration appears after 1.5 years at 14.5°N, and, in the following years, intensifies and shifts toward the equator. Hence the equatorial minimum of the troposphere is overlaid by an equatorial maximum in the stratosphere. This is a consequence of shorter stratosphere removal HR times away from the equator.

In both troposphere and stratosphere, latitudinal mixing is rapid at middle and higher latitudes and slow at low latitudes. This is a direct consequence of longer horizontal HR times at low latitudes.

Figure 4a depicts the time variation of stratospheric inventories for the total stratosphere \( S_T \), Northern Hemisphere \( S_N \), Southern Hemisphere \( S_S \), and Northern–Southern Hemispheres given by the model. Figure 4b depicts observed inventories for the period January 1963–February 1967 (Leifer et al., 1979; Staley, 1982), a period when there was no atmospheric testing. The observed and model curves are similar when it is recognized that radioactivity was injected at various times at various latitudes prior to 1963 and that the model source was an arbitrary \( 1.1 \times 10^4 \) confined to the Northern Hemisphere. Thus, for example, at the beginning of 1963 there was already
considerable activity in the Southern Hemisphere. One feature of the observations not shared by the model results is the slow decay in early 1963 of stratospheric burdens. If error bars were known, this feature might be questioned, but if the slow decay is real, a possible interpretation is that the center of mass of Sr-90 in the stratosphere lowered in the first few months after injection.

The calculated curves are more detailed and show seasonal variations which cannot appear reliably in quarterly observation means subject to sampling errors and other unknown effects. Figure 4a shows that $S_T$, $S_N$ and $S_N - S_S$ have initially the same seasonal variation, which must, of course, be the case, since the burst was well to the north of the equator and interhemispheric mixing is slow. With increasing time and transport across the equator, the oscillations of $S_T$ dampen, while those of $S_N - S_S$ increase (relative to the long-term approximately exponential decay). Eventually, if the integration is carried beyond 5
years, $S_N - S_S$ will range between positive and negative values. Of course, $S_N$ and $S_S$ are 6 months out of phase.

The departure of Northern Hemisphere stratospheric inventory from the long term (5 year) exponential decay (used to compute the average 5-year HR time in Table 2) is a maximum in late winter (January–March) and a minimum in summer (June or July). This is consistent with the variable removal that leads to the spring peak and fall minimum in the troposphere.

For a quantitative comparison with observations, various HR times can be computed from the inventory changes. Four of these are tabulated in Table 2 and compared with HR times inferred by Staley (1982) from 1963–67 measurements (parenthetical values). The HR times for the Northern Hemisphere ($\tau_{NH}$) and total stratosphere ($\tau_{ST}$) both increase by almost 2 months over the first 5-year period. The HR times near the end of the period agree well with the observed values. The increase with time reflects the initial heavy burden in the region of rapid transport to the troposphere, and the subsequent depletion in higher latitudes relative to lower latitudes. The 5-year values of $\tau_{NH}$ and $\tau_{ST}$ are a little lower than the observed values and reflect the rapid initial transport to the troposphere. The interhemispheric HR times $\tau_{NH}$ and $\tau_{ST}$ decrease with time as would be expected for a midlatitude injection that must first be transported to the equator. The HR time for decay of $S_N - S_S$ by transfer to the Southern Hemisphere alone $\tau_{NH}$ is particularly sensitive to the latitude of injection and decreases rapidly as the activity spreads equatorward.

It should be noted that $\tau_{NH}$ and $\tau_{ST}$ not only vary with time, but also are always smaller than the average of the box HR times. The latter is $(12 + 10 + 8 + 7 + 19 + 24)/6 = 13.3$ months. This difference is understandable when it is recognized that the HR

---

**Figure 4.** (a) Stratospheric burdens of Sr-90 in the five years following the detonation in Case 1. Subscripts N, S, T refer to northern, southern, and total stratosphere, respectively. (b) Observed stratospheric burdens (after Leifer et al., 1979; see Staley, 1982).

**Table 2.** Half-residence times in Case 1 for Sr-90 in the Northern Hemisphere stratosphere $\tau_{NH}$, the total stratosphere $\tau_{ST}$, the decay of the Northern–Southern Hemisphere inventory difference $\tau_{NH}$, and decay of the Northern–Southern Hemisphere inventory difference by transfer to the Southern Hemisphere alone $\tau_{NH}$. Yearly and five-year values are given, based on the tabulated inventories (and additional data in the case of $\tau_{NH}$). Half-residence times in months. Parenthetical values were calculated from observed inventories (Staley, 1982).

<table>
<thead>
<tr>
<th>$t$ (years)</th>
<th>$S_N$</th>
<th>$S_S - S_N$</th>
<th>$S_T$</th>
<th>$\tau_{NH}$</th>
<th>$\tau_{ST}$</th>
<th>$\tau_{NH}$</th>
<th>$\tau_{ST}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>0</td>
<td>10⁴</td>
<td>10⁴</td>
<td>10⁴</td>
<td>7.80</td>
<td>8.72</td>
<td>6.98</td>
<td>70.0</td>
</tr>
<tr>
<td>1</td>
<td>1255</td>
<td>3036</td>
<td>3852</td>
<td>8.24</td>
<td>10.0</td>
<td>6.43</td>
<td>36.0</td>
</tr>
<tr>
<td>2</td>
<td>832.8</td>
<td>1677</td>
<td>755.0</td>
<td>8.81</td>
<td>10.4</td>
<td>6.28</td>
<td>31.6</td>
</tr>
<tr>
<td>3</td>
<td>221.6</td>
<td>342.2</td>
<td>5.65</td>
<td>9.29</td>
<td>10.5</td>
<td>6.09</td>
<td>29.0</td>
</tr>
<tr>
<td>4</td>
<td>56.61</td>
<td>155.2</td>
<td>9.65</td>
<td>10.5</td>
<td>5.76</td>
<td>25.5</td>
<td></td>
</tr>
<tr>
<td>5</td>
<td>13.38</td>
<td>9.92</td>
<td>10.5</td>
<td>5.12</td>
<td>19.9</td>
<td></td>
<td></td>
</tr>
<tr>
<td>6</td>
<td>2.632</td>
<td>10.1</td>
<td>10.5</td>
<td>3.44</td>
<td>10.4</td>
<td></td>
<td></td>
</tr>
<tr>
<td>7</td>
<td>0.235</td>
<td>7.111</td>
<td>10.2</td>
<td>—</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td>8</td>
<td>-0.159</td>
<td>0.103</td>
<td>10.5</td>
<td>—</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td>9</td>
<td>-0.146</td>
<td>6.502</td>
<td>6.09</td>
<td>—</td>
<td>—</td>
<td></td>
<td></td>
</tr>
<tr>
<td>First five years</td>
<td>8.71</td>
<td>9.98</td>
<td>6.28</td>
<td>33.9</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Nine years</td>
<td>9.29</td>
<td>10.2</td>
<td>—</td>
<td>—</td>
<td></td>
<td></td>
<td></td>
</tr>
<tr>
<td>Observed</td>
<td>(9-10)</td>
<td>(10-11)</td>
<td>(6-7)</td>
<td>(30-42)</td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>
time for transfer to the troposphere should be more nearly a quantity $x$ given by $x^{-1} = \left(\tau_{1/2}\right)^{-1}$, where the term on the right is the average inverse of the individual HR times. We find $x = 11.0$ months, a value closer to the values of $\tau_{90N}$ and $\tau_{90T}$ in Table 2. Time variations of the individual HR times and latitudinally varying stratospheric concentration prevent a closer match.

The formulation of the transfer across the tropopause as being proportional to the difference between stratospheric and tropospheric burdens goes far toward assuring that the transfer agrees with observations in Fig. 4 and Table 2. Why the complex processes of transfer within the stratosphere and across the tropopause should lead to an approximately exponential decay of stratospheric burden is an unsolved problem not addressed here. In the present investigation, the primary purpose is to understand the latitudinal and seasonal variations of Sr-90 at the surface (assuming a well-mixed troposphere). To this extent, the transfer across the tropopause is a sort of boundary condition essential to the determination of the tropospheric Sr-90 evolution. The properties of that evolution, including magnitude and seasonal variations, differ profoundly from those of the stratosphere because of precipitation scavenging.

Figure 5 shows the model tropospheric burdens and their sum as functions of time. The burden in the Southern Hemisphere rises rapidly during the first year following the burst, then oscillates seasonally with little amplitude variation for the next two years, as the larger Northern Hemisphere burden acts as a source. The curves are similar to those for the corresponding stratospheres, but with large (percentage) seasonal oscillations. The seasonal oscillations are only approximately 6 months out of phase across the hemispheres, despite exact 6-month phase differences assumed for diffusion and scavenging processes in corresponding boxes. Also shown in Fig. 5 is the observed tropospheric burden in the Northern Hemisphere, computed from the data used to construct Fig. 3c. (The numerical values are only proportional to the actual burdens and the vertical position of the curve is arbitrary.) It is clear that the model describes the observations fairly well.

In the previous paper, it was shown that the relationship of surface Sr-90 concentration $C$ in low latitudes to precipitation rate $P(t)$ could be roughly reproduced by integration of

$$\frac{\partial C}{\partial t} = -kP(t)C + S(t),$$  \hspace{1cm} (11)

where the source $S(t)$ was taken as a constant and $C$ was regarded as a departure from annual means with interannual variation removed. It is of interest to plot, for Case 1, the variation of burden in a low-

![Graph](https://example.com/graph.png)

**Fig. 5.** As in Fig. 4 but for the troposphere. Shaded area denotes the envelope of the Northern Hemisphere burden. $OT_{N}$ is the observed tropospheric burden in the Northern Hemisphere.
latitude tropospheric box versus the assumed precipitation variation \( P(t) \), and to compare it with (11) using the same \( P(t) \) and a long-term exponential decay of the source. These are compared in Figs. 6a, b and, in turn, with observations in Figs. 6b, c.

Figure 6a shows concentration versus assumed precipitation rate for the box centered at 14.53°N. Times 0, 12, 24 month etc. are August or September, the time of assumed maximum precipitation rate. The detonation occurred at time 0. The variations with time are very smooth, and the character of the long-term variation is established after a very few months. Maximum and minimum concentrations occur one to two months after minimum and maximum precipitation, respectively.

Figure 6b shows the result of forward integration of (11) for

\[ S(t) = S_0 e^{-\lambda t}, \]

where \( \lambda = 0.693/10 \) with \( r_{1/2} = 10 \) month. The time increment was 0.1 month, \( S_0 = 100 \) arbitrary units and \( C = 83.33 \) (from arbitrarily setting \( \partial C/\partial t = 0 \) at \( t = 0 \)). The curve in Fig. 6b is similar to that in Fig. 7a, but with loop area reduced.

Figure 6c shows Sr-90 observations (Feely et al., 1980) at San Juan, Puerto Rico (18°26'N) versus average precipitation rate (1931–60) for the period from January 1963 to February 1968. Both Sr-90 and precipitation have been smoothed by 3-month running means. The early 1963 and late 1967–early 1968 data are affected by bomb testing in 1962 and 1967.

Figure 6c also shows looping, but the loops are much larger than in both 6a and 6b. The precipitation variation is not sinusoidal, and most of the decrease of Sr-90 occurs not during the rapid increase of precipitation rate, from March to May, but during the long period, from May to October, of large precipitation rate. Possibly, from March through May, there is an increased transport from higher latitudes that largely offsets the increasing precipitation scavenging. Another possibility is increased mixing with the lower stratosphere during this time of increasing convection. On the other hand, the Sr-90 surface air concentrations at a particular location reflect rain scavenging over some unknown region including the location rather than scavenging at the observing site only. Note also that precipitation rates during the specific years of the Sr-90 measurements could have been significantly at variance with the 30-year averages used in the figure. Seasonal variations of mean circulation can also affect local Sr-90 concentrations.

Figure 6d depicts the variations of concentration and precipitation rate (1951–60 average) at Balboa (8°58'N), again with both smoothed by 3-month running means. The tilt of the loops resembles those of the model somewhat more closely than those at San Juan. However, every year shows a small counterclockwise loop during the autumn months.

The agreement between observations in Figs. 6c, d and the model results, 6a, is limited by the fact that precipitation does not vary sinusoidally at San Juan and Balboa. In summary of Fig. 6, it may be argued that the importance of seasonal variation of precipitation scavenging is confirmed, but that transport mechanisms other than diffusion are probably necessary to explain many of the variations of concentration.

The source in Case 1 was primarily stratospheric, corresponding to a megaton-yield burst at the surface.

---

**Fig. 6.** Sr-90 concentration as a function of precipitation rate: (a) concentration in the tropospheric box centered at 14.53°N for Case 1; (b) from (7) and (8) (see text for parameters); (c) at San Juan, Puerto Rico (18°26'N); (d) at Miraflores or Balboa (9°00'N, 8°58'N, respectively). Units of concentration are arbitrary in (a, b) and |Ci m⁻³| in (c, d).
For a kiloton-yield burst at the surface, most of the burden is put into the troposphere.

b. Case 2

In this case, we assume a kiloton-yield burst with a source of $10^4$ in the tropospheric box between 30 and 41.8° of the Northern Hemisphere, no source in the stratosphere, and all other conditions the same as in Case 1. Figure 7 depicts the burdens in the Northern Hemisphere troposphere and stratosphere and the total stratospheric burden as functions of time. The first three months are characterized by transfer of a small amount of debris to the stratosphere by mixing and a large amount to the surface by scavenging. Thereafter, the stratospheric burden exceeds that in the troposphere, and their relative magnitudes and variations resemble those subsequent to an instantaneous stratospheric source. The radioactivity initially diffused to the stratosphere becomes a long-lasting source for tropospheric radioactivity.

c. Case 3

This is the same as Case 2, except that the source is in the Northern Hemisphere box adjacent to the equator. The results are also depicted in Fig. 7. The tropospheric burden is more rapidly reduced than in Case 2 because of higher precipitation rates, which also reduce the debris that escapes to the stratosphere. After a few months, the stratosphere holds most of the debris remaining in the atmosphere, as in Case 2.

5. Concluding remarks

Mean circulations have not been included in the simple model described here. It is useful to distinguish between those circulations that are confined either to the stratosphere or the troposphere and those that intersect the tropopause. Examples of the former type are the stratospheric circulations identified by Holton (1981) and Kida (1983). The effect can be to bring high Sr-90 concentrations to the lower stratosphere or to carry low concentrations up from the lower stratosphere. In either case, the transport between boxes will not be accurately expressible as constant times burden differences.

Circulations intersecting the tropopause have been identified by Palmén and Vuorela (1963), Mahlman and Moxim (1978), Oort (1983) and others. These may greatly increase downward transport to the troposphere, where the circulation velocity is downward, or greatly reduce downward diffusive transport where the circulation is upward across the troposphere. In a simple experiment, a mean upward transport across the tropopause was assumed in the first two pairs of boxes poleward from the equator, and a downward transport across the tropopause in the next two pairs of boxes. These were connected by horizontal transports. This circulation resembles the Hadley-type circulation depicted by Palmén and Vuorela. An upstream (upbox) differencing was used that preserved Sr-90 mass and mass squared. The product of circulation velocities and wall areas per volume of box yield transport coefficients and corresponding half-lives. Numerical estimates for reasonable mean circulation speeds suggested half-lives of the order of a year. This simple transport was superimposed on the diffusion model. The principal effect was a significant (1–3 months for reasonable circulation speeds) reduction of stratospheric residence time. Apparently the Sr-90 transported to the troposphere is rapidly scavenged, while an insignificant amount of Sr-90 is transported to the stratosphere. The pattern of Sr-90 in tropospheric air was not significantly altered from that depicted in Fig. 3a in the absence of mean circulation. The principal inference is that mean circulations intersecting the tropopause can reduce the diffusion required to yield observed residence times.

An ambitious attempt to simulate (among other things) tropospheric tracer variations in a GCM was
made by Mahlman and Moxim (1978). An instantaneous source was introduced on 1 January within a volume centered at 36°N, 180°E, and 65 mb in a model with 11 levels. Although the position in nature of actual injections seems not to be crucial for long-term tracer evolution, the position of injections in their model led to an almost completely equatorward initial transport and a first-year tropospheric spring maximum at 15°N.

A number of features in the Mahlman-Moxim simulation of the surface tracer field do not agree with observations. For example, the April maximum in the first year occurs not only in the Northern but also in the Southern hemisphere. A small maximum appears in September. In the second year, the spring maximum is evident only at 48 and 60°N, and the seasonal variation is too small. In the third and fourth years of the simulation, very little seasonal variation occurs.

Mahlman and Moxim attribute the observed spring peak to the action of the winter circulation producing a tracer buildup in the lower stratosphere, but their model fails to simulate the late-winter breakdowns (sudden warmings). They attribute the failure to simulate a strong fall minimum to a GCM defect that causes delay of onset of the summertime easterlies and associated weakening, small extent and short duration, compared to observations. This, in turn, they suggest, leads to a simulated summertime transport efficiency greater than observed; this high efficiency obscures the fall minimum that appears to depend on a weak summertime transport rate.

In contrast, the present simple model attributes the spring peak and fall minimum in middle and high latitudes to a combination of transport variation (stratosphere to troposphere) and precipitation variation, and the peak and minimum at low latitudes to precipitation variation. At middle and high latitudes, the assumed maximum of transport out of the stratosphere may in fact result mostly from a late winter tracer buildup in the lower stratosphere, as argued by Mahlman and Moxim, and from an assimilation of stratospheric mass as the tropopause rises (Staley, 1962).

The model described here is extremely simple and is proposed not as an improvement over a properly formulated tracer experiment in the context of an impeccable general circulation model, but rather as a "toy" that attempts to provide a simplified Sr-90 evolution for arbitrary times and several locations of injections. The processes used in the model, scavenging and large-scale diffusion, are given simplified distributions, and what is accomplished in the model by diffusion is no doubt at times and places in the atmosphere accomplished by processes more accurately described as advective or as part of a mean circulation. It is the task of a general circulation tracer experiment to generate from the fundamental equations the appropriate transports, precipitation, and scavenging. Some of the difficulties with such experiments might be averted by experimenting first with an instantaneous annular source rather than a point source.

The usefulness of a box model elaboration, such as the introduction of more boxes of limited latitudinal extent, is probably limited. On the other hand, the spatially-varied transport in the stratosphere and the absence of precipitation scavenging suggest the possible usefulness of more detailed stratospheric resolution.

Acknowledgments. The author thanks Margaret Sanderson Rae for editing the final manuscript.

REFERENCES


