On the Mean Meridional Circulation of the Middle Atmosphere

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ABSTRACT

A zonally averaged, quasi-geostrophic residual Eulerian model is used to illustrate how the adjustment of the middle atmosphere to externally imposed forcing depends on internal dissipative properties (parameterized as Newtonian cooling $\alpha$ and Rayleigh friction $K_r$) and on the periodicity of the forcing. It is shown that when the problem is formulated in this manner, many well-known properties of the stratosphere/mesosphere system (e.g., the relative efficiency of wave versus diabatic driving of the meridional circulation, and the near radiative equilibrium of much of the stratosphere) are succinctly expressed in terms of the governing elliptic differential equation and its solutions. Despite its simplicity, the model is a useful heuristic tool for studying the response of the middle atmosphere to external forcing.

1. Introduction

The mean meridional circulation of the Earth's atmosphere is of interest because it forms a basis for understanding how quantities such as angular momentum, temperature and chemical constituents are distributed in latitude and altitude. Although diagnostic and modeling studies of the mean meridional circulation are numerous, relatively few works have addressed the question of what fundamental processes determine its structure and strength. For the scales of motion typical of the mean circulation, the geostrophic approximation is valid except very near the equator, and above $\sim 100$ km (where ion drag becomes larger than the Coriolis torque.) Eliassen (1951) used a quasi-geostrophic set of equations to show how the meridional circulation restores the thermal wind balance upset by externally imposed thermal or mechanical forcing. He demonstrated how gravitational and inertial forces influence the shape of the mean meridional streamfunction, and how the ratio of the strength of these forces accounts for the quasi-horizontal, elliptical shape of the streamlines in the meridional plane. Eliassen also showed that a point source of heat produces a "thermally direct" meridional circulation (i.e., from regions of net diabatic heating to regions of net warming), while a point source of easterly (westerly) momentum gives rise to a poleward (equatorward) circulation cell. In a recent study, Plumb (1982) extended Eliassen's work by considering the response to more realistic forcing. In particular, Plumb showed how the position of the forcing in the meridional plane affects the extent of the meridional circulation: The response to forcing in the vicinity of the equator is largely confined to the tropics, but forcing at middle and high latitudes drives a global circulation, extending into the opposite hemisphere.

The studies of Eliassen and Plumb provide many useful insights into the processes controlling the mean meridional circulation. However, an important question not addressed in these works concerns the adjustment of the atmosphere to externally imposed forcing. When a stratified, rotating atmosphere in quasi-geostrophic equilibrium is subject to external forcing, it can respond to such forcing by a change in its zonally averaged temperature and wind structure, or through a mean meridional circulation. Specifically, the thermodynamic budget can be balanced by infrared relaxation, or by vertical motions that produce adiabatic cooling and warming, while the momentum budget can adjust by means of frictional dissipation of zonal momentum or through the acceleration produced by the Coriolis torque. The nature of this adjustment process is of importance for understanding such issues as the thermodynamic budget of the middle atmosphere, its zonal mean temperature and wind structure, and its response to perturbations in external forcing. For example, Fels et al. (1980) have shown that the response of a general circulation model to changes in stratospheric heating resulting from ozone reductions proceeds through infrared relaxation except near the equator, where a mean meridional circulation is generated.

The response of the tropical troposphere to mechanical and thermal forcing was considered by Dickinson (1971a,b), who pointed out that the rates of in-

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frared dissipation and frictional damping play a central role in determining the relative efficiency of the two types of forcing. In particular, Dickinson demonstrated that when frictional damping of zonal wind perturbations is weak, the response to diabatic heating takes place mainly through infrared relaxation because the large Coriolis torques associated with a strong meridional circulation cannot be balanced in the steady state. Conversely, when frictional damping is fast compared to thermal relaxation, diabatic heating is balanced principally by a mean meridional circulation. Dickinson (1968) has also shown that the time dependence of external forcing influences the response in a manner similar to thermal and frictional damping. When time dependence is important, the momentum and thermodynamic budgets can be balanced by the time tendencies of the zonal wind and temperature fields. The fundamental idea elucidated by Dickinson’s work is that departures from radiative equilibrium in a stratiﬁed, rotating atmosphere require some mechanism to balance the Coriolis torque produced by the mean meridional circulation. Some of these concepts were also discussed by Leovy (1964), who showed how the circulation driven by external heating depends on thermal and mechanical dissipation rates.

It appears that, apart from the work of Dickinson and Leovy, previous studies of the mean meridional circulation have concentrated on diagnosing the response given the net diabatic heating rates and wave driving without entering into considerations of how the dissipative properties of the atmosphere affect the nature of this response. The purpose of the present study is to examine the adjustment problem for the middle atmosphere in terms of the transformed Eulerian zonal mean equations (Boyd, 1976; Andrews and McIntyre, 1976). The transformed Eulerian formalism allows a clearer separation between thermal (e.g., solar heating) and mechanical (e.g., planetary wave) drives, and highlights the fundamentally different effects of the two kinds of forcing. Although the emphasis is on the mean meridional circulation, the latter is closely related to the zonal wind structure through the meridional temperature gradient and the thermal wind relationship. Therefore, we also discuss where appropriate the mean zonal wind distributions obtained for various choices of external forcing and internal parameters of the atmosphere. Section 2 reviews brieﬂy the solution of the quasi-geostrophic set of zonally averaging equations, following the method of Plumb (1982). Sections 3 and 4 illustrate by means of simple numerical examples the response to steady-state and periodic, time-dependent forcing, respectively. The results are summarized in the last section.

2. Governing equations

We use the transformed Eulerian quasi-geostrophic equations in log-pressure coordinates on a spherical atmosphere:

\[
i\omega \tilde{u} - 2\Omega \mu \tilde{v} = -K_R \tilde{u} + \tilde{M}
\]  
\[
2\Omega \mu \tilde{u} = -\frac{1}{\mu} (1 - \mu^2)^{1/2} \frac{\partial \tilde{\phi}}{\partial \mu} \tag{1.1}
\]  
\[
\frac{\partial \tilde{\phi}}{\partial \tau} = \frac{\tilde{g} \tilde{T}}{T_0} \tag{1.2}
\]  
\[
i\omega \tilde{T} + \tilde{v} * \tilde{S} = \tilde{H}_s - (B + \alpha \tilde{T}) \tag{1.4}
\]  
\[
\frac{1}{\sigma} \frac{\partial}{\partial \mu} [\tilde{v} * (1 - \mu^2)^{1/2}] + \frac{1}{\rho} \frac{\partial (\tilde{v} * \rho)}{\partial z} = 0 \tag{1.5}
\]  

where \( \tilde{u} \) is the zonal mean wind, \( \tilde{v} \) and \( \tilde{v} * \) define the residual Eulerian meridional circulation, \( \tilde{\phi} \) is the geopotential, \( \tilde{T} \) is the deviation of the zonally averaged temperature from its global mean \( T_0 \), and \( S = HN^2/R \) is a global mean static stability parameter. The horizontal coordinate is \( \mu = \sin(\phi) \), where \( \phi \) is the latitude, and \( \rho_0 = \rho_0 \exp(-z/H) \), where \( \rho_0 \) is a standard density and \( H \) is the atmospheric scale height. The angular frequency and radius of Earth are denoted by \( \Omega \) and \( a \), respectively. These equations are similar to those used by Plumb (1982), but we have departed from his formulation in several respects:

The diabatic heating term has been written as the sum of shortwave heating \( \tilde{H}_s \) and infrared cooling \( B + \alpha \tilde{T} \), where \( \alpha \) is a Newtonian cooling coefficient and \( B \) is the cooling averaged globally on a pressure surface (Schoeberl and Strobel, 1978);

Frictional dissipation has been included in the zonal momentum equation as Rayleigh friction with coefficient \( K_R \), in addition to the momentum source term \( \tilde{M} \), which in the stratosphere is due principally to the Eliassen–Palm flux divergence of planetary waves;

The time-dependent terms in the zonal momentum and thermodynamic equations have been written as \( i\omega \tilde{u} \) and \( i\omega \tilde{T} \), respectively, where \( \omega \) corresponds to the frequency of the forcing. For steady forcing, \( \omega = 0 \). This representation does not entail any loss of generality, since arbitrary time dependence can be obtained by suitable Fourier synthesis of frequency components. Dickinson (1969; 1971a,b) used an analogous formalism to study how external forcing determines the structure of the zonal wind in the tropics and the transient evolution of zonal wind systems in the tropical and extratropical stratosphere. Volland (1983) has applied a similar method to study the inverse problem of determining the external heating distribution that drives a specified mean meridional circulation.

We proceed by combining (1.1)–(1.5) into an elliptic partial differential equation for the vertical velocity

\[
L_z(\tilde{\omega}^*) + \left( \frac{K_R + i\omega}{\alpha + i\omega} \right) \frac{N^2}{4\Omega^2a^2}L_\mu(\tilde{\omega}^*) = \frac{1}{S} \left( \frac{K_R + i\omega}{\alpha + i\omega} \right) \times \frac{N^2}{4\Omega^2a^2}L_\mu(\tilde{H}_s) + \frac{1}{2\Omega a} \frac{\partial}{\partial \mu} \left[ \frac{(1 - \mu^2)^{1/2}}{\mu} \frac{\partial \tilde{M}}{\partial z} \right] \tag{2}
\]
where

\[ L_{\mu}(\cdot) = \frac{\partial}{\partial \mu} \left[ (1 - \mu^2) \frac{\partial}{\partial \mu} (\cdot) \right] \]  

(3.1)

\[ L_z(\cdot) = \frac{\partial}{\partial z} \left[ \frac{1}{\rho} \frac{\partial}{\partial z} (\rho \cdot) \right] \]  

(3.2)

Equation (2) is essentially the same as Eq. (2.10) in Plumb, except for the factor \((K_R + i\omega)/(\alpha + i\omega)\) multiplying the last term on the lhs and first term on the rhs. According to (2), the relative efficiency of thermal versus mechanical forcing depends on the ratio \((K_R + i\omega)/(\alpha + i\omega)\). In particular, for steady-state conditions or sufficiently slow forcing \((\omega \to 0)\), the relevant ratio becomes \(K_R/\alpha\). If \(K_R/\alpha \ll 1\), thermal forcing is relatively inefficient, and the atmosphere responds to heating by changing its temperature through infrared cooling, while for \(K_R/\alpha \gg 1\) the adjustment is through the mean meridional circulation. Note also that thermal forcing depends not on the heating rate \(H_z\), but on its meridional gradient, while mechanical forcing is given by the vertical gradient of \(M\). Physically, a horizontally invariant heat source will affect the global mean temperature distribution, but it will not drive a meridional circulation since it cannot produce meridional gradients of geopotential. Similarly, a momentum source which is constant in height will produce a uniform acceleration of the mean zonal wind but no meridional circulation.

The foregoing properties can be illustrated more explicitly by considering the solution to (2). This equation is separable, i.e., the solution can be written as

\[ \tilde{w}^* = \sum_n W_n(z)\theta_n(\mu) \]  

(4)

Substitution of (4) into (2) yields a homogeneous horizontal eigenfunction equation

\[ L_{\mu}(\theta_n) = \epsilon_n \theta_n \]  

(5.1)

and a forced vertical structure equation

\[ L_z(W_n) - \Lambda_n^2 W_n = -\Lambda_n^2 \left( \frac{H_{sn}}{S} - \frac{1}{2\Omega a \Lambda_n^2} \frac{\partial M_n}{\partial z} \right) \]  

(5.2)

where

\[ \Lambda_n^2 = -\frac{N^2}{4\Omega^2 \alpha^2} \epsilon_n \left( \frac{i\omega + K_R}{i\omega + \alpha} \right); \]  

(5.3)

\(H_{sn}\) and \(M_n\) are the projections of the thermal and mechanical forcing onto the relevant eigenfunctions, and \(\epsilon_n\) is the separation (Lamb's) parameter.

The set (5), together with suitable boundary conditions can be solved by numerical methods. The homogeneous solutions of (5.2), i.e., the solutions outside the forcing region, can be obtained analytically and are of the form

\[ W_n \sim \exp(\kappa_n^\pm z) \]  

(6)

where

\[ \kappa_n^\pm = \frac{1}{2H} \left( 1 \pm \sqrt{1 + 4H^2\Lambda_n^2} \right) \]  

(7)

As noted by Plumb (1982), \(\epsilon_n < 0\) for all \(n\) in the present, zonally symmetric problem. However, the homogeneous solutions described by (6) and (7) are in general damped waves rather than the purely external motions obtained by Plumb. Plumb's solutions are recovered in the case of very high frequency forcing, where the thermal and mechanical damping rates become irrelevant. This limit has been called the "adiabatic regime" by Dickinson (1968). Purely external solutions are also obtained in the steady state, although under these conditions their behavior is sensitive to the relative magnitude of frictional dissipation and thermal relaxation (cf. Dickinson's, 1968, "steady regime").

The homogeneous solutions describe the behavior below and above the forcing, respectively, as illustrated schematically in Fig. 1 for the steady-state case. The solutions decay more rapidly below the forcing than above it as a consequence of the density stratification of the atmosphere (the difference disappears as \(H \to \infty\)). The rate of decay also depends, through \(\Lambda_n^2\), on the ratios \(N^2/\Omega^2\) and \((K_R + i\omega)/(\alpha + i\omega)\). The dependence on \(N^2/\Omega^2\) corresponds to Eliassen's result regarding the relationship between the "aspect ratio" of the meridional circulation and the ratio of gravitational to inertial stability, i.e., vertical motions are constrained by the stratification of the atmosphere and horizontal motions by its rate of rotation. Thus, a shallow (deep) mean meridional circulation obtains for large (small) \(N^2/\Omega^2\). The dependence on \((K_R + i\omega)/(\alpha + i\omega)\) arises

![Fig. 1. Schematic diagram of the behavior of the solutions $W_n$ for various choices of $K_R/\alpha$.](image-url)
from similar considerations: strong frictional damping and/or rapidly varying forcing will readily balance the Coriolis acceleration due to meridional motion, thus confining the circulation to a narrow layer about the forcing. On the other hand, when \( (K_R + i\omega) \) is small, the circulation must be weaker and spread over a deep layer if the momentum budget is to be balanced. An analogous argument can be made for the vertical velocity from the thermodynamic equation.

In all the examples presented in sections 3 and 4, we follow Plumb (1982) in imposing the boundary conditions \( W_n(z) = 0 \) at \( z = 0 \) and \( W_n(z) \sim \exp(\kappa_n z) \) at \( z = z_{\text{top}} \) (here taken to be 80 km). Once the \( W_n \) are determined, \( \bar{w}^* \) is obtained from (4) and the remaining fields are computed using the quasi-geostrophic relationships (1.1)–(1.5).

3. Steady-state forcing

In this section we consider the response of an atmosphere governed by (1.1)–(1.5) to thermal and mechanical forcing for various choices of the parameters \( \alpha \) and \( K_R \). Although the mechanisms that force the meridional circulation usually vary in time, it is nevertheless useful to illustrate some basic features of the response by considering the simple case of steady-state forcing, wherein \( \omega \rightarrow 0 \) in (2).

a. Ozone heating in the upper stratosphere

We examine first the response to heating given by

\[
Q_0 = \bar{H}_z - B = 10^{-4} \exp \left( \frac{(z-50 \text{ km})^2}{15 \text{ km}} \right) (-\mu^{1/2}) \text{K s}^{-1}.
\]

This represents a large-scale heating distribution with a "summer to winter" heating differential of approximately 20 K day\(^{-1}\). As noted previously, the response of the atmosphere to such external forcing will depend on the ratio \( K_R/\alpha \). In the stratosphere, the radiative relaxation time scale varies from a few days at the stratosphere to several weeks near the tropopause; we have chosen \( \alpha = 0.1 \text{ day}^{-1} \) as a representative value for the entire region.\(^1\) The Rayleigh friction coefficient, which can be thought of as representing the effect of "wave drag" produced by dissipating stationary gravity waves, is expected to be small in the stratosphere (e.g., Holton and Wehbein, 1980). We adopt a value \( K_R = 0.02 \text{ day}^{-1} \) which, together with our choice of \( \alpha \) gives \( K_R/\alpha = 0.2 \). The resulting mean meridional circulation is illustrated in Fig. 2. The meridional and vertical velocities have typical magnitudes of 0.3 m s\(^{-1}\) and 0.5 mm s\(^{-1}\), respectively. These values are consistent with

\(^1\) Because the derivation of Eq. (2) involves only horizontal differentiation of the thermodynamic equation (1.4), it is possible to specify a vertically varying Newtonian cooling coefficient, \( \alpha(z) \), without affecting the separability of the solutions. However, as long as the ratio \( K_R/\alpha(z) \) remains small, these solutions do not differ qualitatively from the results for constant \( \alpha \) presented in this section.

(although somewhat smaller than) estimates derived from diagnostic studies of the diabatic circulation (e.g., Murgatroyd and Singleton, 1961; Solomon et al., 1986), but they are much smaller than the maximum values that could be realized if the externally imposed heating were balanced solely by the meridional circulation. Indeed, for the small value of \( K_R/\alpha \) used in this example, the external heating is nearly balanced by Newtonian infrared cooling, and the steady-state temperature distribution is rather close to radiative equilibrium (see Fig. 6). Figure 3 presents the solutions \( W_n \) of (5.2) corresponding to the first three antisymmetric eigenvalues \( \kappa_n \). [The heating distribution (8) does not project onto the symmetric eigenfunctions.] The \( W_n \) decay slowly away from the forcing, as expected from the smallness of the ratio \( K_R/\alpha \), and this is reflected in the large depth occupied by the meridional circulation. For comparison, the response of the system to the same heating distribution when \( K_R = 0.2 \text{ day}^{-1} \) \((K_R/\alpha = 2.0)\) is shown in Fig. 4. Typical values of \( \bar{w}^* \) and \( \bar{w}^* \) are now larger by a factor of 4 or 5 and the meridional circulation is considerably shallower.

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**Fig. 2.** Horizontal and vertical residual Eulerian mean velocities driven by steady-state stratospheric shortwave heating when \( K_R/\alpha = 0.2 \).
It is perhaps worth noting that the strong sensitivity of the meridional circulation forced by the heating distribution (8) to the ratio $K_R/\alpha$ is in part due to the fact that the heating projects strongly onto only the first few (antisymmetric) modes (see Fig. 3). The value of the separation parameter $\epsilon_n$ increases rapidly with the mode number $n$ and, for sufficiently large $\epsilon_n$, the factor $A_n^2$ in (5.2) will become large regardless of the magnitude of $K_R/\alpha$. Thus, the mean meridional circulation forced by a heating distribution with significant projection onto the higher-order modes will be less affected by the ratio $K_R/\alpha$ than that in the present example. Given the latitudinal structure of the eigenfunctions $\Theta_n$, strong projection onto high-order modes will occur most readily for heat sources located in the tropics (cf. Plumb, 1982). This implies that tropical heating will tend to be balanced by a mean meridional circulation rather than by radiative relaxation, in agreement with the conclusions of Fels et al. (1980). Physically, this effect can be understood by noting that the constraint imposed on the momentum budget by the Earth's rotation is relaxed near the equator, where the Coriolis torque vanishes.

Figure 5 shows the zonal wind distributions obtained from Eq. (1.1) for $K_R/\alpha = 0.2$ and $K_R/\alpha = 2.0$. In the first case the zonal winds are very strong, and the jets do not "close off" above the stratopause. When $K_R/\alpha = 2.0$, on the other hand, the strength of the zonal wind is reduced by a factor of 2 and decreases rapidly.
above 65 km, as first demonstrated by Leovy (1964). The strong jets of Fig. 5a are representative of conditions in the stratosphere, where the ratio $K_R/\alpha$ is expected to be small, while the rapid decrease in amplitude seen in Fig. 5b resembles the behavior in the mesosphere, where the “wave drag” due to breaking gravity waves, and hence the effective $K_R$, is known to become large (Lindzen, 1981; Holton, 1982; Garcia and Solomon, 1985). The temperature distribution corresponding to the zonal wind of Fig. 5b departs sharply from radiative equilibrium, as illustrated in Fig. 6.

The result that the zonal wind decreases with increasing $K_R/\alpha$ is perhaps not entirely obvious. From Eq. (1.1)

$$\bar{u} = \frac{f \bar{v}^*}{K_R}$$

in the thermally forced, steady-state case. If an increase in $K_R$ (and hence $K_R/\alpha$) results in a larger $\bar{v}^*$, it is not immediately clear whether (9) then implies a decrease or an increase in $\bar{u}$. The reason is that the equilibrium value of $\bar{u}$ cannot be determined from (9) alone; the remaining governing equations must also be taken into account.

With a scale analysis, Eqs. (1.1)–(1.5) can be combined to yield:

$$\bar{u} \sim \left( \frac{L_z R}{L_y H} \frac{1}{f \alpha} \frac{\bar{H}_z}{1 + \frac{L_z^2 N^2 K_R}{L_y f^2 \alpha}} + \frac{\bar{M}}{K_R} \frac{L_z^2 N^2 K_R}{1 + \frac{L_z^2 N^2 K_R}{L_y f^2 \alpha}} \right)$$

where $L_y$ and $L_z$ are characteristic horizontal and vertical scales. Further, using (1.3) and (1.4), we can write

$$\frac{L_z R}{L_y H} \frac{1}{f \alpha} \bar{H}_z \sim \bar{u}_e$$

where $\bar{u}_e$ is the radiative equilibrium wind, i.e., the zonal wind in geostrophic balance with the radiative equilibrium temperature gradient.

Equation (10) provides a simple way of estimating the magnitude of the steady-state zonal wind produced by specified thermal and/or mechanical forcing in a quasi-geostrophic atmosphere of known $K_R/\alpha$. In particular, as $K_R/\alpha \to 0$ in the absence of mechanical forcing $\bar{M}, \bar{u} \to \bar{u}_e$, while for $K_R/\alpha > 0$

$$\bar{u} \sim \bar{u}_e \left( 1 + \frac{L_z^2 N^2 K_R}{L_y f^2 \alpha} \right)$$

When expressed in terms of the radiative equilibrium wind $\bar{u}_e$, the result is independent of whether $K_R/\alpha$ is increased by increasing $K_R$ or by decreasing $\alpha$. However, in the latter case $\bar{u}_e$ will be larger.

b. Eliassen-Palm flux divergence

We consider next the steady-state response to mechanical forcing such as the divergence of the Eliassen-Palm (EP) flux of planetary waves. We specify the following simple pattern of EP flux divergence

$$-3 \times 10^{-5} \exp \left[ -\frac{(z - 35 \text{ km})^2}{15 \text{ km}} \right]$$

$$M_0 = -2 \mu (1 - \mu^2)^{1/2} \text{ m s}^{-2}, \quad \mu > 0$$

which corresponds to a mean flow acceleration of about $-3 \text{ m s}^{-1} \text{/day}$ centered at 35 km and 45°N. The mean meridional velocities are shown in Fig. 7. Note that the meridional circulation extends well into the tropics of the Southern Hemisphere even though the forcing is located at 45°N (cf. Plumb, 1982). The magnitudes of the horizontal and vertical velocities produced by this small mechanical forcing are comparable to their counterparts due to the heating differential discussed in section 3a. This is an example of the inefficiency of thermal relative to mechanical forcing for small $K_R/\alpha$ and implies that the mean meridional circulation of the winter stratosphere must be strongly influenced by the EP flux divergence of planetary waves. Of course, the equator-to-pole circulation characteristic of the lower stratosphere (Brewer, 1949; Dobson, 1956; Murgatroyd and Singleton, 1961; etc.) cannot be produced by stratospheric ozone heating in any case since the horizontal distribution of such heating forces mainly a single pole-to-pole cell.

The zonal wind distribution resulting from the wave forcing (Fig. 8) is interesting in several respects. The largest easterly winds occur in the tropics of the winter hemisphere, even though the forcing is centered at 45°N. This is a consequence of the westerly Coriolis torque on the mean meridional circulation which opposes the easterly acceleration due to the EP flux divergence. South of the equator the Coriolis torque is
easterly and produces an easterly wind maximum at \( \sim 15^\circ \)S. There is no contribution here from wave EP flux divergence, since this has been set to zero in the Southern Hemisphere. The magnitude of this easterly wind (\( \sim 10 \) m s\(^{-1}\) at 15\(^\circ\)S and 45 km) is considerable, and suggests a role for planetary waves in the easterly phase of the semiannual oscillation (SAO) beyond that due to the direct effect of the EP flux divergence. Of course, the easterly winds computed with our quasi-geostrophic model vanish at the equator, where \( f \rightarrow 0 \), but the inclusion of nonlinear advection of zonal momentum would have the effect of spreading the area of easterly accelerations equatorward. The importance of the meridional circulation in the easterly phase of the SAO has been noted previously (e.g., Holton and Wehrbein, 1980; Takahashi, 1984), but usually in the context of diabatic forcing. However, the present results suggest that the circulation forced by waves in the winter hemisphere can extend into the tropics of both hemispheres with enough amplitude to produce substantial easterly accelerations there. Further, a wave-driven circulation may help explain such aspects of the easterly phase of the SAO as the altitude of maximum easterlies (which is lower than can be accounted for by the thermally driven circulation; e.g., see Holton and Wehrbein, 1980; Hirot a, 1980) and their short-term variability in the summer hemisphere (Hopkins, 1975).

The foregoing conclusions must be tempered by the observation that nonlinear momentum advection will produce inertially unstable regions near the equator (e.g., Dunkerton, 1981). This implies that the nonlinear counterpart of (2) is hyperbolic in this neighborhood, and casts doubt on the validity of our results in the tropics. However, the idea that circulations forced in the winter hemisphere extend into the tropics of both hemispheres is supported empirically by the almost simultaneous occurrence of tropical stratospheric cooling and high-latitude stratospheric warming (e.g., Quir oz, 1979).

The zonal mean wind distribution produced by the combined effect of thermal and mechanical forcing specified above (with \( K_R/\alpha = 0.2 \)) is shown in Fig. 9. (Note that since the governing equation (2) is linear, the effects are simply additive). Although the maximum winds in the upper stratosphere are somewhat weaker than obtained for thermal forcing alone, the jets still do not close off in the mesosphere. In the tropical middle and upper stratosphere a region of easterly winds now encroaches into the winter hemisphere. These easterlies correspond roughly to the situation observed in winter climatologies of the stratosphere.

4. Time-dependent forcing

Forcing of the stratospheric mean meridional circulation varies on annual (e.g., solar heating) and intraseasonal (e.g., EP flux divergence) time scales. Here we examine briefly the effect of such time dependence.

a. The annual heating cycle

The largest variation of the stratospheric global heating distribution is due to the annual cycle. We
study the response of the stratosphere to such variation by imposing the heating distribution \( \dot{Q}(t) = Q_0 \cos(\omega t) \), with \( Q_0 \) given by (8) and \( \omega = 2 \pi / 365 \) day\(^{-1}\). All other parameters are the same as in the steady-state calculations of the previous section. The problem is thus identical to that of steady thermal forcing, except that the factor \( K_R / \alpha \) is replaced by \( (K_R + i \omega) / (\alpha + i \omega) \). Because the forcing frequency \( \omega \) is small compared to the radiative relaxation time scale, the ratio \( (K_R + i \omega) / (\alpha + i \omega) \) itself remains small and the solution for the mean meridional circulation (not shown) resembles closely those obtained in the steady-state case (Fig. 2). The stratosphere remains near radiative equilibrium and the temperature response lags the thermal forcing by only a small fraction of a cycle. These relationships are illustrated in Fig. 10, which shows the evolution of the terms in the thermodynamic equation (1.4) at 40 km and 60°N through one cycle of thermal forcing. Except near equinox, the temperature tendency \( \partial T / \partial t \) is small compared to both the forcing \( \dot{Q} \) and the thermal relaxation \( -\alpha \dot{T} \). Adiabatic effects due to the mean vertical velocity are likewise small, and the temperature response lags the heating by \( \sim 0.05 \) cycles (i.e., less than three weeks) as can be ascertained by comparing the phases of the curves for \( \dot{Q} \) and \( -\alpha \dot{T} \). Note, incidentally, that near solstice the vertical velocity \( \bar{w}^* \) could be determined with small error from the net diabatic heating rate even if \( \partial T / \partial t \) is ignored (cf. Dunkerton, 1978). Near equinox, on the other hand, \( \partial T / \partial t \) is as large as the diabatic heating terms and therefore must be included in the calculation of \( \bar{w}^* \).

\[ b. \textbf{Short-term variations in EP flux divergence} \]

We examine next the response to rapidly varying planetary wave activity. For example, Madden and Labitzke (1981) have shown that during January of 1979 the amplitude of planetary wave 1 in the stratosphere underwent regular variations with a period of roughly 16 days. We represent such a situation by imposing an EP flux divergence \( \dot{M}(t) = M_0 \cos(\omega t) \), where \( \omega = 2 \pi / 15 \) day\(^{-1}\), and \( M_0 \) has the same horizontal and vertical dependence as (13), but its amplitude is now \( \sim 40 \) m s\(^{-1}\)/day, consistent with magnitudes observed during large-amplitude wave events (e.g., Palmer, 1981). If the variation of EP flux divergence is assumed to arise from interference between standing and traveling waves (e.g., Salby and Garcia, 1987), the spatial structure of the EP flux is much more complex than the simple pattern represented by (15), but the latter will suffice for our present purposes.

Figure 11 shows the meridional circulation (at \( t = 0 \)) induced by the fluctuating EP flux divergence. The circulation consists of a clockwise lower cell and a counterclockwise upper cell. The lower cell, by far the strongest one, dominates the entire winter stratosphere and both cells extend into the subtropics of the summer hemisphere. The adiabatic effects associated with the stratospheric cell produce strong warming in the high-latitude winter stratosphere and smaller, but by no means negligible cooling in the tropics (Fritts and Soules, 1970; Barnett, 1975; Quirroz, 1979). The mesospheric cell has the opposite effect, producing mesospheric cooling at high latitudes and warming in the tropics (Fig. 12).

The zonal wind distribution induced by the forcing (at \( t = 0.25 \) cycles, near the time of maximum response) is shown in Fig. 13. Strongest easterlies occur at \( \sim 35^\circ \)N, equatorward of the maximum EP flux divergence. In the southern hemisphere, there is a region of easterlies centered at \( \sim 10^\circ \)S. As discussed in the previous section, these effects are due to the varying action of the Coriolis force on the (northward) mean meridional velocity.

Figure 14 illustrates the zonal momentum budget at 40 km in northern high latitudes. The zonal wind tendency is almost in phase with the EP flux divergence, but its magnitude is less than half that of the forcing. The difference is made up by the Coriolis force on the mean meridional velocity. This situation is similar to

\[ \text{FIG. 10. Thermodynamic budget for annual shortwave stratospheric heating with } \alpha = 0.1 \text{ day}^{-1}. \]
is $O(1)$, and we may write
\begin{equation}
\tilde{u} \sim \left( \frac{\tilde{M}}{K_R + i\omega} \right) \frac{K_R + i\omega}{\alpha + i\omega} \frac{\alpha + i\omega}{1 + \frac{K_R + i\omega}{\alpha + i\omega}}
\end{equation}

For time-dependent forcing that varies rapidly with respect to both $K_R$ and $\alpha$, $i\omega\tilde{u} = \frac{\alpha + i\omega}{\alpha + i\omega}$, which is consistent with the results depicted in Fig. 14. Thus, in this high-frequency limit, time-dependent wave driving will produce a tendency in the zonal wind roughly one half the value of the EP flux divergence, the excess EP flux divergence being balanced by the Coriolis torque on the meridional velocity. Of course, this argument breaks down in the tropics, where $f \rightarrow 0$ and—in the absence of nonlinear advection—wave driving must be balanced by the zonal wind tendency and zonal momentum dissipation.

Well away from the forcing the zonal acceleration is dominated by the Coriolis effect of the wave-driven circulation. For example, in the tropics the easterly acceleration (not shown) follows very closely the behavior of the term $f v^*$. In regions where both the EP flux divergence and $f v^*$ are important, the relationship between $\tilde{\alpha} u/\tilde{t}$ and either of these terms will not be straightforward, as noted by O'Neill and Youngblut (1982) in connection with EP flux diagnostics of a stratospheric sudden warming.

The varying zonal winds induced by the Coriolis torque in the summer hemisphere are reminiscent of the region of high variance in the tropics of the summer hemisphere associated with the stratosphere SAO (Hopkins, 1975). Although Hopkins attributed such variability to the direct effect of planetary waves at the tropical zero-wind line, it would appear that at least some of it must be due to Coriolis torques acting on

that found in observations (Palmer, 1981; O'Neill and Youngblut, 1982) and numerical models (Dunkerton et al., 1981; Andrews et al., 1983), where $\tilde{\alpha} u/\tilde{t}$ follows the EP flux divergence only approximately. A simple interpretation of this result can be given by recourse to the same scale analysis that lead to Eq. (10) in the previous section. Assuming now that the forcing is time dependent and due to planetary wave EP flux divergence, we obtain
\begin{equation}
\tilde{u} \sim \left( \frac{\tilde{M}}{K_R + i\omega} \right) \frac{L_x^2 N^2 K_R + i\omega}{L_y^2 f^2 \alpha + i\omega} \frac{\alpha + i\omega}{1 + \frac{L_x^2 N^2 K_R + i\omega}{L_y^2 f^2 \alpha + i\omega}}
\end{equation}

In extratropical latitudes and for typical values of $L_y$ and $L_x$, the factor
\begin{equation}
\left( \frac{L_x^2}{L_y} \right)^2 \left( \frac{N}{f} \right)^2
\end{equation}
the time-varying circulation induced by nonlocal EP flux divergence.

The behavior of the temperature response associated with forcing by planetary waves is also of interest. The temperature tendency (not shown) is dominated by the effect of the mean vertical velocity, $\overline{\vec{w}} S$, and is almost in phase with this term. Because the frequency associated with the wave forcing ($2\pi/15 \text{ day}^{-1} \approx 1/2.5 \text{ day}^{-1}$) is considerably faster than the Newtonian cooling coefficient, ($\alpha = 1/10 \text{ day}^{-1}$), the thermal response to rapidly varying wave forcing is essentially adiabatic, as noted by Dunkerton et al. (1981).

5. Summary

A zonally averaged, quasi-geostrophic meridional Eulerian model has been used to illustrate how the adjustment of the middle atmosphere to externally imposed forcing depends on internal dissipative properties of the atmosphere (parameterized as linear damping), and on the periodicity of the forcing. In general, the middle atmosphere can respond to external forcing by changing its mean temperature and wind structure, or through a mean meridional circulation. Which of these limiting cases is realized depends principally on the extent to which the Coriolis torque and adiabatic cooling/warming generated by the meridional circulation can be balanced through dissipative processes, or by the time tendencies of the zonal mean wind and temperature fields.

For steady or very slowly varying forcing, the strength of the meridional circulation depends on the ratio of Rayleigh friction to Newtonian cooling coefficients, $K_R/\alpha$. In particular, if $K_R/\alpha \ll 1$, thermal forcing (e.g., shortwave heating) does not generate a strong meridional circulation because the Coriolis torque cannot be readily balanced (cf. Dickinson, 1971a). Instead, the atmosphere adjusts by changing its zonal mean temperature gradient and, consequently, the zonal wind distribution. Although the circulation produced by ozone heating in the stratosphere is not negligible, it is much weaker than could be realized were the heating balanced solely by vertical motions. Thus the temperature structure of the stratosphere is, generally speaking, close to radiative equilibrium. The most important exceptions to this statement occur in the extratropical winter stratosphere, where meridional motions forced by divergence of planetary wave EP fluxes drive the temperature away from radiative equilibrium, and in the tropics, where the constraint imposed by the Coriolis torque on the zonal mean momentum budget is relaxed as $f \to 0$.

For the typically small values of $K_R/\alpha$ prevailing throughout most of the stratosphere, low frequency mechanical forcing (e.g., planetary wave EP flux divergence) is an efficient drive of the mean meridional circulation. On seasonal time scales, net EP flux divergence as small as $-3 \text{ m s}^{-1}/\text{day}$ produces meridional motions comparable to those due to stratospheric ozone heating. Mechanical forcing centered in mid-latitudes of the winter hemisphere drives an extensive circulation that extends into the subtropics of the summer hemisphere. In the lower stratosphere, this circulation leads to warming at high latitudes and cooling in the tropics, while the reverse is true in the upper stratosphere. Easterly accelerations are produced in the winter hemisphere through the direct effect of the EP flux divergence and, in the tropical regions of the summer hemisphere, through the Coriolis torque on the equatorward meridional velocity.

When mechanical forcing varies on subseasonal time scales (as in the case of EP flux divergence produced by interference between stationary and traveling waves), high-latitude stratospheric warming and tropical cooling are produced by a strong circulation extending below the forcing level, with opposite behavior at higher altitudes. Near the forcing, the evolution of the zonal wind follows the wave driving, but in regions remote from the forcing (as in the tropics and summer hemisphere) the zonal wind responds to the Coriolis...
torque acting on the meridional wind. In particular, the short-term variability associated with the easterly winds of the SAO may be attributable in part to the indirect effect of unsteady wave forcing in the winter hemisphere.

The examples presented here illustrate how a simple dynamical model can be used to examine the fundamental processes that control the zonally averaged structure and meridional circulation of the middle atmosphere. Similar arguments have been invoked to explain the effect of stratospheric perturbations, e.g., decreases in ozone and attendant shortwave heating (Fels et al., 1980). Of course, it must be borne in mind that the results are predicated on the use of quasi-geostrophic dynamics (and linearized frictional and thermal damping) to allow a straightforward solution of the governing equations. Thus, the calculations must be considered qualitative rather than accurate estimates of the adjustment of the middle atmosphere to external forcing. Nevertheless, it is clear that even a simple formulation of the momentum and thermodynamic constraints imposed by rotation and vertical stratification can provide powerful insights into the adjustment of the middle atmosphere to externally imposed forcing.

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