Effects of Convection Cell Geometry on Simulated Tornadoes

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ABSTRACT

Experimental data and theory suggest that certain convection cell scale factors must be specified for correct interpretation of tornado simulations. The most important nondimensional number appears to be the ratio of convection cell depth (relating to height of vortex terminal region) to radius of the updraft. This scale factor has been ignored by most investigators, but it is especially important when comparing experimental results between two simulators or when relating simulator results to real tornadoes. Implications are discussed for possible cell geometry effects on tornadoes.

1. Introduction

The tornado simulator at Purdue University (PU) is closely patterned after the original Ward simulator at the University of Oklahoma (OU), except that it has incorporated several improvements, including a size that is about 25 percent larger. We assumed that none of the departures from Ward's design would have an important influence on vortex dynamics. However, when we joined our colleagues at Purdue in some coordinated experiments to compare simulator characteristics we found that this was not the case. Differences were found in the relationships of vortex central pressure drop and vortex radius to swirl ratio, and in the critical swirl ratios for multiple-vortex transitions.

The comparison experiments were conducted at identical values of the important parameters of flow and aspect ratio. The parameter found to be most descriptive is the swirl ratio, $S \equiv (\tan\theta)/2a$, where θ is the inflow angle and a is the aspect ratio, the ratio of the inflow layer height (h) to the radius (r_0) of the updraft "hole" at the top of the inflow layer.

The important dimensions of the simulator are shown in Fig. 1. The other flow parameter controlled for the experiments is the radial Reynolds number, Re_r, given by $Re_r = Q/\nu h$, where $2\pi Q$ is the volume flow rate through the simulator and ν is the kinematic viscosity of air. Within the range of Re_r used in our experiments, $10^4 < Re_r < 10^5$, we do not observe a considerable Reynolds number effect (Leslie, 1979a). The comparison experiments convinced us that at least one other flow parameter must be important, even though none of the simulation experiments reported in the literature to date have taken it into account.

Davies-Jones (1973) pointed out that the flow is a function of r_0, r_s, R, h, Z (all defined in Fig. 1) $2\pi Q$ and $2\pi M$, the latter defined as the criculation at the screen, or periphery of the inflow at radius r_s. He also used the Buckingham pi-theorem to determine that no more than six independent nondimensional numbers exist. The R and Z are the radius and height of the convection cell, and these, along with r_s, are fixed dimensions for any particular simulator. The other two dimensions, r_0 and h, are subject to variation, but are often fixed for a given set of experiments. Davies-Jones selected the following set of nondimensional numbers: r_0/r_s, R/r_s, Z/r_s, h/r_0 ($\equiv a$), hM/Q ($\equiv \tan\theta$) and $Q/\nu h$. A seventh (dependent) nondimensional number is the swirl ratio, S, which is the dominant parameter in vortex breakdown phenomena, multiple-vortex transitions, vortex core radius, r_c, and the relationship between central pressure drop and maximum tangential velocity (Davies-Jones, 1973; Leslie, 1979b).

Because of the dominant influence of swirl ratio on vortex dynamics and partly because most of the simulator dimensions are either fixed or seldom varied, the possible roles of cell height Z and cell radius R were not examined until after the comparison experiments.

2. Vortex core radius theories

The experimentally observed increase in vortex core radius with increasing swirl ratio has inspired several investigators (Ward, 1972; Davies-Jones, 1973; Jischke and Parang, 1974; Leslie, 1979b; Baker and Church, 1979; Gall, 1982; Walko and Gall, 1986) to theorize about the dynamics of this relationship. All of the theories make a reasonable fit to the available experimental data. Interestingly, Rotunno's (1977, 1979) numerical

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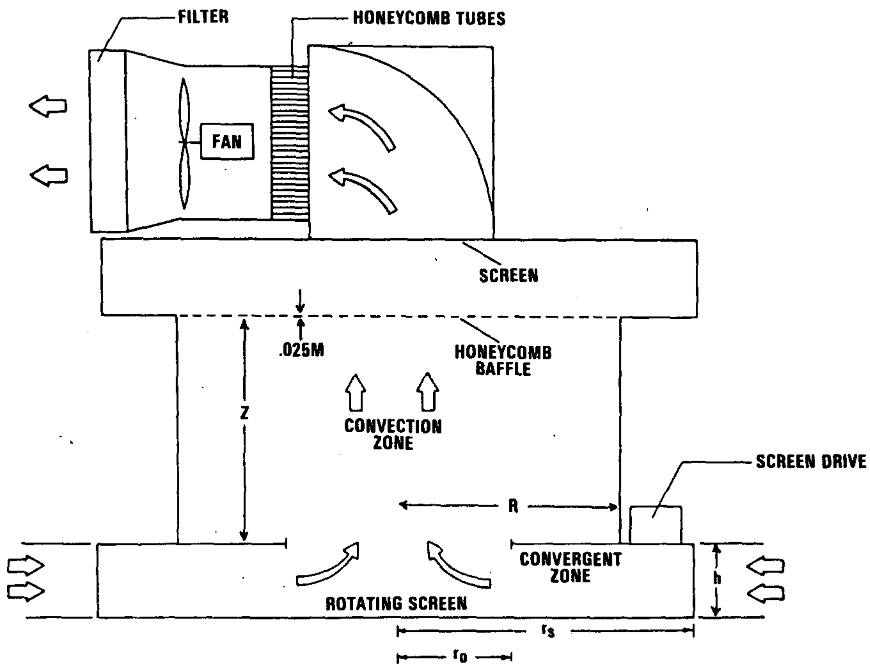

FIG. 1. Schematic of the Ward tornado simulator.

model of the tornado simulator also describes this phenomenon rather well. Only three of these theories are concerned with the dimensions of the convection cell, Davies-Jones (1973), Baker and Church (1979) and Gall (1982).

Davies-Jones (1973) used a potential flow model with a stagnant core, typical of two-cell vortex models. He derived the equation for the central pressure deficit and then solved for the value of the core radius which minimizes the pressure drop. The results are

$$r_1 = \left[\frac{1}{r_2^2} + \frac{r_0^2}{S^2(R^2 - r_2^2)^2}\right]^{-1/2} \tag{1}$$

where r_1 is the core radius at the surface and r_2 is the radius at the top of the vortex (terminated in the simulator by a baffle which spoils the swirl). The appropriate value of r_2 satisfies the equation

$$\frac{S^2}{2r_0^2} = \frac{r_2^4}{(R^2 - r_2^2)^3}. \tag{2}$$

The nondimensional number indicated by this theory is R/r_0, which for one set of comparative data was 55 percent larger in the Purdue simulator than in the OU simulator. In another dataset it was 24 percent larger. As noted by Davies-Jones, the theory does not fit the available data very well and, hence, does not explain the observed differences between the curves of core radius versus swirl ratio in the two simulators. These curves invariably show the rate of increase of core radius with swirl ratio to be greater in the OU simulator.

The Z/R value is the same for both simulators, and so cannot account for the observed differences in vortex dynamics. Cell radius R might be an important simulator dimension if the torque delivered to the cell wall by the rotating flow is an appreciable fraction of the torque introduced to the inflow by the simulator's rotating screen. We performed an analysis to examine this possibility and found this torque ratio to be less than 0.001 for either simulator.

The vortex simulator model derived by Gall (1982) is a version of the two-cell vortex configuration very similar to that of Davies-Jones (1973) with respect to the distribution of tangential velocity [his Eq. (25)]. He couples that equation to his model for pressure drop, divergence and volume flow rate through various proportionalities, and finally to the reciprocal square of the swirl ratio through his proportionality constant, b. His Eq. (29) for nondimensional core radius is

$$\frac{r_c}{r_0} = bS(b^2S^2 + 1)^{-1/2} \tag{3}$$

which can be made to fit any of the existing datasets by adjusting the constant b.

From Gall's reasoning in the derivation of Eq. (3) we may infer that b is related to simulator geometry as

$$b \propto (Z/h)^{-1/2}. \tag{4}$$

Gall (1982) showed that $b = 0.42$ and 0.36 fits Eq. (3) very nicely to OU data (his Fig. 8) and Purdue data (his Fig. 9), respectively. Since $Z/h = 3.0$ at OU and 4.36 at PU, the adjustment to b is in the correct sense.

However, the two datasets were obtained in greatly different ranges of swirl ratio, and this clouds the issue. Also, h does not appear to be the best dimension for nondimensionalizing cell height Z, since Davies-Jones (1973) pointed out from the Ward (1972) data that vortex core radius is not especially sensitive to changes in h.

Figure 2 has been constructed in order to provide a closer examination of data in the range of S applicable to the two-cell single vortex. The OU data are from Diamond (1982) and Leslie (1979a) and the Purdue data are from Snow et al. (1980) and Pauley (1980). These datasets were taken at the same aspect ratio ($a = 0.5$) and approximately the same Reynolds number ($Re_r \approx 2 \times 10^4$). They also include the nondimensional central pressure drops, which will be used later for comparison with other core radius theories. The solid curves in Fig. 2 are Gall's equation for $b = 0.42$ (OU data) and $b = 0.36$ (PU data) as before. The dash curve for a b-value of 0.26 is seen to make a better fit to the Purdue data within this range of S. The misfits at small S are within the laminar vortex regime. Thus b appears to be as sensitive to the range of S as to Z/h. If we restricted the comparison to the two-cell single-vortex regime, then no comparison would be possible, since the S values do not overlap for the two simulators.

This poses a question as to the range of S over which the two-cell vortex model is valid. By definition, the lower limit would be the S-value at which the two-cell vortex is formed. The upper limit is not well defined, since the multiple vortex configuration is also two-celled. For the OU simulator the lower limit is $S = 0.12$ and for the PU simulator it is $S = 0.3$. It is not unusual to extend the analysis beyond the single vortex range, as was done by Baker and Church (1979) and by Gall (1982). Indeed, the fit may be as good as in the single vortex range, although in the multiple vortex mode the velocity maximum defining r_c must be extracted as an average of wildly fluctuating values, as multiple vortexes pass the velocity probe. We may be stretching the point when we consider the multiple vortex modes to be an extension of the turbulent two-cell single vortex, but the excellent fit to the data by the simplified two-cell vortex theories certainly supports this idea.

Baker and Church (1979) constructed a potential flow vortex model with a core analogous to turbulent flow in a duct. Their derived equation is a sixth-order polynomial in r_c/r_0, as is that of Davies-Jones (1973) when his variational analysis is carried to completion. Neglecting terms of order $(r_c/r_0)^2$ or higher reduces their equation to

$$\frac{r_c}{r_0} = \frac{1.78S^2}{\lambda} \frac{r_0}{Z} \tag{5}$$

where λ is a nondimensional coefficient of flow resistance. Baker and Church also find convection cell height Z to be a critical geometric parameter, except that it is here nondimensionalized with respect to updraft radius r_0 rather than inflow layer depth h. (Note that in Gall's simulator model r_0 and R are the same, in contrast with the tornado simulators where $R > r_0$.) The variation is in the correct sense (smaller Z gives larger r_c/r_0 for a given swirl ratio). The magnitude restriction on r_c/r_0 confines Eq. (5) to the laminar vortex

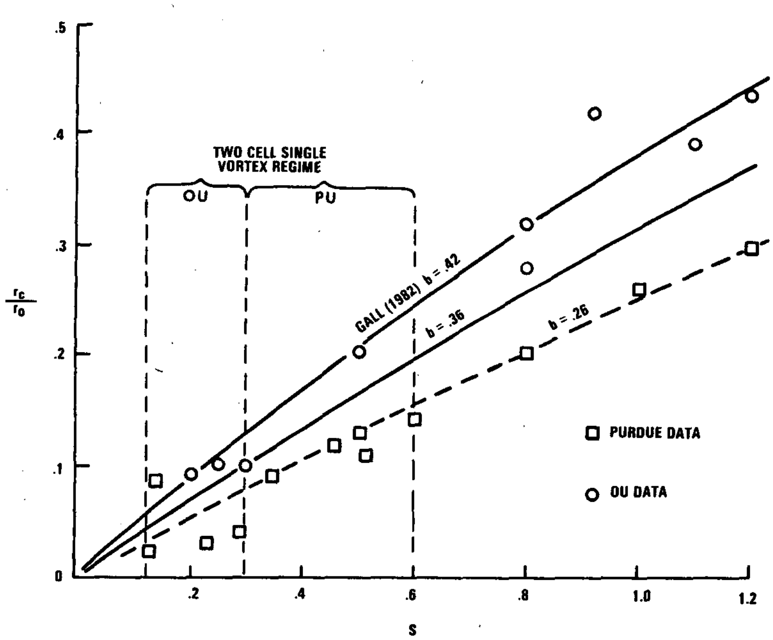

FIG. 2. Core radius vs swirl ratio for comparable datasets obtained in the Purdue and OU tornado simulators.

regime unless errors greater than about 10 percent are to be accepted; however, the polynomial solution fits existing data very well.

It is of interest that the theories of Gall (1982) and of Baker and Church (1979) contain a similar dependence on nondimensional convective cell height, and as shown in Gall's Fig. 9, they both show good agreement with the Purdue experimental data.

The cell geometry parameter Z/r_0 is preferred to Z/h for several reasons. As mentioned earlier, vortex core radius is not especially sensitive to h. The maximum pressure drop that can be maintained at the base of the vortex is proportional to $(Z + h)/r_c$, and this number is largely controlled by Z/r_0. The parameter Z/r_0 is a measure of vortex shape; large Z/r_0 implies a tall, narrow vortex, which experimentally exhibits the largest pressure drop of all of the vortex modes. The parameter Z/r_0 was 1.5 in the OU simulator and 2.18 in the Purdue simulator, 45 percent greater. Just as h/r_0 is the inflow region aspect ratio, Z/r_0 is the "convection cell aspect ratio."

Two other formulas predict core radius as a function of the tangent of the inflow angle ($\equiv 2aS$) and nondimensional pressure deficit p^*. Leslie (1979b) assumed a solid rotation core with a potential outer flow (Rankine) type vortex and integrated the kinetic energy versus pressure drop in four separate regions of the vortex between $r = 0$ and $r = r_s$. The resulting equation is

$$\frac{r_c}{r_0} = \frac{4aS\sqrt{2}}{[(2aS)^2 + p^* + 1]^{1/2}} \tag{6}$$

where $p^* = \Delta p/(\frac{1}{2}\rho u_s^2)$, ρ is air density and u_s is radial inflow velocity at the screen (periphery of the inflow layer).

Walko and Gall (1986) argue that the core radius of the theoretical inviscid vortex may be derived by assuming a stagnant core and balancing the kinetic energy of the potential flow region against the central pressure drop. The result is

$$\frac{r_c}{r_0} = \frac{4aS}{p^{*1/2}}, \tag{7}$$

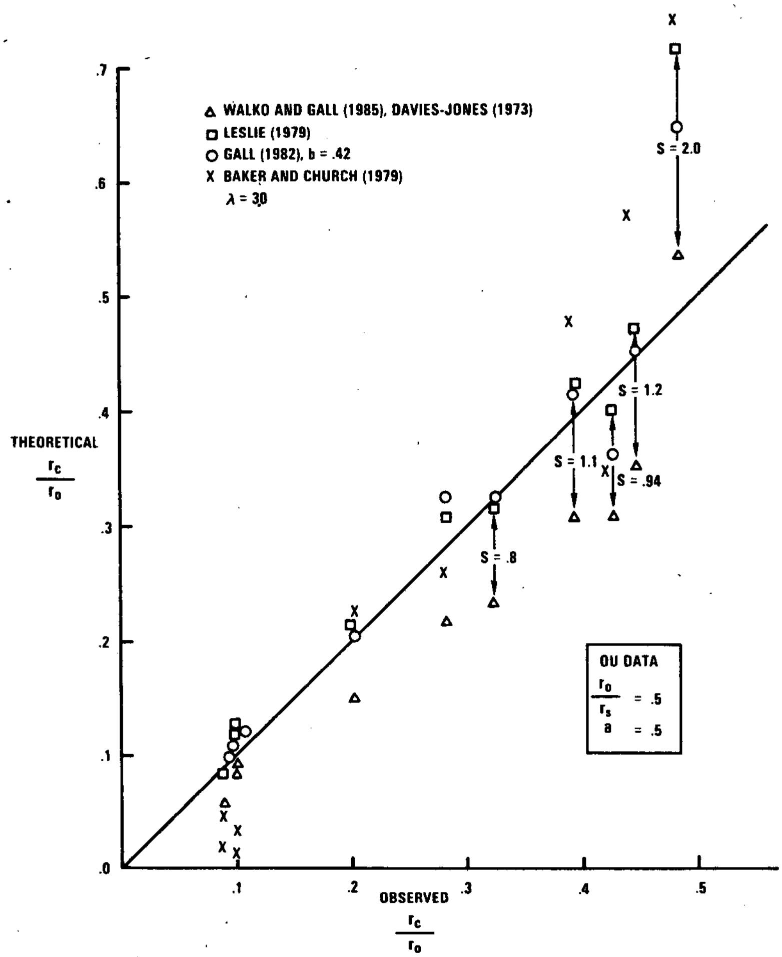

FIG. 3. Comparison of theoretical and observed values of vortex core radius with OU tornado simulator.

a slightly simpler formula due to the simpler inviscid flow model. The formulation given by Davies-Jones (1973) reduces to the same form for the case $r_0/r_s = 0.5$, which was true for our original comparison experiments.

Figures 3 and 4 compare the p^* theories with those of Gall and Baker and Church for OU data and PU data, respectively. The data points for large S are labeled; they are characterized by a larger disparity between the three methods of prediction. These S values all lie in the multiple vortex regime, in any case. All four equations make a good fit to the OU data, and the predictions of Leslie and Gall are almost identical. The simpler model of Walko and Gall, although not intended for core radius prediction, appears to do well at small core radii (Fig. 3), but underpredicts the radius more with increasing radius. For the Purdue data in Fig. 4 we see that the simpler model of Walko and Gall (1986) performs better than that of Leslie (1979b) or Gall (1982) with $b = 0.36$. However, as before, Gall's equation may be tuned to an excellent fit by adjusting b. The equation of Baker and Church gives less scatter over the full range of S values and makes an excellent fit to the Purdue data.

These comparisons show that satisfactory estimates of vortex core radius are obtainable from formulas that take into account convection cell depth. The results are as good as with the less approximate p^* theories, which require pressure measurements.

3. Experiments on cell geometry effects

For our purpose the upper boundary of the convective cell may be considered as the terminal region of the vortex. Until very recently, no experiments had been made to assess the effect of cell height on vortex dynamics. In the tornado simulators at OU and Purdue this dimension is determined by the placement of a flow baffle at the top of the cell. This baffle tends to reduce or destroy the tangential component of the circulation. Changing the dimension Z in these simulators is very difficult. We arranged some experiments in the OU simulator in which the baffle could be lowered by about ten percent. At a swirl ratio near 0.1, this change

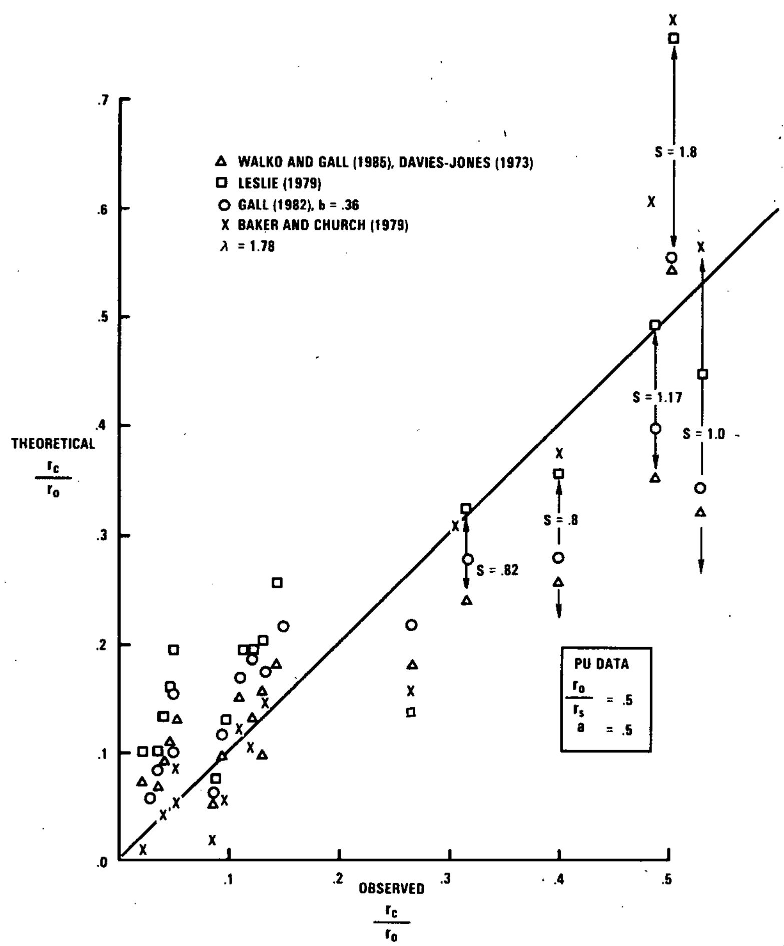

FIG. 4. As in Fig. 3, but with the Purdue tornado simulator.

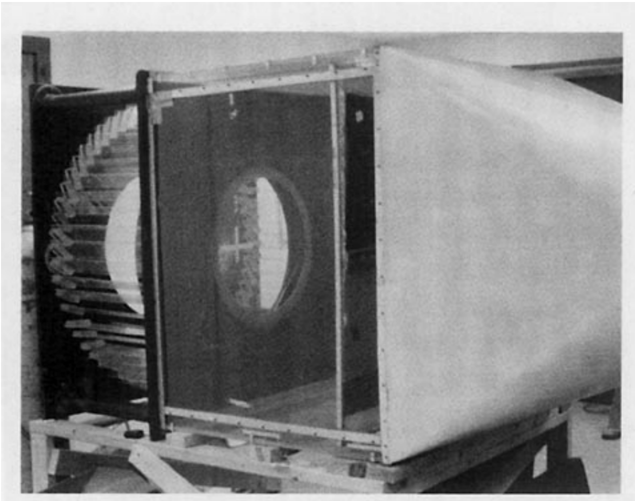

FIG. 5. The Viney tornado simulator at the University of Oklahoma.

in Z caused the breakdown bubble to descend from midheight to the floor of the simulator, giving a complete transition to the two-cell vortex. A similar experiment was conducted by Pauley (1986) in the Purdue simulator, using a very crude arrangement for varying Z. Within the very limited range of parameters possible under these circumstances, Pauley could detect only a small change in vortex radius. We shall see later that within certain ranges of swirl ratio the response to changes in Z/r_0 is very small.

Smith (1986) performed numerical experiments using the Rotunno (1984) model which simulated both the presence and absence of the baffle in the upper part of the convection cell. These experiments showed a profound impact on the two-vortex mode at $S = 0.87$. Pressure field changes occurred which intensified the vortexes when the "baffle" was removed. At higher swirl ratios (up to $S = 2.61$) he found very little effect of the upper boundary. He interpreted this to mean that the larger diameter (larger volume) vortex which was present absorbed the effects of pressure change with a less obvious impact on the velocity field. The asymmetric model of Rotunno (1984) reproduces faithfully most of the vortex phenomena observed in Ward-type simulators.

Recently we reassembled the Viney simulator (named for the graduate student who designed it). It is a horizontal version of the Ward simulator, as shown in Fig. 5. The convection cell is square instead of round, and the baffle is easily adjustable over a wide range of Z values. Swirl is introduced via guide vanes. We made a transparent floor so that vortex radius can be measured very accurately from a bottom view of the vortex. With the Viney simulator it is possible to reproduce

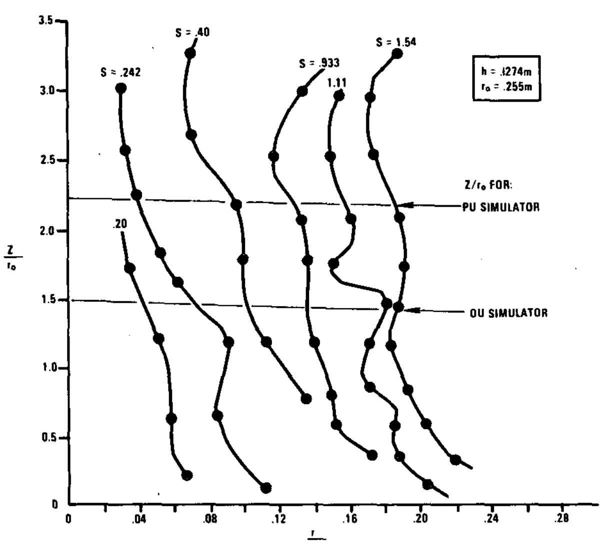

FIG. 6. Core radius vs Z/r_0 for six different swirl ratios.

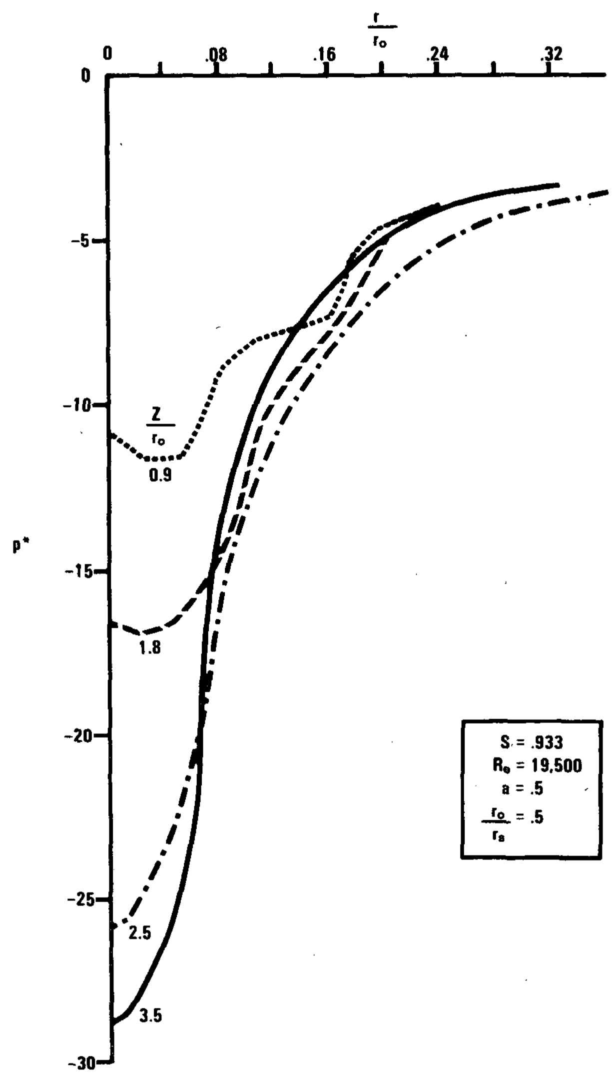

FIG. 7. Radial profiles of surface pressure for four values of Z/r_0.

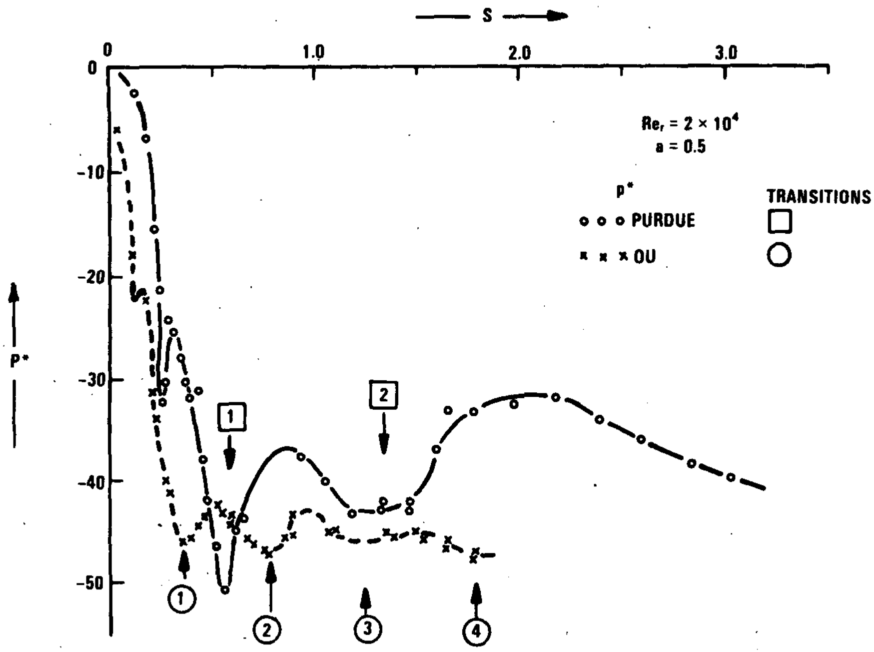

FIG. 8. Comparison of OU and Purdue central pressure drop and transition occurrences as a function of swirl ratio.

all of the nondimensional parameters available to both the OU and PU simulators, including Z/r_0.

Several experiments were conducted to examine the effect of varying Z/r_0. The other simulator parameters were set to the values used in the comparison experiments: $a = 0.5$, $r_0/r_s = 0.5$, $\mathrm{Re}_r = 19\,500$ unless stated otherwise.

Figure 6 shows the variation of dimensionless core radius with Z/r_0 for six different swirl ratios. As suspected, the radius tends to decrease with increasing Z/r_0 within the range of the OU and PU simulators. However, this decrease is much less pronounced at higher swirl ratios, which tends to agree with the numerical experiment of Smith (1986). The decrease reverses to an increase above about $Z/r_0 = 2.5$ for the higher swirl ratios. The fluctuations in some of the curves are due to multiple-vortex transitions that occurred over this range of Z/r_0. Two transitions occurred for $S = 1.11$: a 1–2 transition and a 2–3 transition. All such curves that we have generated exhibit at least one range of Z/r_0 values in which changes in Z/r_0 have essentially no effect on vortex radius. Someone experimenting in this range might conclude that Z/r_0 is not a critical parameter.

Figure 7 gives radial profiles of pressure drop for four values of Z/r_0. The swirl ratio was held at 0.933 for these curves. Off-center pressure minima occur for the smaller Z/r_0 values, indicative of the multiple-vortex mode, whereas a strong single vortex was present for the two larger Z/r_0 values. These profiles are consistent with our observations during the OU/PU simulator comparison experiments. Figure 8 compares the central pressure drop and transition occurrences as a function of swirl ratio for the two simulators, showing the larger critical swirl ratios required for a given transition to occur in the PU simulator. These transition points are transposed onto Fig. 9 in order to check for agreement with data taken with the Viney simulator.

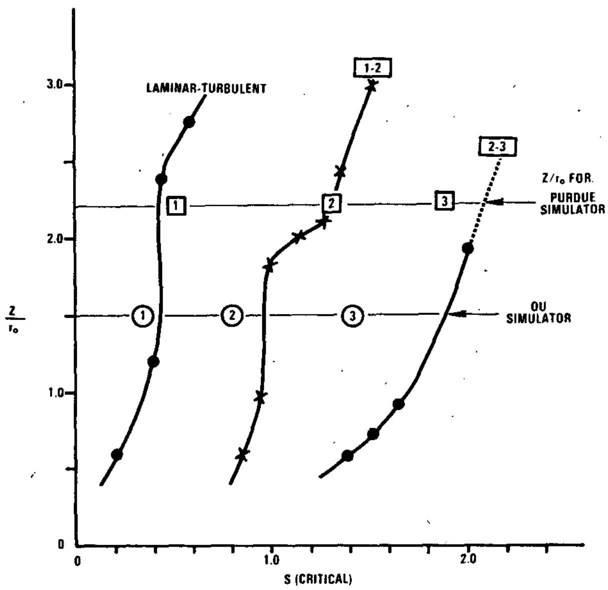

FIG. 9. Critical swirl ratio vs Z/r_0 for three transitions in the Viney simulator. $\mathrm{Re}_r = 10\,600$, $a = 0.5$, $r_0/r_s = 0.5$.

Although the match is less than perfect, the trend is unmistakable. Differences occur due to experimental error, and possibly also due to different criteria for defining the completion of a transition. Existing theories on multiple vortex transitions do not address directly the possible effects of convection cell geometry.

4. Discussion

The effects of varying Z/r_0 observed with the Viney simulator verify that this parameter is of primary importance for interpreting tornado simulator experiments, and especially for comparing experimental results between different simulators. Furthermore, the results are consistent with the findings of Smith (1986) and Pauley (1986). It is clear that the occurrences of larger vortex radii and transitions at smaller swirl ratios in the OU simulator have a common cause: smaller convection cell aspect ratio Z/r_0. This smaller Z/r_0 produces a shorter, wider vortex when all other simulator parameters are equal. The wider vortex will more readily accommodate the downflow from above, and this promotes transition to the two-cell mode at lower swirl ratio. The effect of Z/r_0 is less pronounced for the large diameter system of multiple vortexes, but still quite noticeable within the range of swirl ratio available to the Viney simulator (up to $S = 1.54$).

The implication to real tornadoes is that supercells with small aspect ratio may proceed to the more dangerous multiple vortex tornado modes at relatively small swirl ratio (weak mesocyclone). The destructive Dallas tornado of 13 December 1984 might have been such a storm; echo tops were innocuous, reaching only about 25 000 ft.

The aspect ratios of the convective cell and the inflow layer are related to each other through the updraft radius r_0. When r_0 is relatively small, the large aspect ratio of the cell inhibits multiple vortex formation as shown by this study, as does the large aspect ratio of the inflow layer (Ward, 1972). The implementation of Doppler radars should make it possible to observe these storm parameters in the future and apply them to tornado forecasting.

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