Wave–Mean Flow Interactions in a General Circulation Model of the Troposphere and Stratosphere

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(Manuscript received 13 September 1985, in final form 19 February 1986)

ABSTRACT

Results are studied from a numerical experiment using a version of the NCAR Community Climate Model with high vertical resolution and extending from the surface to the lower mesosphere. The model was integrated for 370 days using external forcing fixed at values appropriate to 15 January (perpetual January), in order to isolate the effects of variability due purely to wave-mean flow interactions from variations due to other sources.

The model gives a reasonably accurate simulation of the mean atmospheric state from the surface to the stratosphere, including the winter stratosphere. Two qualitatively different mean states are found in the winter stratosphere for periods separated in time by a sudden warming. The changes in the atmospheric state between the two periods extend from the surface to the stratosphere.

By use of the refractive index and the EP flux, the zonal mean state in the two periods is shown to affect the vertical propagation of waves quite differently. The momentum balance of the two mean states is examined using transformed Eulerian diagnostics. Substantial changes in the Eliassen–Palm (EP) flux divergence are found between the two periods, indicating that the eddies affect the zonal mean state differently. A positive feedback mechanism appears to exist through which a strong lower stratospheric jet tends to favor weak wave forcing of the jet, while a weak jet favors stronger wave forcing.

1. Introduction

The circulation of the winter stratosphere poses an interesting problem in wave-mean flow interactions because the observed time mean temperatures (e.g., Geller et al., 1984) are so much warmer than the radiation equilibrium temperatures (e.g., Mahlman and Umscheid, 1984). The observed polar night jet is, of course, much weaker than would be required for thermal wind balance with the radiative equilibrium temperatures. The observed departures from the radiative equilibrium temperatures and winds are produced by the interactions of waves with the mean flow which alter both the momentum and the thermal balance of the winter stratosphere. In this study, a general circulation model (GCM) extending from the surface into the mesosphere is used to examine the winter stratospheric momentum balance and the effects of the mean flow on the propagation of waves.

There have been many studies in which the stratosphere has been simulated numerically and, generally speaking, the more comprehensive the numerical model, the worse the results have been. Stratospheric warmings have been studied extensively and successfully with nearly two-dimensional models containing only one or two degrees of freedom in the zonal direc-

* The National Center for Atmospheric Research is sponsored by the National Science Foundation.

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The interactions of the waves with the mean flow are crucial in determining both the zonal mean state of the stratosphere and the wave properties there. The study of these interactions is complicated by the fact that the waves induce mean meridional circulations (MMC) which may largely compensate for the wave fluxes of heat and momentum. The mean flow also acts to refract wave activity away from some regions and into others.

The transformed Eulerian mean (TEM) formalism introduced by Andrews and McIntyre (1976) attempts to isolate the component of the MMC which is driven by zonal mean forcing terms from that which is driven by the waves. In the case of small amplitude waves, the TEM formalism allows one to distinguish precisely between the net effect of waves on the mean circulation and the effect of mean forcing. When finite amplitude eddies are present in the flow, the TEM formalism is less successful at isolating the eddy effects from the mean forcing effects but still appears to work reasonably well in a time mean sense.

Several studies have used the TEM equations to examine the zonal mean momentum balance. Primarily observational studies include Edmon et al. (1980), Palmer (1981), O'Neill and Youngblut (1982) and Smith (1983a,b). Primarily numerical modeling studies include Dunkerton et al. (1981), Bridger and Stevens (1982), Mahlman and Umscheid (1984) and Andrews et al. (1983). One of the principal findings of these studies is that the net effect of waves is to decelerate the zonal mean westerlies, resulting in a weaker polar night jet and warmer polar temperatures than would otherwise be the case.

In this study the TEM formalism is used to examine the momentum balance of the winter stratosphere in a comprehensive GCM which produces a relatively accurate simulation of the zonal mean structure of the troposphere and stratosphere. The model does not appear to have a single equilibrium state with perturbations imposed on it. Instead, there are two quite different states in the model simulation which persist for lengthy periods of time. The TEM diagnostics are computed for the two persistent states in order to look at the differences in their momentum balance and in the influence of the zonal mean state on the propagation of waves in the meridional plane.

The remainder of this paper is organized as follows. Section 2 contains the relevant TEM theory: The GCM used is described briefly in section 3 and its Northern Hemisphere simulation is described in section 4. The effects of the mean states on the propagation of wave activity is discussed in section 5 and the momentum balance of the two states is discussed in section 6. Section 7 contains the conclusions of the study.

2. Transformed Eulerian mean formalism

The TEM formalism which is now commonly used in the study of wave-mean flow interactions derives from the work of Charney and Drazin (1961) and of Eliassen and Palm (1961). Steady conservative waves were found to have no net effect on the mean zonal flow (the nonacceleration theorem) because they induce a meridional circulation which exactly compensates for their heat and momentum fluxes. Andrews and McIntyre (1976) showed that a simple transformation of the zonally averaged equations of motion results in a system of equations in which steady conservative waves do not appear at all. Only those wave effects which are not compensated for by wave induced meridional circulations appear in the TEM equations. Edmon et al. (1980) first applied these equations to diagnose atmospheric data and simple model calculations to show the eddy effects on the mean zonal flow.

Following Dunkerton et al. (1981) the conventional Eulerian mean momentum equation for the primitive equations on a sphere in log pressure coordinates may be written as:

\[
\frac{\partial \bar{u}}{\partial t} + \left( \frac{\bar{u} \cos \phi}{\alpha \cos \phi} - \bar{\beta} \right) \bar{z} + \bar{u}_x \bar{w} = \bar{D} - \frac{1}{\alpha \cos^2 \phi} \frac{\partial}{\partial \phi} \left( \bar{w}' \cos^2 \phi \right) - \frac{1}{\varrho_0} \frac{\partial}{\partial z} \left( \varrho_0 \bar{u}' \bar{w}' \right),
\]

while the transformed Eulerian mean momentum equation is:

\[
\frac{\partial \bar{u}}{\partial t} + \left( \frac{\bar{u} \cos \phi}{\alpha \cos \phi} - \bar{\beta} \right) \bar{z}^* + \bar{u}_x \bar{w}^* = \bar{D} + \frac{\nabla \cdot \mathbf{F}}{\varrho_0 \alpha \cos \phi},
\]

where \( \bar{D} \) is any zonally averaged nonconservative force per unit mass such as diffusion. The subscripts denote partial derivatives, the overbars are zonal averages and the primes are deviations from zonal averages. Other symbols are defined in Table 1.

<table>
<thead>
<tr>
<th>Symbol</th>
<th>Definition</th>
</tr>
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<tbody>
<tr>
<td>( z = -H \ln p/p_0 )</td>
<td>pressure altitude</td>
</tr>
<tr>
<td>( H = RT/g )</td>
<td>scale height</td>
</tr>
<tr>
<td>( R )</td>
<td>gas constant for dry air</td>
</tr>
<tr>
<td>( T_i )</td>
<td>reference temperature (239 K)</td>
</tr>
<tr>
<td>( g )</td>
<td>gravitational acceleration</td>
</tr>
<tr>
<td>( p_0 )</td>
<td>reference pressure (1000 mb)</td>
</tr>
<tr>
<td>( \phi )</td>
<td>latitude</td>
</tr>
<tr>
<td>( w = dz/dt )</td>
<td>vertical wind component</td>
</tr>
<tr>
<td>( a )</td>
<td>radius of the earth</td>
</tr>
<tr>
<td>( u )</td>
<td>zonal wind component</td>
</tr>
<tr>
<td>( v )</td>
<td>meridional wind component</td>
</tr>
<tr>
<td>( F )</td>
<td>Eliassen-Palm flux vector</td>
</tr>
<tr>
<td>( \rho(z) = \rho_0 e^{-\gamma H} )</td>
<td>basic density profile</td>
</tr>
<tr>
<td>( \rho_i = RT/p_i )</td>
<td>reference density</td>
</tr>
<tr>
<td>( f )</td>
<td>Coriolis parameter</td>
</tr>
<tr>
<td>( \theta )</td>
<td>potential temperature</td>
</tr>
<tr>
<td>( q )</td>
<td>potential vorticity</td>
</tr>
<tr>
<td>( N )</td>
<td>Brunt-Väisälä frequency</td>
</tr>
</tbody>
</table>
The left-hand sides of (2.1) and (2.2) contain the force per unit mass exerted on \( \mathbf{v} \) by the MMC and are identical except that the conventional Eulerian MMC in (2.1) is replaced by the transformed (or residual) MMC in (2.2), defined as:

\[
\mathbf{v}^* = \mathbf{v} - \frac{(\rho_0 \psi \alpha \cos \phi)}{\rho_0} \mathbf{u},
\]

\[
\mathbf{w}^* = \mathbf{w} + \frac{(\psi \cos \phi) \alpha}{\alpha \cos \phi} \mathbf{u},
\]

where

\[
\psi = \frac{\partial \theta}{\partial z}.
\]

(2.3)

(2.4)

For steady conservative waves, the transformed MMC is that part of the conventional MMC which was not induced by the presence of the waves, but is forced by mean diabatic heating (through the thermodynamic equation, see Dunkerton et al., 1981). The transformed MMC satisfies a continuity equation identical to that for the conventional MMC. On the right-hand side of (2.2), the momentum flux convergence of (2.1) has been replaced by the divergence of the Eliassen-Palm (EP) flux vector \( \mathbf{F} \), defined as:

\[
\mathbf{F} = \{ F_{(\phi)}, F_{(z)} \}
\]

\[
= \rho_0 \alpha \cos \phi \left\{ \frac{\partial}{\partial z} - \left( \frac{\mathbf{u} \cdot \nabla}{\alpha \cos \phi} \right) \right\} \psi - \frac{\mathbf{w} \cdot \nabla}{\alpha \cos \phi} \psi.
\]

(2.5)

The divergence operator in the meridional plane in (2.2) is

\[
\nabla \cdot \mathbf{F} = \frac{F_{(\phi)} \cos \phi}{\alpha \cos \phi} + \frac{\partial F_{(z)}}{\partial z}.
\]

(2.6)

The properties of the EP flux have been discussed extensively in recent years by a number of authors, including Andrews and McIntyre (1976, 1978), Edmon et al. (1980), Dunkerton et al. (1981) and Palmer (1982). The most important properties of the EP flux in the current context are that it is parallel to the group velocity and its magnitude is an indicator of the flux of wave activity. Therefore, the direction of \( \mathbf{F} \) generally gives the direction of energy propagation by the waves and its divergence gives the degree of interaction with the mean zonal flow. If the waves are quasi-geostrophic, \( \nabla \cdot \mathbf{F} \) in the momentum equation is the only remaining wave term in the transformed mean equations. Ageostrophy introduces wave forcing terms in the thermodynamic equation as well.

While the EP flux is of fundamental importance in determining the wave forcing of the zonal mean state, it is also useful in determining the effect of the mean state on the waves. Matsuno (1970) showed that wave energy is refracted in the meridional plane by the zonal mean wind and stability structure. Palmer (1982) gives a summary of the refraction problem for planetary waves on a sphere. For a stationary linear wave in the WKBJ limit, \( \mathbf{F} \) is parallel to the group velocity and is refracted by the normal component of \( Q_m \), where

\[
Q_m = \frac{1}{\sin^2 \phi} \left( \frac{\partial^2}{\partial \phi} - \frac{\mathbf{u} \cdot \nabla^2}{4H^2N^2} \right) (\cos^2 \phi),
\]

(2.7)

and \( m \) is the zonal wave number. Within the linear stationary WKBJ limit, a wave cannot propagate energy in regions where \( Q_m \) is negative and will tend to propagate energy toward large \( Q_m \). It is important to note that the planetary waves are generally not linear, stationary or rapidly varying compared to the zonally averaged flow. Therefore, the assumptions made in determining the refractive index are all violated for realistic atmospheric states and the index provides only a rough indication of the effect of the mean state on the waves.

3. Model description

The model used in this study is a modified version of the NCAR Community Climate Model described by Pitcher et al. (1983), Ramanathan et al. (1983) and references therein. An independently programmed code with minor changes in some of the numerical algorithms has been used here. This version of the model is described in Williamson (1983).

The model is a primitive equation GCM in \( \sigma \) coordinates with 33 levels in the vertical extending from the surface to approximately 63 km in the lower mesosphere. The locations of the levels are indicated on the ordinate of Fig. 1 along with the approximate height \( (z) \) in kilometers. The value of \( z \) for each \( \sigma \) level is based on a surface pressure of 1000 mb and a 7-km scale height. In the horizontal, the model uses the spectral transform method with a rhomboidal 15 truncation. The associated Gaussian grid has 48 points in longitude with a spacing of 7.5 degrees and 40 points in latitude with an approximate spacing of 4.5 degrees. The actual locations of the latitudinal gridpoints are indicated on the abscissas of Fig. 1.

Aside from vertical resolution, the differences between the standard CCM and the model used in this study lie in the diffusion operators. The original nonlinear form of the vertical diffusion operator described in Bourke et al. (1977) was used, as opposed to the linearized form actually used by Bourke et al. (1977) and in other versions of the Community Climate Model. The operator is applied at all model levels. The horizontal diffusion used a biharmonic \( (\nabla^4) \) operator with a diffusion coefficient of \( 2.3 \times 10^{10} \) m² s⁻¹.

A Rayleigh friction term was added to the zonal momentum equation in order to provide a crude parameterization of the effect of breaking gravity waves in the mesosphere and to prevent reflection of waves from the top boundary. The friction coefficient (Fig. 2) was determined by

\[
K_f = \frac{1}{3} \left[ 1 + \tanh \left( \frac{z - 63 \times 10^3}{7.5 \times 10^3} \right) \right] \text{days}^{-1}
\]
and was set to zero below 40 km. The Rayleigh friction coefficient used by Holton and Wehrbein (1981) is included in Fig. 2 for reference along with the coefficient estimated from satellite data by Smith and Lyjak (1985). The damping time scale is three days at the top of the model, increasing to 50 days at 50 km.

The Rayleigh friction term is not regarded as an accurate representation of the mesospheric dynamics which the model does not explicitly resolve. The magnitude of $K_R$ was chosen to match that of Smith and Lyjak (1985) in the middle mesosphere and to be insignificant in the stratosphere, with smooth variation in the lower mesosphere. Approximately eight levels have been included in the lower mesosphere primarily to separate the region of interest from the model's upper boundary, allowing wave energy to propagate upward out of the stratosphere without being reflected. Wave energy propagating into the mesosphere is absorbed by radiative damping and by the Rayleigh friction.

The model was integrated for 370 days using January insolation and sea surface temperatures. The ozone distribution was specified as a zonal mean mixing ratio on pressure surfaces and the ozone path lengths were evaluated at each grid point when a full radiation calculation was performed (every 12 hours). The ozone distribution was obtained from Dutsch (1978) as described in Chervin (1986) with the above modification. The top level at which the ozone data were available was 0.49 mb and this caused a problem with the mesospheric heating rates. At all levels above 0.49 mb the ozone mixing ratio was set equal to the value at 0.49 mb. The resulting path lengths appear to be somewhat too long and the solar heating rates too large, causing abnormal temperature profiles in the mesosphere (Fig. 1). Only the stratosphere will be examined in this model and the mesospheric temperature profile is not of great importance. A different treatment of mesospheric ozone would be required before the model could be extended upward.

The zonally averaged temperatures and zonal winds for a 30-day period from the integration (days 280 to 310) are shown in Fig. 1 for the full model domain. As previously mentioned, the mesosphere is not well represented by the model although both the westerly and the easterly jets decrease with height above the stratosphere because of the Rayleigh friction. Below the stratosphere, the model reproduces the atmosphere reasonably well. The Southern Hemisphere tropospheric jet has nearly the right magnitude and is in the right
location. The equatorial easterlies are too strong in the upper troposphere and lower stratosphere, in conjunction with a tropical tropopause temperature which is nearly 10 K too cold. The latter feature is similar to the results of the original CCM (Pitcher et al., 1983) but very different from the results of Mahlman and Umscheid (1984). The Northern Hemisphere (winter) simulation is quite variable; it is discussed in the following sections for the region below 1 mb and poleward of 20°N.

All of the data used in the rest of this study were interpolated from \( \sigma \) to pressure surfaces. In order to minimize the vertical interpolation errors, the surfaces were chosen to be the same as the \( \sigma \) levels if the surface pressure were 1000 mb. The lowest two and highest six levels were not interpolated to pressure.

4. Northern Hemisphere winter simulation

The time evolution of the simulation of the winter stratosphere is illustrated in Figs. 3 and 4. The initial condition was taken from a previous test run in which a linear fit was used to the Smith and Lyjak (1985) profile for \( K_R \), resulting in a substantial Rayleigh friction in the upper stratosphere. During the first 90 days of the simulation, the polar temperature at 10 mb declined steadily (Fig. 3) as a result of the elimination of the stratospheric Rayleigh friction. Following day 90, a great deal of transience can be seen in the temperature series with a strong sudden warming occurring between days 200 and 210, when the 10 mb polar temperature reached 248 K. After the warming, the temperature declined slowly for about 60 days, following which a series of minor warmings occurred.

The time series of the zonal wind at 60°N as a function of pressure (Fig. 4) shows similar features to the 10 mb polar temperature series. For the first 90 days, the westerlies increased fairly steadily at all levels although some variability can be seen. Between days 90 and 200, there was a considerable amount of variability, particularly in the upper stratosphere, which corresponds closely with the fluctuations in the 10-mb polar temperature. During the strong warming between days 200 and 210, the zonal wind decreased extremely rapidly in the stratosphere, with easterlies appearing briefly in the upper stratosphere between days 210 and 220. Easterlies also appeared briefly in the middle stratosphere around day 230 so that the strong warming was just at the threshold of the usual definition of a “major” warming. Following day 230, the westerlies strengthened slowly, first in the upper stratosphere, then at all stratospheric levels until about day 260. Beginning about day 280, strong fluctuations in the upper stratospheric wind occurred again, in conjunction with the minor warmings seen in the 10 mb polar temperature. In the lower troposphere, easterlies appeared intermittently following day 280. It is interesting that zonal mean easterlies did not occur at any level prior to the major warming.

A fascinating feature of the time evolution described here is that two persistent periods exist which have quite different time mean 10-mb polar temperatures and in which the mean zonal wind at 60°N is quite different both in the stratosphere and in the lower troposphere. These periods are indicated in Fig. 3 as period I (days 90.5 to 180) and period II (days 280.5 to 370).
and they comprise the last half of the individual panels in Fig. 4. Although the time means are different between periods I and II, the level of variability is similar. The two periods will be examined in considerable detail in the remainder of this article.

The zonally averaged temperatures and zonal winds for 90-day periods before (period I) and after (period II) the warming are shown in Figs. 5 and 6. For comparison, plots of Northern Hemisphere observations averaged over four winters may be found in Geller et al. (1983). Averages over four different years for the zonal wind may be found in Smith (1983a).

There are substantial differences between the four-year averages of Geller et al. (1983) and Smith (1983a) resulting from the large variability of the winter stratosphere. The difference in the jet maximum between Geller et al. (1983) and Smith (1983a) is about 15 m s⁻¹ at 10 mb and 10 m s⁻¹ at 1 mb. The jet axis tilts southward by only 5 degrees between 10 and 1 mb in Geller et al. (1983), while the tilt is 15 degrees in Smith (1983a). The vertical shear is near zero above 7 mb in Geller et al. (1983), while in Smith (1983a) the shear is nearly constant across the middle and upper stratosphere. Individual months often bear little resemblance to the four-year averages (Geller et al., 1984). Ideally, one should simulate both the long-term mean and the interannual variability of the stratosphere; however, the published statistics do not provide a sufficient database for comparing simulated and observed variability. Although it is difficult to assess the accuracy of

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**Fig. 5.** Zonal mean temperatures interpolated to pressure surfaces and averaged over periods I (left) and II (right) for the extratropical winter hemisphere. Contour interval is 10 K.

**Fig. 6.** As in Fig. 5 except that the zonal wind is shown with a contour interval of 10 m s⁻¹.
a simulation of the winter stratosphere, certain strengths and weaknesses of the time mean simulation are obvious.

Examining the temperatures first (Fig. 5), one finds that the stratopause temperature outside of the polar region is about 275 K, perhaps 10 K warmer than the observations. The 200 K isotherm for the tropical tropopause extends to 35°N, as opposed to 20°N in the observations, because of the excessively cold equatorial tropopause mentioned earlier. The polar region is quite well simulated although the vertical gradient in the 220 K isotherm is 10 degrees poleward of the normal position in the observations. The minimum polar temperature of about 185 K in period I is perhaps 10 K colder than found in the observations while the 200 K temperatures in period II are quite realistic although the minimum in both periods is located too high.

The mean zonal winds in Fig. 6 are about what one should expect from the temperature distribution. Accompanying the excessively cold tropopause temperatures are tropical easterlies extending beyond 20°N, while the observations rarely show easterlies north of 10°N. Along with the cold polar temperature in period I is a polar night jet maximum of 80 m s⁻¹ at the stratopause, while period II has a much weaker jet (50 m s⁻¹ at the stratopause). In neither period is the jet outside the range of the observations for single monthly means, although period I is near the upper bound of the observed January wind speeds. In fact, period I more closely resembles the mean December jet magnitude. In both cases the jet is located too close to the pole. When the jet is located at high latitudes in the lower stratosphere, the observations show a strong tendency for the jet axis to tilt southward with height.

There is no evidence of this tilt in period I and only a weak tilt in period II.

The 10 mb height (Fig. 7) has a very different character in period I compared to period II, with period II looking much closer to the long-term mean from observations (e.g., see Newell et al., 1972). The polar vortex is 1.5 km deeper in period I, as should be expected from the colder lower stratospheric temperatures. The vortex also appears much more symmetric and more closely centered on the pole when compared to period II. It is interesting to note that the planetary wave amplitudes in the two periods are very similar and that the difference in the character of the maps is almost entirely due to the excessive depth of the zonal mean vortex in period I.

The 500-mb height (Fig. 8) for periods I and II shows the same characteristic differences as found by Boville (1984) for different mean zonal wind regimes in a model extending only to the middle stratosphere. In period I, when the mean zonal wind in the lower stratosphere is very strong, there is very little ridging in the western North American or Atlantic regions and the 500-mb map is very zonal. In contrast, there are well developed ridges in the 500-mb height during period II, indicating more frequent blocking events in better agreement with observations, although the ridges are actually too well developed. While these features of the middle tropospheric flow are indicative of different stationary and transient wave forcing of the stratosphere, they also show an influence of the lower stratospheric wind structure on the tropospheric flow (see Boville, 1984).

The relative success of the model at simulating the mean state of the troposphere and stratosphere in Jan-

Fig. 7. Ten mb height in hectometers averaged over periods I (left) and II (right) mapped on a polar stereographic projection with the outer latitude at 20°N and a contour interval of 400 m.
uary is interesting in itself, since full GCM simulations have not been very successful in reproducing the structure of the winter stratosphere. The model used in this study is very similar to that used in Boville (1985) to study the effects of wave damping on the lower stratospheric circulation except that levels have been added in the upper stratosphere and lower mesosphere (and Rayleigh friction has been included in the mesosphere). The vertical resolution in the upper troposphere and lower stratosphere has also been greatly increased as compared to the model used in Boville (1984).

Boville (1985) presented a theoretical analysis of the effects of the damping of waves on the momentum balance, followed by a series of practical tests in the model, which were performed by altering the damping rates due to horizontal diffusion and to radiation. It was shown that an extra momentum forcing term in the form of some (nonunique) combination of additional wave damping (in excess of that produced naturally by radiation) and zonal mean diffusion (drag) was required in order to obtain a reasonable simulation of the winter lower stratosphere. Broadly speaking, three possible explanations were advanced for the requirement of additional momentum forcing: the upper boundary of the model was too low; the forcing of waves in the troposphere was too weak; gravity wave breaking produces significant stress divergence in the vicinity of the tropopause and lower stratosphere which is not resolved in the model.

The wave damping effects of the $\nabla^4$ horizontal diffusion operator used in this study are most similar to the $\nabla^2$ rhomboid operator in Boville (1984), which gave excessively cold temperatures in the polar middle stratosphere accompanied by a strong polar night jet. This case was not reproduced in Boville (1985) since the increased vertical resolution in that study had little effect on the zonal mean simulation. The response to the diffusion operating on the zonal mean flow and on the waves was tested separately in Boville (1985).

The success of the current simulation indicates that the principal reason that additional momentum forcing was required in Boville (1984, 1985) was that no levels were included in the upper stratosphere. The usual upper boundary condition in GCMs ($\sigma = 0$, at $\sigma = 0$) is appropriate in the original differential equations but its effect in the vertically differenced equations solved in a model is quite different. Lindzen et al. (1968) and Kirkwood and Derome (1977) have shown that the usual upper boundary condition is similar to the introduction of a rigid lid at finite height. The effects of placing the topmost level of a GCM in the middle stratosphere will be examined in a subsequent manuscript, using exactly comparable simulations.

5. Wave propagation

The wave propagation characteristics of the mean state, as diagnosed by the refractive index defined in (2.7), is a sensitive function of the zonal mean wind which depends on both the wind shear and curvature in both dimensions. The zonal mean wind for the two periods in Fig. 6 is very different in the lower stratosphere and so it is not surprising that the refractive index is also significantly different, as shown in Fig. 9 (for $m = 1$). Wave 1 is the dominant component in the stratosphere and the refractive index for shorter waves differs from $Q_1$ primarily in the expansion of the negative region through the term $m^2/(\sin^2\phi \cos^2\phi)$ subtracted from $Q_0$ in (2.7).
In period I, there is a region of negative refractive index at 200 mb between 70° and 80°N and second region at 30 mb near 40°N. The two regions of negative index are connected by a region of weak positive index extending equatorward and upward from 70° to 45°N. Throughout the lower stratosphere at middle and high latitudes, the mean flow structure poses a strong impediment to vertical propagation of wave activity. Theoretically, the regions of negative index form an effective barrier to vertical propagation, while the strong equatorward component of the refractive index gradient between 45° and 70°N should cause vertically propagating wave activity to be deflected equatorward in the upper troposphere and lower stratosphere.

In period II, the situation is quite different in the lower stratosphere. The region of negative refractive index at 40°N is gone, while the high latitude region is located farther poleward than before. A region of relatively high refractive index has appeared in the lower stratosphere between 60° and 70°N with the result that the refractive index gradient is of a radically different character. Poleward of 55°N the horizontal component of the gradient is poleward instead of equatorward while the vertical component of the gradient is small. There now exists a region in which vertically propagating wave activity originating in the lower troposphere should be able to penetrate more easily into the stratosphere and then be deflected into the polar region instead of the equatorial region.

The wave activity, as represented by the EP flux (top of Fig. 10) behaves approximately as expected from the refractive index, given the fact that wave activity is generated near the surface and initially propagates almost vertically. Equatorward of 50°N during both periods there is a strong equatorward component of the refractive index gradient in the upper troposphere and lower stratosphere and the EP flux vectors deflect strongly equatorward there. Between 50° and 70°N the wave activity continues to deflect equatorward in period I (although more weakly) following the weakened refractive index gradient. The wave activity deflects poleward between 50°N and 70°N in period II when the refractive index gradient is reversed. The magnitude of the EP flux vectors is also larger in period II than in period I, reflecting the decrease in the vertical component of the refractive index gradient.

In Fig. 10, F(eq) has been scaled by the aspect ratio of the plot so that the vectors point in the right direction. The vector lengths are proportional to the flux magnitude except where the flux is larger than a threshold value, such vectors all being plotted with the same length. This method has the advantage that regions where the flux magnitude is negligible can be identified.

The effect of the weaker lower stratospheric wind in period II appears to be the result of the ducting of vertically propagating wave activity into the polar stratosphere. This process is aided by changes in the zonal wind structure extending down into the lower troposphere. On the other hand, the strong lower stratospheric winds in period I appear to deflect wave activity away from the polar stratosphere. Similar results have been found by O'Neill and Youngblut (1982) for the effect of the mean flow on the propagation of wave activity during a sudden warming.

The total flux in the top panels of Fig. 10 was computed by taking the time mean of the flux computed for the instantaneous flow every ½ day. The lower panels of Fig. 10 show the EP flux calculated for the time mean flow in the two periods (stationary flux) and the difference between the total and the stationary flux.
(transient flux). The EP flux is nearly evenly split between stationary and transient components in period I, while the stationary flux dominates in period II. This partitioning emphasizes the effects of the mean flow on the vertical propagation of the stationary waves, to which the derivation of the refractive index actually applies. The features discussed for the total EP flux, are even more apparent for the stationary flux.

The reasons for the lack of vertically propagating transient wave activity in period II are not at all clear. Figure 11 shows the zonal mean of the time standard deviation of the geopotential height at each point for
period I and the difference between periods I and II. This quantity is a root mean square measure of the transient wave amplitude. The difference between the periods is relatively small and is within the sampling variability of 90 day means based on previous experience. Since the transient heat flux \((F_{\text{th}})\) changes more than the mean transient wave amplitude between the periods, the mean wave structure must have changed. This is the same conclusion reached by Boville (1984) in examining the tropospheric changes induced by changes in stratospheric jet structure. Boville (1984) used a model extending only to 10 mb and the changes in the jet structure resulted from changes in the form of the horizontal diffusion; they were not spontaneous as in the current model. One should keep in mind that it is only the first 2 or 3 planetary waves which are important in the current context. The stability analysis of Zhang and Sasamori (1985) provides some indication that planetary waves of different structure are likely to dominate under different mean wind conditions.

6. Transformed Eulerian mean (TEM) momentum balance

Following (2.2), it is the EP flux divergence rather than the flux itself which determines the effect of the waves on the mean flow. In the time mean, when \(\partial u/\partial t\) can be neglected, (2.2) reduces to a balance between the force produced by the MMC on the left-hand side of (2.2) and the force produced by waves \((\nabla \cdot F)\) and mean drag on the right-hand side. For examining the mean momentum balance of 90-day periods, it is sufficient to consider the terms on one side of (2.2). In the following discussion, attention is focused on \(\nabla \cdot F\) and the Rayleigh friction term is also presented for completeness.

The EP flux divergence for the two periods shown in Fig. 12 is a force per unit mass, defined as \(\nabla \cdot F/p\rho a \times \cos \phi\) and scaled to units of m s\(^{-1}\)/day. Almost throughout the winter hemisphere, the waves exert a negative force (acceleration) on the mean flow except at very high latitudes (poleward of 80\(^\circ\)N) where noisy regions of positive values appear. These noisy regions lie poleward of the center of the polar vortex, where the separation of the fields into zonal means and waves has little meaning, the zonal mean winds being small and the waves enormous. Because the EP flux divergence is primarily negative, it will be more convenient to discuss the magnitude of the EP flux convergence below.

Most of the stationary EP flux convergence in Fig. 12 is from zonal wave numbers less than 5 (planetary waves), while the transient component in the troposphere is from wave numbers greater than 5 (synoptic waves). In the stratosphere, the planetary waves are responsible for both the stationary and the transient components of the EP flux convergence.

The remaining term on the left-hand side of (2.2) is the Rayleigh friction which is shown in Fig. 13. The Rayleigh friction is negligible compared to the EP flux divergence in both periods. It is only in the middle mesosphere that this term becomes an important part of the model’s momentum balance.

The MMC force on the right-hand side of (2.2) is dominated by the Coriolis torque \((f \times F)\) although the contributions from the other terms (advection of mean shear) are not completely negligible. Because the difference in the shear is large between periods I and II, the advection terms are quite different but the change...
in the net MMC driving is still dominated by the Coriolis torque.

In the troposphere, there is a distinct increase in the maximum EP flux convergence at 500 mb from period I to period II which is primarily accounted for by the stationary (planetary) waves. This reflects a substantial change in the structure of the stationary waves, as can be seen in Fig. 8, and is similar to the findings of Boville (1984). In period II, the stationary waves are playing a much greater role in the tropospheric momentum balance than in period I, as might be suspected from the changed character of the 500 mb height field (Fig. 8) between the periods. There is also a distinct southward shift and narrowing of the tropospheric conver-
gence maximum by the transient (synoptic) waves reflecting a change in the mean storm track.

The changes in the character of EP flux convergence are as large in the stratosphere as in the troposphere. In conjunction with the ducting of the EP flux into the polar region during period II, the EP flux convergence becomes large considerably lower in the stratosphere than during period I. The EP flux convergence is near zero in the whole region between 70 and 30 mb during period I, while it is about 1 m s\(^{-1}\)/day in that region during period II. The \(-2\) m s\(^{-1}\)/day (divergence) contour in period I is nearly horizontal at 6 mb but descends below 15 mb in the vicinity of the jet in period II. In the upper stratosphere, the convergence is actually slightly weaker in period II, when the 1 mb polar temperatures are slightly colder than in period I (Fig. 5).

The split between stationary and transient components of the convergence in the stratosphere is quite different between the two periods, reflecting the difference in the vertical propagation seen in the EP flux itself. The convergence due to the stationary waves is larger at all levels in period II both because the amount of stationary wave activity reaching the stratosphere is larger and because the radiative damping time for waves is shorter (Fig. 14). The radiative damping time in Fig. 14 is defined as in Boville (1985):

\[ \tau = -\frac{\tau}{Q'/T'} \]

where \( Q \) is the net radiative heating rate. The damping time is strongly dependent on the mean temperature because the long wave radiative cooling rate is a strong function of temperature. The warmer middle stratospheric temperatures in period II result in shorter damping times and would cause stronger EP flux convergence even if the wave activity reaching the stratosphere was fixed.

The combination of the refraction of wave activity by the mean flow structure and the dependence of the wave forcing of the mean flow on the propagation of wave activity appears to represent a positive feedback mechanism. The ducting of wave activity into the polar stratosphere which occurs in conjunction with the weaker lower stratospheric winds in period II results in stronger wave forcing of the mean winds in the lower and middle stratosphere. This wave forcing tends to maintain the weak winds and warm polar stratospheric temperatures, allowing the continued strong propagation of wave activity into the stratosphere. On the other hand, the strong lower stratospheric winds in period I, which seem to shield the polar stratosphere from wave activity, tend to decrease the wave forcing of the mean wind, allowing it to approach the radiative equilibrium state more closely.

The positive feedback described above is not, of course, the only mechanism affecting the stratospheric wind structure. Under the right stratospheric conditions, anomalous wave structures in the troposphere can lead to a sudden warming which destroys the cold state (period I) as occurred at day 200 of this experiment. The warm state (period II) does not appear to
be stable either, since it requires the continuous influx of wave activity from the troposphere. The cold state is recovered after several of the minor warmings seen in Fig. 3, although this requires more than 200 days following the major warming.

7. Conclusions

The model does an adequate job of simulating the mean state of the troposphere and stratosphere in January. This is an interesting result since full GCM simulations have generally not been very successful at reproducing the structure of the winter stratosphere. Previous versions of this model which simulated only the troposphere and lower stratosphere required additional forcing of the zonal mean flow in order to produce reasonable simulations of the winter stratosphere (Boville, 1984, 1985). This forcing was provided by a horizontal diffusion term. The success of the current simulation indicates that the additional forcing of the mean flow was making up for the erroneous effects of the usual GCM upper boundary condition when the upper stratosphere and lower mesosphere are not explicitly resolved. The effects of breaking gravity waves in the vicinity of the tropopause do not appear to be required in order to obtain a reasonable simulation, as was suggested by Boville (1985), Boer et al. (1984) and Palmer et al. (1985). It is important to note that the relative accuracy of the simulation discussed here does not necessarily imply that gravity wave effects on scales not resolved by the model are not important in the atmosphere.

The model produces quite different time mean (90 day) states depending on the period studied, although the variability of the model within the 90-day periods is somewhat less than is common for the winter season in the atmosphere. This is not surprising since the external forcing is fixed at values representative of 15 January. Subsequent integrations of the model with a prescribed annual cycle of external forcing have given more realistic variability. The type of integration examined here is useful for studying the interactions of the waves with the mean flow, since variability on all time scales is exclusively due to this process. It is important to note that, in a GCM, this interaction is not only a stratospheric process, but involves the troposphere as well. In section 4 it was shown that both the troposphere and the stratosphere have quite different stationary wave structures in the two periods examined.

A positive feedback mechanism exists between wave forcing of the zonal mean flow and the refraction of waves by the mean flow. When the polar night jet is strong, particularly in the lower stratosphere, it tends to inhibit the vertical propagation of wave activity into the polar stratosphere. The EP flux divergence then tends to be weak in the lower and middle stratospheres, permitting the jet to remain strong. If the jet is weakened sufficiently by some transient event (such as a sudden warming) the waves can propagate more effectively into the polar stratosphere. The EP flux divergence is then stronger and tends to maintain the weaker jet. The process is tightly coupled to the tropospheric generation of vertically propagating wave activity and, in the case examined here, the changes in refractive properties, wave activity and EP flux divergence extended from the surface to the upper stratosphere.

Acknowledgments. I wish to thank Dr. Anne Smith for providing the original figure from Smith and Lyjak (1985) which was used to construct Fig. 2. Valuable comments on the original manuscript were provided by Drs. R. E. Dickinson, J. T. Kiehl and W. M. Washington and subsequently by Dr. K. E. Trenberth and three anonymous reviewers. I am very grateful to my section head, Dr. W. M. Washington, for the continuing moral support and substantial computer allocation which made the development of this model possible.

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