Time and Space Variability of Spectral Estimates of Atmospheric Pressure

FLAVIO G. CANAVERO* AND FRANCO EINAUDI†

School of Geophysical Sciences, Georgia Institute of Technology, Atlanta, GA 30332

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ABSTRACT

The purpose of this paper is to analyze the temporal and spatial behavior of atmospheric pressure spectra. The literature shows many examples of pressure, wind and temperature spectra whose shapes display a remarkable degree of universality. Theories relying either on turbulence or internal waves have been suggested to account for such spectra. While the former accounts for features at the observed synoptic scales and the latter for the local scales, several difficulties remain especially at the intermediate or the so-called mesoscale range.

As a preliminary step for our understanding of the physical mechanisms underlying the spectral behavior, a detailed analysis of the surface pressure was carried out having in mind to test the temporal and spatial variability of these spectra. The source of the data were two microbarograph stations in the Po Valley, 276 km apart; one in the plains 50 km south of the Alps, the other in the foothills of the Dolomites. The 60-day record was part of the Alpine Experiment (ALPEX) which took place in 1982.

The study reveals new important phenomena concerning the behavior of spectra of atmospheric pressure: 1) Pressure records are intrinsically nonstationary and thus the analysis should more properly be conducted in terms of evolutionary spectra which account for the time variations of the frequency components. 2) A relationship exists between a given frequency range and a corresponding nonstationarity scale, i.e., the minimum time interval needed by nonstationarity to manifest itself fully. 3) A quantitative discussion of the time variability of spectra demonstrates that their shape and energy content depend on different time segments. 4) Important differences of the spectra exist between the two stations, indicating a substantial effect of topography, particularly for periods below 40 min.

The results of this work suggest that nonstationarity of the pressure spectra and their temporal and spatial behavior are important elements for our understanding of atmospheric and oceanic dynamics. It is still possible that universality remains a valid concept, but it must be recognized that averages over large temporal or spatial datasets may produce results that mask the underlying physical processes.

1. Introduction

To explain the behavior of the atmosphere and the ocean, it is essential to understand the energy exchanges among the motions at different spatial and temporal scales. Spectra are suitable tools for such a purpose; in fact, they represent the energy distribution among the sinusoidal components of any atmospheric variable, viewed as a random process. The interest in atmospheric and oceanic spectra has intensified since the early work of Munk et al. (1959) and Gossard (1960), in part because of the development of remote sensing techniques which have substantially improved our ability to measure various atmospheric variables and in part because of the increased recognition of the meteorological significance of subsynoptic scale motions. As examples, the development of the mesosphere, stratosphere, troposphere (MST) radar technique has made it possible to measure radial velocity spectra in the mesosphere, stratosphere and troposphere, while such a research initiative as the Global Atmospheric Sampling Program (GASP) has provided information concerning kinetic energy spectra from a few kilometers to about 14 km above the Earth’s surface.

Spectra have been calculated for a number of variables and for various frequency and wavenumber ranges. Pressure spectra analyzed by Gossard (1960), Herron et al. (1969) and Bull et al. (1981) cover periods from 0.2 s to 1 week, from 30 s to 10 hours and from 2 to 128 min, respectively. Wind spectra are well covered in the literature: the papers by VanZandt et al. (1978), Balsley and Gage (1980), Ruster et al. (1980), Balsley and Carter (1982), VanZandt (1982), Lilly and Petersen (1983), and Nastrom and Gage (1983, 1986) present observations from a variety of sensors distributed in different geographical areas and looking at various altitude ranges. Temperature spectra have also been published (e.g., Mantis and Pepin, 1971; Gage and Nastrom, 1985).

Observed spectra usually have a very similar shape which can be approximated with a few linear segments of somewhat varying slope (see Fig. 5). Spectral estimates of kinetic energy observed by various authors are summarized and discussed by Lilly (1983) in a composite figure in which the spectra obtained in the frequency domain / by Vinnichenko (1970) and Balsley

* Permanent affiliation: Dipartimento di Elettronica, Politecnico di Torino, 10129 Turin, Italy.
† Present address: Laboratory for Atmospheres, NASA/Goddard Space Flight Center, Greenbelt, MD 20771.

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and Carter (1982) have been transferred into the wavenumber domain \( k \) by use of the Taylor hypothesis. Typically, longer wavelength portions of the spectra are steeper than shorter wavelength ones. Thus, for example, a \( k^{-3} \) dependence in the planetary wavenumber range 10 to 30 (with corresponding horizontal wavelengths between 1000 and 3000 km) appears well documented (Winn-Nielsen, 1967; Kao and Wendell, 1970; Kao, 1970; Chen and Winn-Nielsen, 1978) and is explained by the theory of geostrophic turbulence (Charney, 1971). At the opposite end of the wavenumber axis, the \( k^{-5/3} \) dependence is interpreted in terms of isotropic three-dimensional turbulence (Kolmogorov, 1941).

In the intermediate range, the so-called mesoscale, where spectral levels are much higher than predicted by the decay of geostrophic turbulence, the \( k^{-5/3} \) behavior seems appropriate (Gage, 1979; Balsley and Carter, 1982; Lilly and Petersen, 1983; Nastrom and Gage, 1983), but its interpretation is an unsettled issue. Two possibilities have been discussed for the mesoscale range: quasi-two-dimensional turbulence (Gage, 1979), leading to an upscale of energy as discussed by Kraichnan (1967) and by Lilly (1983), and a universal spectrum produced by internal and inertio-gravity waves as proposed by Dewan (1979) and detailed by VanZandt (1982), who extended the results of a theory worked out by Garrett and Munk (1972) for oceanic spectra.

The time scales at which the regime separations occur are somewhat uncertain: Balsley and Carter (1982) estimate the transition from mesoscale to large scale to be of the order of 3 h and that from small scales to mesoscale to be of the order of a few seconds.

The above slopes refer to spectra of the horizontal wind; temperature is shown to display a similar behavior (Nastrom and Gage, 1986). A model of the atmospheric pressure spectrum, deduced according to the physical interpretation of the wind spectra available in the literature, consists of an \( f^{-3} \) behavior at very long periods and of an \( f^{-5/3} \) in the mesoscale range as well as at lower periods where isotropic three-dimensional turbulence dominates (Lumley and Panofsky, 1964). The gravity wave theory predicts an \( f^{-5/3} \) behavior for pressure spectra (VanZandt, 1982).

Although quite similar in appearance, the slopes and the levels of the spectra vary substantially in time and space, probably as a result of the intermittency of the energy injection as well as of the nonlinearity of the atmospheric processes. This paper deals with the question of the temporal and spatial analysis of spectra determined through measurements of atmospheric pressure collected at different sites in northern Italy, in connection with the ALPEX experiment. The records provide information on their harmonic content for periods ranging from 20 s up to about one week. Thus, the variability of the spectra can be analyzed in detail over a very broad range of frequencies.

In section 2, the network and data collection are briefly described. The ad hoc procedure used to analyze the nonstationarity and to calculate the spectra of the data are presented in sections 3 and 4, respectively, and the results discussed for two different stations in sections 5 and 6.

2. Experimental setup and data collection

An experimental campaign was conducted in northern Italy from mid-April to mid-June 1982. The first 15 days overlapped with the ALPEX Intensive Observation Period (IOP). During that period, surface pressure measurements were carried out at seven sites located south of the Alpine ridge with a separation of 50 to 100 km, covering an area of about 100 × 300 km². The map of Fig. 1 shows the topography of northern Italy and the location of the stations. The Po Valley is surrounded by the Alps to the north and west and by a lower mountain ridge, the Apennines, to the south. The mountain heights range from about 800 to more than 3000 m. Six measurement stations were located in the plain at different distances from the mountains, and one was placed in a valley at the southern edge of the Alps. In this paper, comparisons are presented between data from the station near Alessandria (hereafter indicated by A), in the middle of the plain, and the station located near Trento (B), in the mountains; the line of sight distance between these two stations is 276 km.

Each station was equipped with an absolute microbarometer, two temperature sensors and a data logger. The pressure sensor was a sensitive capsule transducer connected to the atmosphere through a large pipe and housed in a box with high thermal capacity. One of the temperature sensors was used to monitor the capsule temperature. The records are very smooth and present only traces of the daily temperature cycle (typical excursions of ±1°C, over a 24-h period). Since the sensitivity to the temperature of the microbarometer is ±1 μbar/°C, the absence of significant inaccuracies due to temperature is assured. In fact, the temperature effect for short time intervals (1 h, say) is much less than the electronic noise level, while for longer periods (6 h or more) the error due to the temperature is negligible with respect to the pressure components at such periods.

The pressure transducer provided an analog output voltage that was sampled every second and converted into a 16-bit value by the data logger. At this point the nominal resolution was 4.5 μbar, comparable with the instrumental errors. The data logger performed an online average over 10 samples and stored the mean on diskettes every 10 s. The averaging process is equivalent to a rudimentary, but easy to implement filter, needed to eliminate the fast varying electronic noise and high frequency turbulence from the sampled signal (see section 3 for more details). Digital pressure data were con-
verted off line to a magnetic tape format and recorded with air temperature data sampled by the same data logger at 10-min intervals. As a consequence of the averaging and conversion, the final resolution of the pressure data is approximately 9 µb.

Each station was equipped with a high precision clock, whose time was checked by the operator once every few days, at the moment of the diskette replacement. No clock failures were reported. The data base contains time references so that comparisons among events at different sites can be done. The average duration of the campaign at each site is approximately 60 days. Two stations only operated approximately 1/3 of the time, due to power failures and a malfunctioning automatic restart. Of the two stations whose data are presented here, one (station A) has only a few missing data; the other (station B) has a week-long gap in mid-May.

3. Nonstationarity analysis

The length and the sampling interval of the available data base allowed us to conduct a spectral analysis ranging from a period of more than a week down to 20 s. A necessary hypothesis for the application of spectral estimation is the statistical stationarity of the random process at hand. A stationary random process is one whose statistical moments, calculated from the ensemble of its realizations, are independent of time (Bendat and Piersol, 1971; chap. 1). Thus, nonstationarity implies a variability of the spectral characteristics of the process. Since a visual inspection of the pressure records reveals the intermittent presence of random amplitude disturbances superimposed on almost daily wavelike fluctuations, it is legitimate to wonder about the stationarity of atmospheric pressure (see Fig. 2).

Because of the wide variety of signal characteristics that can be found in a real time series, a general theory for studying nonstationary random processes is not known. Instead, different methods have been envisaged to explain particular categories of experimental data.

The approach described here relies on Priestley’s (1965) idea of an evolutionary spectrum, i.e., of an “instantaneous” spectrum, S(ω; t), which evolves in time t. The angular frequency ω is the only variable the spectral density S depends on when the process is stationary. The meaning of the evolutionary spectrum becomes clearer for a band-limited process with lower
FIG. 2. Example of a pressure record for station B. The upper plot refers to 15 May 1982 and the lower to 4 June 1982. Note that the pressure scale of the lower part of the figure is double the upper one. The superposition of fluctuations at different scales is evident.

The pressure records collected during the experimental program were tested for nonstationarity. Then, once nonstationarity was ascertained, an attempt was made to determine the time scale $T_2$.

The search for stationarity in our data used a modified version of a scheme developed for the analysis of bursts of acoustic noise. This choice was based on the similarity, though incomplete, of the two processes. Only a brief description of the method is given here; the interested reader is referred to the paper by Tsao (1984) and to the references therein. The main assumption of Tsao’s method is that the spectral estimate $S_{jm}$ can be factorized into a term $B_j$ depending only on the time subinterval, a term $C_m$ depending on the frequency and a term $D_{jm}$, called the interaction term, reflecting a nonseparable time and frequency behavior:

$$S_{jm} = AB_jC_mD_{jm},$$  \hspace{1cm} (3.1)

where $A$ is a constant scale factor. Obviously, for a stationary process, only the $C_m$ term survives, whereas the absence of the $D_{jm}$ term means that the process maintains the same spectrum during its evolution, apart from a scale factor depending on the running time.

Our simplification of the above scheme consisted in grouping $K$ frequencies together to form $M/K$ bands. The ratio $M/K$ is assumed to be an integer, without loss of generality. The assumption here is that (3.1) still holds for the new spectral quantities, i.e.,

$$S_{js} = (1/K) \sum_{k=(s-1)K}^{sK} S_{jk} = A'B_jC_jD_j,$$

$$s = 1, 2, \ldots, M/K,$$  \hspace{1cm} (3.2)

where $s$ is the band index and the primed quantities refer to the new time segment-frequency band classification. Because of the Parseval theorem, the quantity $S_{js}$ represents the variance of the random process in the frequency band considered, and can be directly calculated from the filtered pressure records in the time domain, without the need for any Fourier transform calculations. This simplified procedure is considered to be sufficient at this stage where one deals with a preliminary test of the process stationarity. Moreover, computation time is saved by doing variance calculations, and the matrix $S_{js}$ requires $(M/K) \times (K - 1) \times J$ less storage locations than using all $M$ frequencies. The characteristics of the bands are listed in Table I.

The variance of each time segment was calculated as follows:

$$\sigma_{js}^2 = (1/N) \sum_{n=0}^{N-1} (\xi_{js}(n) - \bar{\xi}_{js})^2,$$  \hspace{1cm} (3.3)

where $\xi_{js}(n)$ represents the $n$-th sample of the $j$-th time segment for the $s$-th band, and it is the result of the band-pass filtering applied to the original time series $x(t)$. The quantity $\bar{\xi}_{js}$ is the mean value of the filtered signal $\xi_{js}(n)$ in the $j$-th segment and for the $s$-th band.
As stated before, \( \sigma_{js}^2 \) is equivalent to \( S'_{js} \). Any changes of the variance \( \sigma_{js}^2 \) given by (3.3) indicate nonstationarity.

So far, the length \( T_s \) of the time segment used in the analysis for band \( s \) has not been specified. Obviously, its choice is unimportant only if the series is stationary. Certainly, \( T_s \) must be greater than \( 1/(f_{ls})_s \), where \( (f_{ls})_s \) is the lower cutoff frequency of band \( s \). For convenience in the variance analysis, the time \( T_s \) was chosen to be independent of \( s \) and equal to 1 day, which ensures a sufficiently large number \( J \) of segments. The limitation in dealing with the bands of Table 1 resides in the inability to test nonstationarity of phenomena with characteristic periods larger than a day, which would require much longer data bases.

The analysis of the variances (3.3) is designed to investigate whether the process is nonstationary, but is unable to predict the nonstationarity time scales \( (T_{2s}) \) for each band. This test was preliminarily conducted over a sample of the entire data base, namely, the time period spanning from 20 April to 3 May, for station A. A standard easy-to-use computer routine was adopted for the variance analysis (Hemmerle, 1982). A matrix \( Y'_{js} \) was generated by taking the logarithm of the spectral density of 14 time segments analyzed in ten bands. This matrix is assumed to satisfy a model like the one given by (3.2):

\[
Y'_{js} = 10 \log_{10} S'_{js} = \alpha' + \beta_j' + \gamma_s' + \delta_{js}',
\]

(3.4)

where \( \alpha = 10 \log_{10} A \), and so on. The variance analysis tests the significance of the \( \beta' \), \( \gamma' \), and \( \delta' \) terms.

A schematic description of the concepts underlying the analysis is given here to help interpret the results of Table 2. The effect of each term \( \beta' \), \( \gamma' \) and \( \delta' \) is evaluated by analyzing the rows and columns of the \( Y'_{js} \) matrix. As an example, let us consider the effect of frequency; \( \dot{M}/K = 10 \) statistical distributions of the spectral values, generated by grouping the matrix values by columns, are compared. If the histograms are similar to each other (i.e., if they show almost equal means and variances), the frequency effect is said to be absent.

Table 1. Bands utilized for the nonstationarity analysis.

<table>
<thead>
<tr>
<th>Band no.</th>
<th>Upper frequency cutoff, ( f_{max} ) (Hz)</th>
<th>Period of the upper cutoff, ( 1/f_{max} ) (s)</th>
<th>Lower frequency cutoff, ( f_{min} ) (Hz)</th>
<th>Decimation ratio in the window</th>
</tr>
</thead>
<tbody>
<tr>
<td>I</td>
<td>( 5.0 \times 10^{-2} )</td>
<td>20</td>
<td>( 2.5 \times 10^{-2} )</td>
<td>1:1</td>
</tr>
<tr>
<td>II</td>
<td>( 2.5 \times 10^{-2} )</td>
<td>40</td>
<td>( 1.25 \times 10^{-2} )</td>
<td>1:1</td>
</tr>
<tr>
<td>III</td>
<td>( 1.25 \times 10^{-3} )</td>
<td>80</td>
<td>( 6.25 \times 10^{-3} )</td>
<td>1:1</td>
</tr>
<tr>
<td>IV</td>
<td>( 6.25 \times 10^{-3} )</td>
<td>160</td>
<td>( 3.13 \times 10^{-3} )</td>
<td>1:2</td>
</tr>
<tr>
<td>V</td>
<td>( 3.13 \times 10^{-3} )</td>
<td>320</td>
<td>( 1.56 \times 10^{-3} )</td>
<td>1:4</td>
</tr>
<tr>
<td>VI</td>
<td>( 1.56 \times 10^{-3} )</td>
<td>640</td>
<td>( 7.81 \times 10^{-4} )</td>
<td>1:8</td>
</tr>
<tr>
<td>VII</td>
<td>( 7.81 \times 10^{-4} )</td>
<td>1280</td>
<td>( 3.91 \times 10^{-4} )</td>
<td>1:16</td>
</tr>
<tr>
<td>VIII</td>
<td>( 3.91 \times 10^{-4} )</td>
<td>2560</td>
<td>( 1.95 \times 10^{-4} )</td>
<td>1:32</td>
</tr>
<tr>
<td>IX</td>
<td>( 1.95 \times 10^{-4} )</td>
<td>5120</td>
<td>( 9.77 \times 10^{-5} )</td>
<td>1:64</td>
</tr>
<tr>
<td>X</td>
<td>( 9.77 \times 10^{-5} )</td>
<td>10240</td>
<td>( 4.88 \times 10^{-5} )</td>
<td>1:128</td>
</tr>
</tbody>
</table>

Quantitatively, the comparison is done by calculating the \( F \)-ratio, which is a statistical parameter giving the variability between the groups with respect to the variability within the groups (Brownlee, 1965). Large \( F \) values suggest a definite variability with frequency. In fact, large \( F \) values imply that the distribution of spectrum values varies from column to column (i.e., frequency by frequency) more than it does along any given column. The significance test for any term of model (3.4) is done by making a null hypothesis for such a term, i.e., by assuming that the log-spectrum \( Y'_{js} \) is independent of the corresponding variable. Then \( F \) is compared with the value that it is supposed to have for a matrix of Gaussian distributed numbers without column or row variability. If the former is larger than the latter, the hypothesis is rejected at a given percent significance level and the spectrum is interpreted as depending on such a variable. In Table 2, the \( F \) values are listed and the results of the test indicate that the null hypothesis for any single term of model (3.4) is significantly rejected with a marginal probability less than 1%. This technique demonstrates that there is nonstationarity for all bands that were analyzed; it does not provide any information on the characteristic nonstationarity time \( T_s \).

The evaluation of the nonstationarity time scale \( (T_{2s}) \) for band \( s \) is crucial for the development of the spectrum analysis technique, as discussed before. The test for stationarity proposed by Bendat and Piersol (1971; chap. 7) was used to determine the quantity \( (T_{2s}) \), because it combines simplicity and robustness. The method consists in generating a new time series from the original process by evaluating the quantity

\[
\sigma_j^2(l) = (1/N_s) \sum_{n=(l-1)N_s}^{lN_s} (\xi_s(n) - \bar{\xi}_s)^2,
\]

(3.5)

which represents the variance of parts of the filtered process \( \xi_s(n) \) for the \( s \)-th band. The quantity \( \sigma_j^2 \) must represent a good estimate of the variance of the process \( \xi_j(n) \). This is accomplished by taking the time length \( T_s = N_s \Delta t \), over which the summation is performed.

Table 2. Results of the nonstationarity analysis. Significance of the time, frequency, and mixed terms in the model of the evolutionary spectrum [see Eq. (3.4)].

<table>
<thead>
<tr>
<th>Cause of variance</th>
<th>Corresponding term in the model</th>
<th>Degrees of freedom for the reduced model</th>
<th>( F ) value</th>
<th>Significance at 1% marginal probability</th>
</tr>
</thead>
<tbody>
<tr>
<td>Time interval</td>
<td>( \beta' )</td>
<td>8</td>
<td>26.01</td>
<td>Yes</td>
</tr>
<tr>
<td>Frequency band</td>
<td>( \gamma' )</td>
<td>13</td>
<td>4.94</td>
<td>Yes</td>
</tr>
<tr>
<td>Interaction term</td>
<td>( \delta' )</td>
<td>104</td>
<td>1.85</td>
<td>Yes</td>
</tr>
</tbody>
</table>

The number of degrees of freedom for the full model equals 1055.
larger than the upper cutoff period \((T_i)_{\sigma} = 1/(f_{\sigma})_{\sigma}\) of the \(s\)-th band; we chose \(T_i = 2(T_i)_{\sigma}\). The "run test" proposed by Bendat and Piersol (1971) was then applied to the new time series \(\sigma^2(t)\). It consists of counting the number of crossings of the median level: nonstationarity is claimed if the figure is significantly different from half the number of subsegments, as it would be in the case of white stationary noise. The time \((T_2)_{\sigma}\) was estimated by increasing the number of terms until the test successfully indicated that nonstationarity has emerged.

The difference between the current approach and Tsao's (1984) method resides in the avoidance of the use of the spectral analysis. Since the stationarity analysis must be taken before the evaluation of the spectrum of the process (in fact, the results of the former analysis will influence the choice of the latter), a method working in the time domain seems more appropriate. Moreover, our technique is computationally faster. The bandpass filtering, while essential for this approach, also becomes unavoidable when the Tsao technique has to be applied to a database like the one under examination, in order to reduce any possible contamination of the high frequencies by the large low-frequency amplitudes. This issue will be discussed in more detail in section 4.

The results of the analysis are presented in Fig. 3 where the time \((T_2)_{\sigma}\) is plotted vs. the characteristic period \((T_i)_{\sigma}\) of the pressure fluctuations in the \(s\)-th band. The test was repeated for the ten bands of Table 1 and for various time segments during the same period (20 April–3 May) used above to determine the nonstationarity. The values of \((T_2)_{\sigma}\) obtained from the repeated trials show a somewhat high scatter (not reported in Fig. 3); this is partly due to the primitiveness of the analysis and, perhaps, to the different meteorological situations corresponding to various time segments. The run test applied to the process filtered in bands IX and X did not evidence nonstationarity. The plotted values of \((T_2)_{\sigma}\) in Fig. 3 are such that 95% of the estimations for the \((T_2)_{\sigma}\) were larger. Such a choice seems appropriate since spectral analysis needs a time segmentation that assures the stationarity (see section 4). The solid curve of Fig. 3 fits the experimental points in the least-squares sense, and can be interpreted as a quantitative estimation of the time one would expect the stationarity to last.

4. Spectral analysis

The available database consists of approximately 60 days of pressure records, sampled every 10 s, so that the spectral analysis can span over more than five orders of magnitude in frequency. It is also well known (e.g., Gossard, 1960) that the spectral density of the atmospheric pressure associated with large periods (greater than 24 h, say) is many orders of magnitude higher than that found at smaller time scales. A wide choice of estimation techniques are available for application, ranging from the conventional approach based on Fast Fourier Transform (FFT) to recent more sophisticated algorithms such as the maximum entropy, autoregressive and maximum likelihood methods. Extensive literature exists on this subject: a tutorial paper by Kay and Marple (1981) gives a unified overview and contains detailed bibliographical references to the various aspects of the problem. In a nutshell, the conventional FFT spectral estimation has the advantage of computational efficiency, but shows two important limitations: one is the inability to resolve close peaks, the other is a possible distortion of the spectral shape due to leakage of energy from high density regions. The recent nonconventional techniques were devised in an attempt to overcome the above limitations of the FFT-based method. Such procedures, in turn, suffer a lack of generality, because they were developed for specific applications in radar techniques, geophysical prospecting, imaging, biomedicine, econometrics, etc., and assume different models for the input time process. The present study is not intended to investigate particular wave events, as was the case in the paper by Stobie et al. (1983) where maximum entropy was proven to be very efficient in detecting short trains of waves. Hence, the authors used the conventional FFT spectral estimation, believing with Tukey (1984) that it is sufficiently general to be adopted when only
the gross features of the spectrum are investigated and no a priori model of the random process is required or known. The way we dealt with the inherent limitations of the FFT spectral estimation technique is explained in this section.

The whole dataset cannot be analyzed at one time. Besides the large amount of samples involved, two problems stand out. First of all, leakage (Koopmans, 1974, chap. 9) will occur: high peaks of the spectrum will influence distant frequencies through the secondary lobes of the transform of the boxcar function which delineates the sample in time. Although this phenomenon can be controlled with tapering windows in the time domain, we believe that the high frequency part of the spectrum would be inevitably distorted by the highly energetic part at lower frequencies. Secondly, but more importantly, the atmospheric pressure is nonstationary. In fact, a single analysis of the complete database would contradict the idea of an evolutionary spectrum, as discussed in section 3.

The strategy devised to overcome such problems consisted in dividing the frequency axis into bands and making an appropriate analysis within each band. The dataset for any band was generated through a convenient filtering of the original records, designed to minimize leakage effects. The length of the segments to be used for spectral analysis was set according to the restrictions imposed by nonstationarity characteristics of any given band. The subdivision of the observable frequency interval is illustrated in Fig. 4 where the filter transfer functions are plotted, and it is clear that the top flat part of the curves represents the pass band, whereas the negative dB values indicate attenuation in the stop band.

A total of nine bands was utilized to cover the portion of the frequency axis for which sufficient data were available to perform the spectral estimation available. The bandwidth between the −3 dB points of the filter transfer function is two octaves for the first five windows, and one octave for the last four. The choice of two different bandwidths is a compromise between having a low number of windows and yet being able to resolve the spectral features. A small number of bands is required in order to reduce the processing time, since the analysis procedure described later must be repeated for every frequency window. On the other hand, it is known that sharp peaks in the spectrum must be expected at time periods longer than 6 h, whereas at shorter periods the spectral behavior is smoother. Slightly larger windows for higher frequencies simplify the analysis, while retaining adequate frequency resolution. The arguments used to set a relationship among the bandwidth, frequency resolution, and data lengths will be discussed later in this section. A summary of the characteristics of the nine frequency bands is given in Table 3.

It is noteworthy that the first band extends for two octaves, beginning with the Nyquist frequency of the sampled process \( f_s = (2 \times 10)^{-1} \text{Hz} \). Negligible aliasing is then expected: in fact, the averaging done by the recording system as mentioned in section 2 and the knowledge that the spectrum presents a marked negative slope in the turbulence region assure that the folded spectrum will quickly die out.

The spectral estimation method deserves some discussion, because of the conflicting requirements of stationarity, uncertainty and resolution. A nonparametric technique was adopted, which estimates the spectrum of a signal from the absolute value squared of its FFT. Basically, this is the time series periodogram, which is known to be a somewhat erratic estimator of the spectrum. A smoother version is obtained by taking a moving average of the raw periodogram values: this is the so-called Daniell estimator (Koopmans, 1974), whose application results in a broadening of the effective resolution bandwidth. Moreover, the finiteness of the data sample causes the Fourier transform to convolve with a sinc-shaped function, so that the form of the spectrum can be distorted. The distortion is eliminated when the sinc-function approaches a \( \delta \)-function. Time windowing is done to approximate this process of convergence to a \( \delta \)-function.

Briefly, the spectral density was estimated as follows:

\[
\hat{S}(\omega) = \frac{1}{m}\left[\frac{1}{N}\sum_{i=-(m/2)}^{i=(m/2)} |X(f_i)|^2\right], \tag{4.1}
\]

where \( m \) is the width of the moving average, and \( f_i \) represents the discrete frequency at which the FFT samples \( X \) are determined. More details about the data processing are given in the Appendix. The choice of the outlined analysis procedure rather than the popular...
Table 3. Characteristics of the spectral analysis in each frequency band.

<table>
<thead>
<tr>
<th>Band no.</th>
<th>Sampling interval (s)</th>
<th>Samples per segment</th>
<th>Lower frequency cutoff (Hz)</th>
<th>Upper frequency cutoff (Hz)</th>
<th>Number of frequencies in the band</th>
<th>Nominal frequency resolution (Hz)</th>
<th>Effective bandwidth (Hz)</th>
<th>Error bars (dB)</th>
</tr>
</thead>
<tbody>
<tr>
<td>1</td>
<td>10</td>
<td>255</td>
<td>$1.25 \times 10^{-2}$</td>
<td>$5.0 \times 10^{-2}$</td>
<td>193</td>
<td>$1.95 \times 10^{-4}$</td>
<td>$6.24 \times 10^{-5}$</td>
<td>$+2.4/-1.9$</td>
</tr>
<tr>
<td>2</td>
<td>10</td>
<td>1019</td>
<td>$3.13 \times 10^{-3}$</td>
<td>$1.25 \times 10^{-2}$</td>
<td>193</td>
<td>$4.88 \times 10^{-5}$</td>
<td>$1.56 \times 10^{-5}$</td>
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</tr>
<tr>
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<td>40</td>
<td>1019</td>
<td>$7.81 \times 10^{-4}$</td>
<td>$3.13 \times 10^{-3}$</td>
<td>193</td>
<td>$1.22 \times 10^{-5}$</td>
<td>$3.90 \times 10^{-5}$</td>
<td>$+2.4/-1.9$</td>
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<tr>
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<td>811</td>
<td>$1.95 \times 10^{-4}$</td>
<td>$7.81 \times 10^{-4}$</td>
<td>193</td>
<td>$3.05 \times 10^{-6}$</td>
<td>$9.76 \times 10^{-6}$</td>
<td>$+2.8/-2.1$</td>
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<tr>
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<td>640</td>
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<td>$4.88 \times 10^{-5}$</td>
<td>$1.95 \times 10^{-4}$</td>
<td>49</td>
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<td>$2.29 \times 10^{-6}$</td>
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<td>$4.88 \times 10^{-5}$</td>
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<td>$1.95 \times 10^{-5}$</td>
<td>39</td>
<td>$3.05 \times 10^{-7}$</td>
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<td>20</td>
<td>$9.76 \times 10^{-8}$</td>
<td>$3.90 \times 10^{-7}$</td>
<td>$+9.2/-4.4$</td>
</tr>
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</table>

Welch method (e.g., see Oppenheim and Schafer, 1975), is justified mainly by the fact that we are dealing with sample sizes that are limited, either by the intrinsic nonstationarity of the data or by the duration of the collection campaign itself. The resulting spectra are individually archived for a subsequent statistical analysis, whose aims and results are the subject of section 5.

5. Results

An overview of the energetic content of the pressure records is presented first; then, a more detailed analysis will disclose spatial and temporal features of the process spectral density.

Figure 5 shows a comparison between the spectra for the two stations under investigation. Each diagram represents the mean spectrum in a log–log scale, obtained from the contributions of the nine separate frequency bands. Each contribution is the average over all calculated spectra in the appropriate band. The number of individual spectra contributing to the average is listed in Table 4 for the two stations examined. The upper and lower curves (not shown for periods higher than approximately 6 h) are the envelopes of all spectra. The individual spectra have been compensated for the attenuation introduced by the band-pass filter:

$$
\hat{S}(f_i) = \frac{1}{|H(f_i)|^2} \hat{S}(f_i),
$$

where $\hat{S}$ is the plotted quantity; $\hat{S}$ is the estimate, according to (4.1); $H$ is the filter transfer function plotted in Fig. 4 for all bands. The quantity $f_i$ represents each

Fig. 5. Estimated spectral density for the pressure records at station A, part a, and station B, part b. The dots on the left-hand side of the plot, and the central line represent the average spectrum for the entire database. The upper and lower lines are the envelopes of the single spectra contributing to the mean. It should be noted that the confidence limits, calculated in Table 3, refer to individual spectral estimations. The average spectrum is expected to have much smaller error bars (see text).
Table 4. Number of spectra contributing to the average.

<table>
<thead>
<tr>
<th>Band</th>
<th>Station A</th>
<th>Station B</th>
</tr>
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<tr>
<td>1</td>
<td>6699</td>
<td>4198</td>
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<tr>
<td>2</td>
<td>1651</td>
<td>1010</td>
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<tr>
<td>3</td>
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<td>213</td>
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<td>3</td>
</tr>
<tr>
<td>9</td>
<td>2</td>
<td></td>
</tr>
</tbody>
</table>

of the discrete frequencies at which the estimate took place. Error bars of the spectral estimates, appropriate for different frequency intervals, are also indicated in Fig. 5 and refer to the confidence intervals of individual spectra, according to Table 3. The central curve, representing the average spectrum, is expected to have error bars reduced by a factor approximately equal to the square root of the number of individual spectra contributing to the mean (see Table 4). This assumes that spectra calculated for separate time segments are statistically independent, which is certainly not far from reality for those bands where there is evidence of non-stationarity.

The mean spectrum shows continuous and smooth behavior for shorter periods, without any apparent structure. The reason is likely to reside in the averaging process itself, which tends to flatten all fluctuations, unless each member of the averaged set has well-defined features at some fixed position on the frequency axis. If one looks at the individual spectra, a substantial variability is present, as evidenced by the large gap between the maximum and minimum values reached by the estimates. Furthermore, this variability is statistically significant since it occurs over a range of values which is much larger than the confidence interval of the power densities. The two lines representing the extreme values are shown for the first five bands, where the departures from the mean are significant. At longer periods, the increased confidence limits and the relatively small number of samples make it difficult to establish the presence of variability in the data. On the other hand, the bumpy nature of the spectrum appears quite clear since the resolution bandwidth was kept low in that part of the frequency axis. The peaks at 24 and 12 h are due to atmospheric tides. There is also evidence of a peak at 8 h.

With one exception, the absence of discontinuities in the spectral diagrams is a clear indication that the adopted processing technique (i.e., signal bandpassing and further separate spectral estimation) works well, since any aliasing problem or defective estimation of the variance in any band would have resulted in an abrupt discontinuity at the band limits. The only exception is represented by the upper envelope of the variation region for band 1 of station B: since the average curve is continuous, it has to be argued that only a small number of time segments must have been biased toward higher values of the estimates.

A comparison of the two stations reveals that the overall shape of the spectra is conserved, apart from the lower period range (less than a few minutes), where station B, located in the mountains, shows spectral densities higher by a factor of 10.

Having established that average spectra display significant temporal and spatial variability, we proceed to study their characteristics in more detail by considering the spectra of the individual time segments. To this end, two quantities are assumed to be most representative of each spectrum: the slope, which provides more information on the shape, and the energy level, defined as the integral of the spectral density over the frequency band. Except for the region containing sharp peaks, the spectral density $\tilde{S}(f)$ is often assumed to be proportional to the $\nu$-th power of the frequency, so that these quantities are linearly related in logarithmic coordinates:

$$S_\nu = \alpha + \nu \log f = \alpha + f_\nu,$$  \hspace{1cm} (5.2)

where $\alpha$ is constant with respect to frequency. The slope $\nu$, within a given frequency band, is calculated by approximating $S$ with a straight line, in a least-squares sense. The likelihood of the straight line approximation implied by (5.2) was tested following Bendat and Piersol (1971, chap. 4): the mean square residual was checked against the error that one would have for a purely random dataset. The null hypothesis (i.e., absence of an obvious relationship between frequency and spectral amplitude) was almost always rejected at the 5% level of significance, which means that, with few exceptions, there is reason to believe that the spectral shape is clearly defined. Next we evaluated the errors involved in the calculation of $\nu$ due to the uncertainties in the spectral estimates. According to Brownlee (1965, section 11), the resulting 95% confidence interval for the slope $\nu$ is $2\epsilon_\nu$, with

$$\epsilon_\nu = \sigma_{S(\nu)} \times t_{M\nu-2.025} \left[ \sum_{i=1}^{M\nu} (f_i - \tilde{f}_\nu)^2 \right]^{-1/2},$$  \hspace{1cm} (5.3)

where $\sigma_{S(\nu)}$ is the sample standard deviation of the spectral values $S_\nu$, given the frequency $f_\nu$, with $S_\nu$ and $f_\nu$ defined by (5.2). The quantity $t_{M\nu-2.025}$ is the Student $t$ parameter with $M\nu - 2$ degrees of freedom and 2.5% marginal probability on both sides of the distribution. $M\nu$ is the total number of discrete frequencies $f_i$ in a given band, and $\tilde{f}_\nu$ is the mean of the $f_i$'s. The error $\epsilon_\nu$ depends on the band, since the number of frequencies $M\nu$ and the standard deviation of the estimated spectral value $\sigma_{S(\nu)}$ vary with the frequency interval. However, $\epsilon_\nu$ is independent of the particular value of the slope $\nu$; hence the uncertainty interval $\epsilon_\nu$ is independent of time.

Figure 6 attempts a classification of the slopes vs time of day for the bands 1, 2, and 3, and for the two
stations examined. The classification refers to the collection time of the original data segment; for simplicity, the daily time scale has been quantized into 1-h intervals. The curve in the middle of each diagram represents the average slope of the spectra of data segments collected at the same local time, during the 60-day campaign. The first curve on both sides of the middle one is placed one standard deviation apart; finally, the top curve indicates the maxima reached by the slope values and the bottom one is the locus of the minima. The shaded area about the average slope curve represents the 95% interval $2e$, given by (5.3). Since the various curves in Fig. 6 are well outside the shaded areas, they correspond to real variations of the spectral slope.

Figure 6 shows that the mean values, the maximum variations, and the temporal evolutions of the slopes...
are substantially different from band to band and from station to station. Band 1 of station B has a striking behavior: the spreading of the curves is larger than in any other diagram. While the average slope is definitely negative, there are a few spectra with a positive (and high) slope. The average slope of band 1 at station B is always lower than station A, but the reverse is true for bands 2 and 3.

A diurnal variation is evident only in band 3 for both stations: the spectra tend to flatten during the afternoon hours, but night and morning hours are characterized by steeper densities.

The other quantity which we take to be representative of spectral behavior is the spectral density integrated over a given band:

$$\Lambda_b = \Delta f \sum_{i=1}^{M_b} \hat{S}(f_i),$$  \hspace{1cm} (5.4)

where $\Delta f$ is the nominal frequency resolution in the band, and the frequencies $f_i$ are those for which the spectrum is estimated; $f_1$ is the lowest frequency of the band, $f_{M_b}$ is the highest, and $f_{i+1} = f_i + \Delta f$.

Equation (5.4) is an approximation to the area under the spectral density in a particular frequency band: for white noise (i.e., flat spectrum), $\Lambda_b$ equals the product of the bandwidth times the noise mean square level. Therefore, higher $\Lambda_b$ values indicate that the process is characterized by wide fluctuations about its mean. The sequence of $\Lambda_b$ values for the examined data segment show a marked variability during the 60 days of data collection; the smaller the band number (i.e., smaller periods), the higher the changes of $\Lambda_b$. An attempt to reveal a diurnal cycle was made, and the results are shown in Fig. 7. The percentages of cases with $(\Lambda_b)^{1/2}$ lesser than a set level are plotted in Fig. 7 vs time of day, for different bands and stations. The following levels were chosen: 7 \(\mu\)b rms represent the minimum detectable signal above the quantization noise, according to the measurement resolution (see section 2); lower levels indicate the absence of a significant pressure signal. Above this level, three thresholds of 15, 50, 100 \(\mu\)bar were selected.

The diagrams in Fig. 7 are composite histograms: each curve represents the percentage of data segments having a variance less than the corresponding threshold, while the difference of the percentages of any two curves indicates the amount of data segments (in percent) with variance higher than the lower threshold, but lower than the upper one.

The percentage of occurrences of higher rms levels increases with the band number. Moreover, station B shows clear evidence of larger percentages of occurrences for higher rms levels. Some curves also display an evident pattern as a function of time of day. Most of the $I_i$ ($i = 0, 1, 2, 3$) curves that have a substantial minimum, reach such a minimum in the early afternoon. The $I_0$ curves, which give the percentage of time without a detectable signal, are present mostly at night, if at all.

Finally, the effect of large scale weather phenomena on the spectral estimates was analyzed in detail for two ALPEx events. Intensive observations were carried out during the intervals from 0900 UTC 24 April to 0900 UTC 26 April and from 0300 UTC 29 April to 2100 UTC 1 May. Both events correspond to deep, moderately strong lee cyclogenesis, fed by intense mesoscale upper-level disturbances embedded in general northerly flow (Buzzi, personal communication). The spectral energies contained in each of the first four frequency bands were evaluated for each of the two cyclogetic events. They were compared with the 24-h periods preceding and following each episode. The results, which we do not show, are different for the two stations and do not enable us to draw any general conclusions. Station A, in the plains, shows energy levels which are consistently higher, by as much as a factor of 3, during cyclogenesis than during the day before or after. Such a characteristic is especially marked for bands 1 and 2. Station B, in the mountains, behaves differently during the two events. Its spectral energy monotonically increases from 23 April through 27 April, during the first cyclogetic event, while it monotonically decreases from 28 April through 2 May, during the second event.

The explanation of the behavior of station A probably resides in an increased wave activity related with the transit of the perturbation. The unsteady nature of the flow is likely to increase the level of turbulence as well as of gravity wave activity. The latter is consistent with the results reported by Gedzelman and Donn (1979). The results of station B seem to imply that the local topographic effects are overwhelming with respect to the large-scale, weather-generated disturbances. On the average, energies in band 1 and 2 are 15 dB higher in station B than in station A just as is the case in Fig. 5.

In both stations, closer inspection of individual time segments reveals the presence of fine structure that seems to have a diurnal cycle. This again suggests some decoupling of the local dynamics from the larger scale motions. The analysis of these two cyclogetic episodes should be taken as further evidence of the spatial as well as temporal variability of pressure spectra.

6. Discussion

In this paper, ground pressure records collected south of the Alpine massif were shown to display a nonstationary character and substantial time and space variability.

Nonstationarity implies that the statistical moments, calculated from the ensemble of the pressure realizations, depend on time. Using a Fourier series representation of the data, we have demonstrated that different frequency bands display nonstationarity time
Fig. 7. Composite histograms for $\lambda_0 = A_0^{1/2}$, where $A_0$ is the spectral density integrated over a frequency band [see (5.4)], vs time of day. Four levels, $I_0, I_1, I_2, I_3$, have been identified corresponding to $\lambda_0$ less than 7, 15, 50, 100 $\mu$b, respectively, with a fifth level $I_4$ corresponding to $\lambda_0 > 100$ $\mu$b. As an example, along the vertical lines corresponding to 0300 and 0700 LT of the bottom right diagram, we have: $I_0$ = percentage of time for which $\lambda_0$ is less than 7 $\mu$b; $I_1 - I_0$ = percentage of time $\lambda_0$ is such that 7 $\mu$b $\leq \lambda_0 < 15$ $\mu$b; $I_2 - I_1$ = percentage of time for which $\lambda_0$ is less than 50 $\mu$b; $I_3 - I_2$ = percentage of time for which 15 $\mu$b $\leq \lambda_0 < 50$ $\mu$b; $I_4 - I_3$ = percentage of time for which 50 $\mu$b $\leq \lambda_0 < 100$ $\mu$b; $I_4$ = 100% means that these components are present all the time with rms amplitude less than 100 $\mu$b.
scales proportional to the corresponding time periods, as indicated in Fig. 3. Such nonstationarity should be viewed as part of the irregular behavior of the atmospheric system. The implication of irregularity for the atmosphere in general and upon weather forecasting in particular was discussed by Lorenz (1984). Here we have confirmed such an intrinsic property of the atmosphere for periods ranging from 20 s to about 1 day and we have provided quantitative estimates of the corresponding nonstationarity scales, based on ground pressure information.

The results of the nonstationarity analysis were utilized in designing the spectral analysis procedure. Rather than let the various band widths be determined by the lengths of different segments of homogeneous data (with stationarity implicitly assumed), we have subdivided the frequency axis in bands of equal width in logarithmic scale and chosen the length of the time segments according to the nonstationarity requirements and the desired error bars.

Clear and independent proof of the nonstationarity of the process is also present in Figs. 5, 6 and 7. In particular, Figs. 6 and 7 show that the spectra have different shapes and energy levels for different time intervals. Furthermore, in sections 3 and 4, statistical arguments have been presented to demonstrate that individual spectra calculated from independent time segments differ from each other by much more than the error width due to the estimation process. Similarly, the slopes of the linear approximation to the individual spectra show a variability well outside the range accounted for by the errors of the spectral amplitudes. All this implies that the slope and energy distribution among the various frequency components are not independent of time.

It should be noted that other authors, for example, Gossard (1960), Bull et al. (1981), Vinnichenko (1970), provide either individual spectra or maximum–minimum variability ranges similar to ours for the power spectra of the atmospheric variables they consider. Many authors, on the other hand, do not provide such information since they focus their attention on the shape and level of the mean spectra and their possible universal nature.

In order to make comparisons with the results of other authors, a mean spectrum was generated for each station (Fig. 5) and the slopes of such mean spectra were also determined via the least-squares method [see (5.2)]. The results for the individual bands are summarized in Fig. 8, for both stations, while those for two or three adjacent bands are displayed in Fig. 9. The uncertainties of the slopes due to the errors of the spectral density estimates were calculated according to (5.3) and are indicated by vertical bars. Where the slope refers to more than one band, the error becomes comparable with the smallest single-band uncertainty; this follows from (5.3), since in such cases the quantity expressing the separation of frequencies from the mean grows more rapidly than the errors of the estimated spectral values. The slopes for the higher bands have wider uncertainty bars due to the larger spectral estimation errors. Moreover, the peaks in such bands make the linear approximation less meaningful.

An analysis of Figs. 8 and 9 indicates that in the period range between approximately 1 day and 6 h (bands 6 and 7), the spectral slopes reach their largest negative values and approach the theoretical value of −5 predicted by geostrophic turbulence. Surprisingly, though, the slope values are substantially less negative at larger periods (band 8 and 9): it is unclear whether this departure calls for a different physical explanation or simply implies poor performance of the least-squares linear fit to data displaying large and well-separated peaks.

The central part of the spectrum (bands 3, 4 and 5 for station A, shown in Fig. 9a, and 4 and 5 for station B, shown in Fig. 9b) presents a slope value close to
\(-7/3\), which is theoretically predicted by the turbulence theory for the mesoscale region. The wave theory predicts a slope of \(-7/3\), which appears to be smaller than the experimental values of Fig. 9. However, from Fig. 8, where the mean values for the slope are plotted for each single band, it seems that some uncertainty bars include the \(-7/3\) values.

At smaller scales the spectra display considerably less negative slopes implying energy levels larger than would exist if the slope for the central part of the spectrum extended to higher frequencies. The change in slope occurs at about 5 and 20 min for station A and B, respectively. Such departure can probably be explained in our case as small-scale turbulence or gravity wave activity related to surface roughness or topographic features. Indeed, the transition in slope begins in station B at periods longer than in station A, with B in the foothills of the mountains and A about 50 km into the plains. It is interesting to note that pressure spectra obtained by Gossard (1960) and by Herron et al. (1969) display a similar behavior with a flattening of the spectrum in the range of about 1 to 6 min and 0.5 to 4 min, respectively. The same can be said of the average spectra presented by Vinnichenko (1970) for the E–W wind components in the free atmosphere. These spectra were obtained by averaging all spectra relative to turbulence conditions deduced from flight reports and reveal an almost flat region in the range of about 1 to 15 min. Bull et al. (1981) also provide evidence of a flattening of the spectrum in the 2 to 5 min period range, though not as intense as ours. Their results are obtained using a network of three microbarographs, with spacing less than 2 km, which allow them to calculate spectral densities in the range from 2 to 128 min and to identify wave events with a median period of 6 min and a median amplitude of 55 \(\mu\)bar. By comparing results from a site in a flat terrain near the Baltic Sea with those from a station 10 to 20 km from the Balkanic mountains, they show substantial increase in wave activity for the station in the foothills. This result is of particular interest here since their two stations are in a position relative to the mountains somewhat similar to that of our stations A and B.

All these results provide evidence for a higher energy content in the range 1–20 min between the mesoscale, at longer periods, and three-dimensional turbulence, at shorter periods, not covered by our data. Such higher energetic content in this transition region can be justified by the presence of an energy source. The theory that explains the mesoscale spectral density in terms of quasi-two-dimensional turbulence (Gage, 1979; Lilly, 1983) needs a small-scale source of turbulence energy, which propagates upscale in the mesoscale region and downwards into the inertial subrange (see also Fig. 1 of Gage and Nastrom, 1986). We suggest that the scales included in our transition region are compatible with the location of the proposed energy source. On the other hand, the wave activity detected by Gossard and Munk (1954), Bull et al. (1981), and others suggests a different possible interpretation for the nature of the energy source in the transition region. Energy in a form of waves is injected in such a range by one of the several mechanisms of wave generation (see Einaudi et al., 1978/79). The downward energy transfer is due to decaying waves, while triad interactions (Orlanski and Cerasoli, 1981; Fritts, 1982; Chimonas and Grant, 1984) are responsible for the energy upscaling.

Finally, the behavior of the pressure spectra appears to depend strongly on geographical position. The flattening of the spectra and the energy levels at station A are quite different from those at station B, as previously discussed. The temporal behavior of the spectra at the two stations, summarized in Figs. 6 and 7, is also substantially different.
The temporal and spatial variability which have been demonstrated in this paper should alert us to the consequences of the averaging process used in the derivation of the mean spectra. Its existence throws some doubt on the concept of universality of the energy distribution, in frequency or wavenumber space, and on its physical origin. This, is, of course, aside from the general concept that more energy exists at larger temporal and spatial scales than at lower ones.

The question of whether the spectral variability is a result of competing physical mechanisms appears to us to be very important. In fact, our understanding of its origin may very well be a key element towards our understanding of spectral behavior of atmospheric or oceanic variables.

Further experimental evidence and, possibly, numerical simulations will be needed in order to validate different theoretical interpretations of atmospheric or oceanic spectra.

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APPENDIX

Data Processing

The relationship among the length N of the data segment under examination, the number m of adjacent spectrum values for the moving average in the frequency domain, and the bandwidth uncertainty ΔB for the Daniell estimator with time tapering is given by Koopmans (1974)

\[ N = m/(ΔBΔt), \]  
(A1)

where the parameter \( k \) represents the effect of the tapering window in the time domain.

The estimated spectrum is shown (Jenkins and Watts, 1968) to approximately follow a \( χ^2 \)-distribution with \( r \) degrees of freedom and is subject to the following inequalities:

\[ [rS(f)/a] \leq S(f) \leq [rS(f)/b], \]  
(A2)

i.e., the "true" spectral value \( S(f) \) for a given frequency \( f \) is bounded by values depending on the estimated \( S(f) \), with a \( q/100 \% \) probability. The quantity \( r \) is related to the moving average width, \( m \), by

\[ r = 2m/k, \]  
(A3)

and the values of \( a \) and \( b \) are set according to

\[ \text{Prob}(a \leq χ^2 \leq b) = q/100. \]  
(A4)

Equation (A2) also holds if the logarithm of all terms is taken, so that the confidence limits for spectra measured in dB are constructed by adding the quantity 10 \( \log_{10}r/b \) to, and subtracting 10\( \log_{10}r/a \) from the estimated value 10 \( \log_{10}S(t) \).

The desire for small error bars (i.e., large \( m \)) and high frequency resolution (i.e., small \( ΔB \)) calls for the analysis of long data records, but it is in contrast with the need for the time series to be short enough to be considered stationary. On the other hand, reducing \( N \) while keeping \( m \) fixed makes the resolution larger and has the effect of smearing out possible peaks of the spectrum. Finally, reducing \( N \) with \( ΔB \) fixed makes the spectrum unreliable, because the confidence limits of the estimates become larger.

The strategy of compromising on the choice of the above parameters resulted in subdividing the frequency scale in two parts. The first five high-frequency bands show a marked nonstationarity as discussed in section 3. The requirement for small error bars was established by postulating that a ±2 dB error bar at 95% confidence level was acceptable. We chose \( r = 32 \) effective degrees of freedom (corresponding to error bars of +2.4 dB and −1.9 dB with respect to the estimated values) and we set the number of points for the moving average according to (A3). The value of \( k \) is assumed to be known: a discussion of the choice of the tapering window is given later. On the other hand, the effective resolution bandwidth was assumed to be at most half the lower frequency \( f_L \) of the band, i.e.,

\[ ΔB \leq f_L/2. \]  
(A5)

The corresponding number of samples for each segment was then calculated from (A1). The time interval \( NΔt \) was compared with the nonstationarity time scale for the appropriate frequency band and a possible necessary reduction was obtained by compromising on \( ΔB \) and by reducing the number of degrees of freedom (i.e., accepting larger error bars).

The remaining four low frequency bands were designed with a smaller effective resolution so that the peaks could be emphasized. Equation (A5) was substituted by the more stringent requirement

\[ ΔB \leq f_L/10. \]  
(A6)

The nonstationarity at these time scales is less severe. Rather, the total length of the data base forces an upper limit for \( N \), so that the confidence interval becomes the dependent parameter: somewhat higher error bars must be expected for the low frequency part of the spectrum.

Table 3 summarizes the characteristics of the nine frequency bands, as they resulted from the application of the outlined strategy. The ratio effective-to-nominal resolution bandwidth (which happens to equal the number of degrees of freedom) ranges from 32 to 4. This, in turn, is reflected into the error bar values.

The analysis of the filtered signal for any single band was performed through the application of the smoothing window to each data segment whose length is discussed here. The particular window used here is one with very good sidelobe behavior (Nuttal, 1981)
\[ w(t) = \left( \frac{1}{W} \right) \sum_{k=0}^{K_w} a_k \cos[2\pi kt/(N\Delta t)], \]
for \(|t| \leq N\Delta t/2, \]
\[ (A7) \]

where \( W \) is a normalizing factor such that \( \int w^2\,dt = 1 \), and \( N\Delta t \) is the segment length. Following Nultal (1981), \( K_w \) was chosen equal to 3 with weights \( a_0 = 0.355768, a_1 = 0.487396, a_2 = 0.144232, \) and \( a_3 = 0.012604 \) in order to minimize the maximum sidelobe. The drawback is a larger main lobe. However, the main lobe width of the window defined by (A7) is comparable with the expected bandwidth resolution (see \( \Delta B \) above).

The second step was to take the FFT of the windowed data segment, doubled in size with padding zeros in order to oversample the spectrum and gain a better approximation to it. Obviously, padding with zeros does not increase the bandwidth resolution of the calculated spectrum.

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