Effects of Eddy Initial Conditions on Nonlinear Forcing of Planetary Scale Waves by Amplifying Baroclinic Eddies

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ABSTRACT

The previous study of Young and Villere concerning growth of planetary scale waves forced by wave–wave interactions of amplifying intermediate scale baroclinic eddies is extended to investigate effects of different eddy initial conditions. A global, spectral, primitive equation model is used for the calculations. For every set of eddy initial conditions considered, growth rates of planetary modes are considerably greater than growth rates computed from linear instability theory for a fixed zonally independent basic state. However, values of growth rates ranged over a factor of 3 depending on the particular set of eddy initial conditions used. Nonlinear forcing of planetary modes via wave–wave coupling becomes more important than baroclinic growth on the basic state at small values of the intermediate-scale modal amplitudes. The relative importance of direct transfer of kinetic energy from intermediate scales of motion to a planetary mode, compared to baroclinic conversion of available potential energy to kinetic energy within that planetary mode, depends on the individual case. In all cases, however, the transfer of either kinetic or available potential energy to the planetary modes was accomplished principally by wave–wave transfer from intermediate scale eddies, rather than from the zonally averaged state. The zonal wavenumber 2 planetary mode was prominent in all solutions, even in those for which eddy initial conditions were such that a different planetary mode was selectively forced at the start. General characteristics of the structural evolution of the planetary wave components of total heat and momentum flux, and modal structures themselves, were relatively insensitive to variations in eddy initial conditions, even though quantitative details varied from case to case.

1. Introduction

In a previous paper Young and Villere (1985, hereafter referred to as YV) investigated the means by which intermediate scale baroclinic waves forced rapid growth of planetary scale waves through wave–wave interactions. Their work was motivated by the discussion of this effect given in Gall et al. (1979), and the possibility that eastward traveling planetary waves observed in the troposphere and stratosphere (Leovy and Webster, 1976; Hartmann, 1976; Mechoso and Hartmann, 1982) were the result of nonlinear forcing by intermediate scale baroclinic modes. It is shown in both Gall et al. and YV that wave–wave coupling between intermediate scale baroclinic modes causes growth rates of planetary modes which are comparable to the linear growth rates of the most unstable intermediate scale baroclinic modes. Such an effect is also apparent in the computations reported by Frederiksen (1981). These results indicate that nonlinear forcing must be considered when trying to understand observations of eastward traveling planetary modes. Planetary wave components of total heat and momentum fluxes computed in YV exhibited relatively large amplitudes near the tropopause. This led to the suggestion in YV that the lack of coherence between the troposphere and stratosphere observed in Southern Hemisphere eastward traveling waves with zonal wavenumbers 1 and 2 (Mechoso and Hartmann, 1982) was the result of nonlinear forcing near the tropopause.

Young and Villere considered to a limited extent the effects of different initial zonal mean states on the growth of planetary scale modes. In order to easily isolate the effects of varying mean zonal jet structure, two initial mean zonal states were considered having realistic vertical distributions but idealized meridional variations. These initial zonal mean states were very similar to the broad 30° jet and the 45° jet mean states used in Simmons and Hoskins (1977, 1978). Generally the growth rates of the planetary modes were larger in the case of the 45° jet than for the 30° jet, and the final kinetic energy per unit mass averaged over the entire atmosphere, KE, was 2–5 times larger for the 45° jet planetary modes. Not directly addressed in YV, however, was the question of the sensitivity of the computed growth of planetary modes to different eddy initial conditions.

The purpose of the present paper is to investigate the sensitivity of planetary-scale mode growth rates, as well as the sensitivity of associated heat and momentum fluxes, to variations of the initial spatial and amplitude distributions of intermediate scale baroclinic modes. The combined results of YV, Gall et al. (1979) and Frederiksen (1981) indicate that the qualitative phenomenon of enhanced growth of planetary modes will occur under a wide variety of circumstances. Each of the above studies involved significantly different initial mean zonal states and eddy distributions, yet enhanced growth of planetary modes was observed in each case. However, the extent to which varying eddy initial conditions can alter the growth rates of the planetary modes, or alter the heat and momentum fluxes which produce the planetary mode amplification, has not been investigated. The linear baroclinic growth rate of a planetary mode amplifying on a zonal mean state is generally much less than either the linear growth rate of intermediate scale modes, or the growth rate of the same planetary mode when forced by intermediate scale baroclinic modes via wave–wave interactions. Such a situation leads one to expect significant dependence of planetary mode growth rate on eddy initial conditions. The present computations confirm this expectation, in that there were significant variations in growth rates and heat and momentum fluxes as eddy initial conditions were varied, although enhanced growth of planetary scale baroclinic modes (relative to linear growth on a zonal mean state) always occurred.

2. Procedure and eddy initial conditions

The numerical model used for the calculations, together with spatial resolution and methods of time integration, are identical to those described in YV. Briefly, the numerical model is a three-dimensional, spectral, primitive equation model on a sphere. Horizontal resolution corresponded to trapezoidal truncation of the spherical harmonic basis functions with maximum zonal wavenumber 20 and total wavenumber 40. A variably spaced vertical grid was used having 17 vertical layers between the surface and 33 km altitude. At the surface the grid spacing was about 0.25 km, while in the upper layers the grid spacing was about 3 km. Each time integration had as initial conditions the 45° jet zonal mean state described in YV, perturbed by disturbances with zonal wavenumbers ranging between 1 and 20. The 45° jet had a meridional dependence, $g(\theta)$, given by $g(\theta) = \sin^3\pi\mu^2$, where θ is colatitude, and $\mu = \cos\theta$. The vertical dependence corresponded to a typical December–February average at 30°N.

Eddy initial conditions were varied primarily to investigate sensitivity of planetary scale mode growth rates to changes in the initial zonal wavenumber spectrum of intermediate scale baroclinic modes. However, there were also other questions we wished to address. One question concerned the evolution of the $m = 2$ planetary mode. The reason for interest in the $m = 2$ mode is that this mode is the most prominent eastward traveling planetary wave in the Southern Hemisphere stratosphere during winter (Leovy and Webster, 1976; Hartmann, 1976; Mechoso and Hartmann, 1982) and it becomes the dominant planetary mode in terms of KE for each of the 30° and 45° jet basic states that were considered by YV. Specifically, we wanted to investigate the situation where initial conditions were such that initially there was no $m = 2$ component in any of the nonlinear terms in the governing equations. It is, of course, possible to have initial conditions such that the solutions would never produce a wavenumber 2 mode. For example, it is possible to force just the $m = 3$ mode if initial conditions consist only of modes which are harmonics of wavenumber 3. However, the more general initial situation would have some mixture of modes which eventually generates all wavenumbers, and this is the type considered in this paper and described below. Another question of interest concerned the effects of multiplying each mode in a given set of initial conditions by a common scale factor to form a new set of initial conditions. The latter set of calculations gives information regarding the amplitude range of the intermediate scale modes above which nonlinear forcing of the planetary modes becomes the dominant forcing mechanism.

In order to keep the number of computations at a reasonable level, each set of initial conditions treated was chosen to specifically address one of the above topics in some way. Hence, the following five distinct sets of eddy initial conditions were considered. The first set, denoted by A, was identically the same as that used in YV, and therefore required no additional calculations. Each zonal wavenumber between 1 and 20 was represented, and at a particular zonal wavenumber the field variables were set to the most unstable baroclinic mode of the previously mentioned 30° jet at that particular zonal wavenumber, suitably scaled so that the maximum amplitude of the surface geopotential for a given zonal wavenumber was approximately 2.5 m. (Further discussion concerning these modes can be found in YV.) The set of initial conditions B was the same as A, except that each mode was multiplied by a scale factor which reduced the initial total kinetic energy, KE, of each mode by a factor of 100. The third set, C, had initial perturbations at every zonal wavenumber the same as that for zonal wavenumber $m = 8$ in set A, i.e., each mode was of the form $f_8(\theta, z)e^{im\phi}$, where ϕ is longitude, z is altitude, and $f_8(\theta, z)$ represents the initial (complex) amplitude function at $m = 8$ for any of the field variables such as geopotential, etc. The purpose of set C was to investigate the effects of an initial distribution of modes in which the amplitude function for each zonal wavenumber had the same spatial distribution in latitude and height, both with regard to magnitude and phase. The fourth set of initial conditions, D, had the same initial perturbations as set A in zonal wavenumbers 5 and 8, but had zero amplitudes for perturbations at all other zonal wavenumbers. One would expect a priori that this set of initial conditions would selectively force planetary mode 3, although in this case the whole spectrum of modes will eventually be produced by wave–wave coupling. This set of initial conditions therefore investigates the evolution of the $m = 2$ planetary mode when wave–wave coupling preferentially forces some other mode at the beginning. The final set of initial conditions considered, set E, was the same as D except that each mode was multiplied by a scale factor which increased the value of KE by 30. This last set of initial conditions was the one yielding the largest planetary mode growth rates.

For ease of reference, the linear growth rate spectrum for the 45° jet basic state is given in Fig. 1. Note the relatively small growth rates of planetary modes 1–3 compared to the intermediate scale modes.

3. Planetary mode growth rates

In order to illustrate the effects of different initial conditions on the growth rates of the planetary modes, the globally averaged kinetic energy per unit mass, KE, was chosen as the quantity to be compared. The temporal variation of KE for the planetary modes being considered here, which are nonlinearly forced by intermediate scale modes, depends not only on the amplification of the mode, but also on the time dependent evolution of the modal structure. This is in contrast to the case of linear modes which can be separated in space and time. However, as will be evident when Fig. 2 is discussed below, during the early growth phases the rate of increase of KE is nearly exponential for the planetary modes even though they are nonlinearly forced. Hence, during this time period, which is the same time period during which comparisons of growth rates will be made, the growth rate of KE for a particular planetary mode is primarily associated with growth in amplitude of the mode and not details of the time evolution of the spatial structure.

The effects of different eddy initial conditions on amplification of kinetic energy are illustrated in Fig. 2 for planetary modes $m = 1$–3. Also shown is the temporal behavior of kinetic energy for $m = 8$. The reader should note that the ordinate scale on Figs. 2a and 2c has been expanded in order to show clearly the details of the curves. Two points apparent from the figure are the following. First, amplification of the planetary modes is always considerably larger than would be the case if growth of the planetary modes was due just to linear growth on the zonally averaged basic state. This can be seen by comparing twice the growth rates given in Fig. 1 (twice because kinetic energy is a quadratic quantity in modal amplitude) to those implied in Fig. 2. Second, it is clear that there is substantial variation in amplification rates of the planetary modes depending on eddy initial conditions. For example, maximum growth rates for wavenumber $m = 2$ kinetic energy varied from about 0.8 per day for initial condition set A, to 2.4 per day for initial condition set E. It is apparent that although different eddy initial conditions produce quantitative differences in the evolution of the planetary modes, enhanced planetary mode growth is generally a feature of the solutions. In YV it was shown for initial condition set A that, when all growing baroclinic modes were allowed to simultaneously interact with and modify just the zonally averaged basic state (no wave–wave coupling allowed), the planetary modes amplified at a slower rate than would be expected on the basis of linear growth (no modification allowed in the basic state). Thus, enhancement in growth of the planetary modes when wave–wave coupling is present, relative to that when there is no wave–wave coupling, is probably larger for all sets of initial conditions than the above comparison with linear growth rates would indicate.

The maximum growth rate of a planetary mode which is being nonlinearly forced by intermediate scale baroclinic modes is approximately the sum of the growth rates of the two most linearly unstable modes which force the planetary mode. In other words, if the $m = 6$ and $m = 8$ modes were the two most unstable modes forcing the $m = 2$ mode, the growth rate of the $m = 2$ mode might be expected to be close to the sum of the growth rates of the $m = 6$ and $m = 8$ modes, at least after some initial time period. As discussed in YV, when all modes are present there can be significant cancellation in the forcing terms, so that the planetary wave component of the heat and momentum fluxes produced by the intermediate scale modes is less than the above simple argument would suggest. This is borne out by Fig. 2, where a comparison of the growth rates for each set of initial conditions shows that only for set E are all planetary mode growth rates close to the growth rates as predicted above. For example, the growth of the $m = 2$ mode for initial condition set A, Fig. 2a, is almost parallel to the middle (1.0 per day) calibration inset line, corresponding to a growth rate about half that predicted, whereas for initial condition set E, Fig. 2e, the growth curve is essentially parallel to the left (2.0 per day) calibration line. The maximum growth rate of the $m = 3$ mode for initial condition set D was also close to that predicted, but modes 1 and 2 had significantly smaller growth rates. Those sets of initial conditions in which all modes were initially present had growth rates which were generally a factor of 2 or more less than the predicted values, an exception

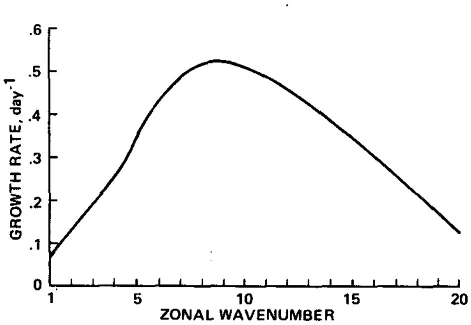

FIG. 1. Growth rate as function of zonal wavenumber for 45° jet.

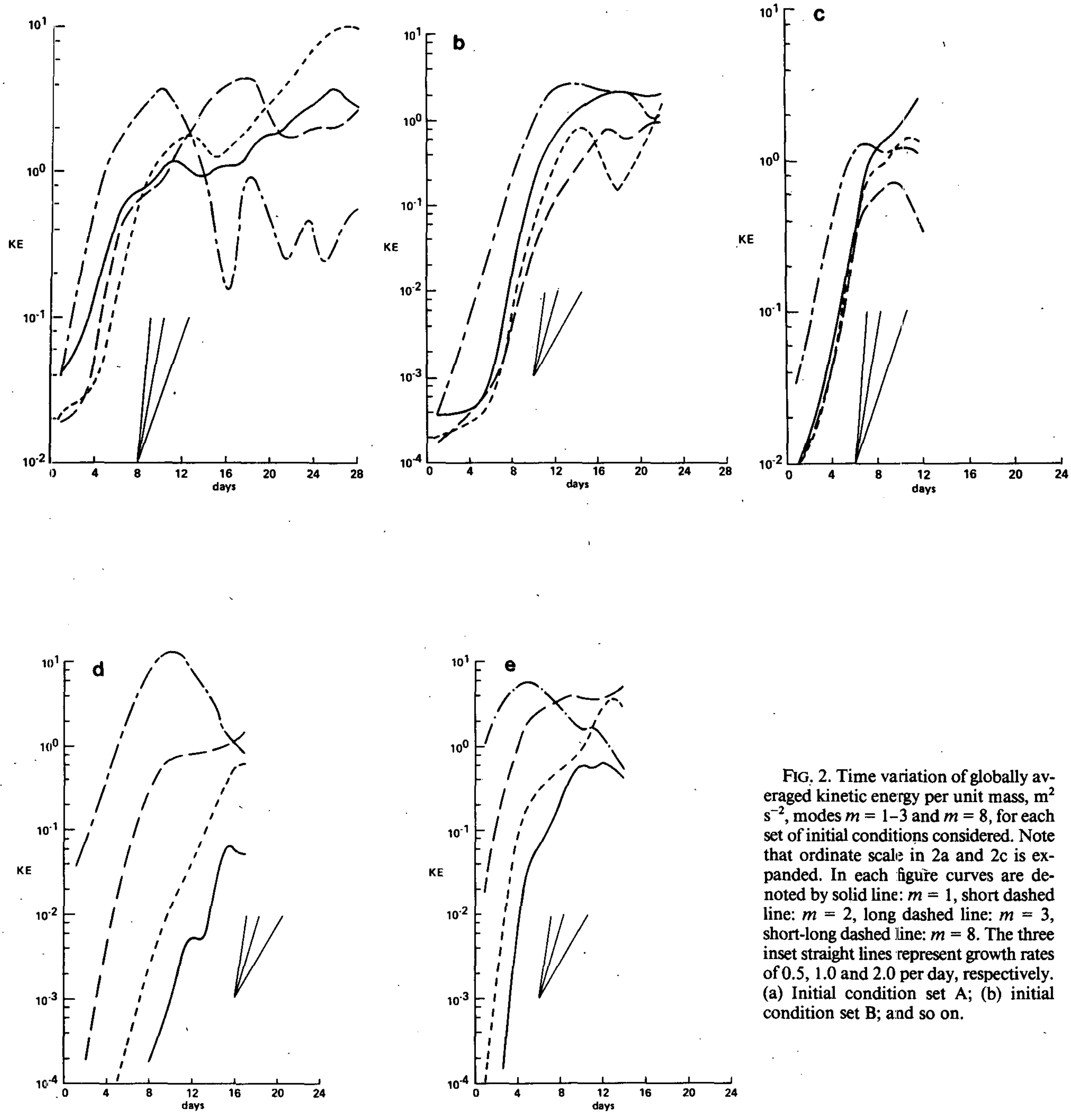

FIG. 2. Time variation of globally averaged kinetic energy per unit mass, $m^2 s^{-2}$, modes $m = 1$–3 and $m = 8$, for each set of initial conditions considered. Note that ordinate scale in 2a and 2c is expanded. In each figure curves are denoted by solid line: $m = 1$, short dashed line: $m = 2$, long dashed line: $m = 3$, short-long dashed line: $m = 8$. The three inset straight lines represent growth rates of 0.5, 1.0 and 2.0 per day, respectively. (a) Initial condition set A; (b) initial condition set B; and so on.

being initial condition set B which had growth rates about 0.7 of those predicted. When the initial eddy spectrum is nearly constant, as was the case for initial condition set C, each planetary mode is forced at essentially the same rate (see Fig. 2c), but one which is substantially smaller than for set E. The above results suggest that the largest planetary mode growth rates can be expected to occur when only a few intermediate scale modes are initially present and, in addition, the initial amplitudes of the intermediate scale modes are relatively large but not so large that amplification due to baroclinic growth is inhibited by nonlinear effects.

It can be observed from Fig. 2 that the relative magnitudes of planetary mode kinetic energy during the first 20 days or so vary with initial conditions, as one might expect. Depending on initial conditions, any of the planetary modes $m = 1$–3 may have the largest amplitude at a given time. A feature which is common, however, is the eventual prominence of the $m = 2$ mode for all sets of initial conditions. In particular it becomes prominent for initial condition sets D and E (Figs. 2d and 2e). Recall that both these sets of initial conditions were such that only modes having zonal wavenumbers 5 and 8 were initially present. In spite of the fact that the only planetary mode to be nonlinearly forced at the start was the $m = 3$ mode, the $m = 1$ and 2 modes rapidly amplify, with the $m = 2$ mode becoming comparable to the $m = 3$ mode after 12–16 days of integration. In fact, the maximum computed growth rate for the $m = 2$ mode occurred for set E. In the two solutions integrated beyond 20 days (which used initial condition sets A and B), the $m = 2$ mode becomes the dominant planetary mode in terms of KE and in terms of amplitude in the stratosphere. Later stages of growth of the $m = 2$ mode for initial condition set A have been attributed by YV to barotropic instability of both the zonally averaged state and other eddy components of the flow (cf. Baines, 1976). The calculations for set A showed modes 1 and 3 to be either stable or less barotropically unstable than mode 2 between days 16 and 28. Thus, since the amplitude of the $m = 2$ mode is comparable to that of the other planetary modes around day 20 in all the solutions, subsequent barotropic instability would probably cause the $m = 2$ mode to become the predominant planetary mode for initial condition sets C–E as well for sets A and B. For initial states other than those considered here and in YV, we cannot say whether the $m = 2$ mode would eventually become the dominant planetary mode. However, in the cases we have treated, the dominance of the $m = 2$ mode is the result of the tendency of wave–wave coupling to produce comparable amplitudes among the planetary modes, coupled with an initial atmospheric state which apparently evolves to one which becomes barotropically unstable to the $m = 2$ mode.

There should be some amplitude range for the intermediate scale baroclinic modes below which nonlinear forcing of planetary modes would not be significant. For example, the rate of transfer of zonal available potential energy to eddy available potential energy at a particular zonal wavenumber m, $R(m)$, involves quadratic products in that modal amplitude times zonally averaged quantities (cf. Saltzman, 1970). However, the rate of transfer of available potential energy to a particular zonal wavenumber m from eddies of all other wavenumbers, $S(m)$, involves triple products of the modal amplitudes of all the other eddies. Similar comments apply to $M(m)$, the conversion of mean zonal kinetic energy to eddy kinetic energy within wavenumber m, and $L(m)$, the transfer of kinetic energy from all other zonal wavenumbers (not equal to zero) to the kinetic energy of wavenumber m. Thus, if initial amplitudes of the intermediate scale modes are sufficiently small, nonlinear forcing of the planetary modes will not be important for some time period, i.e., intermediate scale modes must attain sufficient amplitude before $|S(m)| \geq |R(m)|$, etc. Fig. 2a shows that for initial condition set A, planetary mode $m = 1$ exhibits enhanced growth almost immediately, and modes $m = 2$ and $m = 3$ do so within about the first three days of integration. For initial condition set B, however, which is the same as A except scaled to smaller amplitude, Fig. 2b shows that there is a period of approximately 5–6 days during which growth of the planetary modes is relatively small. In fact, the growth rates during this time are similar to those computed for initial condition set A when no wave–wave coupling is allowed (see YV). A plot of the ratio $|S(m)/R(m)|$ for set B, Fig. 3, shows that $|S/R| \leq 1$ for the planetary modes during the first several days of integration. Thus, during this time period transfer of available potential energy to planetary waves is due to the usual baroclinic conversion of zonal mean available potential energy to planetary wave available potential energy. However, beyond about day 6, $|S/R|$ reaches values much greater than unity, and planetary wave available potential energy is supplied principally by transfer of available potential energy from intermediate scale modes. Moreover, it is not until $|S/R| > 1$ that planetary mode growth rates become relatively large. Note that in contrast to the planetary modes, $|S/R|$ for the intermediate scale mode $m = 8$ never exceeds unity, and is usually

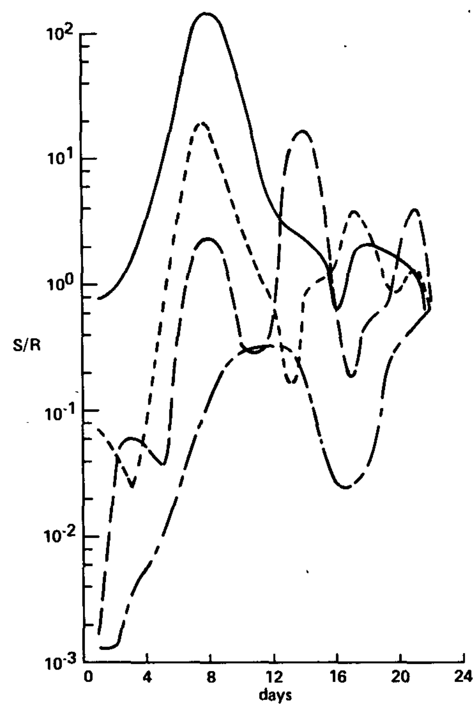

FIG. 3. Temporal behavior of ratio $|S(m)/R(m)|$ for initial condition set B. Curves for different values of m are denoted as in Fig. 2.

much less. Thus, as might be expected, intermediate scale modes derive most of their energy from the zonally averaged state, and subsequently this energy is partially transferred to the planetary modes. The fact that $|S/R|$ exceeds unity for $m = 1$–3 at such relatively small intermediate scale modal amplitudes implies that baroclinic energy conversion between planetary modes and the zonal mean state is so weak that almost any zonal wavenumber spectral distribution of eddies, which includes global distributions of intermediate scales of motion, will predominantly force the planetary modes after some early time. This conclusion is consistent with the fact that enhanced growth of planetary modes occurs for all sets of eddy initial conditions considered (although quantitative details may vary).

4. Energetics

The temporal behavior of kinetic energy for each zonal wavenumber mode depends on the sum of three energy transfer rates: $L(m)$, $M(m)$ and $C(m)$. The $L(m)$ and $M(m)$ were defined previously; $C(m)$ is the conversion rate of available potential energy to kinetic energy within zonal wavenumber m. For the planetary modes it is generally true that $L(m)$ and $C(m)$ are the most important energy transfer rates, with the relative importance of $C(m)$ and $L(m)$ depending on the particular case. Here $C(m)$ is important for the planetary modes because, as discussed previously, available potential energy is transferred to the planetary modes from intermediate scale modes (see Fig. 3) and, subsequently, available potential energy is converted into kinetic energy within each planetary mode. Discussions of the details of this process may be found in Gall et al. (1979) and YV.

Figure 4 illustrates the energy budgets for two cases. The first is for $m = 3$ and initial condition set B, the second is for $m = 2$ and initial condition set D. These two cases clearly illustrate how $C(m)$ and $L(m)$ dominate the energy budget, but also how the relative magnitudes of $C(m)$ and $L(m)$ vary for different situations. Furthermore, a comparison of Fig. 4a with Fig. 7c in YV shows that details of the energy transfers have changed significantly as initial condition set A was simply multiplied by a scale factor to become set B. Thus, it is apparent that detailed energetics during growth of the planetary modes are sensitive to initial conditions. However, this again illustrates the point that the existence of rapid growth of the planetary modes does not depend particularly on details of nonlinear interactions, even if quantitative aspects of the growth do.

5. Structure of modes and heat and momentum fluxes

One aspect of the calculations which remained unchanging as initial conditions were varied was the basic evolution of the structure of the planetary modes and the heat and momentum fluxes which produced them.

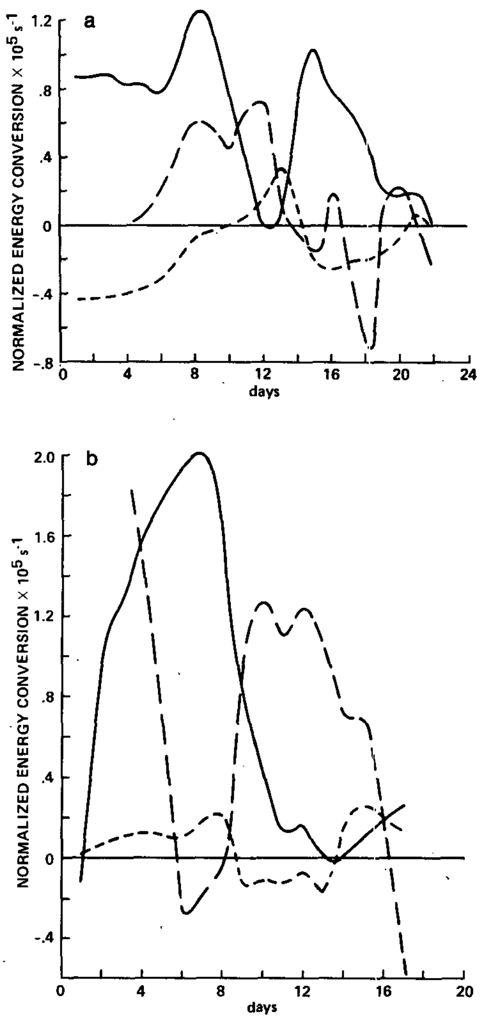

FIG. 4. Kinetic energy budgets for two cases: (a) $m = 3$, initial condition set B; (b) $m = 2$, initial condition set D. Values normalized by total kinetic energy of wave. Solid curve is $C(m)$; long dashed is $L(m)$; short dashed is $M(m)$.

Of course, details of the time evolution of the above quantities depended on initial conditions and zonal wavenumber, etc., but in every instance certain characteristics of the evolution were the same. In order to illustrate these characteristics, we consider the case $m = 2$ and initial condition set B. Unless otherwise noted, each of the features mentioned in the following discussion was typical of all the cases. The times at which events occurred varied depending on the particular situation.

Figure 5 presents latitude–height sections of amplitude for the $m = 2$ component of temperature, total heat flux and total momentum flux, at times $t = 10$ and $t = 14$ days for initial condition set B. At $t = 10$ days the temperature is confined to low levels, as is the $m = 2$ component of the total heat flux. The double maximum in the temperature structure at the lowest

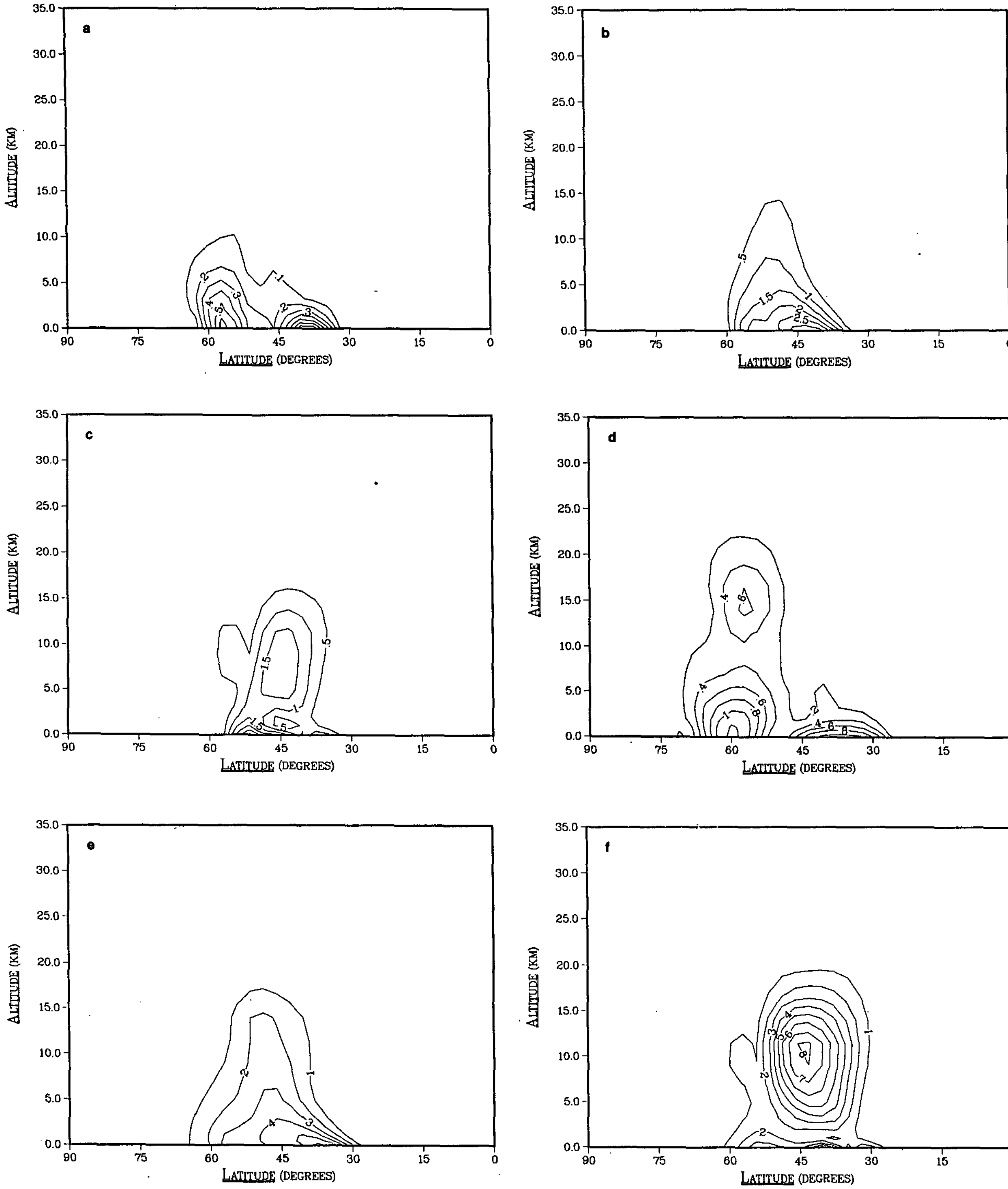

FIG. 5. Amplitudes of $m = 2$ component of temperature, °C, poleward heat flux, m °C s^{-1}, and momentum flux, m^2 s^{-2}, in order. Initial condition set B. Contributions from all zonal wavenumbers are included. (a)–(c) day 10; (d)–(f) day 14.

levels is pronounced; such structure and the reasons for it have been discussed in detail by Gall et al. (1979) and YV. Although temperature and heat flux are confined to low levels, by day 10 the momentum flux has penetrated the tropopause. It was generally the case that momentum flux reached the upper troposphere and lower stratosphere before the heat flux by a few days or so. By day 14 the temperature has penetrated into the stratosphere, although the double maximum structure is still apparent at lower levels. It is almost always the polar part of the planetary mode which propagates into the upper layers first. Between days 10 and 14 the heat flux also penetrates the upper troposphere and lower stratosphere, and continues to do so, such that at later times a local maximum occurs near the tropopause. It is this type of behavior, namely the penetration of the planetary mode into the stratosphere and the existence of upper level maxima in the heat and momentum fluxes, which led YV to suggest the importance of wave–wave coupling for producing certain characteristics of planetary waves $m = 1$ and $m = 2$ observed in the Southern Hemisphere troposphere and stratosphere. For example, the lack of coherence between the troposphere and stratosphere of eastward traveling planetary waves 1 and 2 in the Southern Hemisphere (Mechoso and Hartmann, 1982) may be due to upper level nonlinear forcing of these waves near the tropopause in the manner described above.

Although the planetary modes computed in this paper are principally forced by wave–wave interactions and may have complex time behavior, some qualitative insight into the structural evolution of the modes can be obtained from the theory for linear steady waves. Steady waves should propagate in regions where Q, the "square of the index of refraction" as termed by Matsuno (1970), is positive. In such regions one would anticipate that almost any disturbance being forced at lower levels would propagate to higher levels, how high depending at least partially on details of the distribution of Q. Using the expression for Q given in Schoeberl and Geller (1977), it is found that for steady planetary waves which have phase velocities comparable to the apparent zonal phase velocity of the computed planetary modes (see discussion below), the square of the index of refraction calculated using the instantaneous zonally averaged state is positive at latitudes below about 70°. The fact that the planetary modal structures develop upward with time below about 70° is consistent with Q being positive there. Additionally, as mentioned above, it is the poleward parts of the planetary modes which reach upper levels first. This also seems consistent with steady linear wave theory. Using the instantaneous computed zonally averaged state, the vertical group velocity for steady waves as derived in Palmer (1982) is significantly larger above 45° latitude than below 45°, at least around days 8–14. Thus, planetary mode energy from lower levels first reaches upper levels in the latitude region expected on the basis of the steady wave vertical group velocity.

The $m = 2$ mode illustrated in Fig. 5 has an apparent eastward phase progression of 15–20 degrees per day between days 10 and 22. The phase velocity of the linear baroclinic modes is near 10 degrees per day for zonal wavenumbers 1 through 12. The larger apparent zonal phase progression of the nonlinearly forced mode is due to the fact that the mode is propagating upwards in altitude and its structure is evolving with time. Young and Villere investigated the eastward propagation characteristics of the planetary modes associated with initial condition set A beyond day 20. In that situation they found that the modes tended to propagate eastward at approximately 10 degrees per day, about the rate for linear modes. Although eastward phase progression of the planetary modes was found to be fairly common, because of nonlinearities such phase progression would not necessarily be expected to be regular, and in fact there are instances when there is no eastward phase progression at all (cf. YV).

6. Conclusions and discussion

It seems clear that nonlinear forcing by intermediate scale baroclinic modes is an important source for the generation of traveling planetary scale waves, and must be considered when trying to understand their dynamical behavior. The present computations indicate that for the 45° jet zonally averaged basic state treated here, planetary scale modes will be forced by intermediate scale baroclinic modes for any initial global distribution of intermediate scale eddies which subsequently amplify because of baroclinic instability. These results are generalized when combined with those of YV, Gall et al. (1979), and Frederiksen (1981), in which different mean zonal states were used. In every case, during the time when intermediate scale baroclinic modes are amplifying, planetary mode growth rates were considerably larger than growth rates corresponding to linear baroclinic instability of a fixed zonally averaged basic state. Energy budget calculations for the transfer of both available potential energy and kinetic energy to the planetary modes indicated that transfer from intermediate scale eddies becomes larger than the corresponding energy transfers from the zonal mean state when intermediate scale modal amplitudes are still relatively small. This result is a consequence of the slow linear growth of the planetary modes on the zonally averaged basic state, which implies weak energy conversions between planetary scale motions and the basic state. Thus, after a short time period, energy transfer from other eddies dominates the planetary wave energy budget for almost any initial global distribution of eddies. This is in contrast to the energy budget for intermediate scale modes, for which energy transfer from the zonally averaged state is generally most important.

It has been shown that quantitative details of the growth phase of the planetary modes are sensitive to initial eddy distributions. Depending on eddy initial conditions, growth rates varied by factors of up to 3, and relative amplitudes of the planetary modes during the early growth phases changed significantly. When all zonal wavenumbers were initially present, planetary mode growth rates were significantly smaller than the sum of the linear growth rates of the two most unstable modes forcing a particular planetary mode. Significant cancellation of individual contributions in the nonlinear forcing terms occurred. When only two intermediate scale modes of relatively large amplitude were initially present, the $m = 8$, and $m = 5$ modes for initial condition set E, planetary mode growth rates were close to the sum of the growth rates of the appropriate most unstable modes. Thus, largest planetary mode growth rates appear to be associated with initial conditions for which only a few intermediate scale modes are present, these modes in addition having large amplitudes but not so large that baroclinic growth is suppressed by nonlinear coupling.

Certain features of the solutions obtained here and in YV appear relevant to observations of traveling planetary waves in the Southern Hemisphere winter stratosphere and troposphere. For all sets of initial conditions total kinetic energy of each of the planetary modes grew rapidly. Even though initial conditions might favor selective forcing of a particular planetary mode, which for initial condition set D and E was the $m = 3$ mode, the $m = 2$ mode quickly amplified and became comparable in magnitude to the favored mode. Thus, nonlinear forcing tends to generate more than a single planetary mode. The planetary mode which eventually becomes dominant in later stages of the solutions appears to be determined by factors other than nonlinear forcing. In our calculations the $m = 2$ mode eventually became dominant, at least for initial conditions sets A and B, apparently as a result of barotropic instability of the flow. Hartmann (1983) has shown that zonally independent states representative of that of the winter Southern Hemisphere are barotropically unstable in the stratosphere to global scale waves, with the zonal wavenumber 2 mode having characteristics similar to those observed. Thus, as in our solutions, it may be the case that nonlinear forcing plays a significant role in producing stratospheric eastward traveling waves, with barotropic instability also being important for determining which planetary waves eventually dominate the planetary wavenumber spectrum. For all sets of initial conditions considered, the planetary wave components of total heat and momentum flux developed local maxima, or at least relatively large values, near the tropopause. This strengthens the suggestion put forth in YV that nonlinear forcing is important for causing the lack of coherence between troposphere and stratosphere in planetary waves $m = 1$ and 2 in Southern Hemisphere winter seen by Mechoso and Hartmann (1982). Obviously, realistic Southern Hemisphere zonal mean states must be considered before these possibilities are adequately tested, and that work will be reported in a future paper.

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