Fluxes of Heat and Constituents Due to Convectively Unstable Gravity Waves

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(Manuscript received 19 April 1984, in final form 2 January 1985)

ABSTRACT

We consider the implications of a nonuniform turbulent diffusion due to the local saturation of a gravity wave via convective instabilities. It is found that both wave and turbulence fluxes of heat can be reduced dramatically, depending on the amplitude of the wave motion and the extent to which the turbulent diffusion is localized. These results suggest that previous studies that assumed a uniform turbulent diffusion may have overestimated the heat and constituent fluxes due to gravity wave saturation.

1. Introduction

The turbulent breakdown of gravity wave motions in the middle atmosphere via convective instabilities was first suggested to occur due to the exponential growth of such wave motions with height by Hodges (1967) and Lindzen (1967, 1968). Subsequent studies addressed the consequences of such breakdown in both the atmosphere and the oceans. The turbulent diffusion needed to offset wave growth with height in an unsheared environment was calculated numerically by Hodges (1969) and by Hines (1970) using the viscous dissipation arguments of Pitteway and Hines (1963). Other investigators (Orlanski and Bryan, 1969; McEwan, 1971, 1973, 1983a,b; Orlanski, 1972; Delisi and Orlanski, 1975; Fritts, 1982; Koop and McGee, 1984) used analytic and laboratory techniques to examine the conditions leading to convective instability. These studies observed convective instability rather than dynamical instability of the wave field in most instances, despite the fact that regions of dynamical instability ($Ri < 1/4$) must occur first in the presence of a vertical mean wind shear. Evidence of convective instability in the middle atmosphere is provided by observations of superadiabatic lapse rates throughout the stratosphere and mesosphere obtained using a variety of instrument systems (Theon *et al.*, 1967; Philbrick *et al.*, 1983).

Motivated in part by the atmospheric observations, Lindzen (1981) made the first attempt to describe, in a quantitative manner, both of the principal effects of gravity wave saturation now thought to be important in the middle atmosphere. Like Hodges (1967, 1969), Lindzen assumed that diffusion results from the turbulence required to limit gravity wave amplitudes. However, Lindzen also recognized that the vertical convergence of momentum flux due to a saturated wave motion must produce a deceleration of the mean flow towards the phase speed of the gravity wave. For simplicity, the turbulent diffusion resulting from wave saturation was assumed by Lindzen to be uniform throughout the wave field. The same assumption was employed by Holton (1982, 1983), Schoeberl *et al.* (1983) and Walterscheid (1984) in numerical studies of the effects of gravity wave saturation on the thermal and momentum budgets of the middle atmosphere.

In practice, however, the level of turbulence associated with gravity wave saturation is both expected and observed to vary throughout the wave field (Fritts, 1984). Atmospheric observations have revealed the presence of thin turbulent layers due to internal gravity waves (Cadet, 1977; Sato and Woodman, 1982; Wand *et al.*, 1983), while laboratory studies have shown turbulence to be due to the local convective instability of such waves (Koop, 1981; McEwan, 1983a; Koop and McGee, 1984). Numerical studies have employed Richardson number-based turbulence parameterization schemes, local convective adjustment, or direct simulation to represent the effects of convective instability (Orlanski and Ross, 1973; Durran and Klemp, 1982; Fritts, 1982; Dunkerton and Fritts, 1984; Fritts and Dunkerton, 1984). Results of the numerical studies employing local convective adjustment, in particular, were found to be in reasonable agreement with certain predictions of linear and quasi-linear saturation theory (Lindzen, 1981; Dunkerton, 1982; Coy, 1983).

The purpose of this paper is to examine the implications of a nonuniform turbulent diffusion in the transport of heat and constituents due to convectively unstable gravity waves. In a related study, Chao and Schoeberl (1984) argued that the dissipation accompanying gravity wave saturation occurs primarily in regions in which $\partial(\bar{\theta} + \theta')/\partial z \approx 0$, consistent with the observational and numerical studies cited above. As a consequence, these authors suggested that wave fluxes of heat are small and that the induced diffusion operates most effectively in the perturbation momentum equation, resulting in a large turbulent Prandtl number. These authors did not address the effects of a localized turbulent diffusion in the mean thermodynamic energy equation, however, arguing instead that thermal diffusion will be large and will drive the mean state towards an adiabatic lapse rate. Our approach is more systematic and recognizes that local diffusion resulting from local turbulence production must be included in both the mean and perturbation equations in order to anticipate the overall effects.

As noted by Schoeberl *et al.* (1983), the total transport will be due to both wave and turbulence fluxes. In the presence of fluctuating diffusion, however, both wave and turbulence transports can include downgradient and countergradient terms. In order to illustrate this point, we demonstrate in Section 2 that a truly local turbulent diffusion within regions that are statically unstable due to internal gravity wave motions must lead to turbulent heat and constituent fluxes that are countergradient with respect to the mean profiles. In Section 3, we show that the total turbulence flux may be either up- or down-gradient depending upon the exact distribution of turbulent diffusion within the wave field. The wave transport is also determined and is found to partially offset the countergradient contribution of the fluctuating turbulence transport. The result, for reasonable distributions of turbulent diffusion, is a total flux of heat and constituents that remains downgradient, but that may be substantially less than anticipated solely on the basis of a mean turbulent diffusion. The conclusions of this study are presented in Section 4.

2. Effects of local diffusion

We consider, for simplicity, two-dimensional, hydrostatic internal gravity wave motions in a stably stratified environment, though similar arguments may also apply to locally unstable regions associated with any wave field. We then assume that the gravity wave results in the formation of a region in which the local static stability is negative due to the differential advection of heavier over lighter fluid. This situation is illustrated schematically in Fig. 1. The solid lines represent surfaces of constant potential temperature (or constituent concentration) displaced from equilibrium by the wave motion. Up to this point we have assumed no dissipation.

Now recognizing that the region within the closed contour is statically unstable, we impose a local

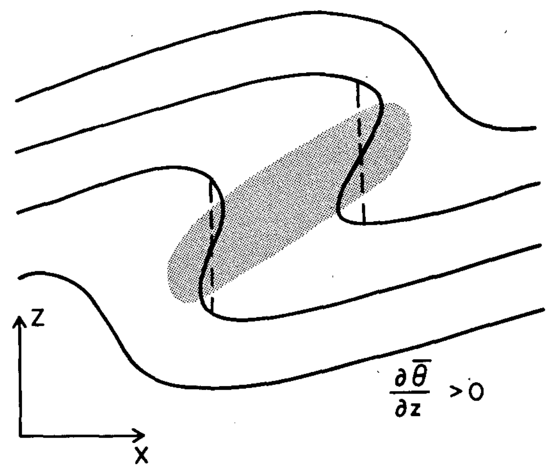

FIG. 1. Region of convective instability (shaded) formed by internal gravity wave motion. Note that convective adjustment requires the alteration of the field outside the region of instability as well.

turbulent diffusion that acts vertically and horizontally to restore the local lapse rate to neutral stability. If turbulence is nearly isotropic and the wave amplitude is sufficiently large that vertical gradients dominate horizontal gradients,

$$|m\theta_0 - \bar{\theta}_z| > k\theta_0, \tag{1}$$

then the adjustment is dominated by vertical transports. Here m and k are the vertical and horizontal wavenumbers of the gravity wave, θ_0 is the amplitude of the potential temperature perturbation, and $\bar{\theta}_z$ is the mean potential temperature gradient. Notice that this restoration of an adiabatic lapse rate requires a turbulent diffusion and adjustment of the local potential temperature in regions that are outside of the convectively unstable zone as well (Dunkerton and Fritts, 1984). The result of this adjustment of each potential temperature surface is shown by the dashed vertical lines extending across the region of convective instability in Fig. 1.

The local consequences of our assumed turbulent adjustment are now easy to see. At each horizontal position, in order to achieve an adiabatic lapse rate

$$(\bar{\theta} + \theta')_z = 0, \tag{2}$$

we have mixed more dense fluid from the upper portion of the unstable region with less dense fluid from below. The effect, at every location, has been a "turbulent" transport of heat upward that removes the convective instability produced by the wave. This countergradient flux of heat (with respect to the mean gradient) is not the only result, however. The adjustment has also altered and reduced the amplitude of the heat flux due to the wave motion itself. The overall result is a combination of the two effects. Nevertheless, this example suggests the possible significance of local turbulent diffusion. In the following section, we will obtain expressions for the turbulence and wave fluxes of potential temperature for various assumed distributions of turbulent diffusion within a saturating gravity wave field.

3. Effects of a nonuniform turbulent diffusion

In reality, of course, we should not expect to find that turbulence and diffusion are confined to the immediate zones of convective instability. However, there is experimental and observational evidence that the turbulence generated by a breaking internal gravity wave is not uniform throughout the wave field. The laboratory studies by McEwan (1983b) and Koop and McGee (1984), for example, suggest active and energetic turbulence within the region of generation and substantially less turbulence activity in the strongly stable portions of the wave field. Likewise, high-resolution balloon and radar observations of turbulence in the troposphere and stratosphere suggest that turbulence typically occurs in thin, intense layers (Cadet, 1977; Sato and Woodman, 1982). On some occasions, multiple thin, turbulent layers are observed (Wand *et al.*, 1983). Finally, rocket and radar observations of the mesosphere provide strong evidence of large variations in turbulence intensities due to locally unstable gravity waves (Philbrick, personal communication, 1983; Fritts *et al.*, 1985). Thus it appears reasonable to assume that turbulence and the diffusion that it produces may undergo substantial variations throughout a field of convectively unstable gravity waves.

In order to determine the wave and turbulence fluxes due to a nonuniform distribution of turbulent diffusion, we use the thermodynamic energy equation and the continuity equation for incompressible flow,

$$\frac{\partial}{\partial t}\theta + \mathbf{u}\cdot\nabla\theta = \nabla(\nu\nabla\theta), \qquad (3)$$

$$\nabla\cdot\mathbf{u} = 0, \qquad (4)$$

where θ, $\mathbf{u}$, and ν represent the total potential temperature, velocity, and turbulent diffusion. We then assume that all of these fields can be written as the sum of a horizontal mean plus a perturbation as

$$\psi = \bar{\psi} + \psi'. \qquad (5)$$

Using the continuity equation (4), the mean and perturbation equations for potential temperature may be written

$$\bar{\theta}_t + (\overline{w'\theta'})_z = (\bar{\nu}\bar{\theta}_z + \overline{\nu'\theta'_z})_z, \qquad (6)$$

$$\theta'_t + \bar{u}\theta'_x + w'\bar{\theta}_z + u'\theta'_x + w'\theta'_z - (\overline{w'\theta'})_z$$
$$= (\nu'\bar{\theta}_z + \bar{\nu}\theta'_z + \nu'\theta'_z - \overline{\nu'\theta'_z})_z + (\bar{\nu}_h\theta'_x + \nu'_h\theta'_x)_x, \qquad (7)$$

where subscripts denote partial differentiation and ν_h is the horizontal component of turbulent diffusion, assumed proportional to ν. Note that all nonlinear terms have been retained. Referring to (6), we see that the mean potential temperature changes in response to three terms. These represent, respectively, a divergence of the flux of potential temperature by various wave motions, a divergence of the diffusion of mean potential temperature by the mean turbulent diffusivity and a divergence of the perturbation potential temperature by the fluctuating turbulent diffusivity.

In order to evaluate the effects of a nonuniform turbulent diffusion, we must specify the distributions of perturbation potential temperature and turbulent diffusivity. The perturbation potential temperature gradient is taken to be that of a monochromatic gravity wave,

$$\theta'_z(x, z, t) = \alpha\bar{\theta}_z\cos\phi, \qquad (8)$$

where

$$\phi = kx + mz - kct. \qquad (9)$$

Here k and m are horizontal and vertical wavenumbers, c is horizontal phase speed, $\bar{\theta}_z$ the mean potential temperature gradient and α an amplitude factor that may vary on slow time and space scales (Andrews and McIntyre, 1976). Then the total potential temperature gradient is

$$\theta_z = \bar{\theta}_z(1 + \alpha\cos\phi). \qquad (10)$$

For $\alpha < 1$, the wave field is statically stable everywhere and no turbulence is produced. For $\alpha > 1$, however, the wave field is locally convectively unstable and turbulence production and diffusion result. Motivated by laboratory and atmospheric observations, we assume a distribution of turbulent diffusion given by

$$\nu = \nu_0 f(\cos\phi), \qquad (11)$$

where ν_0 is the maximum level of turbulent diffusion and $f(\cos\phi)$ is assumed to be negatively correlated with θ'_z and expresses the degree of localization of turbulent diffusion. This general expression, together with a specific functional form to be introduced later, will permit us to examine the consequences of a localized turbulent diffusion in some detail.

We now obtain an expression for the wave flux by multiplying (7) by θ' and averaging horizontally,

$$\overline{w'\theta'} = \frac{1}{\bar{\theta}_z}(\overline{\nu'\theta'}\bar{\theta}_{zz} + \overline{\nu'_z\theta'}\bar{\theta}_z + \bar{\nu}_z\overline{\theta'\theta'_z}$$
$$+ \bar{\nu}\overline{\theta'_{zz}\theta'} + \bar{\nu}_h\overline{\theta'_{xx}\theta'} + \overline{\nu'_{h_x}\theta'\theta'_x} + \overline{\nu'_h\theta'_{xx}\theta'} + \overline{\nu'\theta'\theta'_{zz}}$$
$$+ \overline{\nu'_z\theta'\theta'_z} - \overline{u'\theta'\theta'_x} - \overline{w'\theta'\theta'_z} - \overline{\theta'\theta'_t}). \qquad (12)$$

This expression may be considerably simplified by using the distributions for ν and θ' given by (8) and (11) and assuming that turbulent diffusion does not alter the wave structure in any significant way. It can be shown that this is a reasonable approximation for saturating gravity waves by using the linear form of (7) and the value of turbulent diffusion inferred by Lindzen (1981), provided that the WKB approximation [$m^2 \gg 1/(4H^2)$] is justified. Then the first term on the right side of (12) is small because ν' and θ' are in approximate quadrature. Likewise, the third term and all of the triple-correlation nonlinear terms may be neglected because they include wave and/or turbulence fluctuations that are also largely in quadrature. The last term on the right side of (12) is the contribution to the wave flux due to wave transience. This term contributes a downward heat flux for growing waves and an upward flux for decaying waves. We are concerned, however, with wave motions that have already achieved saturation amplitudes, with turbulent diffusion preventing further wave growth. Thus (12) reduces to

$$\overline{w'\theta'} = \overline{\nu'_z\theta'} + \frac{\bar{\nu}}{\bar{\theta}_z}\overline{\theta'_{zz}\theta'} + \frac{\bar{\nu}_h}{\bar{\theta}_z}\overline{\theta'_{xx}\theta'}, \qquad (13)$$

and the rate of change of the mean potential temperature becomes

$$\bar{\theta}_t = \frac{\partial}{\partial z}\left(\bar{\nu}\bar{\theta}_z + \overline{\nu'\theta'_z} - \overline{\nu'_z\theta'} - \frac{\bar{\nu}}{\bar{\theta}_z}\overline{\theta'_{zz}\theta'} - \frac{\bar{\nu}_h}{\bar{\theta}_z}\overline{\theta'_{xx}\theta'}\right). \qquad (14)$$

The first two terms on the right side of (14) represent the turbulence contributions; the last three are the surviving wave fluxes from (12).

Using integration by parts and the quadrature of wave and turbulence fields, the remaining wave fluxes may be rewritten as

$$-\overline{\nu'_z\theta'} - \frac{\bar{\nu}}{\bar{\theta}_z}\overline{\theta'_{zz}\theta'} - \frac{\bar{\nu}_h}{\bar{\theta}_z}\overline{\theta'_{xx}\theta'} = \overline{\nu'\theta'_z} + \frac{\bar{\nu}}{\bar{\theta}_z}\overline{\theta'^2_z} + \frac{\bar{\nu}_h}{\bar{\theta}_z}\overline{\theta'^2_x}. \qquad (15)$$

We now write

$$\bar{\nu} = \nu_0\bar{f}(\cos\phi), \qquad (16)$$

and define a coefficient β which expresses the degree of localization of turbulent diffusion,

$$\beta \equiv \frac{-\overline{\cos\phi f(\cos\phi)}}{\overline{f(\cos\phi)}}. \qquad (17)$$

Note that β is positive because of the assumed negative correlation of ν and θ'_z. Then using (8), (9) and (15)–(17), Eq. (14) may be written as

$$\bar{\theta}_t = \frac{\partial}{\partial z}\left\{(\bar{\nu}\bar{\theta}_z)\left[1 - 2\alpha\beta + \frac{\alpha^2}{2}\left(1 + \frac{k^2}{m^2}\frac{\bar{\nu}_h}{\bar{\nu}}\right)\right]\right\}. \qquad (18)$$

Thus, it is possible to relate each of the turbulence and wave contributions to the mean diffusion term. The first term on the right side of (18) is the mean downgradient turbulence transport retained by previous authors. The second term represents equal countergradient wave and turbulence fluxes due to the localization of turbulent diffusion. The final two terms are the downgradient wave fluxes due to the mean vertical and horizontal turbulent diffusivities.

In order to illustrate the behavior of the wave and turbulence fluxes with wave amplitude and degree of localization of turbulent diffusion, we assume a diffusion of the form

$$f(\cos\phi) = \left(\frac{1-\cos\phi}{2}\right)^n \qquad (19)$$

for $n \geqslant 0$ so that (17) becomes

$$\beta = \frac{n}{n+1}. \qquad (20)$$

The total turbulence contribution is then

$$\bar{\theta}_t = \frac{\partial}{\partial z}\left[(\bar{\nu}\bar{\theta}_z)\left(1 - \frac{\alpha n}{n+1}\right)\right], \qquad (21)$$

which is countergradient for

$$\alpha > \frac{n+1}{n}. \qquad (22)$$

Thus the degree of supersaturation ($\alpha > 1$) required for countergradient turbulence fluxes decreases as the degree of turbulence localization increases. The distributions of the perturbation potential temperature gradient (for $\alpha = 1.3$) and total turbulent diffusion coefficient (for $n = 1$) are shown in Fig. 2. The turbulence flux as a function of α for various n is shown with dashed lines in Fig. 3. The motivation for a turbulent diffusion extending beyond the convectively unstable layer is provided by the need for an adjustment in the adjacent stable regions (see Fig. 1). Like the turbulence contribution, the wave transport includes both down-gradient and countergradient terms. The former are independent of the distribution of turbulent diffusion and increase quadratically with wave amplitude; the latter is equal to the countergradient turbulence flux and requires some degree of turbulence localization.

To evaluate the wave flux, we must also assume the relative size of the horizontal and vertical turbulent diffusivities. The value $\bar{\nu}_h = 0$, together with the assumptions of uniform turbulent diffusion ($\beta = 0$) and marginal saturation ($\alpha = 1$) yields

$$\bar{\theta}_t = \frac{1}{2}\frac{\partial}{\partial z}(\bar{\nu}\bar{\theta}_z). \qquad (23)$$

This is equal to half the turbulence flux obtained under the same assumptions, in agreement with the result obtained by Schoeberl *et al.* (1983). Because of the dependence of the wave flux on amplitude, however, the transport in the case of uniform turbulent diffusion increases rapidly as α increases. Attention to supersaturated wave motions ($\alpha > 1$) seems warranted based on atmospheric observations of superadiabatic lapse rates due to large-amplitude gravity waves (Theon *et al.*, 1967; Philbrick *et al.*, 1983) and numerical studies of gravity wave saturation via convective adjustment showing ~20% ($\alpha \sim 1.2$) supersaturation (Fritts and Dunkerton, 1984).

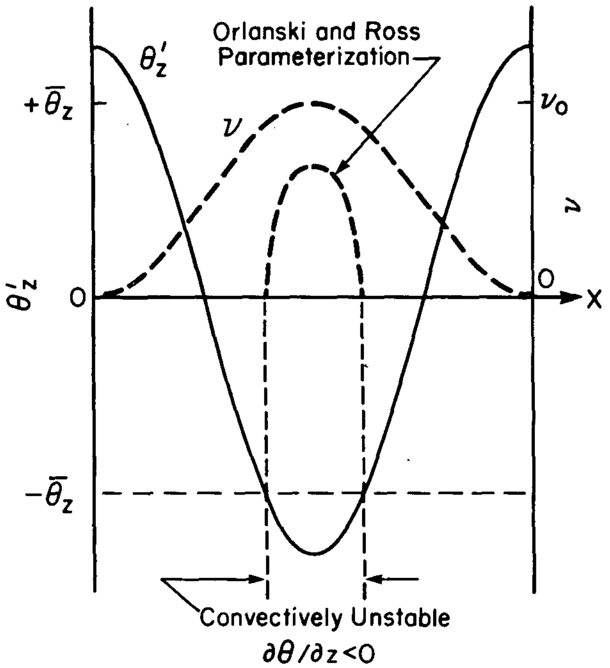

FIG. 2. Perturbation potential temperature gradient for a convectively unstable (α = 1.3) gravity wave. Also shown are the turbulent diffusion distributions given by (10) with n = 1 and that assumed by Orlanski and Ross (1973).

As turbulence becomes more localized about the regions of convective instability (increasing n), the total (wave plus turbulence) flux decreases dramatically at all saturation amplitudes ($\alpha \geq 1$). Assuming nearly isotropic turbulent diffusion and hydrostatic wave motion,

$$\frac{k^2}{m^2}\bar{\nu}_h \ll \bar{\nu}, \tag{24}$$

and the total (wave plus turbulence) flux is

$$\bar{\theta}_t = \frac{\partial}{\partial z}\left[(\bar{\nu}\bar{\theta}_z)\left(1 - \frac{2\alpha n}{n+1} + \frac{\alpha^2}{2}\right)\right]. \tag{25}$$

This total flux is shown, for various values of n, by the solid lines in Fig. 3. Note that, except at large n and small α, the total (downgradient) flux is greater than the turbulence flux alone.

The countergradient terms cause the net flux (for negligible $\bar{\nu}_h$) to experience a minimum value at

$$\alpha = 2\beta = \frac{2n}{n+1}, \tag{26}$$

which ranges from 1 to 2 for saturation amplitudes. It is even possible, in principle, for the countergradient terms to dominate the mean downgradient transports for some range of α, provided that the turbulent diffusion is sufficiently localized. However, we do not expect this situation to arise in reality for several reasons. First, the generation of a very localized turbulent diffusion in response to a wave field with extended superadiabatic layers (large α) violates the argument concerning the effects of convective adjustment presented in the previous section. Second, we have not yet accounted for a possibly significant horizontal turbulent diffusivity. The nonlinear saturation theory advanced by Weinstock (1982) suggests a relation of the form

$$\bar{\nu}_h = \frac{m^2}{k^2}\bar{\nu}. \tag{27}$$

If we use this value in (18), we see that horizontal and vertical diffusivities contribute equally to the downgradient wave flux,

$$\bar{\theta}_t = \frac{\partial}{\partial z}\left[(\bar{\nu}\bar{\theta}_z)\left(1 - \frac{2\alpha n}{n+1} + \alpha^2\right)\right]. \tag{28}$$

The total flux in this case is downgradient for all $\alpha \geq 1$ and $n \geq 0$. Note also that the wave contribution due to the mean diffusivity (for α = 1) is consistent with that obtained by Weinstock (1983).

The total flux of heat (and constituents) may, nevertheless, be significantly reduced through the effects of a localized turbulent diffusion. The important point to be made here is that even a relatively small degree of localization can result in an appreciable reduction of the downgradient heat flux due to saturating gravity wave motions. Indeed, the laboratory study by Delisi and Orlanski (1975) revealed no detectable changes of the mean density profile due to gravity wave breaking despite several detailed searches for such modifications. In a similar experiment, McEwan (1983b) observed only very gradual weak-

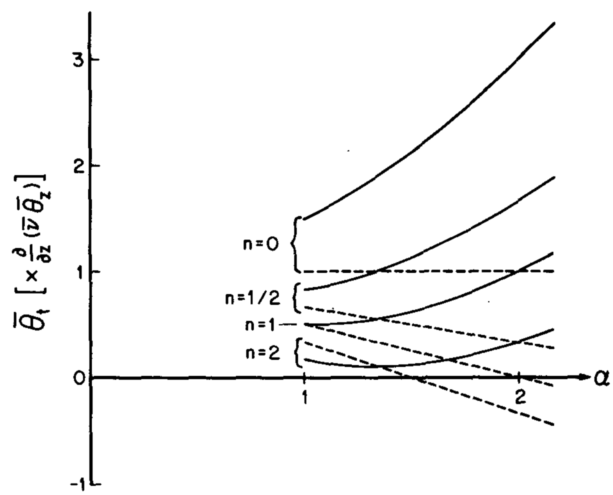

FIG. 3. Variation of the turbulent (dashed) and net (solid) rate of change of mean potential temperature normalized by the turbulent transport of the mean for the case of a vertical turbulent diffusivity alone. Note that even small values of n cause major reductions in the net flux.

ening of the stratification due to wave breaking. Also, recent numerical studies of radiative effects by Apruzese *et al.* (1984) suggest that a uniform turbulent diffusion may overestimate the downward heat flux due to gravity wave saturation (Strobel, private communication, 1983). Additional evidence of this is provided by the vertical diffusion estimates of Vincent (1984), based on the predictions of linear and nonlinear saturation theories, that are larger than required by photochemical equilibrium considerations (Allen *et al.*, 1981).

The assumption of a localized turbulent diffusion implies a turbulent Prandtl number that may achieve large values. Because the velocity and potential temperature fields are in approximate quadrature, the fluctuating turbulent diffusion contributes to dissipation only in the potential temperature equations. Thus, with $\bar{\nu}_h = \bar{\nu}$, the turbulent Prandtl numbers appropriate for the mean and perturbation equations are approximately

$$\mathrm{Pr}_{\mathrm{mean}} \approx \frac{1}{1 - \alpha\beta} \tag{29}$$

and

$$\mathrm{Pr}_{\mathrm{pert}} \approx \frac{1}{1 - \beta/\alpha}. \tag{30}$$

The Prandtl number appropriate for the mean equations, in particular, may become large for large-amplitude waves experiencing local dissipation. Thus a large turbulent Prandtl number, as inferred in the study by McEwan (1983b), provides additional evidence of the localized nature and effects of wave breaking.

It is interesting to note that previous eddy diffusion schemes employed in numerical models assumed distributions of turbulent diffusion that were, in many cases, even more confined than that given by (19). In weakly sheared environments, the turbulent diffusion assumed by Priestly (1954) and Lilly (1962) varied as

$$\nu \sim \left(-\frac{g}{\theta}\frac{\partial\theta}{\partial z}\right)^{1/2} \tag{31}$$

for $\partial\theta/\partial z < 0$ and was zero for $\partial\theta/\partial z > 0$. Other studies suggested that an exponent of ⅓ might be more appropriate (Globe and Dropkin, 1959; Kraichnan, 1962; Ingersoll, 1966). The latter dependence was also employed by Orlanski and Ross (1973) and found to provide satisfactory results for a field of breaking internal gravity waves. A turbulent diffusion of the form used by these authors is denoted by a dashed line in Fig. 2. A more recent study by Durran and Klemp (1982) used a Richardson number parameterization with eddy diffusion occurring for $\mathrm{Ri} < 1/3$. Significantly, however, all of these schemes have confined turbulent diffusion within or very near the regions of convective instability and thus imply small downgradient fluxes of heat and constituents.

4. Conclusions

We have presented a simple model of the effects of localized turbulent diffusion due to the convective instability of internal gravity waves. Both the wave and turbulence fluxes were found to include downgradient and countergradient terms for steady state wave motions. Results suggest that localized diffusion may cause a significant reduction of the heat and constituent fluxes relative to those anticipated using a uniform turbulent diffusion. The magnitude of the departure from the uniform diffusion result depends on both the degree of turbulence localization and the amplitude of the wave motion. Importantly, however, a localized turbulent diffusion can cause a significant reduction in heat and constituent fluxes for gravity waves that are at or slightly exceed nominal saturation amplitudes ($\alpha \approx 1$). Reasonable values for these quantities suggest that the heat fluxes due to gravity wave saturation inferred in several studies assuming uniform turbulent diffusion may substantially overestimate these effects. Thus, it appears to be important to obtain measurements of both the degree of supersaturation of gravity waves in the middle atmosphere and the distribution of turbulence within the wave fields. A greatly reduced flux of heat would also increase the relative importance of gravity wave heating due to turbulent dissipation (Clark and Morone, 1981).

There are, of course, other processes that will contribute to downgradient fluxes of heat and constituents due to convectively unstable gravity wave motions. As noted earlier, a downward flux of heat is associated with the transient growth of a gravity wave motion prior to saturation. This effect may be offset, at least partially, however, by an upward heat transport due to wave decay. But provided that gravity wave saturation has contributed to a reduction of $\bar{\theta}_z$, this upward heat flux will be relatively less important. It is also likely that the continued propagation of the gravity wave (and the region of active convective instability) away from the induced turbulence layer (which is assumed to move with the local flow) will introduce a phase shift between the maximum of ν and the minimum of θ_z, reducing the tendency for countergradient wave and turbulence fluxes. However, we expect this effect to be small for wave motions for which the time scale appropriate for turbulent collapse, the mean Brunt-Väisälä period, is substantially less than the half-period of the wave motion.

Finally, it is important to recognize that other mechanisms may contribute to the saturation of a large-amplitude gravity wave spectrum. These include nonlinear interactions among individual wave motions (Weinstock, 1982) and the dynamical instability of low-frequency gravity waves (Fritts, 1984). Dynamical (most commonly Kelvin-Helmholtz) instabilities, in particular, arise as the result of an unstable shear flow in which the local static stability is positive. While small-scale convective instabilities and turbulence often occur as a result of dynamical instability (Metcalf and Atlas, 1973; Thorpe, 1973; Maxworthy and Browand, 1975; Sykes and Lewellen, 1982), the initial instabilities (convective and dynamical) and their consequences for heat and constituent transport may be somewhat different. This is because the dynamical instability acts locally to mix a stable gradient of potential temperature, $\theta_z > 0$, while the reverse is true for the convective instability. Nevertheless, because both instabilities are expected to occur in approximately the same phase of the wave field (i.e., near a minimum of the local static stability rather than at a maximum of the velocity shear), their effects should be qualitatively the same.

Acknowledgments. The authors are grateful to W. Chao, A. Plumb, M. Schoeberl, D. Strobel, and R. Walterscheid for helpful comments and discussions. This research was sponsored by the Air Force Office of Scientific Research (AFSC) under Grants AFOSR-82-0125 and F49620-83-C-0061.

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