Stable and Unstable Air–Sea Interactions in the Equatorial Region

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ABSTRACT

The linear stability of two coupled shallow water models of the equatorial atmosphere and ocean is investigated analytically. The ocean-to-atmosphere coupling is parameterized in terms of the sea surface temperature anomaly, T. In one model T is generated by zonal advection, while in the other model it is parameterized in terms of the thermocline depth anomaly, h. In both models, the behavior of a small amplitude wave disturbance is found to be extremely sensitive to the atmospheric first baroclinic mode Kelvin wave speed, C_a, and mean zonal wind, $\bar{U}$. Most importantly, the growth rates, phase speeds and meridional structures of the disturbances (and their dependence on the above basic state parameters) are sensitive to the form of the atmosphere–ocean coupling. This sensitivity is due to the fact that, for an oceanic Kelvin wave, the two methods of computing T result in different feedback effects. According to the simple analytic models used in this study, in which the meridional component of motion is neglected in both the atmosphere and ocean, equatorially trapped unstable (growing) modes occur only when the ocean-to-atmosphere coupling is parameterized in terms of the advectively produced sea surface temperature anomaly. The resulting growth rates and phase speeds of the growing modes can perhaps account for the onset of an ENSO (El Niño–Southern Oscillation) event, but only in the unusual case in which $C_a \sim |\bar{U}|$.

1. Introduction

The problem of the initiation and subsequent growth of large-scale anomalies associated with the El Niño–Southern Oscillation (ENSO) phenomenon has been the subject of very intense investigation during the past few years. These investigations have been further stimulated by the occurrence in 1982–83 of perhaps the strongest, and best documented, ENSO event on record. While the actual triggering mechanism for the ENSO event must still be considered an open question, several recent studies have suggested unstable air–sea interactions in the equatorial region as a viable working hypothesis. For example, Philander (1983) and Gill and Rasmusson (1983) present convincing arguments describing how an unstable interaction between the ocean and the atmosphere may have contributed to the development of the unusually strong 1982–83 ENSO event. Their arguments concerning the behavior of the coupled ocean–atmosphere system are based on our present understanding of how the ocean responds to specified changes in the equatorial surface winds (e.g., Wyrtki, 1975) and how the atmosphere responds to specified changes in heat sources in the tropics (e.g., Gill, 1980).

The unstable air–sea interaction hypothesis is also supported by recent theoretical studies which show that certain small-amplitude wave disturbances in coupled shallow water models of the atmosphere and ocean are unstable. Lau (1981) first identified such an instability in a coupled ocean–atmosphere model which allowed no meridional motion. More recently, Philander et al. (1984) also found an instability using a more realistic dynamical model of the coupled system. In both of the above studies the atmospheric heating is assumed to be proportional to the thermocline depth anomaly in the ocean, and the instability arises because the altered surface winds associated with a heat source in the atmosphere modify the ocean in such a way as to induce further heating of the atmosphere.

In the present study we analyze and compare the results of two different coupled models of the equatorial ocean-atmosphere system. The heating of the atmosphere is assumed to be proportional to the sea surface temperature anomaly T. In one model, T is assumed to be proportional to the thermocline depth anomaly (Lau, 1981; Philander et al., 1984), while in the other model, T is assumed to be produced by anomalous zonal advection in the ocean (Rennick, 1983). A mean zonal wind is included in the atmosphere in all cases. Our analytic results show that, within the restricted framework of equatorial Kelvin wave dynamics (i.e., no meridional motion in the ocean or atmosphere), the form of the ocean-to-atmosphere coupling and the values used for certain atmospheric model parameters significantly alter the stability properties of wave disturbances in the coupled system.

2. The models

The feasibility of positive feedback between disturbances in the ocean and atmosphere as a mechanism for the initiation and development of an ENSO event is investigated within the context of a simple analytic model. The model is based on the linearized shallow water equations on an unbounded equatorial β-plane. Following Matsuno (1966) the equations are applied to a single vertical mode of a stratified fluid by a judicious choice of equivalent depth.

The system is constrained to allow interactions only between Kelvin wavelike disturbances in the atmosphere and ocean by requiring the meridional velocities to vanish. This restriction limits the generality of the model results. However, atmosphere-only models (e.g., Gill, 1980; Lim and Chang, 1983) have shown that the atmospheric response to prescribed forcing similar to that associated with an ENSO event is dominated by Kelvin and Rossby waves of roughly equal amplitude (The inertial gravity wave response is very weak). In fact, most of the qualitative features of the response are contained in the Kelvin wave signal. Furthermore, ocean-only models (e.g., McCreary, 1976) have shown that the oceanic response to ENSO-like forcing is completely dominated by an eastward propagating Kelvin wave throughout the initiation phase of an event. Thus, the only significant interaction which is eliminated by the model restriction is that between the Kelvin wave in the ocean and the Rossby wave in the atmosphere. Philander et al. (1984) suggest that the elimination of meridional motion is equivalent to ignoring the effects of rotation. Indeed, it can be shown that the dispersion relation for a rotating fluid with $v = 0$ (derived below), is the same as that of the lowest meridional mode in a nonrotating system. However, the amplitude of the lowest meridional mode in a nonrotating fluid is constant with latitude whereas the disturbances in a rotating fluid with $v = 0$ are trapped near the equator, as required by the β-plane geometry.

The advantage of considering only coupled Kelvin waves is that it allows a simple analytic solution which provides insight into the sensitivity of the coupled system to a mean zonal wind in the atmosphere, simple thermodynamics in the ocean, and alternative coupling strategies.

The model equations are the following:

$$U_t + \bar{U}U_x + gH_x = 0 \tag{1a}$$

$$\beta y U + gH_y = 0 \tag{1b}$$

$$H_t + \bar{U}H_x + DU_x = Q \tag{1c}$$

$$u_t + gh_x = S \tag{1d}$$

$$\beta y u + gh_y = 0 \tag{1e}$$

$$h_t + du_x = 0. \tag{1f}$$

In (1), the coordinates (x, y) measure distance in the eastward and northward directions, and derivatives are represented by subscripts. The atmosphere equations (1a–c) are linearized about a barotropic basic state with constant zonal velocity $\bar{U}$ and equivalent depth D. Deviations from the atmosphere basic state are given by U, H. The ocean equations (1d–f) are linearized about a motionless basic state with equivalent depth d.

In order to satisfy the undisturbed equations, $\bar{U}$ and D must be related geostrophically, so that $D = D_0 - (\bar{U}/2g)\beta y^2$, where D_0 is a constant. For $y^2 \ll 4\Omega a L^2/|\bar{U}|$, where L is the Rossby radius of deformation, the y-dependent contribution is negligible. Thus, for $|\bar{U}| \leqslant 30$ m s^{-1} the meridional variation in D is negligible over distances of a few times L. Therefore, in the following discussion D is treated as a constant. Also, the equivalent depths of the atmosphere and ocean are expressed in terms of the corresponding free Kelvin wave speeds, $C_a = (gD)^{1/2}$ and $C_o = (gd)^{1/2}$.

The atmosphere–ocean coupling is accomplished through a heating term, Q, in (1c) and a surface wind stress term, S, in (1d). The specification of Q and S in terms of oceanic and atmospheric variables, respectively, is of central importance. Since the system (1) is overdetermined due to the elimination of meridional motions, only certain forms for Q and S will yield a solution [see (3) below].

The physical grounds for the coupling are very similar to that called upon by Lau (1981) and Philander et al. (1984). The surface wind stress is treated as a body force, proportional to the low level atmospheric wind. Since U actually represents the zonal wind disturbance for the first baroclinic mode of the atmosphere, $U > 0$ represents a positive vertical wind shear and thus a negative zonal stress at the sea surface. Then S is proportional to $-U$. Heating is assumed to take place wherever the sea surface temperature (SST) is above normal. Thus, Q is proportional to the SST anomaly.

In order to compute an SST anomaly field, a thermodynamic equation for the ocean must be added to the strictly dynamical system (1). Two different thermodynamic equations, representing two different physical mechanisms for determining the SST, are used:

$$T - \mu h = 0 \tag{2a}$$

$$T_t - Gu = 0. \tag{2b}$$

Calculations using (2a) and (2b) are referred to as "Model 1" and "Model 2", respectively. In (2), T is the disturbance SST field, μ is a constant of proportionality, and $G = -T_x > 0$ represents the (constant) zonal gradient of mean SST. Equation (2a) is based on the assumption that anomalously warm ocean temperatures are associated with an anomalously deep upper layer and vice versa. This is the traditional interpretation of SST in shallow water models of the ocean, and it can be justified if changes in T are mainly due to changes in the rate of entrainment mixing at the base of the mixed layer associated with upwelling and downwelling. The second form of the thermodynamic equation (2b) is based on the assumption that the zonal advection of a mean temperature gradient is the principal mechanism by which SST anomalies develop. The importance of this mechanism in producing SST anomalies in the equatorial Pacific Ocean is pointed out by Gill (1983) and by Harrison and Schopf (1984).

3. Stability analysis

The behavior of a small amplitude wave disturbance in models 1 and 2 is analyzed and compared for the case of fully time dependent atmospheric response and for quasi-steady atmospheric response. In all cases, we focus on the unstable modes which are trapped near the equator.

a. Time dependent atmospheric response

We assume a zonally propagating wave solution of the form

$$q(x,y,t)=\tilde{q}(y)e^{i(kx-\omega t)}$$

for each variable, and substitute these into (1). The meridional structure of the atmospheric and oceanic variables is found from (1a, b) and (1e, f), respectively:

$$(U,H)\alpha\ \exp[-y^2/2L_a^2] \quad (3a)$$

$$(u,h,T)\alpha\ \exp[-y^2/2L_o^2] \quad (3b)$$

where $L_a^2 \equiv (\omega/k - \bar{U})/\beta$ and $L_o^2 \equiv kC_o^2/\omega\beta$; L_a and L_o represent the meridional scales of the atmospheric and oceanic disturbances.

Since the β-plane formulation is valid only in the equatorial region, physically meaningful solutions must be trapped near the equator. That is, the real parts of both L_a^2 and L_o^2 must be positive. The condition on L_o requires that the wave disturbance propagates towards the east $[\mathrm{Re}(\omega)/k > 0]$. Similarly, the condition on L_a means that the Doppler shifted phase speed $[\mathrm{Re}(\omega)/k - \bar{U}]$ must also be eastward. In the central and western Pacific, the climatological mean zonal wind $\bar{U}$ is westward throughout the year, with a typical magnitude of 5–10 m s^{-1} (Sadler, 1975). Therefore, eastward propagating modes will normally satisfy both trapping conditions.

As mentioned earlier, only certain forms for Q and S will yield a solution to (1). The problem is that Q, which forces the atmosphere, is proportional to an oceanic quantity, whereas S, which forces the ocean, is proportional to an atmospheric quantity. Due to the neglect of meridional motion, (3) allows only one meridional mode in each field. Since in general, $L_a \neq L_o$, consistency requires that the coupling terms be written

$$Q=\eta(y)\alpha T \quad (4a)$$

$$S=\zeta(y)\gamma U \quad (4b)$$

where η and ζ are projection operators, projecting the oceanic temperature disturbance T onto the meridional structure of the atmospheric variables, and the atmospheric wind disturbance onto the meridional structure of the oceanic variables. The form of these operators is derived in the Appendix. Here, we simply note that the product, $\eta\zeta$, must be independent of y.

In order to evaluate the results quantatively, numerical estimates of the model parameters are needed. The values used in this study are shown in Table 1. The value of γ is based on an average surface windspeed of 10 m s^{-1}, a drag coefficient of 10^{-3}, and a typical mixed layer depth of 50 m. The value of G is based on a 5° change in SST across the entire equatorial Pacific. The value of α is based on the assumption that an anomalous surface heat flux, Q_1T, heats a tropospheric column of thickness D. We then find that

$$\alpha=\frac{RQ_1}{\rho_a C_p g D}$$

where R is the gas constant for dry air, ρ_a is the air density, and C_p is the specific heat of air. Using Q_1 = 25 W m^{-2} K^{-1} (Haney, 1971) and D = 5 km, we obtain the value of α in Table 1. The value of μ is chosen to be consistent with the heating used by Philander et al. (1984). These values, as well as the free Kelvin wave speed for the ocean (C_o) are held fixed for all numerical calculations. Because of the variability of atmospheric conditions, solutions were computed for a range of values for the free atmosphere Kelvin wave speed (C_a) and the mean atmospheric wind speed ($\bar{U}$). We will find that the model results depend weakly on the product of the coupling parameters, $\gamma\alpha\mu$ (model 1) and $\gamma\alpha G$ (model 2), and strongly on the mean wind speed $\bar{U}$ and the free Kelvin wave speed C_a.

The dispersion relations obtained from (1) are

Model 1:

$$[\omega-kC_o][\omega+kC_o][\omega-k(C_a+\bar{U})][\omega+k(C_a-\bar{U})] - k^2C_o^2\eta\zeta\gamma\alpha\mu=0 \quad (5a)$$

Model 2:

$$[\omega-kC_o][\omega+kC_o][\omega-k(C_a+\bar{U})][\omega+k(C_a-\bar{U})] - ikg\eta\zeta\gamma\alpha G=0. \quad (5b)$$

Certain features of the roots of (5) can be anticipated without actually solving the equations. It is clear that if the ocean–atmosphere coupling is broken, i.e., α or

TABLE 1. The values of parameters which are taken as constant throughout this study.

Parameter	Value
C_o	2 m s^{-1}
γ	2.0×10^{-7} s^{-1}
μ	6.7 K m^{-1}
α	1.5×10^{-4} m s^{-1} K^{-1}
G	4.0×10^{-7} K m^{-1}
C_a	0–50 m s^{-1}
$\bar{U}$	−15–15 m s^{-1}

$\gamma = 0$, solutions to (5) are neutral Kelvin waves [$\omega = \pm kC_o$ and $\omega = \pm k(C_a + \bar{U})$] with the eastward propagating modes satisfying the trapping condition. However, these waves are neutral and cannot contribute to unstable growth. For the more interesting case with α, $\gamma \neq 0$, the possibility of complex roots arises. It can be anticipated that the conditions for the existence of unstable modes [$\mathrm{Im}(\omega) > 0$] depend critically on the choice of heating parameterization, since the unfactored term on the left-hand sides of (5a, b) is pure real for the coupling of model 1, and pure imaginary for the coupling of model 2.

Numerical solutions to (5) were obtained using the parameter values indicated in Table 1. Of the four roots to (5a), two are real and the other two are a complex conjugate pair. The real roots cannot contribute to an instability. The member of the complex conjugate pair with positive imaginary part is shown in Fig. 1 for $U = -10$ m s^{-1}. This is the mode discussed by Lau (1981). The growth rate of this mode suggests that it may contribute to an observable phenomenon. However, for all reasonable values of C_a and k the wave is either stationary, or westward propagating. (This result holds for other values of $\bar{U}$, as well.) Thus, the β-plane solution is not equatorially trapped, so that its physical significance is highly questionable.

For model 2, (5b), with atmospheric heating proportional to the advectively determined SST perturbation, the situation is rather different. Each of the solutions to (5b) is complex. Two of the modes are damped, $\mathrm{Im}(\omega) < 0$, and two are growing, $\mathrm{Im}(\omega) > 0$. Of the two growing modes, one propagates westward, and therefore is untrapped, while the other propagates eastward, and therefore is trapped near the equator. The phase speed and growth rate of this latter mode, the only equatorially trapped unstable mode allowed by (5) is shown in Fig. 2a, b. This mode propagates rapidly toward the east with phase speed given approximately by $C_a + \bar{U}$. It has a positive growth rate for most values of the parameters. With $\bar{U} = -10$ m s^{-1}, the maximum growth rate, which occurs for $C_a \approx C_o - \bar{U} = 12$ m s^{-1}, corresponds to an e-folding time of 17 days. However, for more realistic values of C_a, say 30 m s^{-1}, the growth rates are much smaller. The existance of this mode illustrates the possibility for unstable interactions between the atmosphere and ocean. However, it is unlikely that this particular mode is of physical significance because its phase speed ($\sim C_a + \bar{U}$) is about an order of magnitude greater than those typical of ENSO disturbances.

b. *Quasi-steady atmospheric response*

Here we consider the case in which the atmospheric motion (which feeds back to the ocean) is the instantaneous *steady state* response to the heat sources determined by the ocean (Philander et al., 1984). The governing equations for model 1 and model 2 are the same as before except the terms U_t and H_t in (1a) and (1c) are dropped. Substituting the wave solution into the new governing equations we obtain

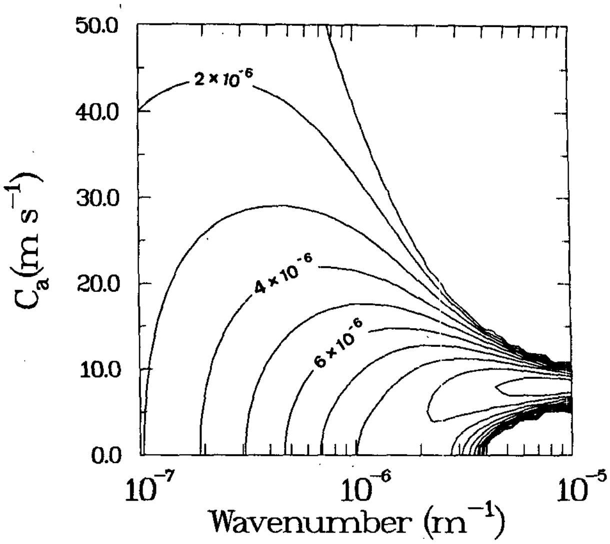

FIG. 1. Growth rates, $\mathrm{Im}(\omega)$, as a function of C_a and k with $\bar{U} = -10$ m s^{-1} in the case of the time dependent atmospheric response using the coupling of model 1. Contour interval = 1.0×10^{-6} s^{-1}.

Model 1:

$$\omega^2 = k^2C_o^2 - \gamma\alpha\mu C_o^2/(C_a^2 - \bar{U}^2), \tag{6a}$$

Model 2:

$$\omega^2 = k^2C_o^2 - ig\gamma\alpha G/k(C_a^2 - \bar{U}^2), \tag{6b}$$

and, for both models,

$$(U, H)\alpha \exp\left[-\frac{1}{2}y^2/L_a^2\right] \tag{7a}$$

$$(u, h, T)\alpha \exp\left[-\frac{1}{2}y^2/L_o^2\right]. \tag{7b}$$

Here $L_a^2 \equiv -\bar{U}/\beta$ and $L_o^2 \equiv kC_o^2/\omega\beta$, as before.

From (7) we see that an equatorially trapped solution requires both $\mathrm{Re}(\omega/k) > 0$ and $\bar{U} < 0$. As noted previously, the latter condition is generally satisfied in the equatorial Pacific west of the dateline.

As in the case of the fully time dependent atmosphere, the solutions to (6) correspond to neutral Kelvin waves if any of the coupling coefficients are zero. For nonzero coupling the presence of the imaginary term on the right-hand side of (6b) makes the solutions to models 1 and 2 quite different. For model 1 (6a), ω^2 is real. Thus, ω is either pure real or pure imaginary. This is the mode considered by Lau (1981)—except that he only treated the case with $\bar{U} = 0$. A stationary growing mode [$\mathrm{Im}(\omega) > 0$] exists, but since $\mathrm{Re}(\omega) = 0$, it is not equatorially trapped. The large growth rate of this mode is similar to that found by Philander et al., (1984). However, the exact correspondence between the present solution and that of Philander et al. (1984) is not

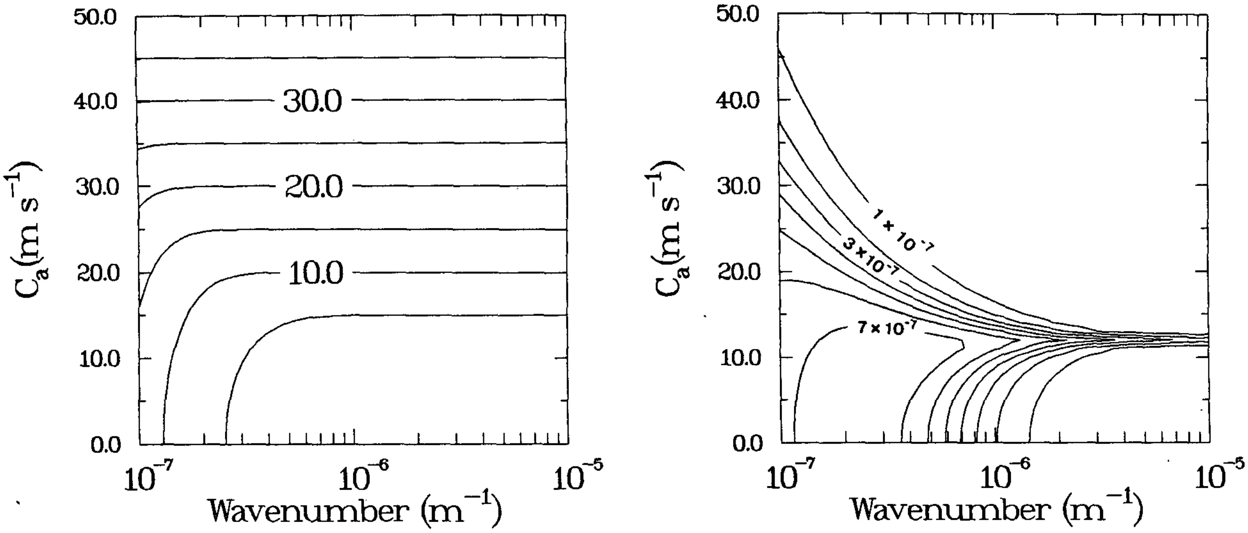

FIG. 2. Phase speeds, Re(ω/k) (left, with contour interval = 5 m s^{-1}) and growth rates (b), Im(ω) (right, with contour interval = 1.0 × 10^{-7} s^{-1}), for $U = -10$ m s^{-1}, as in Fig. 1 except using the coupling of model 2.

clear, due to differences between the two dynamical models. In particular, Philander et al. included more meridional modes (Rossby waves) in their solution.

The heating parameterization of model 2 (6b) allows an eastward propagating (equatorially trapped) solution which is unstable for at least some regions of parameter space. The growth rates for this mode are shown in Fig. 3. This mode has a minimum *e*-folding time of about 6 days and an eastward phase speed of about 3 m s^{-1}. These values suggest a possibly active mechanism for unstable interactions between the ocean and atmosphere, but one which exists only for very small values of C_a.

The correspondence between the solutions for the fully time dependent atmosphere and those for the quasi-steady atmosphere is clear when it is noted that (5) reduces to (6) in the limit as $\omega^2/k^2(C_a^2 - \bar{U}^2) \rightarrow 0$. The two rapidly propagating modes for each model in the fully time dependent case are eliminated by the quasi-steady assumption. The two slowly propagating modes survive. The characteristics of the quasi-steady, slowly propagating modes are quite similar to those of the analogous modes of the fully time dependent system in those regions of parameter space for which $(\omega/k)^2 \ll C_a^2 - \bar{U}^2$. Significant differences are found only where $C_a^2 \sim \bar{U}^2$. For the unusual (in the real atmosphere) case of $C_a^2 \sim \bar{U}^2$, the low frequency condition required by the quasi-steady approximation is violated.

4. Summary and conclusions

We have investigated the behavior of a small amplitude wave disturbance in two versions of two different coupled shallow water models of the ocean–atmosphere system. Our most important result is that the particular form of the ocean-to-atmosphere coupling used in the model, i.e., whether the SST anomaly is parameterized in terms of *h* (model 1) or whether it is produced by zonal advection (model 2), has a very significant effect on the model results. This difference is due to the fact that the SST anomaly in model 1 is in quadrature with that in model 2. These results suggest that the growth of a disturbance in the coupled ocean–atmosphere system will be sensitive to the exact mechanism by which the equatorial SST anomaly is formed. Recently, Hirst (1986) reached a similar con-

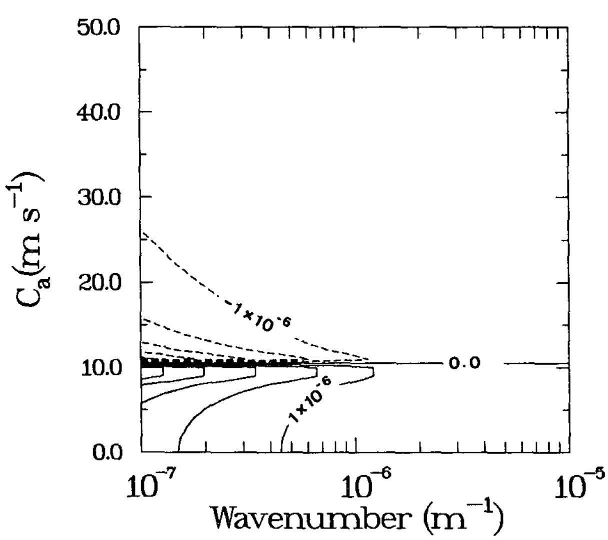

FIG. 3. Growth rates, Im(ω), as in Fig. 1 except for the quasi-steady atmospheric response using the coupling of model 2. Contour interval = 1.0 × 10^{-6} s^{-1}. Negative values are dashed.

clusion regarding sensitivity to the physical parameterization of heating in a numerical study of coupled modes in a less restricted dynamical system. Further research on this important topic is clearly needed.

We have found that the stability and maximum growth rates of the unstable modes in the coupled system are sensitive to the values used for the atmospheric free Kelvin wave speed, C_a, and mean zonal wind, $\bar{U}$. This sensitivity to the values of C_a and $\bar{U}$, together with the fact that the most realistic values of these quantities (i.e., $C_a \approx 30$ m s^{-1}, $\bar{U} \approx -10$ m s^{-1}, and $C_o \approx 2$ m s^{-1}) are not very close (in parameter space) to the values required for maximum growth rate, provides a *potential* mechanism for explaining the infrequent occurrence of ENSO events. For example, of all the unstable modes found in this study, only two are equatorially trapped (Figs. 2 and 3), and neither of these can be considered potential candidates for explaining the sudden development of an ENSO event unless $C_a^2 \sim \bar{U}^2$. Thus it is possible that the seasonal and interannual variations in C_a and $\bar{U}$ may be such as to only rarely bring these quantities into the range where significant growth rates occur. While the exact nature of the above solutions depends on the rather restrictive model assumption of no meridional motion, our results serve to indicate the kind of subtleties involved in the application of simple model solutions to the real world.

We would like to conclude this summary by referring the reader to the discussion section of the recent paper by Philander et al. (1984) in which the many limitations of the coupled shallow water models used there and here are nicely described. In order to obtain insight, we have further simplified the models used in this study by assuming no meridional motion. We have also assumed the ocean model to be unbounded in the zonal direction. These assumptions serve to restrict the model's applicability to the earliest part of the development of a coupled ocean-atmosphere disturbance. These restrictions notwithstanding, our basic results concerning the sensitivity of commonly used models to the values of the important parameters that go into them, and to the assumed form of the ocean-to-atmosphere coupling, are very likely to hold in more realistic shallow water models in which meridional motion is included. They may extend to more complex dynamical and thermodynamical ocean-atmosphere models as well. We therefore believe that the solutions to idealized models as used in this study can serve as a useful guide for understanding more complex and realistic models of the actual ocean–atmosphere system.

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APPENDIX

Operator Forms

In order to find the projection functions $\eta(y)$ and $\zeta(y)$, the meridional structures of the atmospheric and oceanic variables must be expressed in terms of each other. This is easily done in terms of the Hermite polynomials. According to (3a), the meridionally varying part of any atmospheric quantity may be written

$$A(y) = BH_0(y/L_a)e^{-1/2(y/L_a)^2} \tag{A1}$$

where B is a constant and H_0 is the zero order Hermite polynomial (=1). Similarly, the meridionally varying part of any oceanic quantity may be written

$$C(y) = DH_0(y/L_o)e^{-1/2(y/L_o)^2} \tag{A2}$$

using (3b).

Since the Hermite polynomials $H_n(y/L_a)$ constitute a complete set of orthogonal basis functions, $C(y)$ can be expanded to give

$$C(y) = \sum_{n=0}^{\infty} \hat{C}_n H_n(y/L_a)e^{-1/2(y/L_a)^2} \tag{A3}$$

where

$$\hat{C}_n = \frac{1}{\sqrt{\pi}}\frac{1}{2^n n!}\int_{-\infty}^{\infty} H_n(y/L_a)C(y)e^{-1/2(y/L_a)^2}d(y/L_a); \tag{A4}$$

(3a) limits $C'(y)$, the atmospheric representation of $C(y)$, to the $n = 0$ term in (A3). Then

$$C'(y) = \hat{C}_o e^{-1/2(y/L_a)^2}.$$

From (A4),

$$\hat{C}_o = D\frac{\sqrt{2}L_o}{(L_a^2 + L_o^2)^{1/2}} = C(y)e^{1/2(y/L_o)^2}\frac{\sqrt{2}L_o}{(L_a^2 + L_o^2)^{1/2}}$$

so that

$$C'(y) = \eta(y)C(y)$$
$$= \frac{\sqrt{2}L_o}{(L_a^2 + L_o^2)^{1/2}}e^{1/2(y/L_o)^2}e^{-1/2(y/L_a)^2}C(y)$$

or

$$\eta(y) = \frac{\sqrt{2}L_o}{(L_a^2 + L_o^2)^{1/2}}e^{1/2(y/L_o)^2}e^{-1/2(y/L_a)^2}. \tag{A5}$$

In a similar manner, an atmospheric quantity can be expanded in terms of the oceanic basis functions $H_n(y/L_o)$ to obtain

$$A(y) = \sum_{n=0}^{\infty} \hat{A}_n H_n(y/L_o)e^{-1/2(y/L_o)^2}. \tag{A6}$$

Since (3b) restricts oceanic variables to the $n = 0$ term of (A6), the oceanic representation of $A(y)$ is given by

$$A'(y) = \hat{A}_0 e^{-1/2(y/L_o)^2} = \zeta(y)A(y).$$

Then

$$A'(y) = B\frac{\sqrt{2}L_a}{(L_a{}^2 + L_o{}^2)^{1/2}} e^{-1/2(y/L_o)^2}$$

$$= \frac{\sqrt{2}L_a}{(L_a{}^2 + L_o{}^2)^{1/2}} e^{1/2(y/L_a)^2} e^{-1/2(y/L_o)^2} A(y)$$

$$\zeta(y) = \frac{\sqrt{2}L_a}{(L_a{}^2 + L_o{}^2)^{1/2}} e^{1/2(y/L_a)^2} e^{-1/2(y/L_o)^2}. \qquad \text{(A7)}$$

Clearly, the product $\zeta\eta = \dfrac{2L_aL_o}{(L_a{}^2 + L_o{}^2)}$ is independent of y, as required by (5).

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