Transient Eddies and the Seasonal Mean Rotational Flow

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ABSTRACT

Virtually all investigations of transient-eddy effects on the large-scale mean vorticity start from the premise that only the rotational transient motion need be considered. In this paper, the seasonal mean vorticity balance at 250 mb is examined, with particular emphasis on those transient terms that are associated with the horizontally divergent transient motion. The largest transient terms are, in fact, found to be the advection of vorticity by the divergent flow and the stretching term. These are only a factor of 2 smaller than the mean flow terms. However, these transient terms have a strong mutual cancellation. Their residual, the convergence of the vorticity flux associated with the divergent motion, although much smaller, is comparable on the planetary scale with the similar term associated with the rotational motion. These properties are interpreted using simple models. It is concluded that a representation of the vorticity flux by the transient divergent flow may be necessary in an accurate parameterization of transient eddies in global-scale climate models, and that any analysis of transient effects must include the divergent motions in a consistent manner.

1. Introduction

The contribution of the transient eddies to the time-mean flow and the possibility of parameterizing this contribution has been the subject of much discussion in recent years (Hoskins, 1983, and references therein). Implicit in analyses based on quasi-geostrophic theory such as that of Green (1970), Lau and Holopainen (1984) and Plumb (1986), and explicit in other investigations such as the *E*-vector work of Hoskins et al. (1983, hereafter HJW) is the assumption that the time-averaged vorticity flux by the transient eddies is predominantly associated with the rotational portion of the horizontal flow. However, as discussed by HJW, whereas the poleward and upward heat flux is an integral part of the existence of synoptic time-scale transients, the time-averaged vorticity flux by the rotational flow is associated merely with the elongation of the transients in some direction: isotropic eddies produce no such vorticity flux. Thus, even though the divergent component of the horizontal velocity is small compared with the rotational component for individual synoptic systems, there is no a priori reason to believe that the time-averaged flux of vorticity by the divergent component of the transient flow should be negligible compared with that by the rotational component.

This point is given added emphasis by Fig. 1, which shows the fluxes of vorticity by the horizontally rotational and divergent flow at 250 mb for the season December 1983–February 1984 for the total transients and for the band-pass, synoptic time-scale transients alone. It is clear that the flux by the divergent flow is generally not much smaller than that by the rotational flow and, if anything, shows a more coherent structure. This behavior appears to be quite general and has been verified for other seasons and also for an ensemble of 5-day forecasts made with the operational forecast model at the European Centre for Medium-range Weather Forecasts (ECMWF), which suggests that it is not an artifact of the data analysis. The band-pass divergent flux is predominantly westward in the Northern Hemisphere and eastward in the Southern Hemisphere, with maximum amplitude in the storm-track regions. The band-pass flux associated with the rotational motion is much noisier, but does exhibit a poleward component in the storm tracks.

The object of this paper is to reexamine the time-mean vorticity balance at an upper tropospheric level, concentrating on the role of the transient divergent motion, and to construct simple models that illustrate the basic nature of the terms connected with this motion. The examples will be drawn, as above, from the 250 mb flow during the December 1983–February 1984 season as described by the 6-hourly, initialized operational analyses on a global 5° latitude–longitude grid archived at ECMWF. These archives include the vertical velocity produced by the normal-mode initialization scheme. As discussed by Leith (1980), these will be an improvement in middle latitudes over those given by the quasi-geostrophic omega equation and should be sufficiently accurate for the present purposes. The contribution from small-scale gravity wave motion will be negligible, given the spatial and temporal sampling of the data. The band-pass synoptic time-scale transient motions are isolated by a 31-point Lanczos filter (Duchon, 1979), applied to the daily 1200 UTC values, that passes periods from 2.16 to 6.15 days.

Section 2 gives a brief summary of the various ways

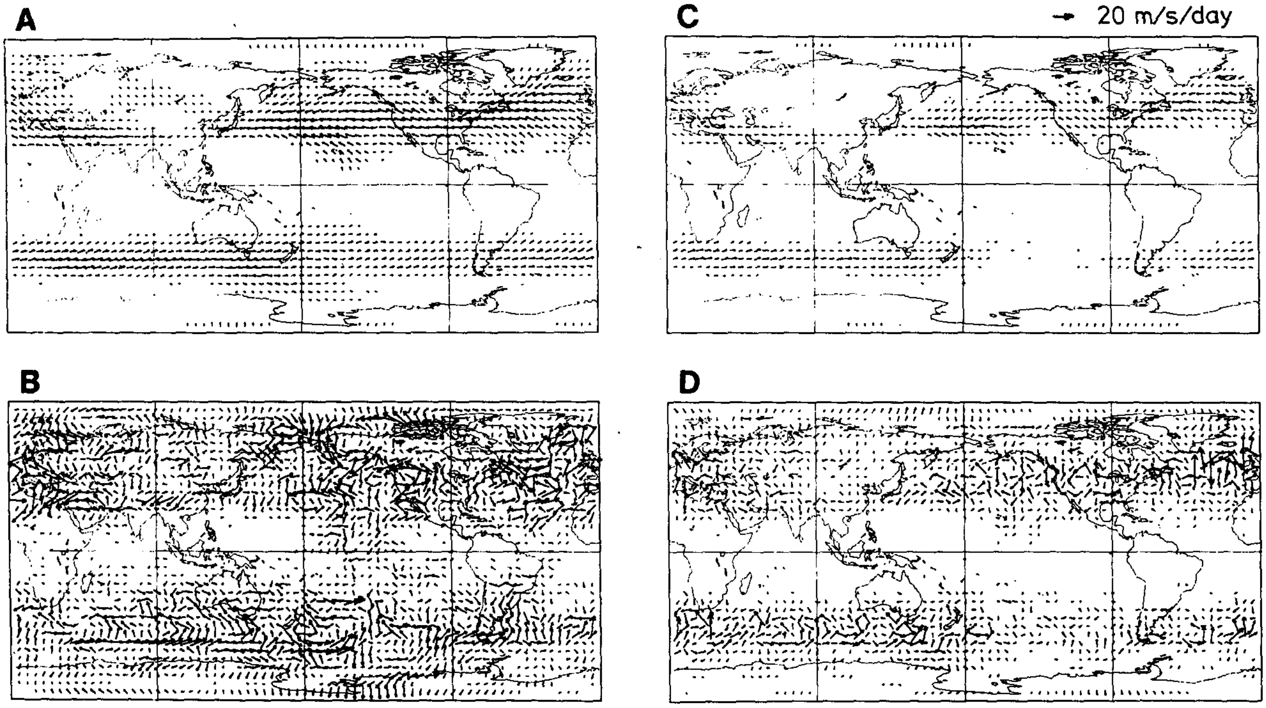

FIG. 1. Vorticity fluxes at 250 mb during December 1983–February 1984 associated with (a) the transient divergent flow, $\overline{\mathbf{v}'_\chi\zeta'}$, and (b) the transient rotational flow, $\overline{\mathbf{v}'_\psi\zeta'}$. The contribution to these fluxes from synoptic-scale transients is shown in (c) and (d), respectively. (a) and (b) are based on data analysed every 6 hours, (c) and (d) on daily 1200 UTC values operated on by a 31-point Lanczos filter to isolate eddies in the 2.16 to 6.15 day band. The arrow scale is the same in all four panels.

in which the transient contribution to the mean vorticity equation can be displayed. Section 3 then uses the data for December 1983–February 1984 to illustrate the seasonal-mean vorticity balance. Section 4 provides an interpretation of the terms involving the transient divergent motion in terms of simple models, and some concluding remarks are made in section 5.

2. Transient terms in the mean vorticity equation

Ignoring frictional and small-scale processes, the seasonal-mean vertical component of the vorticity equation in pressure coordinates may be written

$$\frac{\partial\bar{\zeta}}{\partial t}=0=-\nabla\cdot\bar{\mathbf{v}}\bar{\zeta}-\mathbf{k}\cdot\nabla\times\bar{\omega}\frac{\partial\bar{\mathbf{v}}}{\partial p}-\nabla\cdot\overline{\mathbf{v}'\zeta'}-\mathbf{k}\cdot\nabla\times\overline{\omega'\frac{\partial\mathbf{v}'}{\partial p}}. \quad (1)$$

Here the mean absolute vorticity $\bar{\zeta}=\bar{\xi}+f$, where $\bar{\xi}$ is the mean relative vorticity and f the Coriolis parameter. Scale analysis suggests, and actual budgets confirm, that the ω' term is extremely small. The convergence of the horizontal flux by the mean flow may be split into advection and stretching contributions:

$$-\nabla\cdot\bar{\mathbf{v}}\bar{\zeta}=-\bar{\mathbf{v}}\cdot\nabla\bar{\zeta}-\bar{\zeta}\bar{D}, \quad (2)$$

where $\bar{D}$ represents the horizontal divergence, $\nabla\cdot\bar{\mathbf{v}}$. Of course, $\bar{D}$ will, in general, include a portion due to the existence of transient fluxes of heat and vorticity as well as diabatic heating associated with the transients. However, the direct transient-eddy effect in (1) is

$$\left(\frac{\partial\bar{\zeta}}{\partial t}\right)_e=-\nabla\cdot\overline{\mathbf{v}'\zeta'}-\mathbf{k}\cdot\nabla\times\overline{\omega'\frac{\partial\mathbf{v}'}{\partial p}}. \quad (3)$$

Holopainen (1978), Holopainen and Oort (1981) and HJW have introduced the anisotropic eddy correlation components $M=\frac{1}{2}(\overline{u'^2-v'^2})$, $N=\overline{u'v'}$. In terms of these, the convergence of the transient horizontal flux of vorticity may be written:

$$-\nabla\cdot\overline{\mathbf{v}'\zeta'}=G(M,N)+\mathbf{k}\cdot\nabla\times\overline{\mathbf{v}'D'}, \quad (4a)$$

where

$$G(M,N)=2\frac{\partial^2M}{\partial x\partial y}+\frac{\partial^2N}{\partial y^2}-\frac{\partial^2N}{\partial x^2}.$$

The curl of the divergence flux is thus the error made in estimating this quantity from the eddy velocity correlations alone. This divergence flux term when combined with the very small ω' term in (3) becomes the curl of the vertical momentum-flux convergence

$$-\mathbf{k}\cdot\nabla\times\frac{\partial}{\partial p}\overline{\omega'\mathbf{v}'}. \quad (5)$$

For ω' zero at upper and lower boundaries, this quantity vanishes in the vertical integral. This is the reason given by Holopainen and Oort (1982) for neglecting it and considering only the M and N terms. The importance of the divergence flux term in (4) at any one level is, however, less apparent.

Alternative ways of writing the convergence of the horizontal vorticity flux, (4a), in terms of the rotational (suffix ψ) and divergent (suffix χ) motion of the transients are

$$-\nabla\cdot\overline{\mathbf{v}'\zeta'}=-\overline{\mathbf{v}'\cdot\nabla\zeta'}-\overline{\zeta'D'} \tag{4b}$$

$$=-\nabla\cdot\overline{\mathbf{v}'_\psi\zeta'}-\nabla\cdot\overline{\mathbf{v}'_\chi\zeta'} \tag{4c}$$

$$=G(M_\psi,N_\psi)-\nabla\cdot\overline{\mathbf{v}'_\chi\zeta'}. \tag{4d}$$

Equation (4b) is a form involving advection of vorticity by the full transient velocity $\mathbf{v}' = \mathbf{v}'_\psi + \mathbf{v}'_\chi$ and stretching. Form (4c) is a split into the convergences of the fluxes due to the rotational and divergent motions, and in (4d) the former is written exactly in terms of the eddy rotational-flow correlations.

In (3) the transient motions are considered in their role of forcing the mean vorticity, but sometimes it is convenient, following Holopainen et al. (1982) and HJW, to discuss their equivalent forcing of the mean streamfunction or rotational momentum. Thus, if a term Z contributes to the forcing of the mean vorticity and $S = \nabla^{-2}Z$, then

$$\left(\frac{\partial\bar{\psi}}{\partial t}\right)_e=S, \tag{6}$$

$$\left(\frac{\partial\bar{\mathbf{v}}_\psi}{\partial t}\right)_e=\mathbf{k}\times\nabla S. \tag{7}$$

As a particular example, if the horizontal eddy-vorticity flux is split into its rotational and divergent parts:

$$\overline{\mathbf{v}'\zeta'}=\mathbf{k}\times\nabla A+\nabla B, \tag{8}$$

then

$$Z=-\nabla^2 B,$$

$$\left(\frac{\partial\bar{\psi}}{\partial t}\right)_e=-B, \tag{9}$$

$$\left(\frac{\partial\bar{\mathbf{v}}_\psi}{\partial t}\right)_e=-\mathbf{k}\times\nabla B. \tag{10}$$

It should be noted that a local interpretation of (6) and (7) is not strictly valid, as is also true for ψ, χ, $\mathbf{v}_\psi$ and $\mathbf{v}_\chi$. An alternative view of $-S$ is that it is a form of Z which is spatially smoothed to expose its structure on the largest planetary length scales.

3. The mean vorticity balance at 250 mb for December 1983–February 1984

The mean flow terms in the mean vorticity budget (1) will be discussed first. As stated in section 2, the mean vertical advection and twisting terms are negligible on the planetary scale of interest in this paper. Further, as in (2), the mean flow vorticity flux convergence, $-\nabla\cdot\bar{\mathbf{v}}\bar{\zeta}$, may be split into the mean flow advection, $-\bar{\mathbf{v}}\cdot\nabla\bar{\zeta}$, and mean stretching, $-\bar{\zeta}\bar{D}$. The mean horizontal wind is nearly parallel to contours of the mean absolute vorticity $\bar{\zeta}$, and it is tempting to think of it, to a first approximation, as a steady nonlinear solution of the unforced barotropic vorticity equation. However, actual computation shows that the magnitude of the mean advection term $-\bar{\mathbf{v}}\cdot\nabla\bar{\zeta}$ can be as large as $4\times10^{-10}\ \mathrm{s}^{-2}$ on the poleward side of the East Asian jet (not shown), implying a vorticity tendency of Ω in only 2–3 days. This large tendency is counterbalanced mainly by the mean stretching $-\bar{\zeta}\bar{D}$, leaving a relatively small residual to be balanced by the combined transient-eddy terms (3). Both the mean advection and the mean stretching can be seriously misrepresented in this balance by replacing $\bar{\zeta}$ by its zonal mean value $[\bar{\zeta}]$ or the Coriolis parameter f. The zonal variations in $\bar{\zeta}$ and in its gradient are significant on even the largest scales and imply a varying restoring force for Rossby-wave motions that is appreciably different from β. Another complication, which will be discussed elsewhere, is that the mean divergent flow $\bar{\mathbf{v}}_\chi$ contributes significantly to the mean advection $-\bar{\mathbf{v}}\cdot\nabla\bar{\zeta}$. This appears less surprising upon recognizing that although the mean divergent flow is much weaker than the mean rotational flow, it is directed almost perpendicular to the mean $\bar{\zeta}$ contours, particularly in the jet entrance and exit regions.

As in Eq. (4b), the transient-eddy vorticity flux convergence may be split into stretching and advection terms. These two terms are shown in Figs. 2a and 2b with a contour interval of $2.5\times10^{-11}\ \mathrm{s}^{-2}$. As will be the case for all the terms in the vorticity budget, these fields have been smoothed for presentation purposes in the manner of Sardeshmukh and Hoskins (1984) by performing a spectral truncation at $n = 24$ and multiplying the spectral coefficients by a function of the form $\exp\{-K[n(n+1)]^2\}$, where K is chosen such that the function is 0.1 at $n = 24$. The values of the individual transient-eddy terms are only a factor of 2 or 3 smaller than their mean counterparts. The stretching term, which is usually neglected, is in fact comparable to the advection term. The two terms exhibit a large cancellation, particularly along the axes of the major storm tracks in the two hemispheres. Plots of $\overline{\zeta'^2}$ and $\overline{D'^2}$ are shown in Figs. 2c and 2d to give an idea of the location and intensity of these storm tracks; note the smaller contouring interval for $\overline{D'^2}$. Although $-\overline{\zeta'D'}$ is an important term in the vorticity budget, it still represents only a weak correlation between ζ' and D', generally on the order of 0.2 or less. This weak correlation is nonetheless crucial for the existence of baroclinic instability, and the opposite signs of $-\overline{\zeta'D'}$ in the two hemispheres are entirely consistent with this fact, as will be discussed in section 4.

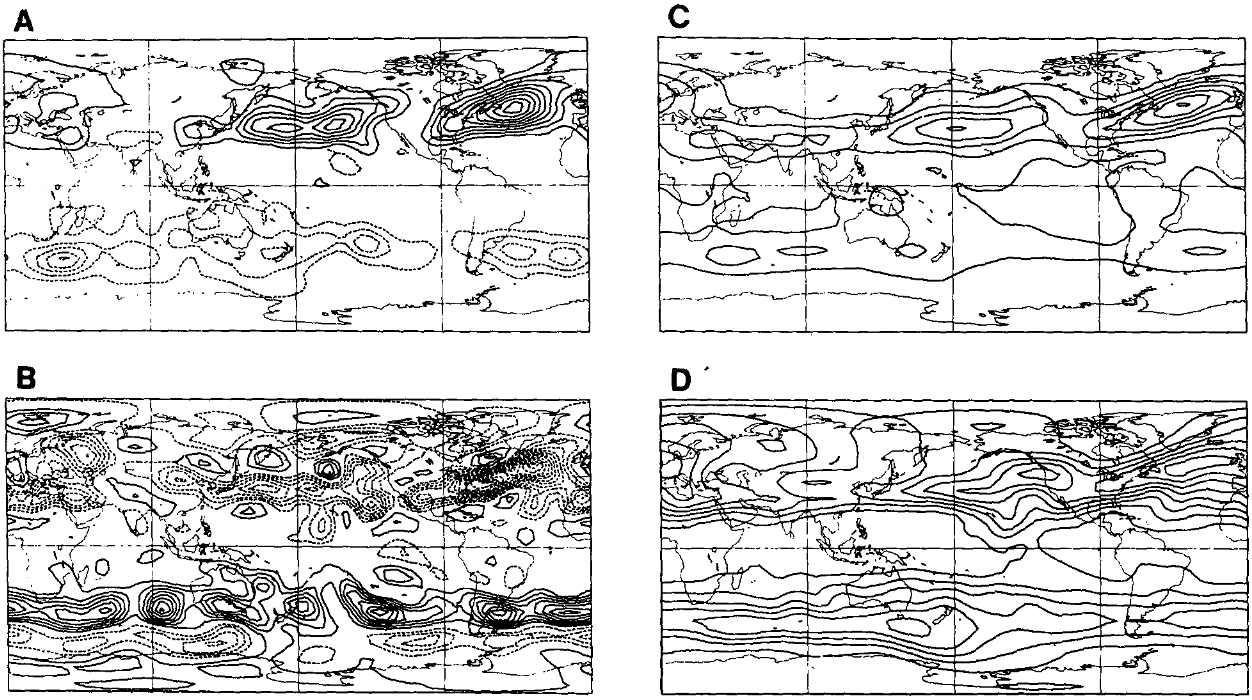

FIG. 2. (a) Transient stretching, $-\overline{\zeta' D'}$, and (b) transient vorticity advection, $-\overline{\mathbf{v}' \cdot \nabla \zeta'}$, at 250 mb, both contoured with an interval of 2.5 $\times 10^{-11}$ s^{-2}. (c) Variance of divergence, $\overline{D'^2}$, and (d) variance of vorticity, $\overline{\zeta'^2}$. The contour interval is 5×10^{-11} s^{-2} in (c) and 5×10^{-10} s^{-2} in (d). Negative values in (a) and (b) are indicated by dashed contours. The zero contour is not shown.

If the transient divergent and rotational winds are scaled as $|\mathbf{v}'_\chi| \sim L(\overline{D'^2})^{1/2}$ and $|\mathbf{v}'_\psi| \sim L(\overline{\zeta'^2})^{1/2}$, then, using typical values, we get $|\mathbf{v}'_\psi| \sim 3|\mathbf{v}'_\chi|$ in the Pacific and Atlantic storm tracks and $\sim 5|\mathbf{v}'_\chi|$ in the Southern Hemisphere track. The assumption of horizontal nondivergence for the eddies appears even less satisfactory upon recalling that in Fig. 1 $|\overline{\mathbf{v}'_\psi \zeta'}|$ and $|\overline{\mathbf{v}'_\chi \zeta'}|$ were comparable rather than different by a factor of 3 or 5 as this scaling would suggest. As stressed in the Introduction, the correlation between $\mathbf{v}'_\psi$ and ζ' is subtle and depends on derivatives of M, N and thus upon eddy anisotropy; it is zero for isotropic eddies. However, as will be discussed below, $\overline{\mathbf{v}'_\chi \zeta'}$ is nonzero even for neutrally stable ($-\overline{\zeta' D'} \equiv 0$) waves.

The transient advection has a more detailed meridional structure than the transient stretching, which results in their sum, $-\nabla \cdot \overline{\mathbf{v}'\zeta'}$, being noisy (Fig. 3a). The figure, including as it does the effects of all transient eddies, is difficult to interpret as it stands; also, the strong cancellation between the contributing terms makes it vulnerable to observation and analysis errors. But it is still possible to discern the forcing of cyclonic vorticity poleward and anticyclonic vorticity equatorward of the axes of the storm tracks in either hemisphere, consistent with the forcing of mean westerly flow along the axes by the eddies.

The transient advection $-\overline{\mathbf{v}' \cdot \nabla \zeta'}$ includes a contribution from the divergent flow $-\overline{\mathbf{v}'_\chi \cdot \nabla \zeta'}$ which, when combined with the transient stretching, becomes $-\nabla \cdot \overline{\mathbf{v}'_\chi \zeta'}$ in Eq. (4c). This term, which is the convergence of the flux in Fig. 1a, is shown in Fig. 3b. It represents about a 20 percent error in approximating $-\nabla \cdot \overline{\mathbf{v}'\zeta'}$ as $-\nabla \cdot \overline{\mathbf{v}'_\psi \zeta'}$ on length scales on the order of 1000 km. One could have anticipated this from Fig. 1; the $\overline{\mathbf{v}'_\chi \zeta'}$ fluxes, though comparable to the $\overline{\mathbf{v}'_\psi \zeta'}$ fluxes, showed a much smoother horizontal variation. Although it is not clear why this should be so, it is clearly consistent with the large cancellation between the transient advection and stretching.

The curl of the divergence flux, $\overline{\mathbf{v}'D'}$, and of the vertical momentum flux convergence, $-\partial(\overline{\omega'\mathbf{v}'})/\partial p$, introduced in Eqs. (4a) and (5), are shown in Figs. 3c and 3d. Their similarity confirms the negligible value of the transient vertical advection and twisting terms. The close correspondence between Figs. 3b and 3c, arises from Eqs. (4a) and (4d), with the eddy correlation tensor components M and N being quite well approximated by the values M_ψ and N_ψ calculated from the rotational flow alone.

Figure 3 suggests that, on the 1000 km length scale at least, the assumptions normally made in assessing the effect of the transient eddies in (1) are not unreasonable. However, a different view is obtained from considering the equivalent forcing of the mean streamfunction and the mean rotational flow in the manner of (6) and (7). Figures 4a and 4b give S for the transient

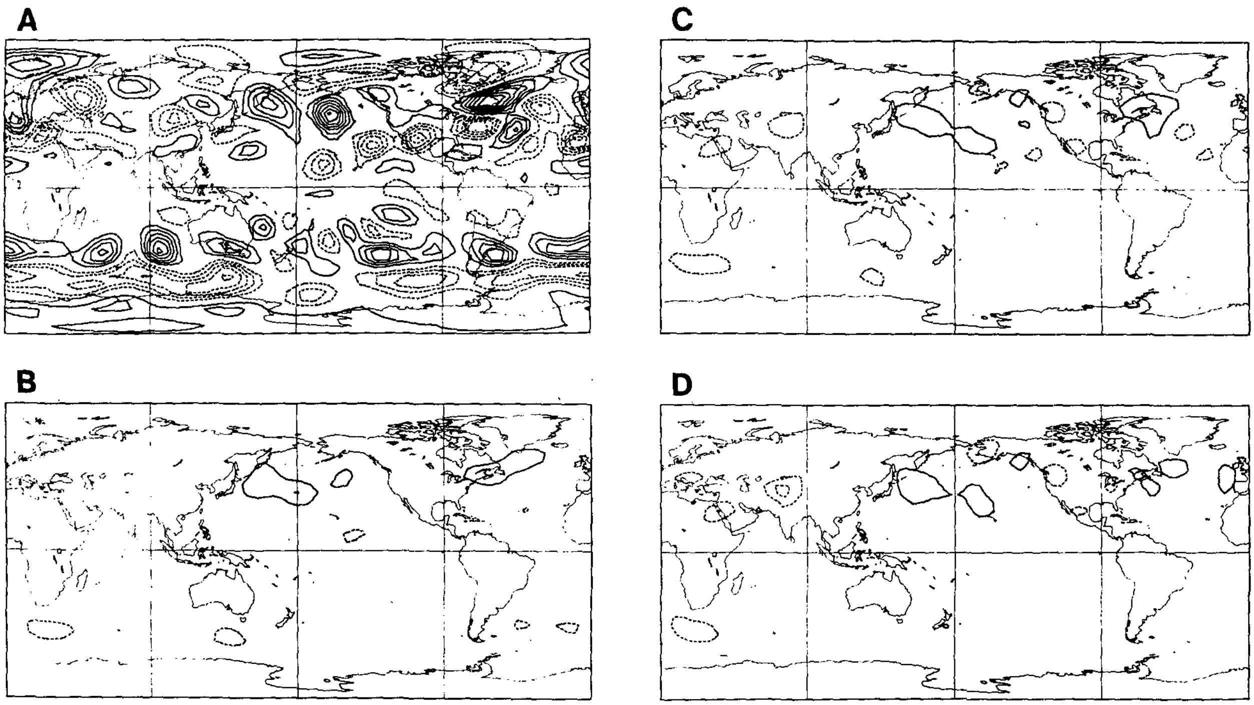

FIG. 3. (a) Convergence of the transient vorticity flux, $-\nabla\cdot\overline{\mathbf{v}'\zeta'}$; (b) convergence of the vorticity flux by the transient divergent flow, $-\nabla\cdot\overline{\mathbf{v}'_{\chi}\zeta'}$; (c) curl of the transient divergence flux, $\mathbf{k}\cdot\nabla\times\overline{\mathbf{v}'D'}$; and (d) curl of the transient vertical momentum flux convergence, $-\mathbf{k}\cdot\nabla\times\partial(\overline{\omega'\mathbf{v}'})/\partial p$. The contour interval is 2.5×10^{-11} s^{-2} in all four panels, and the zero contour is not shown.

stretching and vorticity advection terms, which were themselves shown in Figs. 2a and 2b. The cancellation between them is striking and was in fact the starting point for the investigation described in this paper. Their sum, shown in Fig. 4c, is S for the transient eddy vorticity-flux convergence, which was itself shown in Fig. 3a. Apart from the small ω' term in (3), it is this field that gives the direct transient-eddy forcing of the planetary-scale mean rotational flow. An approximation by the transient-eddy advection, Fig. 4b for example, would be disastrously wrong. Figure 4c does indicate, rather more clearly than Fig. 3a, the tendency of the transient eddies to accelerate the mean westerlies along the storm-track regions.

The contribution made to Fig. 4c by the horizontal flux associated with the divergent motion is shown in Fig. 4d. In Fig. 1, the flux $\overline{\mathbf{v}'_{\chi}\zeta'}$ had looked very important, whereas in the vorticity budget in Fig. 3 it appeared relatively unimportant. In Fig. 4, concentrating on the planetary scale, it again appears to be important. These remarks are clearly consistent with the different length scales apparent in Fig. 1.

Since $-\nabla\cdot\overline{\mathbf{v}'_{\chi}\zeta'} = -\overline{\mathbf{v}'_{\chi}\cdot\nabla\zeta'} - \overline{\zeta'D'}$, comparing Figs. 4a and 4d shows that on planetary scales there is a dramatic cancellation between the transient eddy terms associated with stretching and with advection by the divergent flow. The latter term is dominant in the total advection term in Fig. 4b, and this is the source of the cancellation between it and the stretching term. Advection by the rotational flow is responsible for the smaller part of the term in Fig. 4b and would be equal to Fig. 4d subtracted from Fig. 4c. Smoothed versions of the terms $\mathbf{k}\cdot\nabla\times\overline{\mathbf{v}'D'}$ and $-\mathbf{k}\cdot\nabla\times\partial(\overline{\omega'\mathbf{v}'})/\partial p$ shown in Figs. 3c and 3d are almost identical with Fig. 4d and are not shown.

For transient eddies on the synoptic time scale, almost all of the above remarks apply equally well. There is again a strong cancellation between the stretching term (Fig. 5a) and the advection term (Fig. 5b). Their sum, the eddy vorticity flux convergence (Fig. 5c), gives a clearer view than Fig. 3a of the mean flow forcing by the eddies in the storm tracks. The curl of the divergence flux (Fig. 5d) is very similar to the terms $-\nabla\cdot\overline{\mathbf{v}'_{\chi}\zeta'}$ and $-\mathbf{k}\cdot\nabla\times\partial(\overline{\omega'\mathbf{v}'})/\partial p$ (not shown) and is hardly changed from its unfiltered picture (Fig. 3c), so that the error in neglecting these terms are relatively slightly more serious for these band-pass eddies. Maps of the terms smoothed by the ∇^{-2} operator (not shown) confirm the points already made.

A final view of the relative importance of the transient vorticity advections by the divergent and rotational motions for unfiltered and filtered eddies is given in Fig. 6, which shows $(\partial\bar{\mathbf{v}}_{\psi}/\partial t)_e$ implied by the four panels of Fig. 1, determined as in (8) and (10). Again, the contribution associated with the transient divergent wind is clearly not negligible for either the total or the

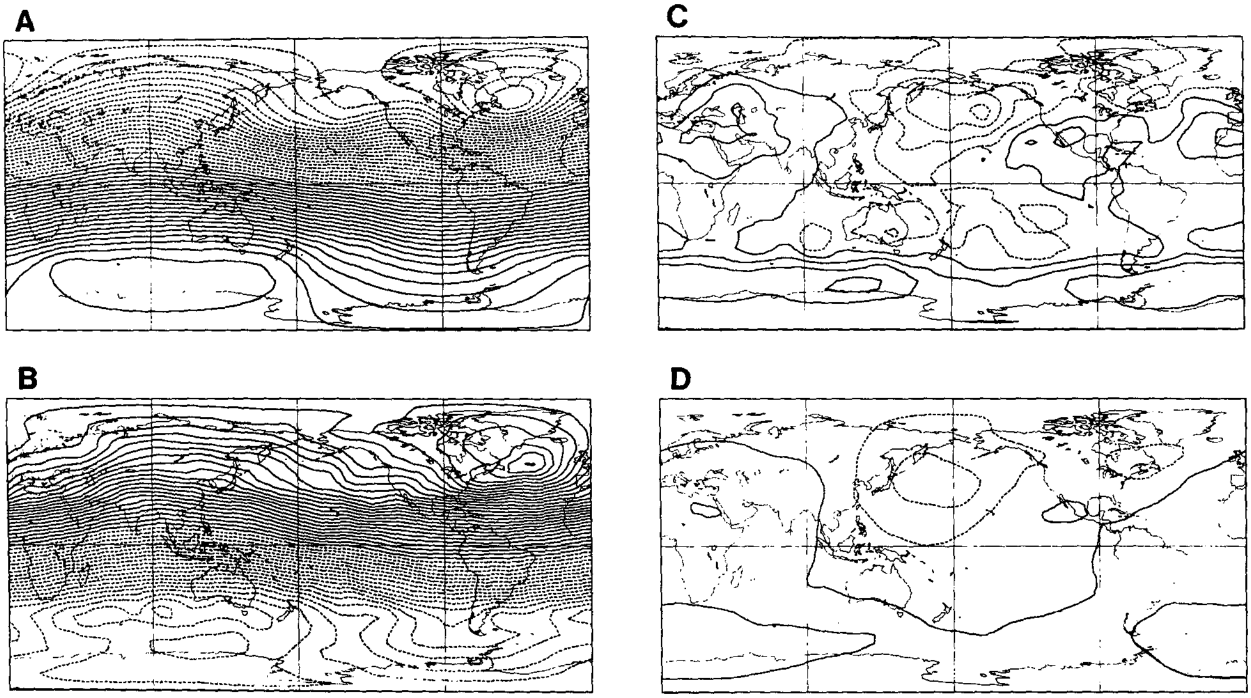

FIG. 4. The streamfunction forcing associated with (a) the transient stretching, $-\overline{\zeta' D'}$; (b) transient advection, $-\overline{\mathbf{v}\cdot\nabla\zeta'}$; (c) their sum, $-\nabla\cdot\overline{\mathbf{v}'\zeta'}$; and (d) the convergence of the vorticity flux by the transient divergent flow, $-\nabla\cdot\overline{\mathbf{v}_\chi'\zeta'}$. The contour interval is $a^2 \times 10^{-12}$ m^2 s^{-2}, where a is the radius of the earth. Negative values are indicated by dashed contours, and the zero contour is shown. Viewed as a smoothing of their vorticity forcing, the sign is reversed and the contour interval 1/25th of that used previously.

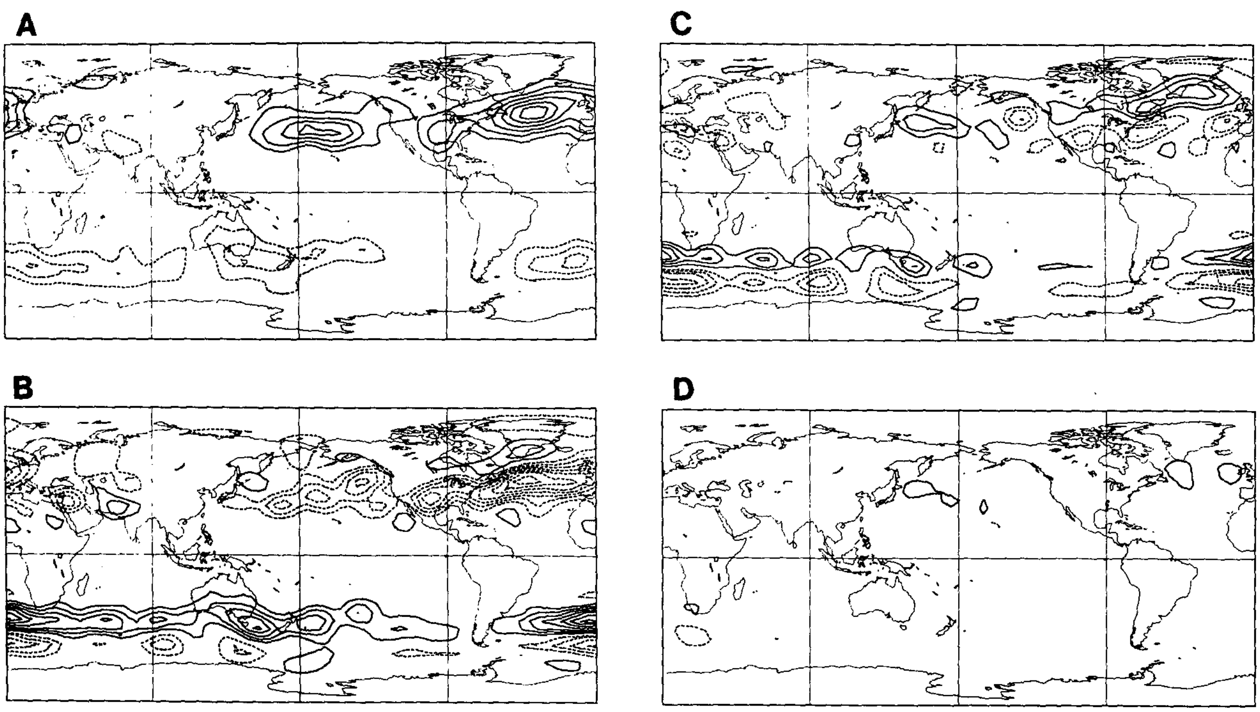

FIG. 5. (a) Transient stretching, $-\overline{\zeta' D'}$; (b) transient advection, $-\overline{\mathbf{v}\cdot\nabla\zeta'}$; (c) their sum, $-\nabla\cdot\overline{\mathbf{v}'\zeta'}$; and (d) the curl of the divergence flux $\mathbf{k}\cdot\nabla\times\overline{\mathbf{v}'D'}$, all associated with eddies in the 2.16–6.15 day band. Contouring as in Fig. 3.

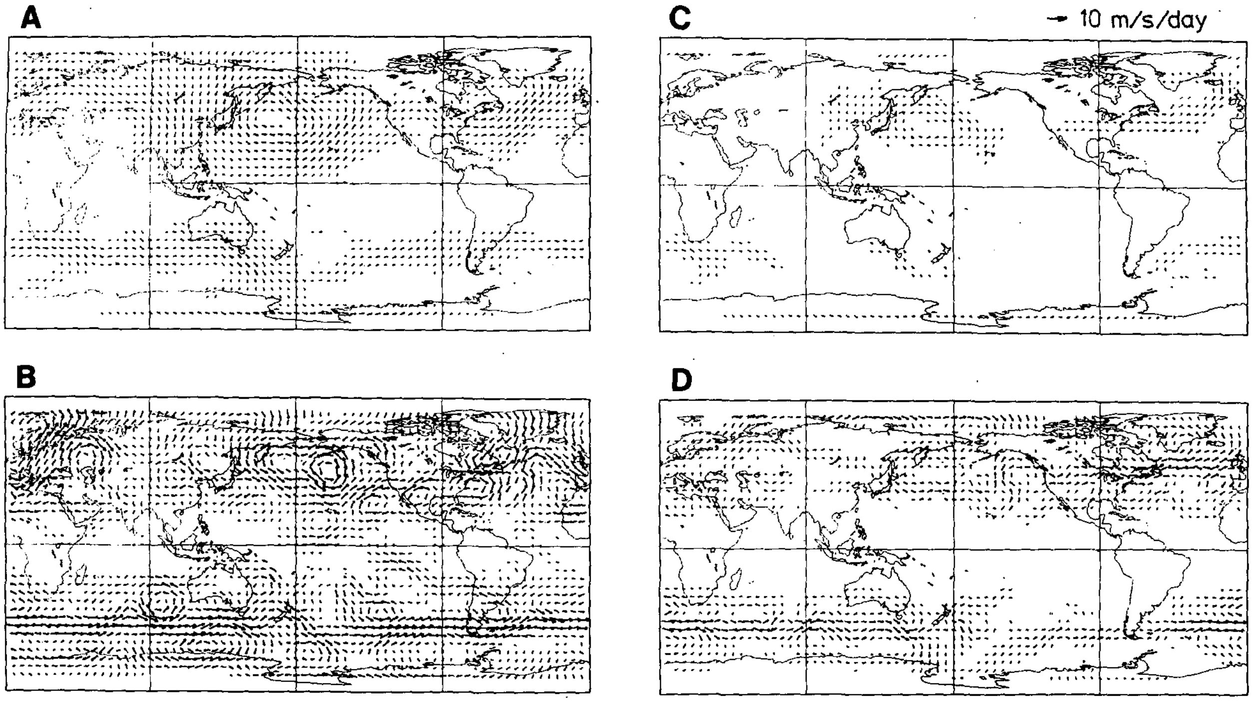

FIG. 6. The forcing of the rotational flow $(\partial\bar{\mathbf{v}}_\psi/\partial t)_e$, defined in (10), by (a) $\overline{\mathbf{v}'_\chi\zeta'}$, (b) $\overline{\mathbf{v}'_\psi\zeta'}$, (c) and (d) as in (a) and (b) but for band-pass eddies.

band-pass eddies. Figures 6a and 6c stress the role of the divergent wind flux in forcing the mean circulation in the sense of westerlies in the Northern Hemisphere storm tracks, though it is of less importance in the Southern Hemisphere.

The vertical structure of $\overline{\mathbf{v}'_\psi\zeta'}$ is essentially a simple one, with maximum amplitude in the region of the tropopause. The $\mathbf{v}'_\chi\zeta'$ flux is much weaker at 850 mb, with its direction reversed. The convergence $-\nabla\cdot\mathbf{v}'_\chi\zeta'$ therefore has a sign opposite to that at 250 mb, but its smaller magnitude suggests that term does not vanish in a vertical average. Splitting this convergence into the transient stretching and advection terms, no reversal in sign of either term is found between 250 and 850 mb, and the cancellation remains just as strong.

4. Interpretation

Some aspects of the terms associated with the transient horizontal divergent flow may be understood from relatively simple arguments based upon linear theory and interpreting observational time averages in terms of spatial averages over a wave.

To fix ideas, we consider first the most unstable zonal wavenumber 6 normal mode for a jet of 45°N discussed by Simmons and Hoskins (1977). Its phase structure at 52°N, which is the latitude of maximum geopotential amplitude, is shown in Fig. 7. The phase speed, c_r, of the mode is about 12 m s^{-1} at this latitude, which gives a steering level near 700 mb. The wave grows exponentially with an e-folding time $\sigma^{-1} \sim 1.4$ days so that the imaginary phase speed $c_i = \sigma/k$ is about 4 m s^{-1}. It is apparent from Fig. 7 that in both the upper and lower troposphere the correlation between ζ' and D' is weak but negative. The correlation between u'_χ and ζ' is, on the other hand, strong. It is negative in the upper troposphere and positive in the lower troposphere. At the latitude of maximum ζ', v'_χ is very weak

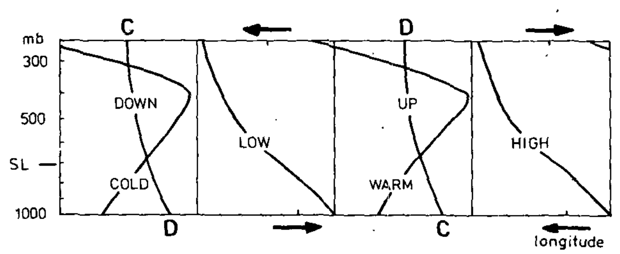

FIG. 7. Vertical phase structure of the most unstable wavenumber 6 disturbance to the 45°N jet as discussed by Simmons and Hoskins (1977). The section is at 52°N, which is the latitude of the maximum geopotential amplitude. The positions of the minimum and maximum geopotential, temperature and vertical velocity perturbations at each level are indicated. The vertical lines are drawn every 90° in longitudinal phase. The letters "C" and "D" refer to the maximum horizontal convergence and divergence of the perturbation wind near the surface and the tropopause, and the arrows to the approximate location of the implied zonal divergent wind maximum. The phases of the maximum and minimum vorticity are very similar to those of the low and high geopotential.

and the correlation in longitude is poor. Thus, $\overline{\mathbf{v}'_\chi\zeta'}$ is predominantly westward and eastward in the upper and lower troposphere, respectively. The nature of the observed transient quantities $\overline{\zeta'D'}$ and $\overline{\mathbf{v}'_\chi\zeta'}$ is therefore consistent with that apparent from the spatial structure of this baroclinically unstable wave.

To show that these properties are quite general and not necessarily restricted to growing synoptic-scale motions, we consider a normal mode of the form

$$\zeta' = \mathrm{Re}\langle\hat{\zeta}\exp\{ik[x-(c_r+ic_i)t]\}\rangle\cos ly, \quad (11)$$

and a simplified eddy vorticity equation:

$$\frac{\partial\zeta'}{\partial t}+\bar{u}\frac{\partial\zeta'}{\partial x}+\beta v'_\psi = -fD'. \quad (12)$$

Then

$$D' = -\frac{k}{f}(c_i+i\tilde{u})\zeta', \quad (13)$$

where

$$\tilde{u} = \bar{u}-c_r-\beta L^2, \quad (14)$$

and the total length scale

$$L = (k^2+l^2)^{-1/2}. \quad (15)$$

This implies that the magnitudes of the divergent and rotational flows are in the ratio

$$\mathcal{R} = \frac{|\overline{D'^2}|^{1/2}}{|\overline{\zeta'^2}|^{1/2}} = \frac{k}{|f|}(\tilde{u}^2+c_i^2)^{1/2}. \quad (16)$$

Since c_i is generally on the order of a few meters per second, except near the level at which the mode moves as a free Rossby wave,

$$\mathcal{R} \approx \frac{|\tilde{u}|}{|f|L_x}, \quad (17)$$

a Rossby number with $L_x = k^{-1}$ as the relevant zonal length scale. Since $\tilde{u}$ is the difference between the actual flow speed and the flow speed at which the mode would be a free Rossby wave, $\tilde{u}$ may be expected to be in the range 10–20 m s^{-1}. For zonal length scales in the range $2000/2\pi$–$5000/2\pi$ km, the ratio $\mathcal{R}$ may be expected to be in the range 0.1–0.7. The observed values of 0.3 in the Northern Hemisphere and 0.2 in the Southern Hemisphere are consistent with (17).

The correlation coefficient of $f\zeta'$ and D' is, from (13),

$$\alpha = -\frac{c_i}{(\tilde{u}^2+c_i^2)}. \quad (18)$$

As discussed above, this is small except near the free Rossby-wave level. Equation (13) also gives

$$\overline{\zeta'D'} = -\frac{\sigma}{f}\overline{\zeta'^2}, \quad (19a)$$

where $\sigma = kc_i$ is the growth rate. Note that with $\mathcal{R}$ and α defined as above,

$$\frac{\sigma}{|f|} = -\mathcal{R}\alpha. \quad (19b)$$

Further, from (13),

$$u'_\chi = -\frac{k^2}{k^2+l^2}\frac{1}{f}(\tilde{u}-ic_i)\zeta', \quad (20)$$

so that the correlation coefficient between u'_χ and ζ' has the sign of $-\tilde{u}/f$ and a generally large magnitude $(1-\alpha^2)^{1/2}$. Then,

$$\overline{u'_\chi\zeta'} = -\frac{L^2}{L_x^2}\frac{\tilde{u}}{f}\overline{\zeta'^2}. \quad (21)$$

The small negative value of the correlation between $f\zeta'$ and D' is seen, from (17) and (18), to be associated with the growth of the wave. The larger correlation between u'_χ and ζ', generally changing sign in the vertical, is dependent on normal-mode motion at a greater or lesser speed than that of free Rossby waves at that level, growth being irrelevant.

The correlation between u'_χ and ζ' is much larger in magnitude than that between $f\zeta'$ and D', but the contributions to the mean vorticity budget are $-\nabla\cdot\overline{\mathbf{v}'_\chi\zeta'}$ and $-\overline{\zeta'D'}$. The relevant zonal length scale, L_s, in the former term is seen from Figs. 1a and 1c to be very large, which is to be expected since, as shown above, the term depends merely on the existence of transients moving at other than the free Rossby-wave speed at that level. From (19) and (21) the ratio of the two terms is

$$\gamma \approx \frac{|\nabla\cdot\overline{\mathbf{v}'_\chi\zeta'}|}{|\overline{\zeta'D'}|} \approx \frac{|\overline{u'_\chi\zeta'}|/L_s}{|\overline{\zeta'D'}|} = \frac{L^2\tilde{u}}{L_x^2L_s\sigma}. \quad (22)$$

Introducing the Rossby number $\mathcal{R}$ from (17),

$$\gamma \approx \mathcal{R}\frac{L_x}{L_s}\frac{L^2}{L_x^2}\frac{|f|}{\sigma}. \quad (23)$$

Typical values are $|f|/\sigma \sim 10$,

$$\frac{L^2}{L_x^2} = \frac{\delta^2}{1+\delta^2} \sim \frac{2}{3},$$

where the meridional elongation of the eddy $\delta = k/l \approx \sqrt{2}$,

$$L_x/L_s \sim 1/10,$$

$$\mathcal{R} \sim 0.3,$$

giving $\gamma \sim 0.2$, as observed. The crucial element in the cancellation between $\overline{\zeta'D'}$ and $\overline{\mathbf{v}'_\chi\cdot\nabla\zeta'}$ allowing $\nabla\cdot\overline{\mathbf{v}'_\chi\zeta'}$ to be smaller, despite the good correlation of u'_χ and ζ' for an individual eddy, is thus seen to be the planetary scale, L_s, of the flux $\overline{u'_\chi\zeta'}$. This large value of L_s is consistent with the fact that transients, whether growing, neutral or decaying may be expected to contribute to such a flux. Despite this cancellation, the flux convergence $-\nabla\cdot\overline{\mathbf{v}'_\chi\zeta'}$ was shown, in particular in Figs. 4c and 4d, to be a crucial part of the transient eddy-effect on the planetary scale.

For very anisotropic eddies, $\delta \gg 1$, the above analysis is easily repeated for non-normal mode form, with the assumption of cyclic behavior in the zonal direction. Multiplying (12) by ζ' and taking an average in x removes the zonal advection term. In the anisotropic limit, $\zeta' \sim \partial v'/\partial x$ and the β term becomes $\beta\partial(\overline{v'^2}/2)/\partial x$ and also disappears. Hence

$$-\overline{\zeta' D'} = \frac{1}{f}\frac{\partial}{\partial t}\frac{1}{2}\overline{\zeta'^2}. \tag{24}$$

This is consistent with the normal-mode result (19a). Notice also that for a neutral wave in the presence of damping, a term of the form $-\lambda\zeta'$ would be included on the right-hand side of (12), and $(\lambda/f)\ \overline{\zeta'^2}$ would replace the right-hand side of (24).

For $\delta \gg 1$, $u'_x \gg v'_x$, which gives a third reason why the poleward component of $\overline{\mathbf{v}'_x\zeta'}$ should be negligible. The zonal component may be obtained from (12), replacing D' by $\partial(u'_x)/\partial x$ and using a frame of reference moving with the eddy:

$$\frac{\partial\zeta'}{\partial t} + (\bar{u} - c_r)\frac{\partial\zeta'}{\partial x} + \beta\frac{\partial\psi'}{\partial x} = -f\frac{\partial u'_x}{\partial x}.$$

Integration over x gives

$$\frac{\partial v'}{\partial t} + (\bar{u} - c_r)\zeta' + \beta\psi' = -fu'_x.$$

Multiplication by ζ' and averaging in x gives

$$\overline{u'_x\zeta'} = -\frac{1}{f}[(\bar{u} - c_r)\overline{\zeta'^2} - \beta\overline{v'^2}] = -\frac{\tilde{u}}{f}\overline{\zeta'^2}, \tag{25}$$

where $\tilde{u}$ is defined in (14) with L^2 now taken to be $\overline{v'^2}/\overline{\zeta'^2}$. In the limit $\delta \gg 1$, (25) is the same as the normal-mode result (21).

Finally, in this section, we use the data to establish typical values in the Pacific, Atlantic and Southern Hemisphere storm tracks of many of the transient quantities discussed in this section. The results are summarized in Table 1. For band-pass eddies, the total length scale defined as

$$L = [(\overline{v'^2} + \overline{u'^2})/\overline{\zeta'^2}]^{1/2}, \tag{26}$$

which is consistent with (15), gives values close to 400 km in all cases. This suggests a total wavelength $2\pi L$ of about 2500 km and a global total wavenumber n of about 16. This compares reasonably well with the normal modes on the jet of Simmons and Hoskins (1977), which, as discussed by Hoskins and Revell (1981), had a total wavenumber of about 12 at zonal wavenumber 6, rising to 15 at zonal wavenumber 9. The eddy meridional anisotropy defined as

$$\delta = (\overline{v'^2}/\overline{u'^2})^{1/2}, \tag{27}$$

which is consistent with the normal-mode definition of k/l, gives values from 1.3 to 1.5. These are similar to those found in HJW and with the normal-mode values that can be shown to vary from about 1.2 at zonal wavenumber 6 to 1.6 at zonal wavenumber 9. The Rossby numbers are on the order of 0.3 in the Northern Hemisphere but rather smaller in the Southern Hemisphere, and the correlations of $f\zeta'$ and D' are all on the order of −0.2. The equivalent growth rates, from (19a) or (19b), give e-folding times of near 2 days in the Northern Hemisphere but about double this in the Southern Hemisphere summer. These times are rather longer than the normal-mode values, which are in the range of 1.2 to 1.6 days. However, this is scarcely surprising, since these are initial values for growth on a very unstable flow.

For unfiltered transients, entries for 250 and 850 mb are also given in Table 1. Given the wide variety of transient behavior on subseasonal time scales, care must be exercised in interpreting these numbers in terms of a single mode. However, the above discussion on the structure and importance of $\overline{\mathbf{v}'_x\zeta'}$ should still be applicable.

5. Discussion

The poleward and upward transports of heat are a necessary consequence of the existence of transient eddies. The same is not true of the flux of vorticity by the rotational part of the motion: it depends on eddy anisotropy in the horizontal. Despite the fact the transient divergent motion is a Rossby number smaller than the rotational portion, the convergence of the flux of vorticity by the divergent motion is important on the largest planetary scales. The correlation between u'_x and ζ' is a good one and is again an integral part of the existence of the transients, growth being irrelevant. The vertical flux of heat and vorticity flux by the divergent motion are both neglected in consistent quasi-geostrophic scaling, but could probably be determined reasonably from a quasi-geostrophic analysis of the eddies. The parameterization of the vorticity flux by the transient divergent motion in a climate model would appear from Fig. 1c and (21) to be certainly no more difficult than the problem of representing the other eddy processes.

According to theory, the zonal length scale of the synoptic time-scale transients may be expected to be the radius of deformation, (NH/f). The latitudinal scale will tend to be the width of the baroclinic regions except for the longest waves (see Hoskins and Revell, 1981). The nature of the observed transient vorticity fluxes by the rotational flow depends crucially on the fact that the baroclinic regions in the present terrestrial climate are significantly broader than the radius of deformation, so that the eddies are predominantly meridionally elongated. A change in parameters could lead to a change in this feature, which should be taken account of in any parameterization of transient motions in a climate model. For a planetary circulation with jet widths comparable to the radius of deformation, the vorticity flux by the divergent transients would

TABLE 1. Typical values of the transient-eddy quantities discussed in the text for the Pacific (P), Atlantic (A) and Southern Hemispheric (SH) storm tracks. The parameters δ, $\mathcal{R}$, α, σ^{-1} and L have been computed using the relations (27), (16), (19b), (19a) and (26), respectively, and presented with, at most, two significant digits to indicate the uncertainty in their values.

	Band pass 250 mb			Total 250 mb			Total 850 mb		
	P	A	SH	P	A	SH	P	A	SH
$\overline{v'^2}$ (m^2 s^{-2})	120	200	170	250	370	300	100	90	50
$\overline{u'^2}$ (m^2 s^{-2})	70	100	70	370	370	200	120	100	50
$\overline{\zeta'^2}$ (10^{-10} s^{-2})	15	20	15	25	40	25	20	15	8
$\overline{D'^2}$ (10^{-10} s^{-2})	1.3	2.0	0.5	2.0	2.5	1.0	1.3	2.0	0.2
$\overline{\zeta' D'}$ (10^{-10} s^{-2})	−1.0	−1.3	0.5	−1.3	−1.5	0.7	−0.7	−2.0	−0.2
δ	1.3	1.4	1.5	0.8	1.0	1.3	0.9	0.9	1.0
$\mathcal{R}$	0.3	0.3	0.2	0.3	0.25	0.2	0.25	0.4	0.15
α	−0.2	−0.2	0.2	−0.2	−0.15	0.15	−0.15	−0.4	0.15
σ^{-1} (days)	2	2	4	2	3	4	3	1	5
L (km)	400	400	400	500	450	450	350	350	350

probably be the dominant direct contribution to the mean vorticity equation.

At any particular level, the large value and strong cancellation of the quantities $\overline{\zeta' D'}$ and $\overline{\mathbf{v}'_\chi \cdot \nabla \zeta'}$ mean that great care has to be taken in discussing the eddy contribution to the mean vorticity equation. For example, as seen in Fig. 4b, $-\overline{\mathbf{v}' \cdot \nabla \zeta'}$ is a disastrous approximation to $-\nabla \cdot \overline{\mathbf{v}'\zeta'}$. Thus the split (4b) is not useful. Using (4a), (4c) and (4d), the approximation by $G(M, N)$ or by $\overline{\mathbf{v}'_\psi \cdot \nabla \zeta'} = G(M_\psi, N_\psi)$ is better, though still not accurate on the planetary scale (e.g., see Figs. 4c and 4d). In the vertical average, however, these latter approximations are good because the errors are all like (5) and tend to cancel in the vertical.

At the level of quasi-geostrophic theory, the mean stretching term $-\bar{\zeta}\bar{D}$ is approximated by $-f\bar{D}$ in the mean vorticity equation (1). This term approximately vanishes in the vertical integral, so that any transient-eddy contribution to $\bar{D}$ does not affect this barotropic mean-flow equation. The only eddy contributions are therefore the explicit eddy terms in (1), and these are well approximated by $G(M, N)$ and $G(M_\psi, N_\psi)$. However, as discussed in section 3, the approximation of $\bar{\zeta}$ by f in the mean stretching term is of doubtful validity. Without this approximation, transient-eddy heat fluxes and vorticity fluxes by the divergent flow at any level will force a mean divergence contribution and may be of significance even for the barotropic mean flow.

Hoskins et al. discussed the eddy velocity correlation tensor and the importance of its anisotropic components M and N. They also introduced the pseudo vector $\mathbf{E} = (-2M, -N)$. The pseudo vector $\mathbf{E}$ is easy to compute from the data and displays the information concerning M and N so that, in simple circumstances, the eddy shape and the sense of the relative group velocity are apparent. Further, when the term N_{xx} is negligible, then

$$G(M, N) \approx -\frac{\partial}{\partial y} \nabla \cdot \mathbf{E}.$$

Thus, the forcing of the mean rotational flow by this term is entirely equivalent to a forcing $\nabla \cdot \mathbf{E}$ of the mean westerly motion. An assumption made from the beginning in HJW was that the eddies could be treated as horizontally nondivergent. At any particular level $G(M, N)$ and $G(M_\psi, N_\psi)$ are the same to order $\mathcal{R}$, so that either can be used for the qualitative shape and group velocity considerations. It is now clear that $\nabla \cdot \mathbf{E}$ or $\nabla \cdot \mathbf{E}_\psi$, which would provide forcings equivalent to those in Figs. 6b and 6d, are not the whole planetary scale forcing at any level. In the vertical average, $\nabla \cdot \mathbf{E}$ or $\nabla \cdot \mathbf{E}_\psi$ approximates the direct eddy contribution to the barotropic mean flow forcing, but the caveat about possible indirect contributions through the nonlinear stretching term $-\bar{\zeta}\bar{D}$ should be noted.

Finally, the difficulty of determining the importance of the transient motions from vorticity budgets should again be stressed. The transient advection and stretching are only a factor of 2 smaller than their mean counterparts. The transient terms cancel strongly in giving the vorticity flux convergence, which is the term usually compared with the individual mean flow terms. Of course, in the absence of large internal friction, the mean vorticity equation implies that the residual between the mean flow terms must be equal to that of the transient terms. The correct yardstick for judging the transient eddy terms is therefore not obvious. Further, budget computations such as these do not assess the real importance of the eddies, whose net impact can only be judged from consistent models with and without their presence.

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