Time-Mean Flow and Variability in a Nonlinear Model of the Atmosphere with Orographic Forcing

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ABSTRACT

The variability and time-mean response to orographic forcing are examined in a nonlinear atmospheric model. Distinct signatures from both high-frequency (synoptic-scale) and low-frequency (periods greater than 10 days) transients are seen in the temporal variance and eddy fluxes. Downstream of the orography, in the region of the time-mean jet stream, high-frequency transients are organized into a storm track and exhibit baroclinic energy conversions. The low-frequency transients, while producing greater variability in the same region as the storm track, exhibit significantly less baroclinic energy generation. The structure of the low-frequency transients downstream of the orography is similar to the observed FN2A pattern. The time scale of the eddies in this region appears to be longer than typical time scales associated with stationary Rossby wave dispersion. These eddies exhibit large local barotropic conversion of mean kinetic energy due to the large zonal gradient of the mean zonal wind. These barotropic processes downstream of the mountain give the appearance of low-frequency waves propagating out of the tropics even though there is no low latitude forcing in this model. Midlatitude orography is shown to influence the tropical time-mean circulation; a weak easterly jet along the equator develops due south of the orography. The influence on the tropical variability is restricted to increased high-frequency variance with limited effects on the low-frequency transients.

1. Introduction

One of the major thrusts of dynamic meteorology in the past 30 years has been the identification and dynamical explanation of the time-mean stationary waves. Orography was identified by Charney and Eliassen (1949) as a major source of the observed stationary waves. Hoskins and Karoly (1981) presented the essential linear dynamics for the orographically forced stationary waves in a stratified atmosphere. They interpreted the local response in terms of the balances of vorticity and heat and the remote response in terms of Rossby wave dispersion.

Not all of the observed stationary wave pattern is due to orographic forcing. (See Wallace, 1983, for an excellent review of the observations.) Thermal forcing (Smagorinsky, 1953) and fluxes of heat and momentum by transient eddies (Lau, 1979; Youngblut and Sasamori, 1980; Holopainen et al., 1982; Lau and Holopainen, 1984) are known to exert considerable influence on the stationary wave structure, particularly in the lower troposphere. The inclusion of the horizontal distribution of diabatic heating and transient fluxes greatly improves linear simulations of the observed stationary waves (Opsteegh and Vernekar, 1982).

While our understanding of the stationary response to large-scale orography is well founded, our knowledge of the temporal variability in the presence of orographic forcing is limited. Detailed observational studies have identified such features of the temporal variability as

1) the organization of synoptic-scale disturbances into storm tracks downstream of the major jet streams;
2) the tendency for the regions of maximum low-frequency variance to be less zonally elongated than their high-frequency counterparts;
3) the dominance of extratropical variance by low-frequency oscillations;
4) the tendency for low-frequency teleconnections to exhibit both geographically fixed and location independent patterns;
5) the tendency for transient eddy fluxes to appear to dissipate the stationary waves;
6) the recurrence of persistent anomalous ridging in the eastern North Atlantic and Pacific oceans which tend to block the eastward progression of synoptic-scale weather systems.

How these aspects of the temporal variability are related to the large-scale orography is unclear. Some of the above features appear to be determined solely by the structure of the mean wind and not by the mechanism responsible for the structure of the mean wind. Fredericksen (1979) and Pierrehumbert (1984) have shown that the fastest growing normal
modes on realistic representations of the observed mean wind exhibit storm track characteristics. Barotropic instability of the upper tropospheric mean wind was shown by Simmons et al. (1983) to be a source of low-frequency variability. The regions of maximum variance produced by the fastest growing barotropic modes closely resemble the observed horizontal distribution of low-frequency variance. Frederiksen (1983) extended the instability calculation to the three-dimensional Northern Hemisphere winter tropospheric flow. Unstable, low-frequency equivalent barotropic modes were found in the presence of time-mean baroclinicity. The spatial and temporal evolution of these modes exhibited remarkable resemblance to some of the observed low-frequency teleconnections patterns.

A limitation of the foregoing studies is the prescription of the mean wind. Interaction of developing unstable modes with the forcing of the mean wind was not allowed. In the present study we wish to relax the constraint of prescribing the mean wind and also include a full spectrum of barotropic and baroclinic waves. The mean wind will be allowed to be internally controlled by the interaction of the eddies, the orographic forcing and the mean wind. We will just concentrate on orographic forcing, realizing that diabatic sources are also of vital importance. By introducing orography into the nonlinear model developed in Hendon and Hartmann (1985; hereafter referred to as HH), we will be able to evaluate the direct effects of orography on the temporal variability, and transient organization and interaction with the stationary waves without the hindrance of complicating physics.

Previous nonlinear interactive simulations have employed severely truncated models. The barotropic model of Charney and DeVore (1979) and subsequent baroclinic extensions allowed multiple stationary solutions to large-scale orographic forcing. These multiple equilibria have been associated with blocking events in the observed atmosphere. The existence of multiple stationary solutions depend on the possibility of resonance in these channel models. Resonance in the observed-troposphere seems unlikely (Held, 1983), and thus the relevance of multiple equilibria is questionable. The existence of multiple equilibria is not expected in this spherical model, and we have seen no evidence of it.

Mountain—no mountain simulations with GCMs (Manabe and Terpstra, 1974; Hahn and Manabe, 1975) elucidated many features of the general circulation that are intimately related to orography. Evaluation of the effects on the variability was hindered by the short integration necessitated by the demand on the computing resources and by complicating internal feedbacks. A recent extended integration of the GFDL GCM with mountains but without prescribed interannual variation (Manabe and Hahn, 1981; Lau, 1981), demonstrated that much of the observed variability at low- and high-frequencies is internally generated. The direct effects of the orography were not examined, however.

We feel that the two-level model (HH) is a powerful tool for investigation of both high- and low-frequency variability and its interaction with the mean flow in the presence of orography. The model has sufficient horizontal resolution to simulate the gross features of baroclinic eddies yet its efficiency permits sensitivity experiments and long integrations relevant to low-frequency variability studies. It is hoped that the results of this study will help integrate previous more simplified numerical studies with the observational and GCM results.

2. Model details

The two-level model is described in detail in HH. The model is formulated in terms of the vertically averaged and vertically differenced vorticity and thermodynamic energy equations and a vertically differenced divergence equation. The upper ($\sigma = 0$ mb) and lower ($\sigma = 1000$ mb) boundary condition is $\omega = 0$. The dependent variables are expanded in a series of spherical harmonics truncated at rhomboidal 15. Diffusion of temperature, vorticity and divergence is included with a diffusion coefficient $K_d = 2.5 \times 10^5$ m$^2$ s$^{-1}$. Linear surface drag, $C_d = (5 \text{ days})^{-1}$, also is included at the lower level.

The incorporation of orographic forcing in a model with fixed surface pressure at first seems contradictory. We will employ the baroclinic extension of the conservation of barotropic potential vorticity as used by Charney and Straus (1980). Held (1983) presents a review of the usefulness of this barotropic representation.

Following Charney and Straus, a vorticity and divergence source due to flow over orography is included at the lowest model level. In our two-level representation this parameterization enters our vertically averaged vorticity $\bar{\zeta} = (\bar{\zeta}_{250 \text{ mb}} + \bar{\zeta}_{750 \text{ mb}})/2$ and vertically differenced vorticity and divergence $\bar{D} = (D_{250 \text{ mb}} - D_{750 \text{ mb}})/2$ equations as additional forcing terms:

\[
\frac{\partial \bar{\zeta}}{\partial t} = -\frac{1}{2} \frac{f h}{H} (\bar{U} - \bar{\bar{U}}), \frac{f h}{H} (\bar{V} - \bar{\bar{V}}),
\]

\[
\frac{\partial \bar{\bar{U}}}{\partial t} = -\frac{1}{2} \frac{f h}{H} (\bar{U} - \bar{\bar{U}}), \frac{f h}{H} (\bar{V} - \bar{\bar{V}}),
\]

\[
\frac{\partial \bar{D}}{\partial t} = -\frac{1}{2} \frac{f h}{H} (\bar{V} - \bar{\bar{V}}), \frac{f h}{H} (\bar{U} - \bar{\bar{U}}).
\]

The definitions of $\bar{U}$, $\bar{V}$, $\bar{\bar{U}}$, $\bar{\bar{V}}$, and $L$ are as in HH:

\[
\bar{U} = \cos \phi (u_{250 \text{ mb}} + u_{750 \text{ mb}})/2,
\]

\[
\bar{\bar{U}} = \cos \phi (u_{250 \text{ mb}} - u_{750 \text{ mb}})/2,
\]

\[
L(A, B) = \frac{1}{a \cos \phi} \left( \frac{\partial A}{\partial \lambda} + \cos \phi \frac{\partial B}{\partial \phi} \right).
\]
The definitions of $\vec{V}$ and $\dot{V}$ are similar to those of $\vec{U}$ and $\dot{U}$. The horizontal distribution of the orography is given by $h$ and the depth of the layer it is affecting is $H$. We investigate magnitudes of $h/H$ ranging from 0.1 to 0.4. Charney and Straus chose $h/H = 0.2$.

The horizontal distribution of $h$ is Gaussian:

$$h(\lambda, \phi) = h \exp\left\{ -\left[ (\phi - 40^\circ)/20^\circ \right]^2 - \left[ (\lambda - 180^\circ)/30^\circ \right]^2 \right\},$$

where $\phi$ is latitude and $\lambda$ is longitude in degrees.

The thermal forcing in these experiments is identical to that in HH; the temperature is relaxed back to a December–February zonally symmetric radiative equilibrium temperature distribution with a relaxation time of 15 days. The integrations are started from rest; the thermal forcing is turned on over a period of 20 days. Model data are taken once per day after an initial 50-day spin-up.

3. Model simulations and analysis

a. Time-mean flow

We describe here the analysis of a 1000-day run with $h/H = 0.4$. The orography is centered at $40^\circ$N, $180^\circ$W. All features discussed below are evident in similar experiments with $h/H = 0.2$. We discuss the run for $h/H = 0.4$ because the relevant features of the response are more clearly identifiable than with smaller values of the orographic height.

The horizontal structure of the 1000-day average zonal wind, eddy vorticity, and eddy temperature at the upper model level are shown in Fig. 1. A well-defined jet stream $45^\circ$ downstream of the orography is seen with a maximum wind speed of 50 m s$^{-1}$. The eddy vorticity (Fig. 1b) exhibits the well-known stationary Rossby wave structure produced by orography. An anticyclone develops just to the west (upstream) of the orography with a much stronger cyclone seen just downstream. A distinct wave train propagates equatorward and eastward away from the orography. The barotropic nature of the response is confirmed by the presence of a cold perturbation (Fig. 1c) in the region of the upper-level cyclone. This horizontal and vertical structure is in close agreement with many previous studies. The use of vorticity to designate the stationary waves in Fig. 1b emphasizes the equatorward component of the wave train. Hoskins and Karoly (1981) present theoretical arguments for this observed structure and the reader is referred there for an excellent discussion.

b. Eddy heat and vorticity budget

To highlight the zonal asymmetries and nonlinearity of the response, the eddy heat and vorticity budgets are examined. The transient eddy fluxes of heat at the lower level are very similar to those at the upper level.

![Fig. 1. The 1000 day mean (a) zonal wind, (b) eddy relative vorticity, and (c) eddy temperature at the upper level. The contour interval in (a) is 5 m s$^{-1}$, in (b) $0.4 \times 10^{-5}$ s$^{-1}$, and in (c) 1 K.](image-url)
Due to the barotropic structure of the stationary waves, we will examine the heat budget at the upper level. All of the features discussed below apply to the lower-level heat budget also. The eddy potential temperature equation at the upper level is

\[
\frac{\partial \tilde{\theta}^*}{\partial t} = -\left[ \left( \tilde{\theta}^* \frac{\partial \tilde{u}^*}{\partial x} + \tilde{u}^* \frac{\partial \tilde{\theta}^*}{\partial y} \right) \right] - \tilde{D}^* \tilde{\theta}^* \\
- \left( \tilde{\nabla}^* \cdot \tilde{\nabla} \tilde{\theta}^* + \tilde{D}^* \tilde{\theta}^* \right) - \tilde{\nabla}^* \cdot \tilde{D}^* \tilde{\theta}^* - K_e \tilde{\theta}^* ,
\]

with \( \partial x = a \cos \phi \partial \lambda \) and \( \partial y = a \partial \phi \). The diffusion is negligible. Brackets denote zonal averages; asterisks denote deviations from the zonal average. Overbars imply time averages; primes are deviations from the time average. The overhat indicates a vertical difference field as defined above. The Newtonian cooling coefficient is \( K_e = (15 \text{ days})^{-1} \).

The transient terms are decomposed into contributions by high-pass and low-pass eddies. The low-pass filter (Blackmon and Lau, 1980) retains fluctuations with periods greater than 10 days. The high-pass filter (HH) retains fluctuations with periods less than 10 days. Most of the variance associated with synoptic-scale eddies is captured by the high-pass filter (HH).

Term a is the linear advection of the perturbation temperature. Term b is the linear vertical flux divergence in this two-level representation. Term c is the
steady nonlinear flux divergence and $d$ is the transient flux divergence. Term $e$ is the eddy Newtonian cooling.

The local heat budget averaged over the 1000-day run is shown in Fig. 2. Following Sardeshmukh and Hoskins (1985), all fields have been smoothed with a spectral filter of the form

$$S_n = \exp(-K\{n(n + 1)\}^2)$$

where $n$ is the total wavenumber on the sphere and $K = 6.4 \times 10^{-6}$. Only the zonal advection is significantly affected; the other fields are not sensitive to the filter.

The upslope cooling and downslope warming (Fig. 2b), due to forced ascent and descent, is balanced to the first order by the steady linear advection (Fig. 2a). The linear advection is dominated by the meridional component which advects cold air southward on the east side of the mountain. This balance between the steady linear divergence and advection is essentially that produced in steady-state linear models. The linear advection is seen to be about two times larger than the linear divergence term, however. Nonlinearity and transients account for this large difference.

The steady nonlinear flux divergence, Fig. 2c, exhibits a dipole structure just downstream of the orography. This structure can be thought of as the nonlinear correction to the linear zonal advection. To the north of the mountain the local zonal wind is larger than the zonal average, while to the south of the mountain the local zonal wind is less than the zonal average. The steady nonlinearity thus adds to the linear zonal advection to the north and offsets it to the south.

The transient term $d$ is decomposed into contributions by low-frequency and high-frequency eddies. Some similarity between the fluxes due to the two frequency bands are observed (Fig. 2d, e). The transients tend to warm the resultant stationary cold cyclone to the east of the orography. The dissipative time scale of this process is about 10 days for the low-frequency eddies and 5 days for the high-frequency eddies. The dissipative nature of the transients has been found in observational studies (Lau, 1979; Lau and Holopainen, 1984) with similar time scales. Compared to the horizontal structure of the Newtonian cooling (Fig. 2f), whose decay time is 15 days, the transient eddy heat has a smaller scale. The horizontal structure is much more similar to a diffusive dissipation, rather than Newtonian cooling, particularly for the low-frequency transients.

Some caution is needed in interpreting the tendency of the transient fluxes as a dissipative one. The problem arises as to whether the transient fluxes can be interpreted as a local forcing of the stationary waves independent of the response, or as a forcing dependent on the response. If the latter is true, i.e., if the transient fluxes depend on the magnitude of the temperature perturbation or its gradient (as seems to be true in these results), then the transient fluxes appear very similar to the Newtonian cooling or diffusion. They simply act to dissipate the stationary temperature field. If, on the other hand, the former is true, then the temperature response to the tendency produced by the transient fluxes needs to be calculated in an interactive manner. Opsteegh and Vernekar (1982) calculated the linear steady response to the observed transient forcing. The transient fluxes produced cyclones and anticyclones which were one-quarter wavelength downstream of both the transient forcing and the observed cyclones and anticyclones. That is, the transient fluxes tended to affect the phase of the stationary waves but not the amplitude. This phase shift is due to the dominance of linear advection in balancing the applied heat (or vorticity) source. The phase shift found by Opsteegh and Vernekar is entirely consistent with the steady linear response to diabatic heating discussed by Hoskins and Karoly (1981). It is important to emphasize here the similarity between the tendency produced by the transient fluxes in this model and in the observed atmosphere. Their interpretation is difficult.

The high-frequency eddy flux shows much more of a dipole structure downstream of the orography than does the low frequency flux. This feature is also observed in the observational studies (Lau, 1979; Lau and Holopainen, 1984). These previous studies have associated this dipole with the meridional convergence of heat along the storm track by synoptic-scale eddies. Warming occurs to the north of the storm track and cooling to the south. This signal is much less clearly seen in the low frequency transient flux.

In summary, downslope warming and upslope cooling (which can be thought of as the applied temperature forcing) are more than offset by linear meridional advection. The cold cyclone to the east of the orography is the result of the linear meridional advection. The balance between the linear advection and adiabatic heating found by Hoskins and Karoly (1981) is not adequate for this nonlinear simulation. Significant warming downstream of the mountain is produced by the nonlinear fluxes which partially offset the trend produced by the linear advection. It is interesting to note that the horizontal structure of the temperature field in steady linear simulations without transient forcing (Hendon and Hartmann, 1982) is very similar to this nonlinear simulation. Apparently the magnitude rather than the horizontal distribution of the temperature perturbation is affected by the transients and nonlinearity.

We now turn our attention to the eddy vorticity budget. At the upper level the eddy vorticity equation is:

$$\frac{\partial \tilde{\zeta}^e}{\partial t} = -\left(\frac{\partial \tilde{\zeta}^e}{\partial x} + \frac{\partial}{\partial y} [f + \tilde{\zeta}]\right) - \tilde{D}^e\{\tilde{\zeta} + f\}
$$

$$- \nabla \cdot (\nabla \times \tilde{\zeta}^e) - \nabla \cdot (\nabla \times \tilde{\zeta}^e).$$

Diffusion and vertical advection and twisting are negligible. The notation is as in (1). Term $a$ is a linear
advection of the vorticity. Term $b$ is the linear generation of vorticity by divergence which can be thought of as the applied forcing due to the flow over the orography. Term $c$ is the steady nonlinear flux divergence and term $d$ is the transient flux divergence.

The local budget of vorticity is displayed in Fig. 3. Again all fields have been filtered. The generation of vorticity by stretching on the downslope and destruction of vorticity by compression on the upslope (Fig. 3b) is balanced to the first order by the linear advection (Fig. 3a). The resulting vorticity perturbation (Fig. 1b) appears to be determined by the tendency of the linear divergence forcing to produce a cyclone downstream of the orography. Due to the large zonal variations of the time-mean zonal wind in the vicinity of the jet, the steady nonlinear zonal advection adds to its linear counterpart to the north of the mountain and offsets it to the south (Fig. 3c).

The tendency produced by the low-frequency transient flux divergence (Fig. 3d) is 180° out of phase with the vorticity of the stationary wave. The time scale of this dissipative effect is about four or five days. The high-frequency transients (Fig. 3e) exhibit similar structure but the tendency is weaker with a decay time of about 11 days. Thus, the mean state can be considered a source of low-frequency vorticity fluctuations. These tendencies, with the low-frequency flux being larger, act to dissipate the stationary wave in a similar manner as is observed (Youngblut and Sasamori, 1980; Holopainen et al., 1982; Lau and Holopainen, 1984).
As with the transient heat fluxes, caution needs to be exercised in strictly interpreting these fluxes as dissipative. They equally well could be thought of as tending to change the phase of the stationary wave due to the linear advection which develops in response to them.

In summary, the linear vorticity balance is seen to be inadequate with an excess positive tendency being produced by the linear advection relative to the divergence forcing. This excess is accounted for by the non-linear fluxes. The horizontal structure (and hence dissipative interpretation) of the transient fluxes agrees well with observations. As in the observed atmosphere, the transient fluxes of heat are dominated by the baroclinic eddies, while the low-pass eddies make a relatively larger contribution to the flux of vorticity (and momentum).

c. Horizontal distribution of variance and covariance

Some insight into the importance of the transients to the stationary wave structure was gained in subsection 3b. Now we would like to concentrate on their horizontal distribution.

Figure 4 displays the streamfunction variance $\psi^2$, for both the high- and low-frequency eddies. Maxima in the variance are seen about 90° downstream of the orography. The low-pass variance maximum is over twice as large as that of the high-pass eddies. This feature is seen in the observed atmosphere roughly in the same position to the large-scale orography (Blackmon et al., 1977). A distinct minimum in the low-frequency variance is seen in the vicinity of the orography. The high-frequency maximum is more elongated in the zonal direction than is the low-frequency maximum. The high-frequency maximum will be associated with the synoptic-scale storm track. All of the above features are clearly seen in the observed troposphere, particularly in the Northern Hemisphere winter (Blackmon et al., 1977).

Compared to the zonally symmetric version of this model (HH), the global mean variance is not affected by the presence of orographic forcing. The wavenumber spectrum and the relative distribution between high and low frequencies of the globally integrated variance, as well as the latitudinal distribution of the zonally averaged variance, are quite similar to that of the zonally symmetric model. The longitudinal distribution, however, is strongly affected as seen in Fig. 4. The existence of time-mean asymmetries in midlatitudes is not necessary for the production of variance (particularly low-frequency variance). The time-mean asymmetries are responsible for creating strong zonal asymmetries in the variance, however.

The horizontal distribution of the meridional heat flux, Fig. 5, reveals that the major contribution is due to the high-frequency eddies in the storm track. The high-frequency maximum is roughly twice as large as that of the low frequencies. As in the zonally symmetric version of this model (HH), the low-frequency eddies clearly dominate the heat flux at high latitudes. The dominance of the meridional heat flux in the storm track by the high-frequency transients agrees well with the assertion that these transients are developing baroclinic waves. The low-frequency eddies are much more barotropic.

The distribution of $\omega \theta'$, the baroclinic conversion term, appears very similar to Fig. 5. This confirms that much of the baroclinic processes are associated with the high-frequency eddies in the storm track east of the orography.

The poleward momentum flux, which is associated with barotropic decay of the transients and equatorward propagation of waves, is shown in Fig. 6. The contributions by both frequency bands are roughly equal. Both high- and low-frequency eddies exhibit distinct maxima on the upstream side of the orography. This aspect of the observed meridional momentum flux is seen especially in the vicinity of the Rocky mountains.
Fig. 5. The 1000 day mean transient meridional heat flux at the upper level for (a) the low-pass filtered data and (b) the high-pass filtered data. The contour interval is 3 m K s\(^{-1}\).

(Blackmon et al., 1977). The smaller fluxes at the end of the storm track (Fig. 6b) appear much less significant in these results. Both frequency bands exhibit equatorward fluxes of momentum downstream of the orography which tail off into the tropics. This feature will be explored further in subsection 3d. It is interesting to note that the large positive and negative momentum fluxes by the low-frequency transients downstream of the orography are associated with large variances (Fig. 4a) but relatively small baroclinic effects (Fig. 5a).

d. **Horizontal eddy propagation**

The strong zonal variations of the momentum flux (Fig. 6) imply zonal variations in the meridional propagation of energy due to the transient eddies. A useful diagnostic for summarizing the horizontal energy propagation is the so called \( E_h \) (Hoskins et al., 1983). We will make use of the slightly different form defined in HH, \( E_h = (v'^2 - u'^2)/(2, -u'v^2) \). Our definition differs from Hoskins et al., by the factor of 2 in the \( x \)-component. In HH, \( E_h \) was shown to be parallel to the intrinsic group velocity of barotropic Rossby waves on a zonal mean basic state. The divergence of \( E_h \) can be interpreted as an eastward acceleration of the zonal mean wind by barotropic processes. As shown in HH and Hendon (1986), the zonal variation of the \( y \)-component gives, perhaps, the most useful information about the propagation of the eddies. Rapid decrease of the \( y \)-component was seen in the vicinity of the critical line for the bulk of the transient eddies. Information about the meridional (zonal) elongation of the transients is given by the positive (negative) sign of the \( x \)-component. Shown in HH and Hoskins et al. (1983), the synoptic eddies are predominantly meridionally elongated, while the low-frequency eddies are zonally elongated.

The \( E_h \)-vectors for the low- and high-frequency transients are shown in Fig. 7, superposed on contours

Fig. 6. The 1000 day mean transient meridional momentum flux at the upper level for (a) the low-pass filtered data and (b) the high-pass filtered data. The contour interval is 10 m s\(^{-2}\).
of the mean zonal wind. The low-frequency transients exhibit more zonal asymmetry than do the high-frequency transients. Enhanced equatorward energy propagation is seen in the vicinity of the orography, upstream of the jet, apparently in response to the equatorward displacement of the critical line. The characteristics of the life cycle of baroclinic waves in the storm track (Simmons and Hoskins, 1978) are seen in Fig. 7b. In the jet region the high-frequency eddies propagate mostly eastward relative to the mean wind and turn equatorward farther downstream as they decay.

In contrast to the high-frequency eddies, the low-frequency eddies have very small $E_{u'}$-vectors in mid-latitudes except near the jet (Fig. 7a). Equatorward energy propagation south of the zonally averaged zonal wind occurs with increased equatorward propagation just upstream of the orography. Strong divergence in the jet exit region and strong convergence where the jet is the strongest is seen. This feature was observed by Hoskins et al. (1983) and was a characteristic of the fastest growing barotropically unstable modes discussed by Simmons et al. (1983). They interpreted the strong divergence as an indication of the source region of the low-frequency eddies. These eddies exert a westerly acceleration at the jet exit and an easterly acceleration where the jet is the strongest. Just east of the jet exit another region of enhanced equatorward propagation exists. Compared to the low-frequency variance (Fig. 4a), the region of strong intrinsic westward energy propagation (and reduced equatorward energy propagation) and zonal elongation coincides with the region of maximum variance. Hoskins et al. (1983) also observed this feature.

The strong asymmetry of the energy propagation of the low-frequency transients was further investigated by calculating one-point correlation maps using the streamfunction. Except in the region 150°W to 90°W (the jet stream region) the one-point correlations exhibit very similar structure to those shown in HH for thezonally symmetric atmosphere. Figure 8 displays a representative map for this region outside of the jet stream. Equatorward energy propagation due to a stationary Rossby wave is seen. Absorption at the critical line ($\mu = 0$) occurs. For base points north of 50°N, poleward propagation through a high latitude turning point (very similar to HH) is observed (not shown).

The correlation patterns for regions outside of the jet stream shift along with the base point as it is moved around a latitude circle. The patterns are also better defined three days prior and at zero lag than they are three days after.

In distinct contrast, the correlation maps in the region of the jet stream (and maximum variance) exhibit a more geographically fixed pattern. All base points in the region 150° to 90°W exhibit vestiges of the same pattern with the high and low centers fixed in space. At 130°W 16°N the pattern is best defined (Fig. 9). At lag-three days a dipole south of the jet is seen. At lag zero, downstream centers exhibit greater correlations and at lag + three, further downstream amplification occurs.

This pattern (Fig. 9), which appears to emanate from low latitudes, is distinctly different from any seen in the zonally symmetric basic state model (HH) or in the presence of tropical diabatic heating (Hendon, 1986). The pattern downstream is better defined as time progresses, suggesting a source of the eddies in the low-latitude region downstream of the orography. Note, however, that there is no latent heat release or low latitude forcing in this model. The appearance of poleward wave propagation results from large-scale, largely barotropic processes. Note that the low frequency $E_{u'}$-vectors (Fig. 8a) also exhibit greatly reduced equatorward energy propagation (i.e., the $y$-component is very small) in this region.

The horizontal structure and temporal evolution of this pattern (Fig. 9) appear similar to the PNA pattern in the observed atmosphere (Wallace and Gutzler, 1981; Blackmon et al., 1984). The instability calcula-
tions of Simmons et al., (1983) and Frederiksen (1983) also find unstable modes which possess similar features. Those results along with the existence of a PNA type teleconnection in the fixed sea surface temperature experiment by Lau (1981) suggest that the PNA-type pattern develops due to the structure of the time-mean flow and not due to low latitude forcing or midlatitude orography directly.

The time scale of the low-frequency eddies, which appear to emanate from low latitudes in the vicinity of the jet, seems to be longer than that for the equatorward propagating eddies which appear to emanate from midlatitudes. One-point correlation maps for the same base grid points used above are shown in Fig. 10 using 30-day average data. The equatorward propagating wavetrain (Fig. 10a) is less clearly defined for 30 day averaged data than for the low-pass filtered data (Fig. 8b). In contrast, the dipole structure in the jet region (Fig. 10b) exhibits greater correlations, as do the downstream centers. Only 33 30-day averages were used in this calculation and thus the statistical significance of these correlations is not as great as for the low-pass filtered correlation maps. However, similar tendencies for the jet exit region to exhibit greater correlations with 30-day averaged data were observed in all other experiments with varying orographic height.

**Fig. 8.** The one-point correlation map for low-pass filtered streamfunction with the base grid at 170°E 15°N. The lag -3 days is shown in (a), lag 0 days in (b) and lag +3 days in (c). The contour interval is 0.1 with the zero contour omitted.
We thus feel that this pattern is reproducible and not just a sampling phenomenon.

The similarity with the unstable modes found by Simmons et al. (1983) suggests that barotropic instability may be an important mechanism here. As a measure of the barotropic conversion of mean zonal kinetic energy to transient kinetic energy, they calculated the local contribution to the global conversion:

$$\frac{\partial KE'}{\partial t} = (v'^2 - u'^2) \frac{\partial \bar{u}}{\partial x} - u'v' \frac{\partial \bar{u}}{\partial y}.$$  

In the region of the maximum low-frequency variance (just downstream of the jet) the unstable modes exhibited strong positive conversions due to a strong correlation of \((v'^2 - u'^2)\) and \(-\partial \bar{u}/\partial x\) (zonally elongated eddies in a region of negative gradient of mean zonal wind). A similar computation for these model results is displayed in Fig. 11.

Just to the north of the jet the low frequency transients exhibit a positive conversion due to the correlation of \(-u'v'\) and \(-\partial \bar{u}/\partial y\) (equatorward propagation of the transients in a region of negative meridional gradient of the zonal wind). The positive conversion just downstream of the jet core is due to the correlation of \(\partial \bar{u}/\partial x\) and \(u'^2 - v'^2\) (zonal elongation of the transients). Both of these positive conversions are clearly
integral. It does seem significant, though, that the horizontal structure of the low-frequency transients and the distribution of barotropic conversions in the region of the jet core are similar to the barotropic unstable modes found by Simmons et al.

The southernmost positive conversion in Fig. 11a appears to be the one relevant to the structure of the low-frequency transients. The conversion on the north side of the jet is typical of all midlatitude transients regardless of whether a zonal gradient in the jet exists. The local barotropic conversion by the high-frequency transients is displayed in Fig. 11b. Note that the jet exit region exhibits mostly negative energy conversion due to meridionally elongated eddies existing in a region of negative gradient of the zonal wind. The northern center, due to the correlation of equatorward propagating high-pass eddies and positive meridional gradient of the zonal wind, is relatively larger in this case.

Fig. 10. (a) As in Fig. 7b and (b) as in Fig. 8b using 30-day average streamfunction.

Fig. 11. The local contribution to the global barotropic converes of mean kinetic energy to transient kinetic energy at the upper le The low-pass filtered transients are shown in (a) and high-pass filters transients in (b). The contour interval is $0.2 \times 10^{-3}$ m$^2$ s$^{-3}$. 
At all longitudes for both high- and low-frequency transients a positive conversion on the north side of the zonal mean jet is seen due to the correlation of negative meridional gradient of the jet and $u'v'$. Local enhancement of this conversion is seen in Fig. 11 to the north of the jet core. The strong conversion in the jet exit by the low-frequency transients appears to be unique to the low-frequency transients and to the region of strong negative zonal gradient of the zonal wind. Everywhere else at this latitude, except in the jet exit, the conversion by all transients is negative due to the correlation of the meridional gradient of the zonal wind and $-u'v'$.

*e. Tropical effects*

The midlatitude orography appears to influence both the time-mean flow and variability of the tropical circulation. A weak easterly jet (Fig. 1a) develops directly south of the midlatitude jet. A region of enhanced high-frequency variance (Fig. 4b) just to the east of the easterly jet is seen, while only a slight hint of enhanced low-frequency variance (Fig. 4a) extends into the tropics.

A closer inspection of the tropical $E_k$-vectors, mean wind and transient kinetic energy (Figs. 12 and 13) reveal many interesting features. Though the energy due to the low-pass eddies appear to propagate strongly equatorward at 90°W (Fig. 12a), the corresponding kinetic energy maximum (Fig. 12b) extends only to about 10°N. This enhanced penetration appears to be due to the weak equatorward depression of the low-frequency transient's critical line ($u = 0$). The high-pass eddies (Fig. 13a) appear to dissipate rapidly in the vicinity of $u = 15$ m s$^{-1}$ except near 90°W. Here a maximum occurs just to the east of the easterly jet. The appearance of enhanced variance in a region of easterly winds is completely opposite both that observed (Webster and Holton, 1982) and that found in this model with strong zonal asymmetries in the tropical winds (Hendon, 1986). In those studies maximum kinetic energy coincided with minimum easterly winds. The convergence of low-frequency transients into the westerly regions was shown to be the major contributor.

In the present case the high frequency transients are solely responsible for the asymmetry of the tropical variance. They appear to emanate in the region where the tropical easterlies bulge northward. The divergence of $E_k$ for the high-pass eddies (Fig. 14) shows strong divergence in this region. This divergence is unusual in the sense that convergence due to midlatitude eddies decaying at their low latitude critical line is observed everywhere else. This region of divergence suggests that

![Fig. 12.](image)

Fig. 12. (a) The low-pass filtered tropical $E_k$-vectors at the upper level superposed on contours of the mean zonal wind, (b) the low-pass filtered transient kinetic energy. The contour in (a) is 4 m s$^{-1}$ and in (b) is 7.5 (m s$^{-1}$)$^2$. The maximum vector in (a) has magnitude 55 (m s$^{-1}$)$^2$. 

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a local source of high-frequency eddies exists. Examination of the power spectrum of the wind components and one-point teleconnection maps indicate that this region is dominated by increased Rossby-gravity wave activity (the structure is similar to that shown in HH). The nonlinear interaction of an incident midlatitude wave and the tropical basic state, as found by Wilson and Mak (1984), could be responsible for the increased variance due to the mixed Rossby-gravity waves. Further study of this phenomenon seems to be warranted.

If one interprets the idealized orography as a representation of the Himalayas, then this high frequency maximum would appear over Indonesia (the winter monsoon region). This has been confirmed by integrating this model with realistic orography (not shown here). Because of their greater height and closer proximity to the equator, the Himalayas produce greater tropical asymmetry than do the Rockies. The ducting of the low-pass eddies and the enhanced high-pass variance just east of the orographically induced easterly

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**Fig. 13.** As in Fig. 12 but for the high-pass filtered transients.

**Fig. 14.** The tropical divergence of $E_q$ at the upper level for the high-pass filtered transients. The contour interval is $0.5 \times 10^{-5}$ m s$^{-2}$.
jet are even more clearly seen over Indonesia than in the idealized case. Observations of this phenomenon are hindered by the presence of strong zonal asymmetries of the basic state due to tropical diabatic heating. The observed variance due to all transients with periods less than a month exhibit maximum variance in the region of time-mean westerlies (west of 180°W, over Indonesia; Webster and Holton, 1982). The maximum variance in regions of westerlies is mostly due to enhanced equatorward propagation of the low-pass eddies (HH). Whether the observed high-frequency variance exhibits a maximum due south of the mid-latitude jets, independent of the low frequency transients, has yet to be determined.

4. Conclusions

As a step toward increasing our knowledge of the importance of orography for the general circulation, the time-mean flow and variability produced by mid-latitude orography in a nonlinear model has been examined. The nonlinear model has sufficient resolution to simulate the nonlinear life cycle of a spectrum of baroclinic and barotropic eddies. The simple physical processes and coarse vertical resolution make it feasible to integrate this model for extended simulations. The model bridges the gap between the steady state and prescribed mean flow models and the sophisticated GCMs.

Many similarities with the observed horizontal structure of the time-mean circulation, transient fluxes and temporal variability were produced by the nonlinear flow over idealized orography. A time-mean jet stream formed 45° downstream of the orography in a position similar to the jet downstream of the Rockies or Himalayas. The time-mean vorticity and heat budgets revealed that steady and transient nonlinear fluxes made major contributions. As in the observed atmosphere, the transient flux of heat by the high-frequency eddies was larger than that by the low frequency eddies. This is because the high-frequency eddies develop baroclinically, while the low-frequency eddies are much more barotropic. The flux of vorticity by the low-frequency eddies was slightly larger than its high-frequency counterpart. This feature is also observed (Holopainen et al. 1982; Lau and Holopainen, 1984) and emphasizes the importance of the barotropic low-frequency eddies to the general circulation. Both the fluxes of heat and vorticity exhibited the tendency to dissipate the stationary wave structure, acting like diffusion with a time scale of around five days. Many previous studies have observed similar tendencies (Lau, 1979; Youngblut and Sasamori, 1980; Holopainen et al., 1982; Lau and Holopainen, 1984). Caution is needed in strictly interpreting this tendency as dissipative. The similarities of these model results with the observed atmosphere suggests that the model is producing transient fluxes which are realistic, regardless of their interpretation.

Distinct maxima of both low- and high-frequency variance developed downstream of the jet. The high-frequency maximum exhibited the characteristics of the observed synoptic-scale storm track: zonally elongated region of the enhanced variance, large transient heat fluxes along the entire storm track, meridionally elongated transients, poleward transient momentum flux and equatorward energy propagation at the end of the storm track (Blackmon et al., 1977). The low frequency variance exhibited a very well-defined minimum in the vicinity of the orography and a maximum whose center coincided with the high-frequency maximum. The magnitude of the low-frequency maximum was twice as large as the high. The eddies in the region of maximum low-pass variance were more meridionally contracted and zonally extended than their high-pass counterparts. These features are also observed (Blackmon et al., 1977). The lack of baroclinic energy conversions despite the large variance suggests that the low frequency eddies, in the region of maximum variance, develop primarily barotropically.

It is somewhat surprising that the overall level of globally integrated transient activity was very similar to the zonally symmetric version of this model (HH). The latitudinal distribution of both the zonally averaged high and low frequency variance was essentially unaffected by the inclusion of mid-latitude orography. Only the longitudinal distribution of the variance was strongly affected by the time-mean asymmetries in the basic state. Regions of enhanced variance coexist with regions of diminished variance.

The horizontal structure of the transients is also affected by the time-mean asymmetries. While the structure of the synoptic-scale transients in the storm track was similar to that in the zonally symmetric model (HH), the structure of the low frequency transients in the vicinity of the jet stream was totally unlike the transients outside of this region or those seen in the zonally symmetric model (HH).

Only in the region of the jet exit did the low-frequency transients appear to emanate from very low latitudes and exhibit poleward energy propagation. This structure was very reminiscent of the PNA pattern discussed by Wallace and Gutzler (1981) and Blackmon et al. (1984). The temporal evolution and barotropic conversion of mean energy into transient energy was similar to the barotropically unstable modes found by Simmons et al. (1983). The strength of the correlations of the downstream wave train were definitely weaker than the observed correlations in the PNA pattern. For smaller values of the orographic height, less zonal asymmetry is produced and even smaller correlations in this pattern are seen. The gradient in the zonal wind may not be as strong as observed and hence results in weaker barotropic conversions.

That the low-frequency variability exhibits many of the observed features and particularly those of the barotropically unstable modes (Simmons et al., 1983) suggests that indeed barotropic instability of the time-
mean flow may be important. It also suggests that the
basic mechanism for the production of this PNA-type
pattern is included in this simple two-level model. La-
tent heat release, which is excluded in this model, is
not necessary for the generation of low-frequency fluc-
tuations that appear to emanate from low latitudes.
The mechanism for the production of these fluctuations
with periods longer than those typical of stationary
Rossby wave dispersion is also included in this model.
Despite the presence of time-mean baroclinicity and
rapid growing baroclinic eddies, the barotropic low-
frequency oscillation in the vicinity of the jet exit was
clearly seen.

The ability of this simple two-level model to simulate
many of the features of both synoptic-scale and low-
frequency transients is encouraging. Much of the vari-
ability appears to be a basic dynamical consequence
of the structure of the mean flow and not directly
related to the orography itself. By incorporation of re-
alistic orography and the horizontal distribution of the
observed diabatic heating, a reasonable simulation of the
time-mean circulation may be achieved. If this is
possible, then climatic studies could be performed with
this efficient model without having to parameterize the
effects of the transients (particularly the low-frequency
transients).

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