Intransitive Multiple Equilibria in Eddy-Active Barotropic Flows

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ABSTRACT

Possible multiple equilibria of large-scale flows over topography have been subjects of many recent investigations. Early suggestions of Charney and DeVore, Hart and Wiin-Nielsen were based upon idealized flows of very few degrees of freedom. Subsequent studies based upon more elaborate systems failed to show intransitive multiple equilibria. On the contrary, we exhibit well-resolved, fully eddy-active model results which show multiple equilibria to be well-established over a range of parameters. "Blocked" regimes are characterized by transient eddy activity. Under steady external forcing, the flow regimes appear to be intransitive.

1. Multiple equilibria: The question

Based upon truncated spectral models with few degrees of freedom, describing barotropic quasi-geostrophic flow over topography, Charney and DeVore (1979, hereafter CDV), Hart (1979) and Wiin-Nielsen (1979) showed that more than one steady solution is possible with steady zonal forcing. When three solutions exist, two are stable and the third is unstable. Typically one stable solution is characterized by strong zonal flow while the other stable solution exhibits large standing waves with weakened zonal flow. The two stable states are sometimes termed "zonal" and "blocked."

The original works have been extended to include baroclinic systems, to include more horizontal modes and to compare spherical and Cartesian geometries. See Charney and Strauss (1980), Davey (1980, 1981), Roads (1980, 1982), Trevisan and Buzzi (1980), Charney et al. (1981), Egger (1981), Källén (1981, 1982, 1983), Pedlosky (1981), Reinhold and Pierrehumbert (1982), Yoden (1983a,b and 1985), Legras and Ghil (1984, 1985), Rambaldi and Mo (1984), Moritz (1984), Speranza (1985), Sutera (1985) and Vallis (1985). In these larger systems, existence of multiple equilibria are less clear. Reinhold and Pierrehumbert (1982) and Legras and Ghil (1984, 1985) find flow regimes that resemble qualitatively the simpler multiple equilibria but which do not exhibit intransitive behavior under steady forcing. In a 25-mode spherical calculation, Legras and Ghil find a CDV-type result only under unrealistically strong zonal forcing.

Recently Källén (1985) and Tung and Rosenthal (1985, hereafter TR) have reexamined the question of multiple equilibria. Using a spherical model with triangular spectral truncation at wavenumber 42 and topography consisting of a single meridionally oriented ridge of height 1600 m, Källén does not find multiple steady states. By slow time variation of zonal forcing, Källén obtains a "hysteresis-like" effect with two branches resembling somewhat the CDV-type multiple states. However, if the rate of change of zonal forcing is sufficiently slow, Källén's results tend toward a single-valued zonal flow.

Tung and Rosenthal investigate multiple equilibria for barotropic flow in a Cartesian channel, retaining 60 modes in the horizontal and performing higher resolution experiments with 240 modes. Two forms of topography are used: a system of periodic ridges after CDV or Hart; and a topography defined by a spectrum of terrestrial topography. For the cases investigated and the range of parameters examined, TR do not find multiple equilibria.

Two parameters of importance are Ekman drag coefficient and strength of zonal forcing (U_*). The physical basis for assigning values to these parameters is unclear. Charney et al. (1981) and Rambaldi and Mo (1984) specify drag corresponding to damping time scale of about 15 days. Källén examines both 5- and 20-day damping. Tung and Rosenthal investigate a range of damping from 5 to 30 days with zonal forcing $U_* = 33$ m s^{-1} to 50 m s^{-1}. At 55°N, Källén's forcing varies from $U_* = 22$ m s^{-1} to 50 m s^{-1}.

Results of the several investigations, for the variety of circumstances and the range of parameters mentioned, suggest that intransitive multiple equilibria do not occur for plausibly realistic parameter values. Earlier accounts of multiple equilibria are attributed to artifacts of severe spectral truncation. In the following section, our purpose is to illustrate by example that this conclusion is unwarranted. We exhibit apparently intransitive multiple equilibria over a range of parameter values in a model that resolves scales of eddy motion.

2. Model results

Following the notation of TR, we consider the barotropic β-plane potential vorticity equation

$$\frac{\partial}{\partial t}\nabla^2\psi + J(\psi, \nabla^2\psi + \beta y) = -J(\psi, h) - k\nabla^2(\psi - \psi_*) - a\nabla^6\psi \quad (1)$$

where ψ is the quasi-geostrophic streamfunction, J the Jacobian determinant with respect to x (east) and y (north), h is topographic elevation as a fraction of scale depth, a is a coefficient of higher order friction which selectively damps small-scale vorticity, k is the Ekman drag coefficient, and ψ_* represents externally applied force. Following Charney et al. (1981) or Rambaldi and Mo (1984) we assume that ψ_* appears as a uniform source of zonal momentum, $\psi_* = -U_* y$, with U_* a prescribed parameter. Evolution of meridionally averaged zonal flow U is given by the zonal momentum budget:

$$\frac{\partial}{\partial t}U = k(U_* - U) + \overline{h\frac{\partial}{\partial x}\psi} \quad (2)$$

where overbar denotes area averaging over the flow domain. Thus $U \equiv -\overline{\partial\psi/\partial y}$.

Motivated by concern for regionalized, as distinct from global, blocking, we consider a square domain of nominal size 6000 km. Topography is a series of meridional ridges of form $\cos(\pi x/1000 \text{ km})$ to which is added broadband spectrum of random relief. Overall rms elevation is specified by a parameter $h_{\rm rms}$. Midlatitude β is 2×10^{-11} m^{-1} s^{-1}. Drag coefficient k has been assigned values from $k^{-1} = 4.4$ days to $k^{-1} = 8.7$ days. Zonal force U_* has been assigned values from 0.67 m s^{-1} to 33.3 m s^{-1}. Boundary conditions are that $\nabla^2\psi$ and h are periodic in both x and y over the 6000-km scale of the domain.

Such choice of physical parameters is, to some extent, arbitrary. For example, a 6000-km box might be viewed as too small. If one chooses, say, a 12 000-km box and takes $\beta = 10^{-11}$ m^{-1} s^{-1}, then reported values of U and U_* need be multiplied by 2 while values of k and of $h_{\rm rms}$ are unaffected.

Numerical integrations of (1) and (2) have been performed using energy and potential enstrophy conserving spectral transform methods (Orszag, 1971), with filtered leapfrog timestepping and exact evaluation of dissipation terms. Spectral truncation is isotropic at radial wavenumber 30, retaining 2870 degrees of freedom.

We have found robust multiple equilibria throughout much of the parameter space that we have explored. However, a complete and systematic exploration would greatly exceed the limits of our efforts. We exhibit some instances of multiple equilibria, remarking only that these exist and that they exist over an extensive range of parameters. An important observation is that the blocked regime appears to be characterized by unsteady flow. Transient eddy activity in the blocked regime induces fluctuations of $\overline{h\partial\psi/\partial x}$, thereby inducing fluctuations of U. Although unsteadiness distinguishes multiple equilibria in this study from the simpler type suggested by CDV and others, our results otherwise resemble the earlier suggestions. An example is shown in Fig. 1, including a map of topography, an instantaneous streamfunction map in the blocked regime, and the steady streamfunction map in the zonal state. In this example $h_{\rm rms} = 629$ m, $U_* = 13.3$ m s^{-1}, and $k^{-1} = 8.7$ days.

Unsteadiness in the blocked regime poses a difficult question: is this regime truly intransitive or only persistent? One approach is to extend the numerical integration to ever greater time, waiting to see if either a spontaneous transition to zonal state occurs or temporal variability wanes, revealing a steady blocked state. To the extent that we've pursued this approach, we observe neither cessation of variability nor transition except for transition when one is very near the parameter space boundary (small $h_{\rm rms}$, large k, and/or large U_*) for extinction of the blocked equilibrium. Time-series shown in Fig. 2 indicate the enduring variability without transition for a blocked regime at $h_{\rm rms} = 650$ m, $U_* = 13.3$ m s^{-1}, and $k^{-1} = 8.7$ days.

An alternative approach to the preceding question is to seek a steady blocked state by means of accelerated convergence of the numerical integrations. We have done two things. First, while integrating (1) as given, we have introduced into (2) a relaxation of U toward a moving time-average of U. Although fluctuations of U are suppressed in this way, temporal variability of ψ is not noticeably affected. Second, we have collected overall time-averaged ψ and U in order to inquire if these time averages are close to a steady state. Initializing (1) and (2) with time-averaged ψ and U, we observe that fluctuations rapidly develop to approximately the levels realized before averaging. The overall impression is that, for the parameters tested, a steady blocked state does not exist or, at least, attractors around one or more steady states are of very small phase volume.

Having not found a stable steady state, the question of transitivity remains open. In a further attempt to address this question, we have perturbed (1) by adding random torques (thermal noise) in a wavenumber band 5 to 7 with a slightly reddened ("pink") frequency spectrum. In this way we have increased temporal variances to more than double their undisturbed values for a case with $h_{\rm rms} = 629$ m, $U_* = 6.7$ m s^{-1}, and $k^{-1} = 17.5$ days. No transition from the blocked regime was observed. When the added noise was subsequently removed, temporal variability relaxed approximately to its previous levels. The indication is that the blocked regime is truly intransitive although this is not so proven.

Finally, one may inquire over what volume of the parameter space multiple equilibria exist. To answer this question by experimentation based upon direct

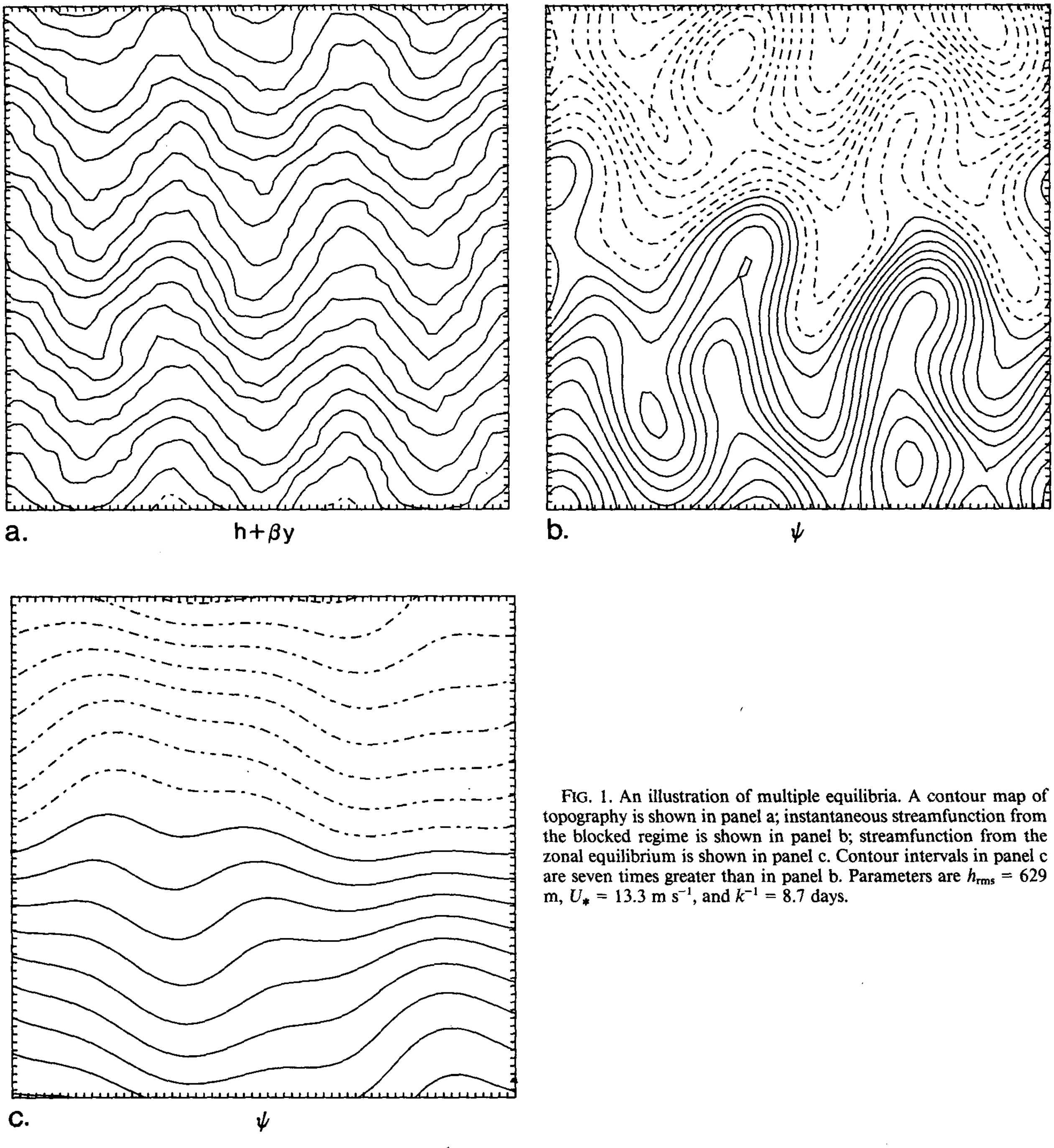

FIG. 1. An illustration of multiple equilibria. A contour map of topography is shown in panel a; instantaneous streamfunction from the blocked regime is shown in panel b; streamfunction from the zonal equilibrium is shown in panel c. Contour intervals in panel c are seven times greater than in panel b. Parameters are h_{rms} = 629 m, U_* = 13.3 m s^{-1}, and k^{-1} = 8.7 days.

integration of (1) and (2) is a daunting challenge. Even if one holds the basic geometry fixed, say as shown in Fig. 1, a complete exploration of the space defined by h_{rms}, U_* and k would be a task considerably outside our current resource.

For the present we only illustrate that multiple equilibria exist over some extended domain. Figure 3 shows U as dependent upon different values of h_{rms} for fixed U_* = 13.3 m s^{-1} and k^{-1} = 8.7 days. Vertical bars indicate rms fluctuation of U during integrations. In this case, multiple equilibria are found in a range from h_{rms} somewhat larger than 500 m to somewhat less than 700 m. The range is not very wide. However, we are motivated to display this case based partly upon plausibly "realistic" choices of parameters and based partly upon interest of Dr. K. K. Tung (personal communication) concerning multiple equilibria near h_{rms} = 600 m.

Concluding, it hardly bears mention that the results reported here are not a thorough study. Neither do we

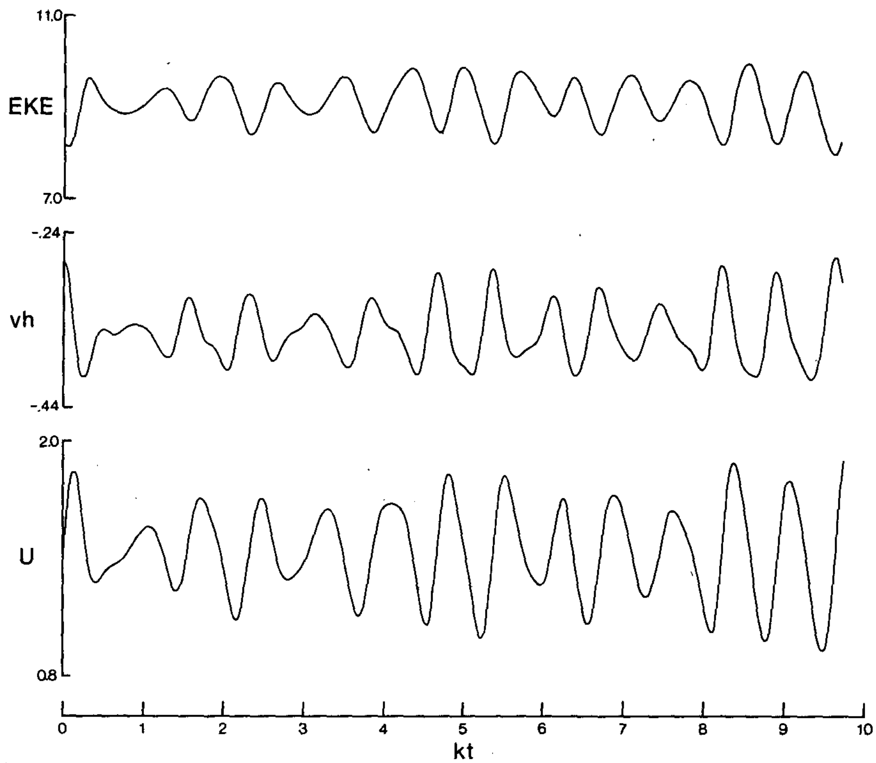

FIG. 2. Time series of eddy kinetic energy (EKE), topographic stress (vh) and mean zonal flow (U) are shown for the blocked regime when h_{rms} = 650 m, U_* = 13.3 m s^{-1}, and k^{-1} = 8.7 days.

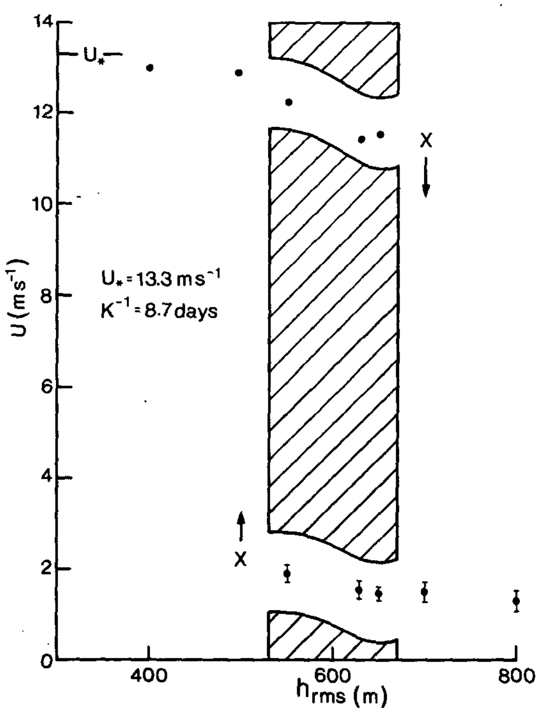

FIG. 3. Mean zonal flow (U) is shown in both blocked and zonal equilibria for a number of values of h_{rms}. Other parameters are U_* = 13.3 m s^{-1}, and k^{-1} = 8.7 days. Vertical bars indicate ± one standard deviation at statistical stationarity. The region supporting multiple equilibria is shown shaded.

address theoretical issues concerning the statistical dynamics of systems supporting multiple equilibria or concerning the dynamical character of the attractor in the blocked regime. The configuration we have examined differs from those examined by Källén or by TR in several regards. Simply, we show that over a range of conditions with some plausible elements of realism, multiple equilibria do exist in a numerical model with "many" degrees of freedom. The blocked regime characteristically remains eddy-active; however, to the extent that we have been able to determine, the equilibria are intransitive over a substantial extent of the parameter space.

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