Measurements of Axial Pressures in Tornado-like Vortices

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ABSTRACT

The results of a series of measurements of centerline pressure deficit in tornado-like vortices are described. These measurements were undertaken for the purpose of determining 1) how the magnitude of the central pressure deficit in a columnar vortex varies with height, and 2) what functional relationships exist between these deficits and the dynamic and geometric parameters characterizing the flow. The results graphically show the complicated variation of central pressure deficit with height in both laminar and turbulent vortices. In low-swirl vortices, the largest deficits are found aloft, not at surface. Further, the low-swirl vortices have generally greater central pressure deficits than moderate-swirl events. The greatest deficits are tied to the approach of the vortex breakdown to the lower surface. The data also indicate a cubic dependence of the central pressure deficit on applied circulation.

1. Introduction

Fundamental questions regarding the relationships between the dynamic structures of tornadoes and the magnitudes of the pressure drops associated with them remain unresolved (*cf.*, Davies-Jones and Kessler, 1974). Pressure data on actual tornadoes are elusive and consist mainly of low-resolution microbarograph traces (*e.g.*, Ward, 1972a) and point value observations (*e.g.*, Agee *et al.*, 1977) obtained fortuitously as tornadoes passed near weather instruments. With a few exceptions (mostly in historical literature), these data indicate that the pressure deficits associated with tornadoes are surprisingly small. Almost all the deficits reported have been in the range 0.5 to 2.5 kPa (5 to 25 mb); very high, unofficial readings remain unsubstantiated. Recent efforts by Bluestein (1983) to obtain surface pressure data in the near vicinity (but outside the core) of tornadoes have also found only small deficits. As a complementary activity to the probing of actual tornadoes in the field, one can consider laboratory simulation of tornadolike vortices. Measurements made in such flows can provide physical insight into columnar vortex behavior, yield quantitative information that may be applicable to actual tornadoes, and provide guidance to be used in the planning and the interpretation of field experiments.

Experiments in fluid dynamics normally involve measurement of either velocity components or static pressure; traditionally investigators have placed more emphasis on velocity rather than on pressure measurements. In investigations of the dynamics of columnar vortices, however, laboratory measurements of the static pressure have often proven more tractable than the comparative velocity measurements. Previous experiments conducted by the authors have concentrated on measurements of surface pressure deficits beneath vortices produced in a Ward-type tornado simulator (Ward, 1972b; Church *et al.*, 1977). Snow *et al.*, (1980) measured radial profiles of time-averaged wall static pressure deficits beneath single and multiple vortex flows. Pauley *et al.* (1982) measured the instantaneous maximum wall static pressure beneath a similar range of flows. Here we continue this work by reporting on findings made during a laboratory investigation of the distribution of static pressure *above the surface* as well as *at the surface*. The specific questions addressed are: 1) how does the magnitude of the central pressure deficit vary with height in the different types of vortices (*e.g.*, one-cell and two-cell), and 2) what functional relationships exist between the measured pressure deficits and the dynamic and geometric flow parameters characterizing the flow (swirl ratio S; Reynolds number, Re_r; aspect ratio a). (See the Appendix for definition of symbols and Fig. 3 in Church *et al.*, 1977 for geometry of the experimental apparatus.)

The experimental techniques employed here to measure pressure differences were an extension of those described in previous publications. To measure the vertical profiles of pressure along the centerline of a vortex, a miniature (1.5 mm diameter) Pitot-static pressure probe was used to sense the local static pressure in the free stream. This probe could be positioned at different fixed heights along the central axis of the experimental chamber. In use the dynamic pressure port of the Pitot tube faced upstream, so that the four static holes (located 15 mm downstream from the dynamic port) were symmetric about the vertical axis. The static pressure port of the Pitot tube was connected to a variable reluctance differential pressure transducer and the output voltage was displayed on the screen of a storage oscilloscope. The reference pressure was taken as the wall static pressure on the lower surface, just inside the rotating screen. As discussed in the Pauley study, a long sampling time (usually 200 s) was required to determine the maximum value of the pressure deficit occurring at a point. Measurements of the vertical distribution of maximum static pressure deficits were made along the centerline in the cores of a large number of laminar (one-cell, low-swirl) and turbulent (two-cell, moderate swirl) vortices, but were not extended to include high-swirl, multiple vortex configurations. The heights indicated below and those shown on the figures refer to the vertical distance from the lower surface to the plane containing the four static holes.

All axial lengths are nondimensionalized by h, the depth of the confluence and convergence zones. Except as noted, this depth h = 30 cm. The nondimensional pressure deficit values reported here are scaled with $\rho\bar{w}^2$, where $\bar{w}$ is the mean vertical velocity component through the updraft hole.

2. Laminar vortex, S = 0.28

The dependence of central pressure deficit on height above surface is shown in Fig. 1 for a typical laminar vortex having a swirl ratio of 0.28. Here h = 40 cm. Our purpose in displaying this vertical profile is to illustrate the scatter in the data resulting from our measurement technique, and to describe features of the pressure distribution common to many of the similar curves presented in the next section. Each data point shown represents the maximum pressure value observed during a 200 s sampling period. Although a fair amount of scatter in the plotted points is evident, a systematic trend in the data (as suggested by the solid line, drawn by eye) can be discerned. The data show that while a small central pressure deficit exists at the surface, a much larger deficit exists in the core above the surface. The nondimensional pressure distribution exhibits the following trends: as the nondimensional height z^* increases from 0 to about 0.2, Δp^* increases from 3 to 38; for $0.2 \leqslant z^* \leqslant 1.1$, the value of Δp^* remains relatively constant, diminishing only gradually with increasing height; for $z^* \approx 1.1$, there is an abrupt decrease in the magnitude of Δp^*.

This behavior can be explained as follows: the low surface value of Δp^* is attributable to the effects of surface friction and strong deceleration of the radial inflow in the "corner" region. This is the region at the base of the vortex in which the surface boundary layer is turned and accelerated upwards to form the laminar core flow. Above this corner region, the dynamic character of the laminar vortex changes very little with height. The gradual decrease of Δp^* with increasing height results from a slight broadening of the vortex core (due to viscous diffusion) as the flow moves downstream. Vortex breakdown (VBD) (for a discussion of VBD in tornadolike vortices, see Snow, 1982) occurs at z^* = 1.1 for this particular vortex, and the pressure deficit within the turbulent, wakelike core downstream of the breakdown is much reduced.

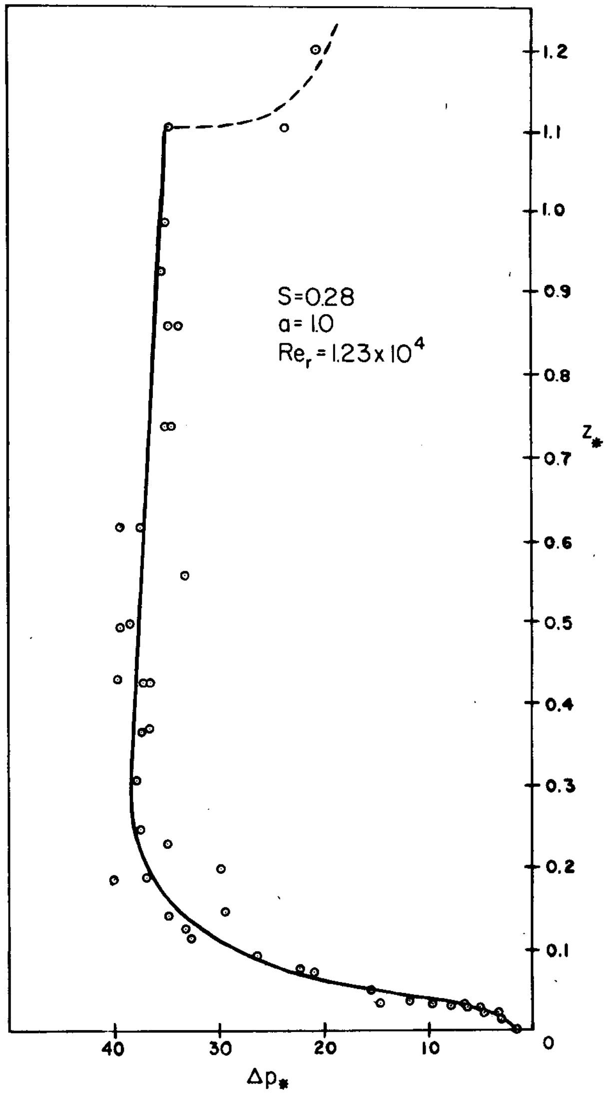

FIG. 1. Dimensionless pressure deficits vs dimensionless height for a vortex with S = 0.28, Re_r = 1.2 × 10^4 and a = 1.0.

3. Laminar and turbulent vortices

A series of measurements of the vertical distribution of central pressure deficit was made on nine vortices (S ranging from 0.18 to 0.70) for fixed geometry (a = 0.6) and fixed radial Reynolds number (Re_r = 2.88 × 10^4). The results of this measurement program are shown in Figs. 2 and 3. The central cores of the low-swirl vortices (0.18 ⩽ S ⩽ 0.31) were laminar in appearance. Core regions for the moderate-swirl vortices (S ⩾ 0.45) were generally turbulent and frequently contained embedded secondary flows. For clarity, the actual data points have been omitted from these figures. The scatter of points about each of the curves was in general similar to that in Fig. 1.

As can be seen in Fig. 2, the pressure deficit *at the surface* beneath a vortex of low swirl can be larger than that beneath a vortex characterized by a larger value of swirl ratio. For example, the surface deficit for S = 0.18 is 10 while that for S = 0.26 is 3. Note however the pressure deficits a short distance *above the surface* are clearly greater for the laminar vortices characterized by larger swirl ratio values. Parallel behavior in the evolution of the surface pressure field with increasing swirl has been reported by Snow *et al.* (1980, see their Fig. 9). Above the surface the low-swirl vortices exhibit behavior in the vertical pressure profiles which is not found in the corresponding profiles of moderate swirl vortices. A local maximum in pressure deficit appears aloft. This evolves toward greater magnitudes as swirl increases (up to S = 0.4). This unusual characteristic is confined below z^* ⩽ 0.35. As noted in the previous section, above this level the central pressure deficit in the core of a given laminar vortex remains nearly constant, decreasing very slowly with height. It can be seen that the magnitude of this deficit increases very rapidly with increasing swirl. For example, increasing S from 0.18 to 0.20 results in an increase of approximately 40% in the central pressure deficit in this region. This trend continues with further increases in swirl ratio, so that a vortex with S = 0.31 is accompanied by

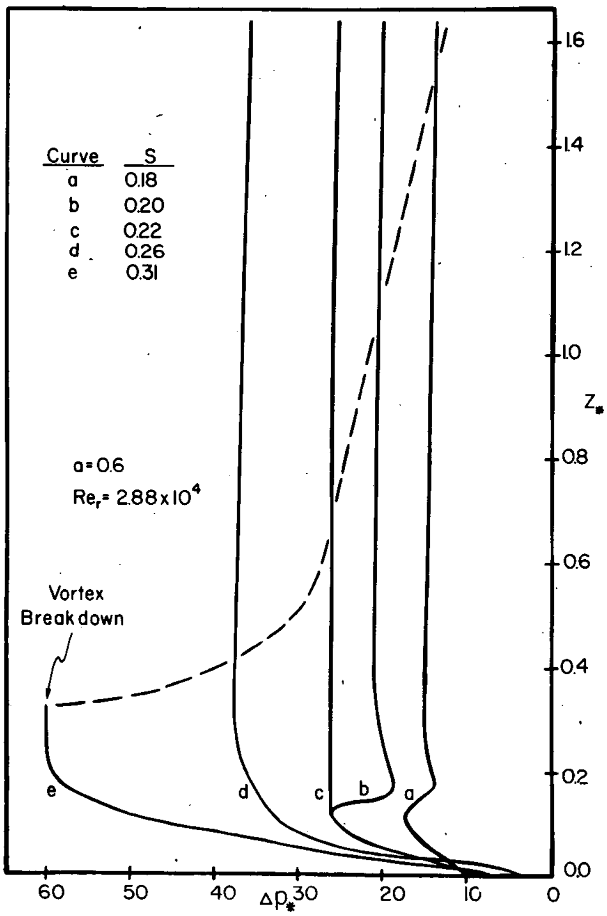

FIG. 2. Dimensionless central pressure deficit vs dimensionless height for five vortices (all laminar at the surface) for the range of swirl ratio 0.18 ⩽ S ⩽ 0.31. These are low-swirl vortices; all are one-celled.

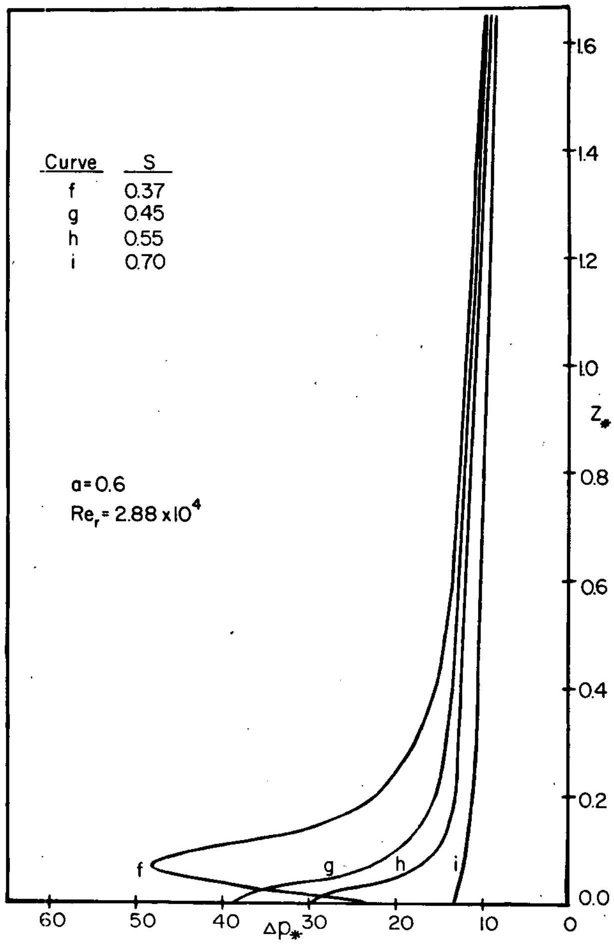

FIG. 3. Dimensionless central pressure deficit vs dimensionless height for four vortices for the range of swirl ratio 0.37 ⩽ S ⩽ 0.70. These are moderate-swirl vortices; these represent the transition from single-celled to two-celled.

about a four-fold increase in central pressure deficit over the value for $S = 0.18$.

The vortex for $S = 0.31$ can be considered a limiting case in that it is laminar throughout the surface inflow and the corner region. Here Δp^* increases rapidly with height, with the VBD being positioned just above the corner region. This configuration produces the largest central pressure deficit found in any vortex at any height, with a value of Δp^* close to 60 in the range $0.22 \leqslant z^* \leqslant 0.32$. The region downstream of the VBD (for $S = 0.31$) has been shown as a dashed line. It should be noted that pressure measurements immediately downstream of the VBD are extremely difficult to make when this feature is aloft because of the unsteady character of the breakdown. Consequently, the amount of data in this region is limited so that the dashed portion of the profile shown here should be considered only approximate. However, it is clear that downstream of the VBD, within the turbulent core, the central pressure deficit is greatly reduced from the value in the surface layer.

Referring now to Fig. 3, a further increase in swirl ratio to $S = 0.37$ creates a vortex with the following characteristics: laminar at the surface, with the VBD located within the corner region. The central pressure deficit still increases with height and the maximum value of central pressure deficit is still found above the surface, but now has a smaller magnitude than for $S = 0.31$. The position of the VBD is now steadier, and pressure measurements in the turbulent portion of the core become easier. Downstream of the VBD the pressure deficit diminishes rapidly with height, up to about $z^* = 0.4$. Above this level, the rate of decrease becomes more gradual.

The vortex for $S = 0.45$ represents the situation where the VBD is just approaching the lower surface and the maximum *surface* pressure deficit is observed. The flow configuration in the corner region is sometimes referred to as being a "drowned vortex jump". It is seen that the corresponding surface Δp^* is consistent with the value of 40 reported by Pauley *et al.* The central pressure deficit diminishes rapidly with height up to $z^* \approx 0.2$, and more slowly above that height.

At higher values of swirl ratio the turbulent core becomes firmly attached to the surface, and the surface pressure deficit diminishes. For a vortex with a swirl ratio of 0.70, the pressure deficit at the surface is not very different from the values at any other level. This profile stands in marked contrast to those of the laminar cores.

Well downstream of the VBD, the magnitude of the central pressure deficit does not appear to be a strong function of swirl ratio. For swirl ratios ranging from 0.37 to 0.70, the central pressure deficit in this region varies by only about 20%, and again in contrast with the laminar cores, the higher the swirl ratio the smaller the central pressure deficit. The value of Δp^* is typically ~ 10, or approximately one-sixth of the maximum value in the laminar core.

4. Empirical relationship for laminar vortices

As noted above, for a fixed geometry and Reynolds number the magnitude of the central pressure deficit in the laminar core (away from effects of the surface) is a strong function of swirl. A second set of experiments was performed to investigate how the core pressure depended on the circulation. The pressure probe was positioned on the centerline of the chamber at a fixed height of 10 cm above the lower surface (so that $z^* = 0.33$). Pressure deficits were then measured as a function of swirl ratio for fourteen different configurations of updraft radius and flow rate. The vortices associated with these configurations were laminar in appearance through the full depth of the convergence zone, the VBD being well aloft in the convection zone.

Figure 4 shows a log–log plot of the observed pressure deficits versus circulation for one value of Reynolds number and three values of aspects ratio. (Note that dimensioned quantities are plotted in Fig. 4.) Examination of this figure reveals that most of the data points are closely distributed about a straight line. This suggests a relationship between Δp and Γ of the form

$$\ln(\Delta p) = A + B \ln(\Gamma). \quad (1)$$

The constants A and B were estimated for each of the three sets of data shown in Fig. 4 using a least squares fit to a straight line. The magnitude of A was found to be in the range 120.3 to 146.9; B in the range 2.58 to 2.87. The normalized linear correlation coefficient ranged from 0.95 to 0.99, indicating good fits to the data. Because of evidence of some systematic errors in the measurement of the pressure deficits (due to small misalignments of the Pitot tube) from run to run, no attempt was made to pool the data. These systematic errors most strongly affect the value of the constant A.

These values, together with those generated from the eleven other cases considered, are summarized in Table 1. Inspection of these results shows that these data strongly support the hypothesis that the magnitude of the central pressure deficit depends (approximately) on the cube of the circulation.

5. Discussion

Pressure data for *laminar* vortices were presented in Figs. 1, 2 and 4. Several conclusions can be drawn from the observed distributions. First, the central pressure deficit on the surface is always smaller than that aloft. In the data presented here, the initial pressure trend at the surface as the swirl ratio is

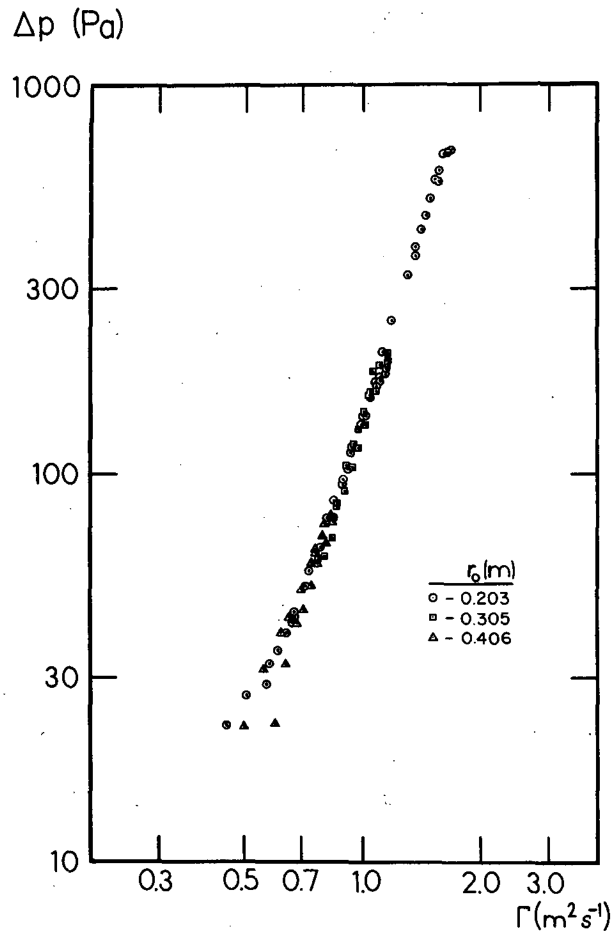

FIG. 4. Central pressure deficit vs circulation at $z^* = 0.33$ for a Reynolds number $Re_r = 2.6 \times 10^4$, and $0.5 \leq a \leq 1.0$. Pressure in Pa, circulation in $m^2 s^{-1}$.

the surface, a feature not found in laminar vortices with intermediate values of swirl. This bears some resemblance to the pressure profiles reported by Ying and Chang (1970). It may be associated with the aforementioned flow separation phenomenon, but here also a clear physical explanation for this feature is not yet available.

Compared with what is found at or near the surface, pressure profiles in laminar cores aloft (*i.e.*, well above the corner region, but upstream of VBD) display relatively uncomplicated behavior. As noted before, the central pressure deficit increases from the surface upwards, attains a maximum value near $z^* = 0.3$, and then decreases only very slowly with height above that level. The small rate of decrease of Δp^* with height above $z^* = 0.3$ can be explained in terms of the observed gradual rate of increase of the core radius with height due to viscious diffusion. The magnitude of the central pressure deficit in the core is seen to be a strong function of swirl ratio, as will be discussed in a following paragraph.

It is interesting to make a comparison between the pressure data in Fig. 1 and an independent set of velocity measurements. Baker (1981) made a detailed series of measurements of the three components of velocity for a laminar vortex ($S = 0.28$). His results for the tangential velocity field showed that the peak velocity as a function of height (at the core radius, $r_m = 2.0$ cm) increased steadily from a surface value of zero to a value of 2.0 m s^{-1} per second at $z^* = 0.3$ and then remained approximately constant as height increased further. The on-axis ($r = 0$) vertical velocity showed a similar trend, increasing from a surface value of zero to a maximum of 4.0 m s^{-1} at the $z^* = 0.3$ level. So these velocity components are increased is "out of phase" with the pressure trend aloft (surface deficit decreases as swirl increases). Thus, at low swirl ratios intensification of the laminar vortex core is accompanied by an increase of the pressure deficit in the core region above the surface, as expected, but at the same time the surface core pressure deficit diminishes. At somewhat higher swirl ratios, before the VBD reaches the surface, the pressure deficit at the surface starts to increase again, reaching a maximum value just as the VBD penetrates to the surface. This bimodal behavior of the pressure deficit at the surface beneath laminar vortices was first reported by Snow *et al.*, and documented in more detail by Pauley *et al.* A clear physical explanation of the initial decrease in deficit observed at low swirl ratio remains elusive, but flow visualization suggests that it is associated with the transition from a separated to a nonseparated surface boundary layer.

Another feature associated with the lowest swirl vortices ($S = 0.18$ or 0.20, for example) is the pressure deficit maximum in the region immediately above

TABLE 1. Pressure deficit Δp(Pa) at $z^* = 0.33$ as a function of circulation $\Gamma(M^2\ s^{-1})$—results of fitting $\log(\Delta p) = \log(A) + B\log(\Gamma)$.

Re_r	a	n	A	B	r
1.2×10^4	1.0	22	65.4	3.66	0.97
1.3×10^4	0.8	6	45.2	2.73	0.98
	1.0	18	59.5	3.47	0.97
	1.33	14	56.3	3.32	0.99
	2.0	10	63.4	2.97	0.98
1.6×10^4	1.0	9	49.4	3.37	0.99
1.7×10^4	1.0	52	70.1	3.60	0.98
2.6×10^4*	0.5	19	120.3	2.58	0.95
	0.67	30	131.1	2.87	0.99
	1.0	39	146.9	2.86	0.99
2.8×10^4	0.6	28	114.4	2.40	0.99
	0.75	17	116.7	3.08	0.99
	1.0	10	86.5	2.78	0.97
	1.5	15	73.7	2.97	0.99

* These 88 data points are shown in Fig. 4.

increasing with height in the region which was identified as that in which the central pressure deficit is increasing.

It was noted earlier that the largest measured pressure deficits are associated with the laminar vortex just upstream of the VBD, with this feature near the $z^* = 0.3$ level. The observed maximum value of Δp^* was about 60. As the breakdown point penetrated into the corner region, the maximum value of Δp^* in this region decreased; at the same time the surface value of Δp^* was increasing. A maximum surface value of about 40 was obtained (as also found in the earlier study of Pauley *et al.*). These new data provide further evidence of the remarkable magnitudes of the pressure drops that can be obtained in the laminar one-cell laboratory vortices. This greatest surface value was achieved just as the VBD penetrated to the surface, signifying the establishment of a vortex which is turbulent along its entire length. For the turbulent vortices with swirl ratios 0.45 and 0.55, pressure deficits were much larger at the surface than aloft.

When the swirl ratio increased to 0.7, pressure was almost constant from the surface up. This suggests that vertical accelerations along the centerline in the turbulent core are small. This contrasts sharply with the results for laminar vortices where upward accelerations in the corner region are very large. Although measurement of the distribution of the vertical velocity component in the turbulent core vortices has not yet been successfully accomplished, flow visualization has shown that vertical velocities in the core region are relatively small. The central pressure deficits in turbulent vortices were only weakly related to swirl ratio. Therefore, although vortex cores containing multiple vortices were not included in this study, we would not expect to find radical differences between the pressure deficits along the centerline of such cores and the single vortex turbulent cores that were studied.

The final set of results deals with the relationship between the central pressure deficit and circulation for laminar vortices. The data presented in Section 4 indicate that the central pressure deficit varies approximately as the cube of the circulation. To see that this result is physically plausible, consider that Baker (1981) has used the boundary layer scaling of Burggraf *et al.* (1971) to relate the minimum central pressure (at the top of the boundary layer) to the circulation. Summarizing Baker's arguments, if the core structure depends primarily on the structure of the boundary layer feeding it (Rotunno, 1980) so that the core radius scales according to the boundary layer thickness, then

$$r_m \sim \delta_0 \sim \left(\frac{\nu}{\Gamma}\right)^{1/2} r_s, \tag{2}$$

the latter relation being taken from the similarity theory of Burggraf *et al.* (It is interesting to note that Granger (1966, 1972), working with a draining vortex in a liquid with a free surface, found a similar dependence of core radius on the circulation.) Assuming that

$$\Delta p_m \sim \rho v_m^2, \tag{3}$$

and further, assuming angular momentum is conserved inward from the screen to the core radius, then

$$\Delta p_m \sim \rho \frac{\Gamma^3}{\nu r_s^2}, \tag{4}$$

in agreement with the experimental results presented in Section 4. Note the inverse dependence on the square of the screen radius. Equation (2) says that the greater the value of r_s, the thicker the boundary layer, and hence the broader the core. In Eq. (4), this broader core results in a smaller value of p_m since the maximum tangential velocity is reduced.

To summarize, the data presented in Fig. 3 for two-celled turbulent vortices show values for Δp^* on the order of 10, with only weak dependence on swirl ratio. The data presented in Figs. 1, 2 and 4 for one-celled laminar vortices show values for Δp^* up to 60, with a strong, approximately cubic dependence on swirl.

Can these results be applied to natural vortices? This depends on the degree of similarity between the natural and laboratory flows. As has been discussed in previous articles there is approximate geometric similarity to a thunderstorm updraft. Further, Barnes (1978) has demonstrated that tornadoes occur in such updrafts at swirl ratios similar to the laboratory values. Applying the results in Fig. 3 to a two-cell tornado forming in an updraft of say 10 m s^{-1}, a central pressure drop of about 1 kPa (10 mb) would be expected—a reasonable value. Considerably higher values could be expected for single-celled tornadoes if the findings from laminar vortices were used. However, the direct applicability of the laboratory data for laminar vortices in this instance is questionable because there was no attempt in these experiments to ensure boundary layer (Re) similarity. In a brief series of preliminary experiments, Pauley (1980) laid carpet over the smooth lower surface of the experimental chamber to roughen it, and found that the central surface pressure deficits in laminar vortices were consequently reduced 50 to 70%. We would similarly expect the earth's surface to normally be "rougher" than the floor of the experimental apparatus, so that using the present laboratory data for laminar vortices may lead to overestimates for the pressure deficits in natural vortices. The question of boundary layer similarity is presently being investigated comprehensively and it should soon be possible to make a more definitive statement on this point.

To return to the original questions raised, we have presented vertical profiles of central pressure deficits for a representative number of laminar and turbulent tornadolike vortices. These show quite clearly that the variation of the central pressure with height is complicated. More importantly, we find that the largest pressure deficits in the low-swirl vortices are a short distance aloft, not at the surface. Further, the low-swirl vortices have, in general, greater central pressure deficits than moderate-swirl events. It would appear that these findings, taken together with the radial profiles of surface pressure given in Snow *et al.* (1980) and Pauley *et al.* (1982), indicate that only small *surface* pressure deficits are likely to be observed beneath tornadoes. Large surface deficits may be found only at certain critical stages in the life cycle of an event, and then only in a small area near the center. This is certainly consistent with the limited number of pressure measurements available from tornadoes.

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APPENDIX

Symbols

- a aspect ratio ($=h/r_0$)
- h height (depth) of the confluence zone (scaling height)
- n number of values in a data set (as used in Table 1)
- p pressure
- Δp central pressure deficit [$= p(0, z) - p(r_s, 0)$]
- Δp_m maximum central pressure deficit
- Δp^* nondimensional central pressure deficit ($=\Delta p/\rho \bar{w}^2$)
- r radial coordinate; correlation coefficient (as used in Table 1)
- r_m core radius; radius of maximum tangential wind
- r_0 radius of the updraft hole (scaling radius)
- r_s radius of the rotating screen
- Re_r radial Reynolds Number ($=\bar{u}r_0/\nu$)
- S swirl ratio (follows Davies-Jones, 1973) ($=\Gamma/4\pi \bar{u}h$)
- $\bar{u}$ mean radial component of velocity at $r = r_0$
- v tangential velocity
- v_m maximum tangential velocity
- v_s tangential velocity at the screen
- $\bar{w}$ mean vertical velocity through the updraft hole (at $z = h$) (scaling velocity)
- z vertical coordinate
- z^* nondimensional vertical coordinate ($=z/h$)
- δ_0 thickness of the inflow boundary layer
- ρ density
- Γ circulation ($=2\pi rv$)
- ν kinematic viscosity.

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