

## A Two-Dimensional Model of the Quasi-Biennial Oscillation

TIMOTHY J. DUNKERTON

*Physical Dynamics, Inc., Bellevue, WA 98009*

(Manuscript received 23 August 1984, in final form 8 February 1985)

### ABSTRACT

The quasi-biennial oscillation is simulated in two dimensions using a WKB approach for the equatorial waves in which analytic approximations to the wave-induced body forces are inserted as forcing terms in a primitive equation model of the zonally-averaged flow. Realistically large amplitude oscillations are obtained with this method.

Examination of one such oscillation in this paper clarifies the linear and nonlinear role of the residual mean meridional circulation: at low latitudes due to its vertical advection of momentum, and at higher latitudes on account of the Coriolis torque. Because this circulation also advects ozone, the primary source of radiative heating in this region, coupling between the ozone and dynamical oscillations will be significant in determining the strength of this meridional circulation.

Also of importance for the waves themselves are the scale-dependent radiative damping rate and, in the Rossby-gravity wave, the effect of latitudinal shear.

### 1. Introduction

It is well known that the mean flow variance of the equatorial lower stratosphere is dominated by the *quasi-biennial oscillation* (QBO; Reed *et al.*, 1961; Veryard and Ebdon, 1961; Nastrom and Belmont, 1975; Hamilton, 1984; Dunkerton and Delisi, 1984). The time-height structure of the QBO was successfully explained by Holton and Lindzen (1972) using a one-dimensional model in which equatorial Kelvin and Rossby-gravity waves propagate vertically and deposit momentum in the mean flow. Subsequent theoretical studies have suggested refinements to the Holton-Lindzen theory, but all have been based on the same, prototype one-dimensional model (Plumb, 1977; Hamilton, 1981; Dunkerton, 1981). Even the remarkable laboratory experiment of Plumb and McEwan (1978) was more analogous to the one-dimensional simulations; their experimental results were well-simulated by the one-dimensional model.

The quasi-biennial oscillation of the real world, however, is two-dimensional. Since Reed's classic study 20 years ago (U.S. Navy Weather Research Facility, 1964), it has been known that the QBO mean zonal wind amplitude is approximately Gaussian about the equator, with  $e$ -folding scale of about  $15^\circ$  latitude. The temperature quasi-biennial oscillation is also centered on the equator, but has a reversed phase at latitudes greater than  $15^\circ$  from the equator (Reed, 1964; Dunkerton and Delisi, 1984).

In view of this two-dimensional structure and the possible role of the residual mean meridional circulation, two-dimensional modeling is a next logical step. But multidimensional modeling of the QBO has

never been very successful; the explanation seems to be that it places stringent requirements on the spatial resolution of a numerical model, and requires accurate simulation of the equatorial wave spectrum. Middle atmosphere GCMs, consequently, have been unable to simulate the QBO. Even the more specialized mechanistic models have had their difficulties. Plumb and Bell (1982), for example, were able to achieve only a small-amplitude stable quasi-biennial oscillation in their two-dimensional mechanistic model. Holton (personal communication, 1979) attempted to integrate an expensive primitive equation model in the presence of two equatorial waves, but found a steady state solution.

The purpose of this paper is to discuss a two-dimensional "waveguide" model of the quasi-biennial oscillation which has resulted in the first realistically large-amplitude simulations of this phenomenon in two dimensions. This model differs from those of Holton (1979) and Plumb and Bell (1982) because the equatorial wave calculation is performed with the same WKB formalism used by Holton and Lindzen (1972). Here, the WKB method is generalized along the lines suggested by Boyd (1978). Boyd retained the two-scaling approximation in time and height, but allowed the latitudinal wave structure to be determined with latitudinal shear at lowest order. The equatorial wave calculation is then split into two parts. First, the latitudinal eigenproblem is solved, giving the eigenvalue, wave fields and vertical group velocity. Second, the latitudinally-integrated wave action equation is used, in time and height, to determine wave amplitude. Details of this method are given in Dunkerton (1983). Once the wave calculation is complete,

the equatorial wave Eliassen-Palm flux convergences are then inserted as forcing terms into the mean zonal momentum equation (Andrews and McIntyre, 1976). Following Holton and Wehrbein (1980), the mean flow equations are implicitly understood to be *transformed Eulerian mean* equations, as defined by Andrews and McIntyre. We note that this method was also used implicitly by Holton and Lindzen, who simplified the procedure by integrating the mean zonal momentum equation in latitude, and subsequently neglecting latitudinal shear and the residual mean meridional circulation.

Andrews and McIntyre give the general WKB expression for the wave-induced body force per unit mass:

$$F = \left( \alpha_M + \frac{1}{2} \frac{\partial}{\partial t} \right) \left[ -(\overline{\eta' u'})_y + \frac{1}{c - \bar{u}} (\overline{u' u'} + \overline{v'^2}) \right] + \left( \alpha_T + \frac{1}{2} \frac{\partial}{\partial t} \right) \frac{\overline{\epsilon \phi'^2}}{c - \bar{u}}. \quad (1.1)$$

In this equation,  $u'$ ,  $v'$  and  $\phi'$  are perturbation zonal wind, meridional wind and geopotential,  $u' = u' + \eta' \bar{u}_y$ , where  $\eta'$  is meridional displacement,  $\epsilon$  is the eigenvalue  $m^2/N^2$ , where  $m$  is vertical wavenumber and  $N$  is static stability, and  $c$  is the phase speed relative to the ground. The body force is attributable to mechanical and thermal damping (rate coefficients  $\alpha_M$  and  $\alpha_T$ ) and wave transience ( $\partial/\partial t$ ). Additional contributions to the mean zonal momentum budget include vertical diffusion, Coriolis torque and nonlinear advection. Although these are small in the WKB approximation<sup>1</sup> (Andrews and McIntyre, 1976), the residual advection is important in other ways, including the ozone QBO. The two-dimensional model now determines this circulation explicitly, together with the latitudinal shear.

## 2. Numerical model

### a. Mean flow

The numerical model consists of two parts. First, the residual mean meridional streamfunction, and the resulting mean flow evolution, are determined with a routine adapted directly from the global, middle atmosphere primitive equation model of Holton and Wehrbein (1980). Their model is zonally symmetric, and solves the transformed Eulerian mean equations with eddy forcing terms included on the right-hand side. The interested reader is referred to their paper for a description of the primitive equations and their solution method. Here, the author will

outline modifications that have been made to their model.

In its original form, the Holton-Wehrbein model utilizes a large  $19 \times 17$  grid domain, extending from pole-to-pole and from 20 to 100 km. In the present calculation the number of grid points remains the same, but the domain size is reduced:

$$\Delta\theta = \frac{5^\circ}{3} \quad (2.1a)$$

$$\Delta z = 1 \text{ km} \quad (2.1b)$$

where  $\theta$  is latitude and  $z$  is height. The model domain now extends from 17 to 33 km and from the equator to  $30^\circ$ . Hemispheric symmetry is assumed.

The model time step has been increased to 1 day, because the mean flow evolution proceeds more slowly than in Holton and Wehrbein. The equatorial wave calculation is the second major ingredient in the model and will be described in Section 2c below.

Boundary conditions on the mean equations are similar or identical to those of Holton and Wehrbein. The mean zonal wind is held constant and equal to zero at the lower boundary while its derivative vanishes at the upper boundary. The side boundary at  $30^\circ$  is a rigid wall for the streamfunction calculation, but is far enough removed from the region of interest. The mean fields are initialized everywhere to zero.

### b. Dissipation

The model includes three dissipative effects in the mean flow equations; vertical momentum diffusion  $\bar{\nu}(z)$ , constant horizontal momentum diffusion  $K = 5 \times 10^3 \text{ m}^2 \text{ s}^{-1}$ , and Newtonian cooling  $\kappa(m, z)$ . The profile of vertical mean momentum diffusivity is of the form

$$\bar{\nu}(z) = 0.3 \text{ m}^2 \text{ s}^{-1} [1 + 5.5 \exp(17 - z)]. \quad (2.2a)$$

The Newtonian cooling rate is adopted from Fels (1982) and is scale-dependent, varying as

$$\kappa(m, z) = \alpha_0(z) \frac{m^2}{\alpha_1(z) + m^{3/2}} + \alpha_2(z) \approx \alpha_0(z) m^{1/2} \quad (2.2b)$$

in Fels'  $m^{1/2}$  regime (where  $m$  is in  $\text{km}^{-1}$ ), which is relevant for the equatorial waves. For the mean state we initially use Newtonian cooling appropriate for radiation to space (i.e.,  $m = 0$ ). This procedure, however, is uncertain on three counts. First, the mean heating includes a time-mean component (e.g., Dopplink, 1979) which is not well-known. Second, the external heating also includes a quasi-biennially varying component due to the ozone QBO. To the author's knowledge no one has yet calculated this component or even estimated its importance (although cf. Dickinson, 1968, p. 269). Finally, the radiation-

<sup>1</sup> In the sense that  $|\bar{u}_z| < N$ . Thus the "internal" diabatic circulation driven by radiative damping or transience in the wave-induced QBO shear zones is also "small." The residual advection is retained here mainly for diagnostic reasons.

to-space approximation is likely to be inaccurate even for the mean state since the vertical “wavelength” of the mean temperature perturbation is  $O(10 \text{ km})$ . For these reasons it will be worthwhile to explore the effect of the mean diabatic heating in Section 3d. The author hopes to update these calculations in a future study of the ozone quasi-biennial oscillation.

c. Equatorial waves

Equation (1.1) determines the body force per unit mass due to equatorial waves. Following Holton and Lindzen (1972) it is assumed as a hypothesis that the westerly acceleration is due to a Kelvin wave (Wallace and Kousky, 1968) while the easterly acceleration is due to a Rossby-gravity wave (Yanai and Maruyama, 1966). As sketched in the Introduction, the WKB wave calculation is split into two parts. In the interests of computational efficiency, both parts of this calculation will receive a simplified semi-analytic treatment in this paper. For the second part (calculation of wave amplitude) the waves are assumed steady, while dissipated by Newtonian cooling (Holton and Lindzen, 1972) and second-order diffusion  $\nu m^2$ . Inspection of the model solutions confirmed *a posteriori* the validity of the steady waves approximation (in the sense that the time rate of change of latitudinally-integrated wave action density is small in the cases discussed here). The vertical component of the latitudinally-integrated Eliassen-Palm flux is, in this case, given by the usual expression

$$B(z) = B(17) \exp\left[\frac{z}{H} - P(z)\right] \quad (2.3)$$

where

$$P(z) = \int_{17}^z D(z') dz' \quad (2.4a)$$

$$D(z) = \begin{cases} (\alpha_M + \alpha_T)W^{-1} & \text{(Kelvin)} \\ \left(\frac{3}{2}\alpha_M + \frac{1}{2}\alpha_T\right)W^{-1} & \text{(Rossby-gravity)}. \end{cases} \quad (2.4b)$$

$W$  is the vertical group velocity and  $H = 6 \text{ km}$  is the density scale height. The effective damping rate for Rossby-gravity waves is derived from the equipartition law (gamma-plane approximation for waves in a hemispherically symmetric mean flow; see Boyd, 1978), and displays the relatively small thermal component of wave action for this wave.

The first part of the wave calculation (determination of the eigensolutions) will also receive a simplified analytic treatment. Although in Dunkerton (1983) the eigenvalue problem was solved exactly, it will be desirable for numerical reasons to have analytic profiles of wave-induced body force per unit mass:

1) KELVIN WAVE

That the Kelvin wave is insensitive to shear in a hemispherically symmetric mean flow, and behaves approximately as if the mean zonal wind were everywhere equal to its value at the equator, is inferred from Boyd’s (1978) perturbation analysis and the numerical solutions of Holton (1979). Thus

$$W = \frac{k(c - \bar{u}_e)^2}{N} \quad (2.5a)$$

where  $k$  is zonal wavenumber and  $\bar{u}_e = \bar{u}(0, z, t)$ . For Kelvin wave parameter values  $k = 1/a$ , where  $a$  is the radius of the earth, and  $c = 30 \text{ m s}^{-1}$ .

The latitudinal profile of Kelvin wave-induced body force per unit mass in this case remains the same as in no shear (i.e. Gaussian), with latitudinal scale

$$y_0 = [(c - \bar{u}_e)/\beta]^{1/2} \quad (2.5b)$$

where  $\beta = 2.29 \cdot 10^{-11} \text{ m}^{-1} \text{ s}^{-1}$ .

2) ROSSBY-GRAVITY WAVE

The so-called “gamma-plane” approximation of Boyd (1978) implies that the Rossby-gravity wave in a hemispherically symmetric mean flow is sensitive to the intrinsic frequency  $\omega_0$  and mean flow curvature  $\delta_0$  within its “potential-well,” defined by the lowest order gamma-plane equation

$$v_{yy} - \epsilon \Delta v = 0 \quad (2.6a)$$

where

$$\Delta \equiv \beta(\beta - \delta_0)y^2 - \omega_0^2. \quad (2.6b)$$

For constant  $\delta_0$ , i.e. parabolic mean flow between the zeroes of  $\Delta$ , the analytic solutions of Boyd (1978) are relevant. For example,

$$W = \frac{|\omega_0|^3}{2N[\beta(\beta - \delta_0)]^{1/2}}. \quad (2.7a)$$

Accuracy of the zeroth-order gamma-plane fields, including this expression, depends on smallness of the parameter  $|\chi| \equiv (k^2|c - \bar{u}|)/\beta$ . The zeroth-order fields are therefore most accurate in the QBO easterly phase, when  $|\chi| < 1/2$ . The accuracy of this approximation deteriorates in the westerly phase, on the other hand, when  $|\chi|$  approaches  $O(1)$ . Primarily for this reason, this simplified treatment of the Rossby-gravity wave is subject to more uncertainty than our treatment of the Kelvin wave outlined above. A lesser consideration is that the mean zonal wind profile is not always parabolic between the zeroes of  $\Delta$ , or that for some reason the mean flow outside the potential well might be influential. It seems, however, that the wind profile departs from a parabolic shape equatorwards of  $10^\circ$  latitude only at the onset on the QBO westerly acceleration phase, when Rossby-gravity

waves are least expected to be present (Dunkerton and Delisi, 1984).

In the gamma-plane approximation, the effect of symmetric, parabolic shear in the QBO easterly (westerly) phase is to decrease (increase) the rate of wave absorption and to expand (contract) the latitudinal scale. The scale factor is

$$y_0 = \frac{|\omega_0|}{[\beta(\beta - \delta_0)]^{1/2}}, \quad (2.7b)$$

into which a constant correction factor is multiplied in order to better approximate the exact scale without shear. Because positive or easterly curvature develops in the easterly phase of the QBO, and vice versa, the latitudinal shear retards the approach to the Rossby-gravity wave critical level, spreading the easterly acceleration over a longer time period. Using the lowest-order gamma-plane fields, the heuristic latitudinal profile of wave-induced body force per unit mass becomes

$$F = \frac{\langle F \rangle}{y_0 \sqrt{\pi} \left( \frac{3}{2} + \frac{1}{2} \lambda^{-1} \right)} \left[ \frac{\beta - \delta_0}{k^2 (c - \bar{u})} \times (2\xi^2 - 1) + 1 + \xi^2 + \lambda^{-1} \xi^2 \right] e^{-\xi^2} \quad (2.8)$$

for the Rossby-gravity wave, where  $\lambda \equiv \alpha_M / \alpha_T$ . In this equation,  $\xi = y/y_0$  and angle brackets denote the latitudinal integral. Parameter values are taken to be  $k = 4/a$  and  $c = -30 \text{ m s}^{-1}$  for this wave.

### 3. A two-dimensional quasi-biennial oscillation

#### a. Forced wave amplitudes

The wave amplitudes are held constant at the lower boundary, as in Holton and Lindzen (1972). One way to specify the forcing is in terms of the maximum

vertical component of Eliassen-Palm flux  $S_{(xz)}$  at the tropopause. For the simulation described in this section,

$$S_{(xz)}(0, 17, t) = 0.0060 \text{ m}^2 \text{ s}^{-1} \quad (\text{Kelvin}) \quad (3.1a)$$

$$S_{(xz)}(y_0, 17, t) = -0.0044 \text{ m}^2 \text{ s}^{-1} \quad (\text{Rossby-gravity}). \quad (3.1b)$$

These choices seem reasonably well-supported by Table 1 of Wallace (1973) which suggests a relatively smaller Rossby-gravity wave forcing (although smaller than 3.1b).

#### b. Time-height cross-section at the equator

Figure 1 shows the time-height cross section of equatorial mean zonal wind for this experiment. The overall structure resembles the one-dimensional solutions. The oscillation period is 31 months, within one standard deviation of the observed average period (27 months). The various model parameters and assumptions have conspired to produce this period, particularly the wave amplitudes, phase speeds and wave dissipation mechanisms  $\alpha_0(z)$  and  $\nu$ . In Fig. 1 we have set  $\nu = 0.05 \text{ m}^2 \text{ s}^{-1}$ , within the range of values recently used by Takahashi (1984) in this region. In fact, reasonable variations of these model parameters can generate quite a large range of periods from 2 to 3 years and beyond. Under some circumstances the two-dimensional model has steady-state solutions like those of Plumb (1977).

The mechanism underlying the oscillation is basically the same as in one dimension. For example, a shear zone is created at upper levels by one of the equatorial waves. Here, the mean flow is shielded from the influence of the opposite wave because that wave has already undergone "self-absorption" at lower levels (by creating its own shear zone). Starting from rest, the whole process may be viewed as a mean

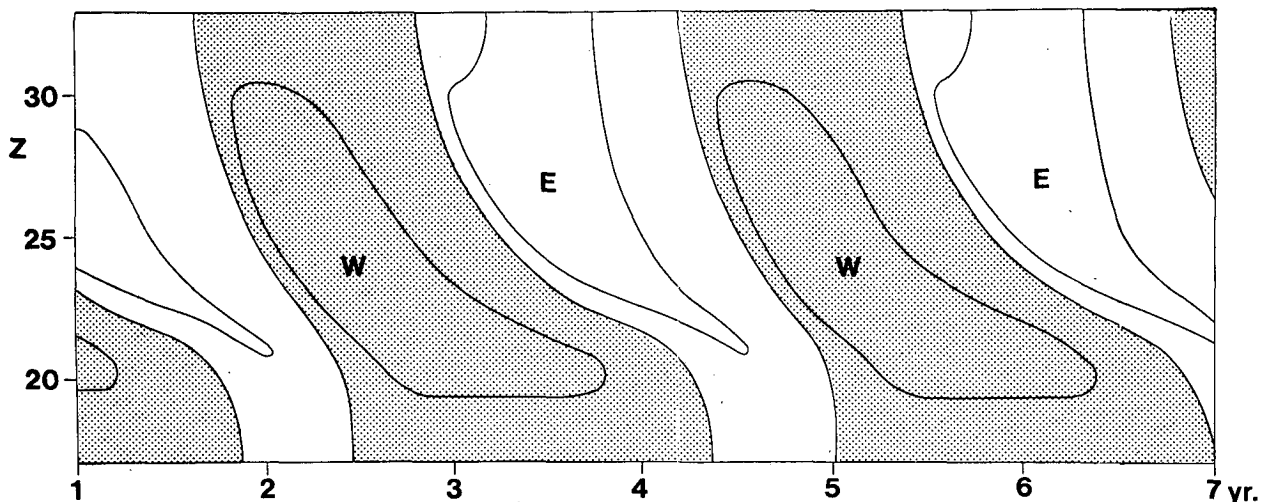


FIG. 1. Time-height cross section of mean zonal wind at the equator. Contour intervals: 15 m s<sup>-1</sup>.

flow instability as noted by Plumb. Wave-induced shear zones descend to the tropopause and are rapidly dissipated. A viscous tropopause is used here, as in Plumb and Bell (1982), and for the same reason, viz. to properly dissipate the shear zones and (in our case to) yield a realistic QBO amplitude profile in height. It seems possible that enhanced diffusion in the lower stratosphere is realistic on account of breaking inertia-gravity waves in this region (e.g. Yamanaka and Tanaka, 1984). A suitable parameterization of this diffusive process lies outside the scope of this paper, however.

The acceleration phases in Fig. 1 agree reasonably well with the observed. Maximum accelerations at 25 km are 12.4 and  $-10.0 \text{ m s}^{-1} \text{ mo}^{-1}$  in the westerly and easterly shear zones, respectively. Since Holton and Lindzen (1972) obtained an incorrect acceleration asymmetry in their one-dimensional solution it is of some interest to ascertain why the correct asymmetry is found in Fig. 1. A combination of factors seems to be involved, viz. the scale-dependent radiative damping, the effect of latitudinal shear on the Rossby-gravity wave, the reduced Rossby-gravity wave amplitude, and the residual mean meridional advection. The latter also has a noticeable effect slowing the easterly shear zone descent in the lower levels of the model (and hastening the westerly descent) (Plumb and Bell, 1982).

### c. Latitudinal structure

Looking at the meridional plane, Figs. 2 and 3 display two phase transitions: day 1650 (descending westerlies) and day 1170 (descending easterlies). There is a noticeable asymmetry in the latitudinal scale of the wave-induced mean flows in Figs. 2 and 3. The descending westerly jet is much narrower, about  $7^\circ$  in scale. Consequently, the  $e$ -folding scale of the mean zonal wind oscillation is  $11^\circ$ , about  $4^\circ$  narrower than the observed ( $15^\circ$ ). In fact, there is a node in the model  $\bar{u}$  just outside of  $20^\circ$  latitude which is not observed. Compared to observations, the easterly phase is about  $10 \text{ m s}^{-1}$  too weak at all latitudes.

Several factors contribute to this unrealistic behavior. First, the Wallace and Goussy (1968) Kelvin wave cannot be expected to yield the observed latitudinal scale of the westerlies, especially under the present assumptions. For the Kelvin wave, and in the absence of all other forces, the mean flow profile would evolve according to

$$\frac{du(y)}{du(0)} = \exp\left[\frac{\beta y^2}{u(0) - c}\right] \quad (3.2)$$

which implies an effective asymptotic  $e$ -folding scale of about  $9^\circ$  latitude, starting from an initial state  $\bar{u}(0) = -25 \text{ m s}^{-1}$ . This general result does not depend

on how wave amplitude is determined (i.e. not dependent on the relative importance of transience and damping), but depends only on the assumption of Gaussian-shaped acceleration, the predominance of  $\bar{u}(0)$ , and the neglect of other forces.

A second cause of the narrow latitudinal scale is the opposing contribution of the Coriolis force at latitudes away from the equator (Figs. 2b and 3b). Since the westerly (easterly) shear zone requires downward (upward) mean motion to balance radiative dissipation, by mass continuity there is equatorward (poleward) meridional flow above the westerly (easterly) shear zone (Plumb and Bell, 1982).

When the linear Coriolis torque is superimposed on the nonlinear vertical advection there results an interesting asymmetry between the two phases. The linear contribution is *diffusive* in height (Dickinson, 1968; Plumb and Bell, 1982) and reinforces the mean flow acceleration below the level of maximum wave driving and away from the equator. The nonlinear vertical advection near the equator, on the other hand, produces westerly accelerations in both phases (Holton and Lindzen, 1972). Consequently it is easy to achieve maximum initial acceleration off the equator in the easterly phase, but not the westerly phase. Figure 4 lends support to this effect, showing the mean zonal wind in the latitude-time plane at 25 km in the model. The phase of maximum westerlies propagates away from the equator, and vice versa, in qualitative accord with observations (Hamilton, 1984; Dunkerton and Delisi, 1984) (although the model simulation does not seem to reproduce the very narrow initial westerly accelerations observed at the equator). As it turns out, however, the primary cause of this behavior is the wavedriving itself—as it should be in the WKB approximation. The westerly Kelvin wave body force always peaks at the equator, whereas the Rossby-gravity wave profile of  $F$  is initially maximum easterly away from the equator on account of the thermal damping. The value of  $\nu$  is important, as wave transience also is, because the ratio  $\lambda$  determines whether  $F$  is peaked at the equator (cf. Fig. 1 of Andrews and McIntyre, 1976). The author explored the range  $0.05 < \nu < 0.30 \text{ m}^2 \text{ s}^{-1}$  and found unrealistically narrow easterly accelerations towards the higher  $\nu$  value, at which point the node in  $\bar{u}$  also moved inside  $20^\circ$  latitude.

Results from a primitive equation model of Rossby-gravity waves, to be reported separately, also indicate a defectively narrow easterly QBO phase, with a more realistic equatorial time-height cross section. Therefore there now seems to be some uncertainty in the wavedriving mechanisms away from the equator, in addition to the question regarding the radiatively-driven mean meridional circulation.

The mean temperature perturbation and the associated residual mean vertical velocity are shown in Figs. 2c, d and 3c, d. The temperature amplitude is

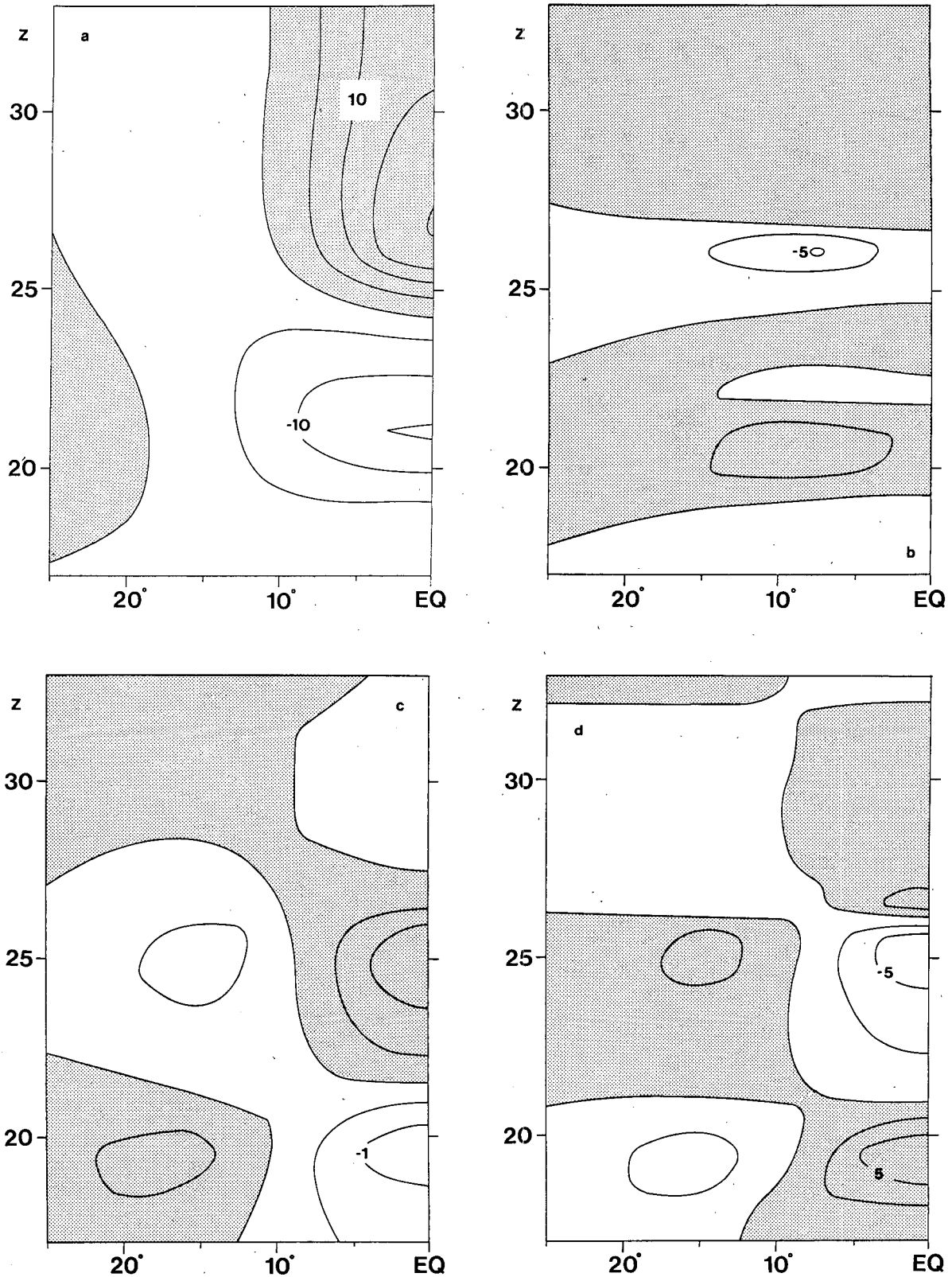


FIG. 2. Model variables in the meridional plane at day 1650. (a)  $\bar{u}$  ( $5 \text{ m s}^{-1}$  intervals); (b)  $\bar{v}^*$  ( $2.5 \text{ cm s}^{-1}$  intervals); (c)  $\bar{T}$  ( $0.5^\circ\text{C}$  intervals); and (d)  $\bar{w}^*$  ( $2.5 \cdot 10^{-5} \text{ m s}^{-1}$  intervals). Positive values are shaded.

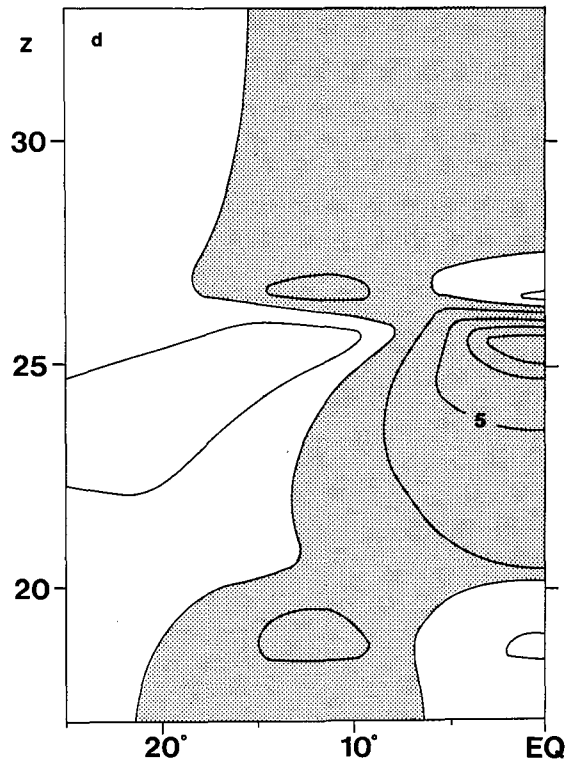
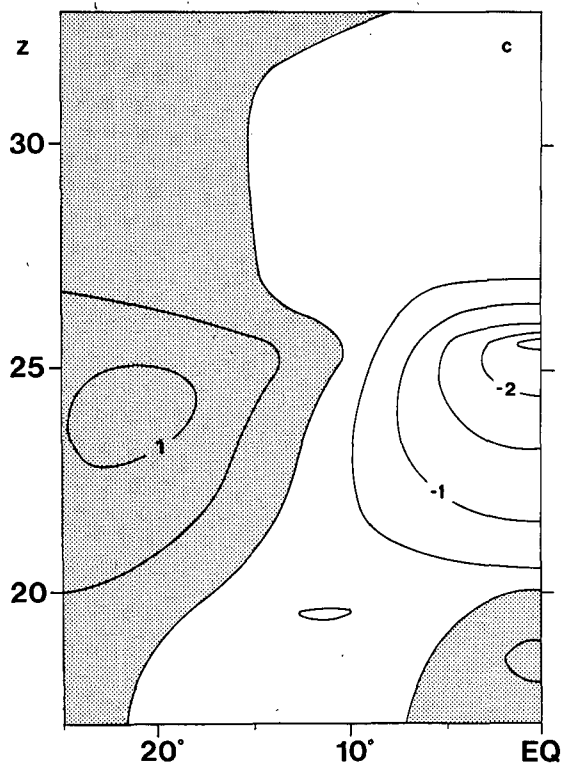
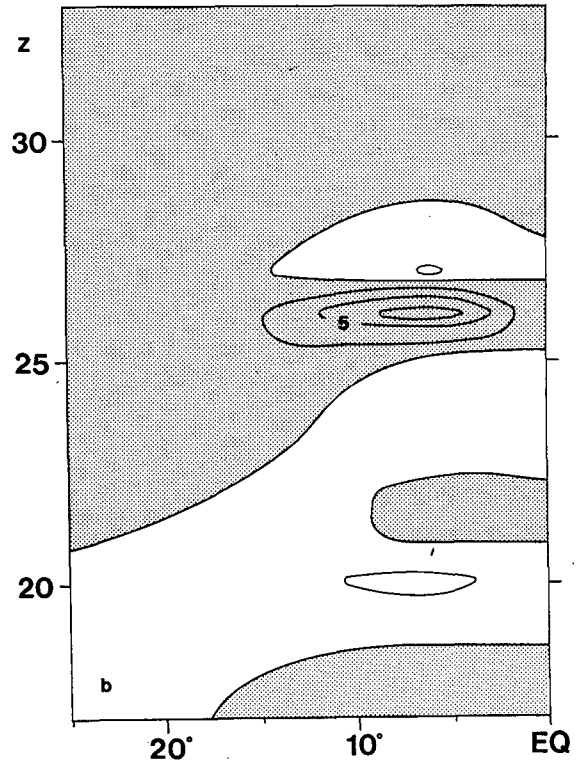
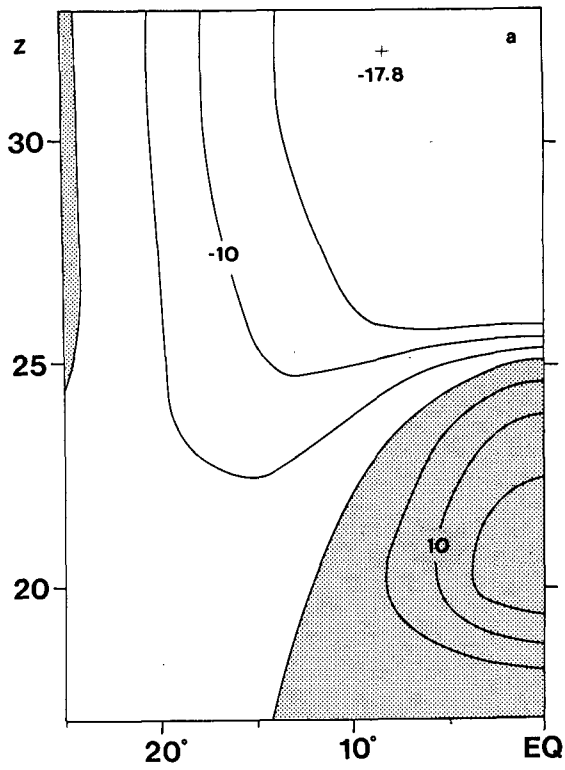


FIG. 3. As in Fig. 2 but for day 1170.

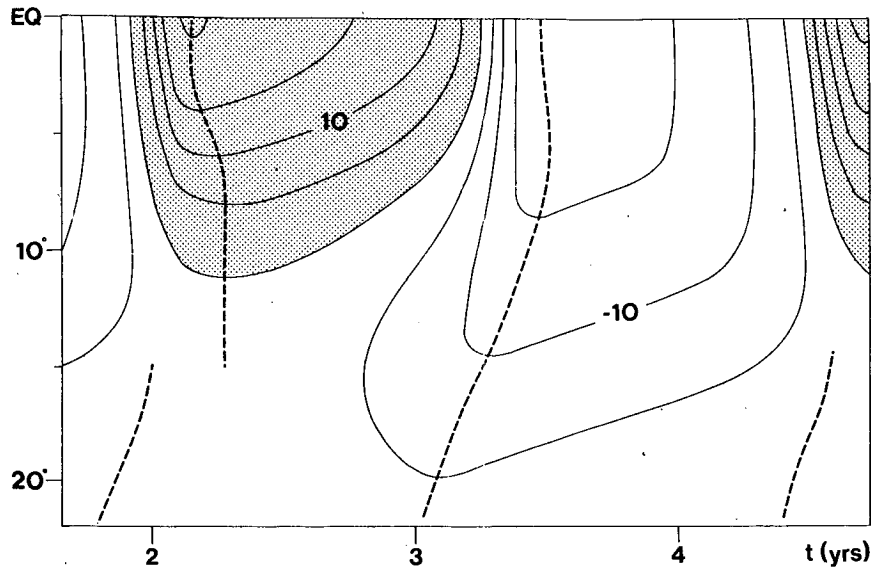


FIG. 4. Mean zonal wind in the latitude-time plane at 25 km. Contour intervals: 5 m s<sup>-1</sup>.

just about 2°C, in fair agreement with earlier observations (e.g. Nastrom and Belmont, 1975), but somewhat lower than what we reported in Dunkerton and Delisi (1984). There the 30 mb temperature amplitude at Singapore was found to be 3 to 4°C (QBO component only). The narrow scale of the simulated QBO is responsible for the smaller temperature oscillation in the model, and not the vertical wind shear. Note that for Gaussian profiles the temperature perturbation by thermal wind balance is proportional to

$$\bar{\phi}_z = \beta L^2 \bar{u}_z \quad (3.3)$$

so that increasing the scale  $L$  could conceivably yield the correct temperature amplitude.

The vertical velocity field, as already mentioned, is characterized by upward (downward) motion in the cold easterly (warm westerly) shear zone. The nonlinear advection  $\bar{w}^* \bar{u}_z$  is  $O(5 \text{ m s}^{-1} \text{ month}^{-1})$  at the peak  $\bar{w}^*$  (near 25 km) in Fig. 3d. Although it is relatively small here, the importance of this contribution increases dramatically in those steady-state solutions mentioned earlier. The typical steady-state solution in this model has an easterly shear zone overlying a low-level westerly jet, with upward residual mean motion preventing descent of the easterly shear zone.

Outside 10° latitude the vertical velocity and temperature oscillations are reversed in phase relative to the equator, although both quantities are typically reduced in amplitude here by a factor of 2. This is in reasonable agreement with the temperature node observed by Dunkerton and Delisi 10–14° from the equator, although the simulated node seems to be at the low end of this range. It would be inferred,

moreover, that a quasi-conserved vertically-stratified tracer, such as ozone, would also have an oscillation node near 10° in this model, if only residual mean advection were taken into account. In contrast, Hasebe (1984) locates the Southern Hemisphere column ozone QBO node at 15° from the equator, with a phase progression away from this point. The reason for these discrepancies is not yet known, although hemispheric asymmetries in lateral diffusion and the Hadley circulation are expected.

#### d. Other simulations

It will be worthwhile to briefly summarize some of the more important variations on this experiment.

##### 1) MEAN RADIATIVE DAMPING

The strength of the meridional circulation is roughly proportional to the magnitude of the Newtonian cooling profile used for the mean state. For example, when enhanced damping appropriate to an  $O(10 \text{ km})$  vertical wavelength perturbation is used (approximately 2.5 times that for radiation-to-space), the circulation strength is doubled, and the westerly jet becomes about 2° narrower. This proportionality is not exact, for when the mean radiative damping is artificially set to zero, we find a more complex meridional cell, about 20% of that observed previously, having a vertically convergent (divergent) behavior in the easterly (westerly) shear zone. This behavior is attributable to mean temperature *transience*, some evidence of which can also be found in Figs. 2 and 3. Other effects due to variation in mean radiative damping were generally small, e.g. the QBO period



varied about a month in either direction, and the larger  $\kappa$  value resulted in a better acceleration asymmetry between the two phases.

## 2) FORCED WAVE AMPLITUDES

In the one-dimensional model the oscillation period is inversely proportional to forced wave amplitude, at least up to the point where the mean flow diffusion is no longer sufficient to cause an oscillation (Plumb, 1977; Dunkerton, 1981). In the two-dimensional model with mean radiative damping, similar behavior is observed, except that (as already noted) the lower limit on the period is now reached on account of the nonlinear advective flux  $\bar{u}\bar{w}^*$ , in addition to the diffusive flux of momentum. Figure 1 seems to be rather close to this limit; increasing the forced-wave amplitude would not serve to lower the oscillation period significantly, without requiring some other change in model parameters. Of course the oscillation period could be lengthened by *reducing* this forcing appreciably.

## 3) LATERAL DIFFUSION COEFFICIENT

The modest value of  $K$  used in this experiment was not an important effect.<sup>2</sup> A larger value  $2 \times 10^5 \text{ m}^2 \text{ s}^{-1}$  quickly reduced the amplitude and period of the simulated oscillation (note that this value implies a momentum decay time comparable to the QBO period). The reason for increasing  $K$  was to determine if the latitudinal scale of the oscillation could be increased significantly; as it turned out, this could be done if the diffusion coefficient  $K$  was cast into a latitudinal profile in which  $K$  rapidly increased away from the equator to this value. In reality the so-called "surf zone" might produce this kind of profile (McIntyre and Palmer, 1984) although strictly speaking the effect diffuses potential vorticity and not momentum. To include such an effect in the model at the time of this writing, however, seemed a little controversial and uncertain.

## 4. Conclusion

Undoubtedly the most satisfactory way to model the quasi-biennial oscillation will involve general circulation modeling—a "brute force" approach. Until such models having adequate resolution can be executed in something less than real time, however, the faster mechanistic models such as that introduced in this paper will provide a dynamically sensible way to simulate this oscillation. An investigation of the ozone quasi-biennial oscillation and its possible coupling with the dynamical QBO is one application that would be useful. The significance of ozone advection

derives from the fact that the external radiative heating is a small residual of two much larger terms. If local enhancement of ozone due to downward advection causes enhanced local heating, the effect will be to lessen the necessary residual mean subsidence in the westerly shear zone, and vice versa. This will in turn lessen the Coriolis torque which opposes the wave drag at latitudes away from the equator. For this and other reasons, the ozone QBO merits further study.

Somewhat more fundamental to the QBO problem are the wavedriving mechanisms themselves. Here, the Holton-Lindzen theory adequately models the flow near the equator (as our more recent primitive equation modelling also confirms) but does not seem to provide enough body force in either acceleration phase at latitudes away from the equator. One cannot even be certain that the Rossby-gravity wave body force is of the correct sign in subtropical latitudes. Therefore, the QBO problem remains partly conceptual as well as numerical. While the success of the present model indicates progress in the latter category, the theory will require better observations before the problem is completely solved.

*Acknowledgments.* The author thanks T. Ackerman and J. R. Holton for helpful discussions. Dr. Holton also kindly provided the source code used for the mean equations. Reviews by K. Hamilton and R. A. Plumb are also acknowledged. This research was supported by the National Science Foundation, NSF Grant ATM-8217503.

## REFERENCES

- Andrews, D. G., and M. E. McIntyre, 1976: Planetary waves in horizontal and vertical shear: the generalized Eliassen-Palm relation and the mean zonal acceleration. *J. Atmos. Sci.*, **33**, 2031-2048.
- Boyd, J. P., 1978: The effects of latitudinal shear on equatorial waves, part 1: theory and methods. *J. Atmos. Sci.*, **35**, 2236-2258.
- Dickinson, R. E., 1968: On the excitation and propagation of zonal winds in an atmosphere with Newtonian cooling. *J. Atmos. Sci.*, **25**, 269-279.
- Dopplack, T. G., 1979: Radiative heating of the global atmosphere: Corrigendum. *J. Atmos. Sci.*, **36**, 1812-1817.
- Dunkerton, T. J., 1981: Wave transience in a compressible atmosphere, part 2: transient equatorial waves in the quasi-biennial oscillation. *J. Atmos. Sci.*, **38**, 298-307.
- , 1983: The evolution of latitudinal shear in Rossby-gravity wave, mean flow interaction. *J. Geophys. Res.*, **88**, 3836-3842.
- , and D. P. Delisi, 1984: Climatology of the equatorial lower stratosphere: An observational study. *J. Atmos. Sci.*, **42**, 376-396.
- Fels, S. B., 1982: A parameterization of scale-dependent radiative damping rates in the middle atmosphere. *J. Atmos. Sci.*, **39**, 1141-1152.
- Hamilton, K., 1981: The vertical structure of the quasi-biennial oscillation: observations and theory. *Atmos.-Ocean*, **19**, 236-250.
- , 1984: Mean wind evolution in the tropical lower stratosphere. *J. Atmos. Sci.*, **41**, 2113-2125.

<sup>2</sup> For this reason analytic profiles of  $F$  were used.

- Hasebe, F., 1984: The global structure of the total ozone fluctuations observed on the time scales of two to several years. *Dynamics of the Middle Atmosphere*, Terra Scientific, 445-464.
- Holton, J. R., 1979: Equatorial wave, mean-flow interaction: a numerical study of the role of latitudinal shear. *J. Atmos. Sci.*, **36**, 1030-1040.
- , and R. S. Lindzen, 1972: An updated theory for the quasi-biennial cycle of the tropical stratosphere. *J. Atmos. Sci.*, **29**, 1076-1080.
- , and W. M. Wehrbein, 1980: A numerical model of the zonal mean circulation of the middle atmosphere. *Pure and Appl. Geophys.*, **118**, 284-306.
- McIntyre, M. E., and T. N. Palmer, 1984: The "surf zone" in the stratosphere. *J. Atmos. Terr. Phys.*, (to appear).
- Nastrom, G. D., and A. D. Belmont, 1975: Periodic variations in stratospheric-mesospheric temperature from 20-65 km at 80°N to 30°S. *J. Atmos. Sci.*, **32**, 1715-1722.
- Plumb, R. A., 1977: The interaction of two internal waves with the mean flow: implications for the theory of the quasi-biennial oscillation. *J. Atmos. Sci.*, **34**, 1847-1858.
- , and A. D. McEwan, 1978: The instability of a forced standing wave in a viscous stratified fluid: A laboratory analog of the quasi-biennial oscillation. *J. Atmos. Sci.*, **35**, 1827-1839.
- , and R. C. Bell, 1982: A model of the quasi-biennial oscillation on an equatorial beta-plane. *Quart. J. Roy. Meteor. Soc.*, **108**, 335-352.
- Reed, R. J., 1964: A tentative model of the 26-month oscillation in tropical latitudes. *Quart. J. Roy. Meteor. Soc.*, **90**, 441-466.
- , W. J. Campbell, L. A. Rasmusson and D. G. Rogers, 1961: Evidence of the downward-propagating annual wind reversal in the equatorial stratosphere. *J. Geophys. Res.*, **66**, 813-818.
- Takahashi, M., 1984: A 2-dimensional numerical model of the semi-annual zonal wind oscillation. *J. Meteor. Soc. Japan*, **62**, 52-68.
- U.S. Navy Weather Research Facility, 1964: *A Climatology of Wind and Temperatures in the Tropical Stratosphere between 100 mb and 10 mb*.
- Veryard, R. G., and R. A. Ebdon, 1961: Fluctuations in tropical stratospheric winds. *Meteor. Mag.*, **90**, 125-143.
- Wallace, J. M., 1973: General circulation of the tropical lower stratosphere. *Rev. Geophys. Space Phys.*, **11**, 191-222.
- , and V. E. Kousky, 1968: Observational evidence of Kelvin waves in the tropical stratosphere. *J. Atmos. Sci.*, **25**, 990-907.
- Yamanaka, M. D., and H. Tanaka, 1984: Propagation and breakdown of internal inertia-gravity waves near critical levels in the middle atmosphere. *J. Meteor. Soc. Japan*, **62**, 1-17.
- Yanai, M., and T. Maruyama, 1966: Stratospheric wave disturbances propagating over the equatorial Pacific. *J. Meteor. Soc. Japan*, **44**, 291-294.