Transient Response to Localized Episodic Heating in the Tropics.
Part II: Far-Field Behavior

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ABSTRACT

In Part I of this investigation, we described the stochastic, near-field behavior of disturbances excited by randomly evolving tropical heating. In the present paper, we examine how those disturbances are modified as they propagate through the far field in the presence of spatially-varying background states. Although the behavior can no longer be broken down into individual Hough modes, it can still be understood in terms of projection and barotropic components of the response.

Responses to fast heating, as may be produced by daily fluctuations in convection, and to slow heating, evolving over seasonal time scales, are studied separately. For fast heating the projection response consists mainly of a spectrum of Kelvin waves which, in the lower stratosphere, is centered at frequencies corresponding to twice the effective depth of the heating. The spectrum shifts to higher frequency with increasing altitude due to differential damping. As a result, the slow, fast and ultrafast Kelvin waves identified in observations all appear in our calculations as manifestations of the same response modified by dissipation. The barotropic response to fast forcing is dominated by the (1,1) Rossby normal mode throughout the tropics and in the stratosphere. In the extratropical troposphere, a transient barotropic wavetrain composed of low frequency Rossby waves of zonal wavenumber 1–3 is also present.

For slowly evolving heating, projection and barotropic components from various modes overlap in the spectrum, coalescing into a continuum near zero frequency. Nevertheless, it is still possible to distinguish projection from barotropic responses because the former are dominant in the tropics while the latter are responsible for the extratropical behavior. The projection response to slow heating does not propagate effectively in the vertical and is largely confined to the troposphere, where its behavior is dictated by the particular part of the solution and assumes the form of a slowly evolving Walker circulation. The barotropic response is dominated by the same transient wavetrain found in the fast forcing case, but its amplitude is larger as a result of the greater amount of power available at low frequencies. Radiation of the barotropic response to higher latitudes is strongly dependent on the presence of westerly shear near the source region. Thus, maximum radiation takes place in the winter hemisphere, where the subtropical jet is closest to the source. The evolution of the wavetrain is also sensitive to the wind within the source region. Given the variability of winds in the tropical troposphere, the extratropical wavetrain can be expected to be a highly variable feature of the response to tropical heating. By contrast, the tropical Walker cell, which is essentially a forced response, is the most robust feature found in our slow heating calculations.

1. Introduction

Latent heat release in organized convective systems constitutes the principal energy input to the atmosphere in the tropics. This energy source is neither spatially uniform nor constant in time, but is better characterized as a distribution of localized centers of convection, each evolving irregularly and comprising a spectrum of spatial and temporal scales.

In Part I of this study (Salby and Garcia, 1987; hereafter SG) we showed that the short-time, near field response to tropical convection could be described analytically in terms of a space–time spectrum of Hough modes. This Hough spectrum is composed of both eastward and westward propagating disturbances (principally the Kelvin wave and an ensemble of Rossby modes, respectively). For both Kelvin and Rossby waves, the spectral response is dominated by two distinct components: a projection response which occurs at large values of Lamb’s parameter and involves a continuum of vertical wavelengths centered on scales approximately twice the effective depth of the heating, and a more discrete barotropic response, concentrated about the eigenfrequencies of normal modes and cor-

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responding to small values of Lamb's parameter. Projection components are trapped about the equator, but radiate vertically out of the troposphere. Barotropic modes are external and thus vertically trapped, but propagate horizontally out of the tropics. In each case, waves generated in the tropical troposphere eventually propagate out of the source region.

The purpose of the present investigation is to examine how the initial disturbances described in SG are modified by refraction and absorption as they propagate through the far field. In the presence of a spatially varying basic state the Hough modes used in SG to describe the near-field behavior lose their individual identity, being replaced by general latitude–height structures at particular wavenumbers and frequencies. In principle, these complex amplitude structures involve combinations of several modes, but individual modes can nevertheless be identified quite clearly in many instances, especially at high frequencies where the projection and barotropic responses are widely separated from each other and from those of other modes. At low frequencies, on the other hand, projection and barotropic components overlap, particularly for Rossby modes. Such behavior is evident in Figs. 15 and 16 of SG, where the barotropic responses of higher order Rossby modes are seen to overlie lower order projection responses. Thus, the low frequency response consists of a superposition of projection and barotropic components, the former being confined to the tropics while the latter dominate the extratropical response. The identification of low frequency barotropic structures with higher order normal modes is further complicated by the fact that phase speeds are comparable to the mean winds in the tropical troposphere and lower stratosphere. In the presence of dissipation, this smears barotropic responses over a range of frequencies so that they coalesce into a continuum (cf. Salby, 1981a,b). Nevertheless, we shall see below that the concepts of projection response (vertically propagating, equatorially trapped) and barotropic response (latitudinally propagating, vertically trapped) remain very useful in interpreting the behavior of the response in the far-field.

As in SG, the problem is formulated in a stochastic framework. Convective heating is prescribed as a second-order process whose structure and evolution vary randomly. Because the process is second order stationary, it is completely specified by its spatial and temporal covariance statistics. The response is then derived in terms of variance spectra which are functions of latitude and height. It is also instructive to generate "cardinal realizations" of the heating and the response. These simple transients, whose power spectra are identical to those of the stochastic process, may be used to generate the random signature, and represent a "snapshot" of that behavior. The cardinal realizations provide a simple view of the randomly-evolving response field and are analogous to statistical composites derived from observations.

Two classes of heating variability are considered: short-term fluctuations having characteristic times of several days, and gradual transitions associated with the seasonal cycle upon which the more rapid fluctuations are superimposed (see SG). We refer to the two forms of heating variability as fast and slow, respectively. The responses to fast and slow heating differ fundamentally from each other. For fast heating, projection responses radiate quickly out of the troposphere, and the barotropic component contains high-frequency Rossby normal modes which are individually distinct. For slowly evolving heating, on the other hand, the projection responses propagate very gradually in the vertical, and projection and barotropic Rossby components overlie one another forming a continuum at low frequencies. Accordingly, we find it convenient to discuss the response to fast and slow heating separately.

The remainder of the paper is organized as follows: Section 2 gives a brief description of the problem and the numerical solution techniques employed; in section 3, we discuss the effect of absorption on disturbances propagating vertically through the stratosphere with particular emphasis on differential absorption of spectral components. With this as background, we examine the response to fast and slow forcing in sections 4 and 5. A summary and discussion of the results are presented in the last section.

2. Formulation

The problem is governed by the linearized primitive equations for an unbounded, baroclinic atmosphere. Since the basic state is invariant in longitude and time, application of the space–time transform δ to the governing equations maps the initial value problem in physical space into a spectrum of boundary value problems in latitude and height over wavenumber–frequency coordinates. (See SG for a detailed discussion.) Because we now have to account for variations in the basic state as disturbances propagate through the far field, the boundary value problems are no longer separable as was the case for the initial near-field behavior studied in SG. Consequently, we must use numerical techniques to derive the various solutions as functions of latitude and height for particular wavenumbers and frequencies.

The transformed system is solved over a restricted range of wavenumber and frequency commensurate with the spectral makeup of the forcing. In the case of fast forcing, the system is solved for frequencies \(-0.5 \leq \sigma \leq 0.5\) cpd/2. For slow forcing, whose characteristic time scales are of order 1 month, it is necessary to range only between frequencies \(-0.1 \leq \sigma \leq 0.1\) cpd/2. Frequency intervals, or bandwidths, of 0.01 cpd/2 for fast forcing and 0.002 cpd/2 for slow forcing are sufficient to resolve the resulting response spectrum. In both cases, six wavenumbers are found to capture adequately the structure of the response.

The numerical scheme used to derive the solutions
is the second order finite element collocation method with B-splines described in Salby (1981a). Radiation and finite energy constraints are imposed at the upper boundary by means of a deep layer of damping which extends between 12 and 20 scale heights (1 scale height = 7.3 km). The vertical velocity is prescribed to vanish at the surface, so that the only forcing is internal heating.

a. Basic state

A simplified seasonal march is constructed from climatologies of zonal-mean winds and temperatures for solstice and equinox conditions. We admit within our seasonal progression the long-term climatological mean, the annual and the semiannual harmonics. Thus, for the mean wind

\[ \tilde{u}(\phi, \xi, t_c) = \tilde{u}_0(\phi, \xi) + \tilde{u}_1(u, \xi) \cos \left( \frac{2\pi t_c}{12} \right) - \tilde{u}_2(\phi, \xi) \cos \left( \frac{4\pi t_c}{12} \right) \]  

where subscripts 0, 1 and 2 denote the various seasonal harmonics, \( \phi \) is latitude, \( \xi \) is altitude in scale heights, and \( t_c \) is the time of the year in months.

The harmonic amplitudes \( \tilde{u}_0, \tilde{u}_1, \) and \( \tilde{u}_2 \) may be derived from climatologies for equinox and solstice conditions (Salby, 1981b) by assuming hemispheric symmetry of the seasonal march. This simple model accounts for a basically antisymmetric structure of the annual cycle and a symmetric structure for the semiannual cycle in \( \tilde{u} \). However, variations in the relative phase between these components such as the descending westerlies of the SAO are not included. Nevertheless, this simplified analytical framework is attractive in that it permits the generation of a smoothly evolving basic flow and has enough degrees of freedom that basic states typical of observed periods can be constructed.

In the lower stratosphere the seasonal march is modified strongly by the QBO. We augment the seasonal behavior in the tropical stratosphere by the following simple description of the QBO, derived from observations (Wallace, 1973; Belmont et al., 1974; Hamilton, 1984a; U.S. Navy Weather Research Facility, 1964):

\[ \tilde{u}_{QBO}(\phi, \xi) = e^{-(\xi - \Delta \phi_{QBO})^2} g(\xi) \text{ (m s}^{-1}) \]  

where

\[ g(\xi) = 21 \cos \left( 2\pi \left( \frac{\xi - 3.2}{7.12} \right) + \theta_{QBO} \right) \]

\[ = \begin{cases} e^{-(\xi - 3.2)^2/0.68^2}, & \xi < 3.2 \\ 1.0, & 3.2 < \xi < 4.1 \\ e^{-(\xi - 4.1)^2/1.37^2}, & \xi > 4.1 \end{cases} \]  

and where \( \Delta \phi_{QBO} \) is 17° and \( \xi \) is in scale heights. The behavior represented by (2) is that of a succession of westerlies and easterlies descending through an envelope centered at 23.5 km. Since the phase of the QBO is irregular, we introduce the arbitrary phase factor \( \theta_{QBO} \) to denote the stage of the QBO progression. We will use values of \( \theta_{QBO} \) of 0 and \( \pi \), corresponding to maximum westerlies and easterlies at 3.2 scale heights, respectively. The combined evolution synthesized from the various components is presented in Figs. 1 and 2 at \( t_c = 1.0 \) (21 January) and \( t_c = 3.0 \) (21 March) for both easterly and westerly phases of the QBO, namely \( \theta_{QBO} = \pi \) and 0. Near solstice, the combination of the seasonal cycle and the easterly phase of the QBO can result in substantial easterlies in the tropical lower stratosphere.

b. Damping mechanisms

Dissipation is included in our calculations in the form of Newtonian cooling and Rayleigh friction. Both are strong functions of altitude, but are independent of latitude. The Newtonian cooling coefficient is given by

\[ \alpha(\xi) = 0.1 + 0.35 \exp \left( \frac{\xi - 8}{3.2} \right) + 3.2 \times 10^{-5} \exp(0.8\xi) \]  

where \( \alpha \) is in units of inverse days. Thus, \( \alpha \) represents a thermal relaxation rate of 1/10 days\(^{-1}\) in the troposphere, increasing to about 1/10 days\(^{-1}\) in the upper stratosphere, and decreasing again to about 1/10 days\(^{-1}\)

![Fig. 1. Zonal wind distribution at \( t_c = 1 \) (January) and \( t_c = 3 \) (March) for the easterly phase of the QBO.](image-url)
in the upper mesosphere. At higher altitudes $\alpha$ increases rapidly, attaining a value of a fraction of a day near the upper boundary.

The Rayleigh friction coefficient is specified as

$$K_a(\xi) = 0.1 + 3.5 \times 10^{-5} \exp(0.8\xi)$$

with $K_a$ in inverse days. This corresponds to a frictional damping time scale of about 10 days throughout the lowest 10 scale heights, decreasing rapidly above $\xi = 10$.

c. Heating

The system is forced by a prescribed heating perturbation $q'$, whose structure and evolution vary randomly. It was shown in SG that, if the random heating signature is assumed to be a second-order stationary stochastic process, then the variance of the response depends only on the power spectrum of the forcing. As a consequence, the stochastic process may be constructed by a random superposition of simple localized transients, or "episodes", having suitable spectral characteristics. The associated cardinal response signature provides a "snapshot" of the complex behavior in physical space and is analogous to a composite derived from a long time series of observations.

As in SG, we consider the class of stochastic heating processes whose power spectra are Gaussian:

$$S_q(\sigma, m, l, \xi) = \frac{1}{(2\pi)^2} \exp\left(-\frac{\sigma^2}{2} + 2\Phi^2l^2 + T^2\sigma^2\right)q_0^2(\xi)$$

(5)

where $m$ and $l$ are zonal and meridional wavenumbers, respectively, and $\sigma$ is linear frequency. The parameters $\Lambda$, $\Phi$ and $T$ determine the distribution of variance over space and time scales. In particular, $1/\Lambda$, $1/\Phi$ and $1/T$ correspond to the e-folding scales of the power spectrum of the heating over horizontal wavenumbers and frequency. The cardinal heating realization corresponding to (5) is also Gaussian:

$$q(\lambda, \phi, \xi, t) = \frac{1}{(2\pi)^{1/2} \Lambda \Phi T} \times \exp\left[-\frac{(\lambda - \lambda_0)^2}{2\Lambda^2} + \frac{(\phi - \phi_0)^2}{2\Phi^2} + \frac{2\pi^2(t - t_0)^2}{T^2}\right]q_0(\xi)$$

(6)

The values $\Lambda$, $\Phi$ and $T$ determine in some mean sense the horizontal "extent" and "duration" of the heating.

The vertical dependence of the forcing, $q_0(\xi)$, is defined simply as

$$q_0(\xi) = \begin{cases} \cos\left[\frac{\xi}{0.75} - 1\right] \pi/2, & 0 \leq \xi \leq 1.5 \\ 0, & \xi > 1.5 \end{cases}$$

(7)

This simple prescription is adequate because the response depends mainly on the depth of the heating and is insensitive to details of its vertical profile as shown in SG (appendix C).

Values assigned to the characteristic scales $\Lambda$, $\Phi$ and $T$ are motivated by previous studies of convective variability in the tropics. For the majority of our calculations, the characteristic horizontal scales $\Lambda$ and $\Phi$ are assigned values of 20° and 10°, respectively, representative of large convective complexes in the tropics, e.g., over Indonesia. As described in SG, large scale convective variability has two basic contributions: a rapidly varying part with most of the heating variance lying at periods of a few days (Orlanski and Polinsky, 1977), and a slowly varying component operating on time scales of order 1 month (Gruber, 1974). Accordingly, we explore the response to heating with relatively large and small values of $T$. In sections 4 and 5 we discuss separately the response to "fast heating" ($T < 5$ days) and "slow heating" ($T > 22$ days). The former may be thought of as representing the heat released by convective complexes with lifetimes of a few days, while the latter can be considered to represent the slower modulation of convective activity resulting from seasonal transitions in the intensity and position of the large-scale convergence pattern.

3. Absorption

We consider here the influence of dissipation, particularly radiative damping in the stratosphere, on disturbances propagating away from the source region. It was shown in SG that, in regions of uniform flow and
constant Newtonian cooling, the vertical structure of a particular mode at frequency $\omega$ has the form $Z_{mn}(\xi) \sim \exp(\sqrt{2} \pm i k_{mn}^w) \xi$, with the vertical wavenumber

$$k_{mn}^w = \frac{\kappa \theta_{mn}^w}{\Gamma} - \frac{1}{4}$$

related to the frequency through

$$\Gamma = 1 + i(\alpha/\gamma \omega).$$

Accordingly, the vertical wavenumber is complex

$$k_{mn}^w = \left(\frac{\kappa \theta_{mn}^w}{[1 + (\alpha/\gamma \omega)^2]} - \frac{1}{4}\right) + i\left(\frac{\kappa \theta_{mn}^w(\alpha/\gamma \omega)}{[1 + (\alpha/\gamma \omega)^2]}\right)$$

with the imaginary part proportional to the ratio of the time scale of the wave to that of the damping ($\alpha/\omega$) and also to $\kappa \theta_{mn}^w$. Since $\kappa \theta_{mn}^w$ is approximately the vertical wavenumber squared in the absence of dissipation, the imaginary component is also inversely proportional to the vertical wavelength. At large values of Lamb's parameter, appropriate to the projection response, vertical wavelength decreases with decreasing frequency. That is, low frequency components have shorter vertical wavelengths than those of higher frequency. Hence, whatever influences are introduced through the imaginary component of $k_{mn}^w$ are compounded for low frequency components.

The effect of introducing an imaginary part into $k_{mn}^w$ is twofold. First, it causes the energy of a particular component to decay vertically away from the source, reducing the $e^{it}$ amplitude growth. This attenuation varies inversely with group velocity and therefore acts differentially on components of smaller intrinsic frequency and shorter vertical wavelength. As a result, low frequency components of the projection continuum are preferentially absorbed relative to those of higher frequency, and the corresponding structures grow more slowly in the vertical. For sufficiently small $\omega$, vertical growth is suppressed altogether, and behavior above the source becomes evanescent (Fig. 3). At very low frequencies, the response collapses to levels of the heating where the behavior reduces to that of the particular solution (SG §5).

Because of the differential absorption acting on the projection continuum (high frequency components growing more rapidly in the vertical than low frequency components), the spectrum will systematically shift to higher frequency with increasing altitude above the source. Occurring simultaneously with this change in frequency is a shift to longer vertical wavelengths. From a WKB perspective, a wavepacket propagating to greater altitudes will undergo a transformation, evolving towards higher speeds and larger vertical scales.

The second effect of an imaginary component of $k_{mn}^w$ is to alter the effective vertical wavelength, which is inversely proportional to the real part of $k_{mn}^w$. For moderate values of $(\alpha/\omega)$, the change in phase tilt is not particularly dramatic; i.e., the vertical wavelength is not altered substantially. For large values of $(\alpha/\omega)$ a greater change in vertical wavelength is expected, but the importance of this effect is unclear. Very slow components decay sharply above the source, so that the response at small $\omega$ collapses to levels of the heating (Fig. 3), where the structure is dictated by the particular part of the solution and the foregoing considerations do not apply. However, damping does introduce a phase tilt in the barotropic components of the response (Salby, 1980, 1981b), for which $k_{mn}^w$ is pure imaginary in the absence of damping. This phase tilt will be greatest in the stratosphere, where Newtonian cooling increases rapidly with altitude.

Although the preceding discussion is largely heuristic, being strictly applicable only in regions of uniform background winds and damping, the same ideas carry over to situations where the WKB approximation is valid (e.g., Lindzen, 1971, 1972; Boyd, 1978). Under such conditions one anticipates similar behavior, except that slow components will experience enhanced dissipation relative to faster components in regions where their intrinsic frequencies are Doppler-shifted to small values. In fact, in the more general cases about to be presented, such behavior remains a robust ingredient of the far-field response at small scales. The tendency of the projection continuum to shift to higher frequencies with increasing altitude emerges as conspicuously in a variety of wind configurations as it does in the absence of winds altogether.

4. Response to fast heating

a. Space–time spectra

In this section we examine the response variance resulting from the heating process defined by (5) for
TABLE 1. Fast forcing.

<table>
<thead>
<tr>
<th>Case</th>
<th>$t_0$, $T$</th>
<th>$\lambda_0$, $\Delta$</th>
<th>$\phi_0$, $\Psi$</th>
<th>$t_c$</th>
<th>$\theta_{QBO}$</th>
</tr>
</thead>
<tbody>
<tr>
<td>1.1</td>
<td>0, 3 d</td>
<td>0°, 20°</td>
<td>0°, 10°</td>
<td>1.0 (Jan. 21)</td>
<td>180° (East)</td>
</tr>
<tr>
<td>1.2</td>
<td>0, 5 d</td>
<td>0°, 20°</td>
<td>0°, 10°</td>
<td>1.0</td>
<td>180° (East)</td>
</tr>
<tr>
<td>1.3</td>
<td>0, 3 d</td>
<td>0°, 20°</td>
<td>0°, 10°</td>
<td>1.0</td>
<td>0° (West)</td>
</tr>
</tbody>
</table>

characteristic time scales $T \leq 5$ days. Table 1 lists the values of all the forcing parameters and the basic state for several cases to be discussed here. These are a representative sample of a larger set of calculations. The first case (1.1) has heating centered on the equator, with characteristic spatial scales ($\Lambda, \Phi$) of 20° longitude and 10° latitude, and a characteristic time scale $T$ of 3 days. The basic state is typical of Northern Hemisphere winter solstice during the easterly phase of the QBO (Fig. 1a).

Figure 4 shows the space–time spectrum of the response at the tropopause (2 scale heights) and stratosphere (7 scale heights) over the equator. At both levels, variance at negative frequencies (westward phase velocities) is concentrated in wavenumber 1 at ~0.1 cph/2. This corresponds to the barotropic response in the gravest symmetric Rossby mode, the 5-day wave (Madden and Julian, 1972; Madden and Stakes, 1975; Madden, 1979; Hirota et al., 1983). Away from the tropics other barotropic and projection Rossby responses become evident (see below), but near the equator the variance carried by the 5-day wave dominates the Rossby contribution to the response. Similar behavior is evident in SG. It is interesting that Hamilton’s (1985) recent study of the correlation between observed amplitudes of the 5-day wave and Eastern Pacific sea surface temperatures suggests that latent heat release in the tropics may be an important excitation mechanism for this mode.

For positive frequencies (eastward phase velocities), the variance at the tropopause is dominated by the Kelvin projection response, which falls off with wavenumber and frequency (Fig. 4a). This behavior is a consequence of the inverse frequency dependence of the response [see SG, Eq. (22.2)] and the redness of the heating spectrum. The projection responses peak at frequencies of approximately 0.04, 0.08 and 0.12 cph/2 for zonal wavenumbers 1, 2 and 3, consistent with the nondispersive nature of Kelvin waves. For $m = 1$ and 2 these values correspond to periods of 12.5 and 6.25 days, in good agreement with Wallace and Kousky’s (1968) observations of slow Kelvin waves and the LIMS observations presented in SG (Fig. 3). The

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1 Recall that in SG space–time spectra were integrated globally to describe the cumulative variance of the initial localized disturbance. Here, spectra are presented locally, i.e., at individual latitudes and heights.

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Fig. 4. Equatorial wavenumber–frequency spectra for Case 1.1 at (a) tropopause and (b) stratosphere levels.
vertical wavelength associated with these periods is approximately 12 km, also in accord with observations at these levels. This vertical scale implies an effective heating depth of 6 km, whereas the value used in the calculations is 11 km. The reduced effective heating depth and shift towards lower frequencies is a consequence of the weak static stability of the tropical troposphere, as shown in SG (appendix D). If the tropical lapse rate is not taken into account, the Kelvin projection response peaks at a period of 7 days for wavenumber 1 which corresponds to a vertical wavelength of 21.5 km, consistent with the actual depth of the forcing.

At the stratopause (Fig. 4b), the Kelvin spectrum has shifted to higher wavenumbers and frequencies as a result of absorption in the stratosphere. Since radiative damping acts preferentially on components of short vertical wavelength and small frequency, low frequency components in each wavenumber are selectively removed relative to those at higher frequencies. As discussed in section 3, this differential absorption causes the entire spectrum to shift systematically to higher frequencies with increasing altitude above the source. Wavenumber 1 variance now peaks at 0.07–0.08 cpd/2, corresponding to periods of 6–7 days, which is within the range (5–10 days) of the fast Kelvin waves reported by Hirota (1978, 1979) and by Salby et al. (1984). The vertical wavelengths and horizontal structures of these disturbances (see below) are also in agreement with observations and with theoretical expectations. Thus, both the slow and the fast Kelvin waves emerge as manifestations of the same response to tropical forcing modified by dissipation. Indeed, the shift of the Kelvin response to higher frequencies occurs continuously throughout the stratosphere as illustrated in Fig. 5, which shows the power spectrum for zonal wavenumber 1 as a function of altitude. This systematic shift of the Kelvin response to higher frequencies is similar to that found by Hayashi et al. (1984, Fig. 2) in their analysis of GCM output. For altitudes greater than 8–9 scale heights, the Kelvin spectrum is centered at periods < 5 days, corresponding approximately to the “ultrafast” Kelvin wave (3.5–4 days for \( m = 1 \)) identified by Salby et al. (1984) in Nimbus-7 LIMS data.

The Kelvin wave behavior diagnosed by Hayashi et al. compares remarkably well with that derived from satellite observations. This is somewhat surprising since the GCM calculations were performed under annual mean conditions, with background winds quite dissimilar from the solstice winds present during the period analyzed by Salby et al. This underscores the robust nature of the shift to higher frequencies, occurring to a great extent independently of the basic wind configuration.

There remains an important discrepancy between our calculations (and for that matter the GCM behavior) and the satellite observations. In the LIMS data, the fast and ultrafast Kelvin waves coexist at the same altitude as distinct spectral peaks (see Salby et al., 1984, Fig. 5), but this is not the case in our calculations or in Hayashi et al.’s analysis, where the spectrum at a given altitude is dominated largely by a single range of frequencies. The presence of a sharp zone of descending westerly shear associated with the SAO, as reported by Hitchman and Leovy (1986), may be responsible for the behavior observed in the data. This shear layer, which is not present in our background winds, would act to Doppler-shift Kelvin waves to low intrinsic frequencies, thereby abruptly increasing the absorption and shifting the Kelvin variance sharply to higher frequencies. In the neighborhood of the shear layer, Kelvin waves would appear at any instant with two distinct frequencies, i.e., one frequency below and another above the layer. Spectral analysis performed over an extended period during which the layer descends appreciably (Salby et al., 1984) would smear this feature over a finite depth and give the appearance of two distinct frequencies coexisting over a range of altitudes.

It should also be noted that, for the fast heating of case I.1., a prominent Kelvin normal mode response (period \( \approx 33 \) h; cf. SG, Fig. 11) is obtained above the stratopause (not shown). The work of Matsuno (1980), Hamilton (1984b), and Hamilton and Garcia (1986) suggests that this mode is indeed present in the at-
atmosphere. However the observed surface pressure variance implies a significantly smaller amplitude at upper levels than is realized for the heating of case I.1. Calculations with slightly slower heating (case I.2; $T = 5$ days) result in a much diminished response at this normal mode frequency. Another possibility is that the damping prescribed in the stratosphere and mesosphere is too weak. A potentially important omission is the rapid diffusion resulting from breaking gravity waves (Lindzen, 1981). Recent investigations of such effects suggest that the damping time for large scale motions may be as short as 1–2 days (Miyahara, 1985). As in SG, because of its isolated appearance in the range of frequencies under consideration, we omit this response from subsequent discussion.

Power spectra at 45°N are shown in Fig. 6 for $z = 2$ and 7 scale heights. In contrast to the spectra over the equator, there is no evidence of Kelvin waves since they are confined to the tropics. At 7 scale heights, variance at negative frequencies is dominated by the 5-day wave, while at 2 scale heights there is low frequency variance in the first four zonal wavenumbers as well. As discussed in the Introduction and in SG, the identification of variance at low negative frequencies with particular Rossby modes is complicated by the tendency of individual response peaks to coalesce, and of barotropic and projection components of various modes to overlie one another. Moreover, the association of the barotropic response with normal modes becomes increasingly dubious because of the pronounced effect of tropical easterlies on Rossby waves of small phase speed (Salby, 1981b). Nevertheless, a great deal can still be learned about the nature of the response by examining the wave structure at particular wavenumber–frequency coordinates.

Figure 7 displays wave structures for $m = 1$ at several frequencies, $\sigma$. For $\sigma = -0.1$ cpd/2 (period $\tau = 5$ days westward) the structure is clearly that of the gravest symmetric Rossby mode (Geisler and Dickinson, 1976; Salby, 1981b), as evidenced by its global extent and symmetry about the equator, and by the lack of phase variation. At lower frequency ($\sigma = -0.03, \tau = 17$ days) the behavior is made up of a combination of projection and barotropic contributions. In the tropics the structure has the character of a Rossby projection response trapped near the equator (note the large phase shift accompanying the node in the tropical midtroposphere). At extratropical latitudes, the structure in the troposphere and lower stratosphere is almost barotropic. A phase tilt with height is evident in the Northern Hemisphere above 5 or 6 scale heights, as the wave veers toward and is absorbed in the tropical easterlies. Absorption of wave activity in the tropical easterlies is also evident in Fig. 8, which shows the Eliassen–Palm (EP) flux magnitude and direction. In the middle and upper winter stratosphere the EP flux vectors turn sharply equatorward, whereas in the extratropical Southern Hemisphere, the structure is evanescent in the strong summer easterlies, and the EP flux decays much more rapidly. It is tempting to identify the response at $\sigma = -0.03$ in Fig. 7 with the second symmetric Rossby normal mode. (Note the sharp meridional phase transition at 30°–40° latitude in both hemispheres.)

At positive frequencies ($\sigma = 0.04, 0.1$ cpd/2; $\tau = 12.5, 5$ days), eastward structures in Fig. 7 are clearly

![Fig. 6. Wavenumber–frequency spectra at 45°N for Case I.1 at (a) tropopause and (b) stratopause levels.](image-url)
Fig. 7. Wave structures for Case 1.1 at various frequencies, $\sigma$ (cpd/2), for zonal wavenumber $m = 1$. Amplitudes are normalized to unity.
those of the Kelvin projection response, decaying monotonically from the equator. The vertical wavelength and the meridional extent of these disturbances increase with frequency. At $\sigma = 0.04$, the phase tilt indicates a vertical wavelength of approximately 15 km while at $\sigma = 0.1$, a wavelength of about 32 km is suggested. These values are consistent with observations of the slow and fast Kelvin waves, respectively (Wallace and Kousky, 1968; Hirota, 1978; 1979; Salby et al., 1984).

Structures for wavenumbers 2 and 3 at several westward frequencies are shown in Fig. 9. At low frequencies these structures exhibit a combination of barotropic and projection components, dominated in the tropics by the Rossby projection response and at extratropical latitudes by the barotropic response, just as in the $m = 1$ case. In particular, the behavior depicted in Figs. 7b and 9 can be ascribed mainly to the projection response of the gravest symmetric mode ($m, 1, 3$) and the barotropic response of the second gravest symmetric mode ($m, 3, 3$). This is also consistent with the spectrum shown in Fig. 13 of SG. It is perhaps worth emphasizing that the barotropic behavior outside the tropics is a fundamental property of meridionally propagating disturbances. Waves able to propagate out of the tropics are associated with small values of Lamb's parameter and, therefore, are external or have very long vertical wavelengths (see also the discussion in SG, and Hoskins and Karoly, 1981).

We consider next how the response is altered as a result of changes in the background state. Case I.3 is identical to I.1 except that the zonal wind configuration in the tropics represents the westerly phase of the QBO (see Fig. 2). At negative frequencies, the response spectrum over the equator (not shown) resembles that of case I.1. However, the Kelvin response is altered significantly throughout the stratosphere, as illustrated in Fig. 10, which may be compared with Fig. 5 for the easterly phase. The changes are most pronounced at low frequencies where the power is reduced by a factor of four. They result from increased damping associated with the westerly phase of the QBO, which Doppler shifts the Kelvin waves to smaller intrinsic frequencies.

Figure 11 shows the structure of the Kelvin projection response at $\sigma = 0.04$ and 0.10 cpd/2 for $m = 1$. At 0.04 cpd/2 ($\tau = 12.5$ days), the response is much reduced compared to that in the easterly phase of the QBO (Fig. 7), but at 0.10 cpd/2 ($\tau = 5$ days) the behavior is essentially unchanged except for a very slight reduction of amplitude in the lower stratosphere. The rapid decay of the slow Kelvin wave in the middle stratosphere implies a strong EP flux convergence during the westerly phase of the QBO (Lindzen and Holton, 1968). Figure 12 illustrates this point by comparing the EP flux due to the Kelvin wave of period 12.5 days for the easterly and westerly phases of the QBO. The vectors point downward, consistent with upward flux of westerly momentum. Much more rapid attenuation of EP flux with altitude is evident in the westerly phase. By comparison, the EP flux associated with the fast Kelvin (Fig. 13) wave penetrates through the mesosphere and spans a much wider belt of latitudes. Because the structure of this fast component is virtually unchanged (cf. Figs. 7d and 11b), the EP flux divergence is almost identical in both phases of the QBO.

\[ b. \text{Cardinal realizations} \]

We have hitherto discussed only the variance of the random wave response. A simple picture of the randomly evolving disturbance may be obtained, as in SG, by considering the response to a cardinal realization, or "snapshot", of the heating process. It should be borne in mind that this represents a greatly simplified picture of the random structure and evolution. Corresponding to the heating spectrum (5) is the cardinal signature (6) which describes a localized pulse of limited duration. The behavior in space and time is recovered by applying the inverse space–time transform to the complex amplitude spectrum resulting from this single heating episode. To illustrate the space–time behavior of the response we consider first a longitude–height cross section at the equator for case I.1 (Fig. 14). At $t = 0$ (the time of peak heating) the disturbance is largely confined to the vicinity of the source; at subsequent times there is rapid dispersion in the vertical and horizontal. High frequency Kelvin waves, which have the largest vertical group velocities (see SG, Fig. 26), arrive at 10 scale heights by $t = 2$ days. These waves also have the longest vertical wavelengths, as is evident from comparison of the cross section at $t = 2$ with that at, say, $t = 8$ days. As indicated in Fig. 14 for $t = 2$ and $t = 8$, the wavelength of the disturbances reaching the upper stratosphere collapses from about 32 to 18 km between these times.
Fig. 9. As in Fig. 7 except for \( m = 2 \) and \( m = 3 \).
It is remarkable how, for forcing on the time scales under consideration, most of the disturbance has left the troposphere by $t = 4$ days, before completing even one traversal of the globe. While the disturbance remains in the troposphere its zonal propagation is more or less nondispersive (cf. $t = 0$ through $t = 4$), in agreement with the distribution of variance in the spectrum at $2$ scale heights shown in Fig. 4a and the longitude-time dispersion characteristics of Kelvin waves. At greater altitudes, however, the spectral content has been strongly modified through absorption (Fig. 4b). Moreover, because of vertical dispersion among different wavenumber components, the disturbance has spread completely around the globe by the time it reaches the middle stratosphere.

The behavior at $45^\circ$ (Fig. 15) in the winter hemisphere is dominated by the contribution from the barotropic Rossby response. Soon after the forcing has peaked ($t = 1$), a barotropic structure emerges at midlatitudes. This wave propagates westward with a period of $5$ days (cf. $t = 1$ and $t = 6$) and dominates the disturbance field in the stratosphere and mesosphere. Because of its high intrinsic frequency, it is affected little by damping. In the troposphere and lower stratosphere, a more complicated pattern emerges after $t = 2$, reflecting the additional contribution from the low frequency Rossby response, which is also barotropic but distributed over several wavenumbers (Fig. 6a).

Fig. 11. Structures of the Kelvin projection response at $\sigma = 0.04$ and $0.10$ cpd/2 in the westerly phase of the QBO. Cf. Fig. 7c, d.
5. Response to slow heating

a. Space–time spectra

For slow forcing, the characteristic time scale $T$ takes on values greater than 22 days. Consequently, the response collapses to a narrow range of frequencies about zero (cf. SG, Fig. 18). Table 2 lists the cases considered in this section. As for the fast forcing cases, these are representative of a larger set of calculations. The first case (II.1) is that of slowly varying heating, centered on the equator and at map longitude $-60^\circ$, and having characteristic scales of $20^\circ$ longitude and $10^\circ$ latitude. If longitude $0^\circ$ is taken to be the dateline, then the heating can be thought to represent convection over Indonesia. The basic state configuration corresponds to Northern Hemisphere winter.

Response spectra in the tropical troposphere and midlatitude stratosphere are shown in Fig. 18. As in SG, variance in the tropics is dominated by the wave-number 1 Kelvin and Rossby contributions. However, the Rossby projection response associated with the gravest symmetric mode is now substantially reduced from that computed in SG in the absence of winds. Easterly shear in the upper troposphere Doppler-shifts the Rossby response to low intrinsic frequencies and thereby increases the effect of damping (§3). Consequently, a larger fraction of the variance now lies in the Kelvin response. For both Kelvin and Rossby components, the variance is confined to very low frequencies, with peaks in the vicinity of ±0.01 cpd/2 ($T = 50$ days). Although for fast forcing the Kelvin response tends to maximize at frequencies corresponding to wavelengths twice the depth of the heating, here this tendency has been overwhelmed by the redness of the forcing spectrum, as described in SG (§5).

We have examined the sensitivity of the tropical response to the time scale of the forcing by varying $T$ between 22 and 45 days, bracketing all of the characteristic time scales indicated by Gruber’s (1974) brightness analysis which lie chiefly in the range 25–30 days.

![Fig. 12](image-url) Eliassen–Palm flux for the Kelvin projection component at $\sigma = 0.04$ cpd/2 in the easterly ($180^\circ$) and westerly ($0^\circ$) phases of the QBO. The contours are proportional to $\log_{10}$ of the magnitude.

The behavior in the latitude–longitude plane at tropopause level ($z = 2$) is illustrated in Fig. 16. Initially ($t = 0$), a Kelvin wavepacket of large meridional extent emerges, but quickly radiates to higher levels and is replaced ($t = 2$) by a narrower Kelvin structure, typical of the slow vertically propagating Kelvin modes. In extratropical latitudes, a complex pattern of highs and lows begins to form at this time and is well established by $t = 4$. The pattern in the winter hemisphere shows evidence of interference between the 5-day wave and the low frequency barotropic components. In the summer hemisphere the amplitude of this quasi-stationary response is much smaller and hence the traveling 5-day wave predominates.

At the stratopause (Fig. 17) the pattern is somewhat simpler. The 5-day wave, strongest in the winter hemisphere, can already be seen at $t = 1$ and is well established by $t = 4$ days. In the tropics, the fastest vertically propagating Kelvin waves are in evidence by $t = 2$. At subsequent times, the more slowly propagating waves arrive, and a dispersive train of Kelvin waves encircles the globe by $t = 6$.

![Fig. 13](image-url) Eliassen–Palm flux for the Kelvin projection component at $\sigma = 0.1$ cpd/2 in the easterly phase of the QBO.
Not surprisingly, the location of the response peak moves to lower frequencies with increasing characteristic time. However, the overall distribution of variance is not affected dramatically. Even though the heating spectra are red, with variance increasing monotonically with decreasing frequency, the response spectra all have peaks well removed from zero. Moreover, it is shown below that the structure and evolution of the response in physical space does not vary greatly between these cases, being dictated by the overall variance distribution rather than the position of its peak.

In midlatitudes (Fig. 18b), significant variance is
found in the first three zonal wavenumbers. Variance is concentrated along a line passing through the origin of the wavenumber–frequency plane, reflecting the background easterly winds in the tropical source region. In our calculations, the zonal wind in the tropical troposphere ranges between \(-3\) and \(-5\) m s\(^{-1}\), which implies that Rossby components in wavenumbers 1, 2 and 3 will be evanescent outside the heating at frequencies more positive than \(-0.004\), \(-0.08\) and \(-0.012\) cpd/2, respectively. The corresponding critical phase speed, \(-4\) m s\(^{-1}\), is indicated in Fig. 18b by the line running from the origin. Phase speeds more positive
than this critical value are suppressed through evanescence. Similar behavior is evident in the response to fast heating (Fig. 6a), but is less pronounced in that case because of the reduced forcing power available at low frequencies. The low-frequency suppression of the Rossby spectrum by the tropical winds can be seen more clearly in Fig. 19, which displays spectra at \( z = 4 \) scale heights for wavenumbers 1 and 2. Frequencies corresponding to a phase speed of \(-4 \text{ m s}^{-1}\) are indicated by the arrows. Response variance decreases sharply at frequencies lower than these values.

The combination of red response and low-frequency suppression by easterlies in the tropical source region thus leads to a concentration of extratropical variance.
about a critical phase speed which is related to the speed of the zonal winds in the tropical troposphere. Since the variability of these winds is comparable to their climatological mean (Lau, 1984; Oort, 1983), one would anticipate that the low frequency extratropical response will itself be subject to substantial variability. Strong easterlies will lead to a concentration of variance about a larger critical phase speed, producing an extratropical response that drifts westward in time, as illustrated in part b of this section. Weak easterlies, on the other hand, will result in a more nearly stationary extratropical response. Such sensitivity distinguishes the low frequency barotropic response from the robust normal mode behavior seen at higher frequencies.

**Fig. 17.** As in Fig. 16 except at the stratopause (z = 7 scale heights).
Structures corresponding to several wavenumbers and frequencies are shown in Fig. 20. For wavenumber 1, an internal Kelvin structure is present at 0.01 cpd/2, with a phase reversal in midtroposphere. The structure in the troposphere implies a vertical wavelength of more than 10 km, in apparent contradiction with the Kelvin dispersion relationship which predicts a much shorter wavelength at this low frequency. As argued in SG, however, the dispersion relation applies only to the homogeneous part of the solution outside the heating and cannot be used in the troposphere, where the equations are inhomogeneous and the particular solution dominates.

Wave structures for wavenumbers 1–3 at −0.01 cpd/2 are also shown in Fig. 20. In the tropics the structures exhibit a phase shift between the lower and upper troposphere, but at middle and high latitudes they are approximately barotropic in the troposphere and lower stratosphere. As for the fast heating cases, this behavior can be attributed to the overlap of projection and barotropic responses of various Rossby modes at low frequencies. For slow forcing, the response is concentrated at very low frequencies and therefore, is strongly damped in easterlies. This accounts for the great asymmetry of the structures, which have much larger amplitudes in the winter hemisphere, where the westerly jet is closer to the heating (see Figs. 1a and 2a) and thus Doppler-shifts the disturbances to higher intrinsic frequency. As a result, the response radiates much more effectively into the winter hemisphere than into the summer hemisphere. Wavenumbers 1 and 2 have large amplitudes in the stratosphere, while wavenumber 3 appears strongly trapped in the troposphere and lower stratosphere by strong stratospheric westerlies. Amplitude maxima in the troposphere move to lower latitude with increasing wavenumber, consistent with the behavior of ray turning points (Hoskins and Karoly, 1981).

We have also computed the response for slow heating located off the equator (case II.2). Heating correlation scales and the zonal mean wind are the same as in case II.1, but the heating envelope is displaced to 10° south. The spectrum of the response (not shown) is similar to that obtained with forcing symmetric about the equator, but the wave structures have larger amplitudes in the Southern Hemisphere due both to the stronger forcing south of the equator and to its proximity to the summer westerly jet. Figure 21 illustrates the enhanced propagation into the summer hemisphere for wavenumber 2 at −0.01 cpd/2 (cf. Fig. 20c). Note the large

<table>
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<th>t₀, T</th>
<th>λ₀, Δ</th>
<th>ϕ₀, Ψ</th>
<th>tₑ</th>
<th>θ_QBO</th>
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<td>0°, 20°</td>
<td>0°, 10°</td>
<td>1.0 (Jan. 21)</td>
<td>180° (East)</td>
</tr>
<tr>
<td>II.2</td>
<td>0, 30 d</td>
<td>0°, 20°</td>
<td>−10°, 10°</td>
<td>1.0</td>
<td>180° (East)</td>
</tr>
<tr>
<td>II.3</td>
<td>0, 30 d</td>
<td>0°, 20°</td>
<td>0°, 10°</td>
<td>3.0 (Mar. 21)</td>
<td>180° (East)</td>
</tr>
</tbody>
</table>

Fig. 18. Wavenumber-frequency spectra for Case II.1 at (a) the equator and z = 1 scale height, and (b) 45°N and z = 7 scale heights.
amplitude in the southern subtropics and midlatitudes. A strong antiphase behavior between the base and the top of the heating is also evident.

As a final example, we consider the response under equinox conditions (case II.3). The response in the tropics once again resembles that of case II.1, but radiation of the Rossby response to higher latitudes (not shown) is much diminished with respect to the winter case. This is apparently due to greater effective damping and stronger evanescence outside the heating, since the westerly jets are weaker and farther from the equator (see Fig. 1b). An interesting feature of the equinocial response is that a distinct peak in the wavenumber 1 spectrum (not shown) occurs at $-0.03$ cpd/2, or approximately 16 days. The corresponding structure is shown in Fig. 22. Except for the out-of-phase behavior between the lower and upper tropical troposphere, where the projection response predominates, the structure is very similar to that of the second symmetric Rossby normal mode (cf. Madden, 1978; Salby, 1981b; Hirooka and Hirota, 1985). Sharp phase transitions are evident at $\pm 35^\circ$, with nearly barotropic behavior at extratropical latitudes. The emergence of this slow normal mode response under equinox conditions is presumably associated with the reduced tropical easterlies typical of this season.

b. Cardinal realizations

The cardinal response signature for case II.1 is shown as a sequence of maps at one scale height in Fig. 23. In addition to contours of geopotential, perturbation velocities are indicated. Tropical behavior is dominated by projection components, whereas a barotropic Rossby wavetrain, developing initially in the form of a wavepacket, emerges conspicuously in the extratropical winter hemisphere. The latter is the manifestation of the low frequency spectral components in wave-numbers 1–3 described above. While the geopotential signal in the tropics is small compared to that in mid-latitudes, the tropical wind anomaly dominates the velocity perturbation throughout. The tropical signal reflects the influence of the Kelvin mode to the east of the heating and the gravest symmetric Rossby mode to the west (cf. Webster, 1972; Gill, 1980; Geisler and Stevens, 1986; Heckley and Gill, 1984). Flow behavior may be described as a surge from the heating region starting at $t = -2.5$ days, building up for the next 10 days, and then decaying over the final 10 days of the map sequence. The surge of westerlies to the east of the heating crosses the Pacific, reaching South America ($\approx 100^\circ$ map longitude) in somewhat less than 10 days, and eventually moves into the Atlantic. After that time, the westerly winds gradually decay without progressing much further east. The center of this surge of westerlies drifts gradually eastward during the decay stage. The main features of the tropical response are not altered significantly when the characteristic time scale of the heating is varied between 22 and 45 days. As an example, Fig. 24 shows the response when $T = 22$ days. Comparison with Fig. 23 reveals the same overall morphology.

To the west of the heating, two subtropical wind gyres associated with positive geopotential perturbations are apparent in Fig. 23. An intense easterly jet forms over the equator as a result of these gyres, which are manifestations of the projection response in the gravest symmetric Rossby mode. A strong subtropical jet, running NW to SE, forms between the northern flank of the tropical Rossby gyre in the winter hemisphere and the cyclonic feature immediately to the north. Because of the smaller phase speed of the Rossby mode, the subtropical gyres drift westward rather slowly in comparison with the eastward movement of the Kelvin response. The eastward advance of the latter is eventually halted as it approaches the region of strong easterlies produced over the equator by the Rossby response.

Outside the tropics, a Rossby wavetrain develops initially in the form of a barotropic wavepacket and propagates poleward and downstream of the forcing
Fig. 20. Wave structures for Case II.1 at various frequencies, \( \sigma \) (cpd/2), and zonal wavenumbers. Amplitudes are normalized to unity.
int into the winter hemisphere. By \( t = 5 \) days, a pattern of highs and lows has been set up, which drifts slowly (\( \approx 1.5^\circ \) per day) westward over the next 15 days. It resembles the teleconnection patterns observed (e.g., Horel and Wallace, 1981; Wallace and Gutzler, 1981) and obtained in both simplified models (Hoskins and Karoly, 1981; Simmons, 1982) and GCM’s (Blackmon et al., 1983). However, it differs from these time-mean pictures in that it develops as a transient wavepacket, subsequently extending downstream along a great circle route and drifting westward. Although the aforementioned studies have examined monthly or seasonal-mean behavior, the computed response is in fact of comparable time scale and therefore would be expected to be only partly captured in such statistics.\(^2\) It should also be borne in mind that the westward drift of the extratropical wavetrain depends on the strength of the easterly winds in the tropical troposphere, and therefore is subject to the same variability as these winds.

Figure 25 shows the behavior in the longitude–height plane over the equator. The low level response is out of phase with that in the middle and upper troposphere,

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\(^2\) Madden (personal communication, 1986) has found similar behavior in the tropics, where the “40-day” signal contaminates monthly-mean fields.
the globe, so a standing oscillation is never truly established. Such behavior involves a spread of frequencies (Fig. 18a), and cannot be reproduced in a monochromatic description. As in SG, unrealistically strong damping is unnecessary to produce plausible behavior in our baroclinic calculations. WESTERLY shear flanking the equator (Figs. 1 and 2) and easterly shear throughout the tropical troposphere act to Doppler shift the
disturbances and thereby accelerate damping already inherent in the response because of its low frequency.

Westerlies associated with the Kelvin response migrate eastward until they intercept the slower moving easterlies of the Rossby response, beyond which point they can propagate no further. The dispersion in opposite directions reflects comparable amounts of power in eastward and westward propagating components, although the proportion of variance is larger at eastward frequencies (Fig. 18a). Thus, the behavior over a majority of the tropical belt to the east of the heating is controlled by the Kelvin response, while the Rossby induced circulation dictates the behavior over a smaller sector on the western flank of the heating. This differs dramatically from the complete response in the absence of winds in SG, where the equatorial Rossby circulation prevailed. Predominance of the Kelvin response is due here to easterly shear within the tropical troposphere which preferentially increases the damping of the Rossby mode relative to that of the Kelvin mode.

If the response shown in Fig. 23 is eastward-filtered, the ensuing behavior resembles closely the simple eastward propagating Walker cell obtained in SG (Figs. 24 and 25) and compared there with Madden and Julian's composite. Several mechanisms could emphasize eastward over westward variance. For example, eastward propagation would be enhanced by stronger easterly shear in the upper troposphere. In such case, low frequency Rossby components would be even more effectively damped, leading to a greater contribution from the Kelvin response. Alternatively, if divergence induced by the disturbances promotes large-scale convection (e.g., Webster, 1981), this might reinforce the Kelvin wave. Although there is a component of cloud brightness which propagates with the circulation pattern (Madden, 1986), it apparently accounts for no
FIG. 25. Space–time evolution for Case II.1 at the equator. The geopotential contours and wind vectors above $z = 6$ scale heights are due to Rossby waves propagating from higher latitudes. (see Fig. 20b–d).
more than 10% of the total variance (e.g., see Weickman et al., 1985), indicating that the majority of convective variability is uncorrelated with this response as we have presumed.

The possibility also exists that the behavior of the 40-day wave is more complex than simple eastward propagation. Madden and Julian note that some of the data suggest that the oscillation is a combination of traveling and standing components. In particular, the phase of the sea level pressure field varies by only 45–60° across the Pacific basin (see Madden and Julian's Fig. 4), whereas a phase shift of about 180° would be expected for a pure propagating disturbance of wave number 1. Evidence of eastward propagation is clearer in the 150 mb zonal wind anomaly, consistent with Lorenc's (1984) analysis of 200 mb velocity potential during the FGGE year. However, Lorenc's analysis filters out the rotational component of the disturbance and hence may emphasize the (eastward propagating) Kelvin component. In fact, other studies of tropical variance in the 30 to 60 day range suggest a predominantly standing oscillation (Maruyama, 1982), or at least a combination of standing and traveling waves (Murakami and Nakazawa, 1985).

The response at midlatitudes (Fig. 26) is barotropic through the lowest four scale heights (i.e., up to \( \approx 28 \) km), above which a westward phase tilt is apparent. We have argued in section 4 that the barotropic nature of the response at extratropical latitudes results from the fact that disturbances most capable of propagating out of the tropics are external (or have very long vertical wavelengths). The westward tilt in the stratosphere can be ascribed to absorption of the waves in the tropical easterlies and to the effects of damping. Also clearly noticeable in Fig. 26 is a change in the zonal dimension of the response between the troposphere, where the first three wavenumbers combine to localize the disturbance zonally in the form of a wavetrain, and the stratosphere, where wavenumber 1 dominates and the disturbance encompasses the entire latitude circle. This

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**FIG. 26.** As in Fig. 25 except at 45°N.
behavior is due to refraction of the shorter waves by the strong winter westerlies which has the effect of filtering the wavenumber spectrum, allowing only the longest waves to penetrate into the middle and upper stratosphere. From a WKB perspective, rays oriented at low levels along great circle routes are Doppler-shifted in the stratosphere by strong westerlies so that they veer parallel to latitude circles, thereby dispersing wave activity completely around the hemisphere.

We examine next the signature of the response to heating located off the equator (case II.2). The forcing parameters are the same as in the previous case, except that the heating is now centered at 10°S. Figure 27 shows map sequences for \( t = 0 \) through 10 days at both \( z = 0.2 \) and \( z = 1 \) scale heights, to simultaneously highlight the horizontal behavior of the response and its vertical structure. The transition from the tropical response, with its antiphase character between the lower and upper troposphere, to the barotropic behavior at middle and high latitudes is readily apparent in this format. The surge of westerlies (easterlies near the surface) to the east of the heating is unchanged from the symmetric forcing case. Similarly, the barotropic wavetrain that radiates into the winter hemisphere is essentially unaltered (cf. Fig. 23). However, there is now a substantial Rossby wavetrain which propagates into the summer hemisphere, consistent with the proximity of the forcing to the summer westerlies. Note also the asymmetry of the response between the Southern Hemisphere (where the forcing is located) and the Northern Hemisphere. In particular, the subtropical gyre associated with the Rossby projection response is vigorous south of the equator, where the heating is located, but much weaker in the north. Consequently, the strongest perturbation in the meridional wind appears in the Southern Hemisphere, and there is cross-equatorial flow, northerly near the surface and southerly aloft. However, for heating located in the Northern Hemisphere during summer, the largest meridional velocity anomaly would appear north of the equator and would have just the opposite sign. Because of such seasonal reversals, the meridional wind signature would be difficult to isolate in annual data, since summer and winter signals would tend to cancel one another. It is apparently for this reason that no meridional velocity signal was found in Madden and Julian's original analysis of the 40-day wave. A recent study using data stratified according to season (Madden, 1986) reveals a highly significant meridional wind perturbation, coherent with that in the zonal wind. Figure 27 also shows that the out of phase relationship between the lower and upper tropospheric response diminishes with meridional distance away from the heating. This behavior is consistent with Madden's (1986) finding that the coherence and out of phase relationship between 850 mb and 150 mb winds is strongest in the summer hemisphere but decreases considerably at latitudes removed from the centers of convection.

Finally, Fig. 28 shows the evolution for case II.3, for which the heating is located on the equator and zonal winds are typical of equinox. The surging behavior of the tropical zonal wind anomaly on either side of the heating is also apparent for this case, although eastward propagation is more evident than for solstice conditions. This pattern is the most robust feature of the response to tropical heating, emerging in all of the calculations we have performed. The extratropical signature is now symmetric, as expected for forcing on the equator in the presence of symmetric background winds. The subtropical Rossby gyres are well developed, but there is no cross-equatorial flow because of the latitudinal symmetry of the pattern. Wavetrains radiate into both hemispheres, with extratropical anomalies in phase between the hemispheres.

6. Summary and discussion

We have examined the stochastic response to localized, random tropical heating in the presence of spatially-varying background states. The calculations presented here illustrate how the near-field disturbances described in our previous study (SG) are modified as they propagate away from the source region. Although, strictly speaking, the far-field behavior cannot be broken down into the individual modes used in SG to describe the near-field response, many of the features seen in SG are nevertheless present here as well. In particular, projection and barotropic components of the response emerge prominently in all ranges of frequency. The former dominate the solution in the tropics, while the latter are predominant at extratropical latitudes.

The distinction between projection and barotropic components of the response in unbounded geometry deserves emphasis. Under the radiation condition the projection component assumes the form of a continuum, with power spectral density everywhere \( \text{finite} \) and distributed over a range of frequency. The barotropic component, on the other hand, embodies normal mode behavior and is associated with \( \text{infinite} \) (or at least very large) spectral density concentrated at a particular frequency (cf. SG, Figs. 6, 10). At very low frequencies, the barotropic response (although not strictly a normal mode) is accentuated by the existence of a critical surface inside the heating which magnifies the effective forcing. Because of its singular nature relative to the baroclinic response, even a small projection onto the barotropic component leads to significant variance. Indeed, in virtually all of the cases considered here and in SG, collective power among projection and barotropic components is comparable.

This may be contrasted with the situation of bounded geometry with a rigid lid. There the projection continuum is replaced by a discrete set of internal normal modes, and hence both internal and external components are associated with singular behavior in the
frequency domain. Comparisons between the external mode and individual internal modes (e.g., Lim and Chang, 1983; Kasahara, 1984) may not be meaningful because the internal normal modes are artifacts of the rigid lid (Lindzen et al. 1968) which have no counterparts in unbounded geometry. Moreover, while the projection continuum arising under the radiation condition involves only those vertical scales that match
the forcing, the discrete spectrum of internal normal modes involves all scales. Consequently, as pointed out by Geisler and Stevens (1982), the role of much of this variance at low frequencies is simply to cancel the solution outside the heating. No such cancellation is necessary in unbounded geometry, as the structures automatically collapse to the heating layer in this limit.

Responses to fast heating, as may be produced by daily fluctuations in tropical convection, and to slowly evolving heating, associated with seasonal transitions in convective activity, have been studied separately. For fast heating, the projection and barotropic components are distinct and widely separated in the spectrum. Therefore, the structure at a particular wavenumber and frequency is dictated by a single component. The response to slow heating is also made up of projection and barotropic components, but in this case contributions from various modes overlap one another in the spectrum, coalescing into more or less of a continuum near zero frequency. Although the behavior at a particular wavenumber and frequency is influenced by both projection and barotropic components, the two remain readily distinguishable because the former dominates the response in the tropics while the latter dictates the behavior at extratropical latitudes.

In the case of fast heating, projection components radiate effectively in the vertical, traversing the stratosphere in only a few days. Barotropic components disperse rapidly out of the tropics, attaining the form of one or more normal modes that encompass the entire globe. Positive frequencies (eastward phase velocities) are dominated by the Kelvin projection continuum centered at frequencies corresponding to twice the effective depth of the heating, i.e., the actual depth reduced by the weak static stability of the tropical troposphere (SG). In the lower stratosphere, the frequency of the Kelvin projection response is consistent with Wallace and Kousky’s (1968) observations of slow
Kelvin waves, and with the 100 mb LIMS observations shown in Fig. 3 of SG. At greater altitudes, the Kelvin response shifts to increasingly higher frequency as a consequence of differential absorption, which acts preferentially on slower components of the spectrum. In the middle and upper stratosphere spectral peaks have shifted to frequencies typical of the fast Kelvin wave observed by Hirota (1978, 1979) and by Salby et al. (1984), while variance in the mesosphere lies at frequencies consistent with the ultrafast Kelvin wave (Salby et al., 1984). Thus, slow, fast and ultrafast Kelvin components all appear as manifestations of the same response modified by dissipation.

At negative frequencies (westward phase velocities) the response to fast heating is dominated by the gravest symmetric wavenumber 1 Rossby normal mode, the 5-day wave. This mode represents by far the greatest barotropic contribution to the response throughout the tropics, and in the extratropical stratosphere and mesosphere. In the extratropical troposphere, substantial variance is also present in zonal wavenumbers 1–3 at periods greater than 20 days. Although it is no longer possible to identify individual Rossby modes unambiguously at such low frequencies, the structures resemble mainly the barotropic response of the second symmetric Rossby mode. From a synoptic perspective this response assumes the form of a barotropic wave train, radiating initially in the form of a wavepacket from the tropical source region to higher latitudes. The barotropic character is a fundamental property of disturbances able to propagate out of the tropics. Such disturbances are associated with small values of Lamb’s parameter and, therefore, must be external or have very long vertical wavelengths. The barotropic component of the response may be difficult to detect in the temperature field, even when it dominates the geopotential and velocity fields, because of the small temperature perturbation associated with the weak vertical phase tilt.

The aforementioned features arise regardless of whether or not the heating is centered on the equator. However, symmetric components are excited to greatest amplitudes when the heating is directly over the equator. The Kelvin mode, in particular, captures a sizeable fraction of the response because its simple structure best matches that of a localized convective center.

Rapid vertical propagation is an integral aspect of the response to all our fast heating cases. For example, the Kelvin wavepacket discussed in section 4 radiates completely out of the troposphere within 4 days, before propagating zonally even once around the globe. Gravity modes radiate vertically even more efficiently. This suggests that barotropic calculations of the response to short-term heating fluctuations (e.g., Silva Dias et al., 1983) must be interpreted with caution. Specifically, persistence of features beyond the forcing duration is inconsistent with the present baroclinic calculations, since the response to fluctuations operating on time scales of a few days radiates freely out of the troposphere.

In the case of slowly evolving heating, the projection response is largely confined to levels of the forcing as a result of dissipation. There, its behavior is dictated by the particular part of the solution to the vertical problem and assumes the form of a transient Walker circulation. Barotropic components radiate laterally into extratropical regions. In contrast to the more dispersive response seen for fast heating, they are confined zonally in the form of a transient wave train which develops from an initially localized wavepacket, extending poleward along a great circle route. The barotropic wave train resembles teleconnection patterns emerging in time-mean statistics of observations (e.g., Horel and Wallace, 1981) and numerical calculations (e.g., Hoskins and Karoly, 1981; Blackmon et al., 1983). However, it is distinguished by its unsteady character, specifically the transient development along a great circle route and westward drift at speeds reflecting the easterly winds within the tropical source region. Such behavior would be only partly captured in monthly or seasonal-mean statistics. Spectrally, the wave train is composed of Rossby variance in zonal wavenumbers 1–3 concentrated at very low frequencies. Radiation of the barotropic response to extratropical latitudes is facilitated by the presence of westerly shear near the source region. Conditions most favorable to extratropical radiation occur in the winter hemisphere, where westerly shear Doppler-shifts disturbances to higher intrinsic frequencies. The extratropical response is then dominated by a large amplitude wave train that propagates into the winter hemisphere. During equinox, equal amounts of wave activity wavetrain into both hemispheres, although the amplitudes are reduced from those in winter. The position of the heating relative to the westerlies also influences the amount and direction of extratropical radiation.

The transient extratropical wave train drifts westward at a rate (typically a few degrees of longitude per day) indicative of easterlies in the tropical source region. Discrimination to a particular phase speed results from the combined influences of red forcing and a critical surface within the heating. Because the effective forcing varies inversely with ω (see SG), low frequency power radiating to extratropical latitudes increases with decreasing frequency, until a critical value is reached where phase speeds approximately match easterlies within the heating. At frequencies below this critical value, Rossby components move eastward relative to the flow and are therefore evanescent. Thus, power in these components falls off sharply away from the source. In combination, these two influences lead to a band-passed extratropical response, components in the far field restricted to a narrow range of phase speeds about the critical value. A parallel may be drawn with the emission of Rossby waves from a critical line when
forcing is located on the evanescent side of the singular region (Dickinson, 1968).

The concentration of extratropical response about a critical phase speed does not depend on the forcing being very slow, as it also occurs for fast heating. However, it is emphasized in the case of slow heating by the predominance of forcing variance at low frequencies. Because of its dependence on tropical winds and their significant variability, the extratropical wavetrain would be expected to have a highly variable feature of the response to tropical convection.

In contrast to the extratropical wavetrain, the tropical response to slowly evolving heating emerges as one of the most robust features of the calculations. It is much the same regardless of the position of the forcing or the background wind configuration. Variance is concentrated at very low frequencies (periods \(\approx 50\) days) and is divided between eastward Kelvin and westward Rossby components. As a result of dissipation, the response is confined to levels of the heating where it is controlled by the particular solution of the vertical problem, which accounts for its robustness.

The overall tropical response is suggestive of a standing oscillation, reflecting comparable amounts of variance at eastward and westward frequencies. However, the two principal components of the response, the Kelvin and gravest symmetric Rossby modes, actually disperse in opposite directions away from the heating. Because they are strongly attenuated as they propagate away from the source, they cannot effectively reinforce one another, and thus a standing oscillation is never truly established. As a result, measurements taken east of the heating will reflect predominantly the influence of the eastward Kelvin response. The amount of variance associated with the Rossby mode is sensitive to easterly shear in the tropical troposphere. Under typical conditions, westward Rossby variance is greatly reduced from that in the absence of winds (SG), and the greater part of the response is captured by the Kelvin mode.

East of the heating, the Kelvin response manifests itself in the zonal wind as a surge of westerlies aloft (easterlies at the surface), which intensifies, advances across the Pacific and collapses, all over a period of about 20 days. West of the heating, there is a less pronounced surge of easterlies aloft (westerlies at the surface), associated with a pair of Rossby gyres located on either side of the equator. The relative strength of these gyres depends on the location of the heating with respect to the equator. Many of these features agree remarkably well with observations of the so-called "40 day wave" (Madden and Julian, 1972; Lorenc, 1984). The behavior west of the heating (i.e., the Rossby gyres) is supported by Madden's (1986) recent analysis.

It would be desirable to affix particular amplitudes to these disturbances, say by quantitatively prescribing the heating. Unfortunately, the precipitation information necessary to do this are unavailable. Required are statistics in the form of spatial and temporal correlations or, equivalently, precipitation power spectra over wavenumber and frequency. These statistics detail the amount of heating taking place on particular space and time scales. In contrast, mean precipitation rates at a single location embody no such information; it is impossible to infer from them the fraction of heating which occurs coherently on large scales and long periods vs that which operates diurnally and on intrinsically smaller scales. Nonetheless, it is instructive to use what information is available to validate the calculated response against known observations.

We relate the response to slow heating to what is perhaps the best documented of the features discussed: the 40-day wave. Thirty years of observations over Truk Island indicate that this disturbance is associated with 150 mb winds of 10-15 m s\(^{-1}\) (Madden, 1986). The peak heating rate that produces a zonal wind amplitude of 10 m s\(^{-1}\) at this location in our calculations is 5.0 K day\(^{-1}\). This is in reasonable agreement with the maximum rate of 6.5 K day\(^{-1}\) estimated by Yanai et al. (1973), which presumably reflects some mean value. The maximum extratropical height anomaly resulting from the aforementioned heating rate, corresponding to the ridge which builds over the northern Pacific (Fig. 23), is 126 gpm. This too is reasonably consistent with geopotential height anomalies associated with observed teleconnection patterns (Wallace and Gutzler, 1981). Using a 15 m s\(^{-1}\) zonal wind amplitude to constrain the solution leads to a heating rate of 7.5 K day\(^{-1}\) and a geopotential height of 190 gpm. However, we emphasize that the precise numerical values attributed to these features, particularly the extratropical wavetrain, will vary with the nature of the heating spectrum, which is poorly known, and with the tropical flow.

Perhaps the most appealing aspect of the present calculations is their ability to capture within a unified framework much of the diversity of the response to tropical convective heating: Propagation of Kelvin waves into the stratosphere and mesosphere, radiation of barotropic wavetrains to extratropical latitudes, and excitation of transient Walker cells (which in the past have been treated more or less individually) can all be understood as different aspects of the response to unsteady latent heat release in the tropics. The concepts of projection and barotropic response, together with simple theoretical arguments regarding the effects of dissipation and Doppler-shifting by background winds, are capable of providing a physically meaningful explanation of the major features of the computed response.

Although these ideas appear to account for phenomena as diverse as mesospheric Kelvin waves and tropospheric Rossby wavetrains, a number of potentially important questions such as nonlinear feedback, instability resulting from the combined presence of stationary and traveling waves in extratropical regions, and the effect of zonally asymmetric basic states have
not been addressed. In addition, all the calculations have been performed with a single heating source. The interaction between multiple centers of convection, especially at long periods for which spatial correlations are likely to be significant, remains to be investigated. Finally, the rich character of the response and its relationship to statistical properties of the heating (e.g., variance spectra and spatial and temporal correlations), underscores the need for a better understanding of these characteristics of tropical convection.

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3 However, instability of the zonally symmetric background flow is embedded in our solutions. It can be shown that only the long-time behavior is not captured when the calculations are performed with finite frequency resolution. Thus, we are led to the conclusion that the initial projections onto unstable modes and/or the growth rates of the latter are small.


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