The Response of the Tropical Atmosphere to Low Frequency Thermal Forcing

JOHN R. ANDERSON* AND DUANE E. STEVENS

Department of Atmospheric Science, Colorado State University, Fort Collins, CO 80523

(Manuscript received 9 December 1985, in final form 13 August 1986)

ABSTRACT

The tropical response to a localized thermal forcing with approximately 45-day period is investigated for several models of increasing complexity consisting of two equivalent shallow water systems and two fully stratified systems. The fully stratified models appear to be able to reproduce a number of observed features of the tropical 40–50 day oscillation including the modulation of the subtropical jet and the eastward and poleward propagation of zonal wind anomalies.

1. Introduction

The goal of this work is to determine the structure of the response of a tropical atmosphere to slow oscillatory forcing with period of approximately 45 days. Our principal motivation for studying this problem is to determine whether the structure of the observed 40–50 day tropical oscillation, first reported by Madden and Julian (1971), is consistent with the response expected from a localized thermal forcing. Observational studies by Yasunari (1980) and Julian and Madden (1981) show that there is a modulation of convective rainfall which presumably provides such forcing with 40–50 day period in the west Pacific/Indonesian region.

We explore the tropical response using a number of successively more complicated models so that we can identify which dynamical effects are critically important. The first model which we use is a simple linear shallow water system. This system is obtained from the fully stratified atmospheric problem by performing a linearization about a resting basic state (Lindzen, 1967; Gill, 1980). This calculation uses different solution techniques on a problem formulation similar to that of Yamagata and Hayashi (1984) and represents the simplest possible approach to the forced problem. The second model which we study is linearized about a zonally symmetric basic state consisting of pure zonal flow that varies with latitude. This is the most general basic state for which a shallow waterlike system which is separable into zonal wavenumbers can be derived. Finally, we employ two fully three-dimensional models. The first of these is linearized about a basic state consisting of a zonal mean flow $\bar{U}(y, z)$; the second is linearized around the same zonal flow and adds the effects of a zonally symmetric Hadley cell, including momentum transport by the cumulus clouds and the advection of perturbation quantities by the mean vertical and meridional winds.

Recent modeling studies (Anderson, 1984a,b) have shown that the inclusion of the Hadley circulation in the basic state results in the formation of zonally symmetric model eigenmodes that represent damped resonances in the 40–50 day period range. In section 3 we will study the effects of the inclusion of the dynamics responsible for these modes on the solution to the forced problem.

It should be emphasized that this work is intended to determine whether the observed motions can be accounted for if we assume the presence of thermal forcing with the 40–50 day period. Any explanation of the 40–50 day oscillation will have to include a means of selecting the oscillation frequency and some sort of feedback closure to drive the forcing. We view the work described here as being a continuation of the work previously reported by Gill (1980), Lim and Chang (1983), and Yamagata and Hayashi (1984) to include the effects of a more general basic state and oscillatory forcing.

2. Equivalent shallow water model calculations

The derivation of the equivalent shallow water model is very similar to the one used by Gill (1980) to study the tropical response to steady-state forcing. It is based on the separation of the problem in the vertical and horizontal directions. The forcing is then projected onto one eigenmode of the resulting vertical structure problem and the evolution of the system can be studied by solving a set of shallow waterlike equations. The most general zonally symmetric basic state for which this separation can be accomplished is given by $\bar{u} = \bar{u}(y)$, $\bar{v} = 0$, $\bar{w} = 0$, $\partial\bar{\theta}/\partial y = 0$ (cf. Stevens, 1983) and yields the following system for the horizontal structure problem;

$$\frac{\partial u}{\partial t} + \bar{u}\frac{\partial u}{\partial x} - fv = -\frac{\partial \phi}{\partial x} - \alpha u \quad \text{(1a)}$$

* Current affiliation: Department of Meteorology, University of Wisconsin, Madison, WI 53706.

$$\frac{\partial v}{\partial t}+\bar{u}\frac{\partial v}{\partial x}+fu=-\frac{\partial \phi}{\partial y}-\alpha v \quad (1b)$$

$$\frac{\partial \phi}{\partial t}+\bar{u}\frac{\partial \phi}{\partial x}=-gH\left(\frac{\partial u}{\partial x}+\frac{\partial v}{\partial y}\right)+Q-\alpha\phi. \quad (1c)$$

Here $u(x, y, t)$, $v(x, y, t)$ and $\phi(x, y, t)$ are the horizontal structure variables defined by $u(x, y, z, t) = u(x, y, t)\cdot u(z)\cdots$, where $u(z)$, $v(z)$, and $\phi(z)$ are the eigenfunctions of the vertical structure problem. H is the equivalent depth which is determined from the eigenvalue of the mode which the forcing is taken to be projected onto (cf. Geisler and Stevens, 1982), $Q(x, y, t)$ is the amplitude of the forcing and α is a simple Rayleigh friction/Newtonian cooling representation for the dissipation. With this choice of heating sign the wind field corresponds to upper tropospheric winds for the original stratified fluid in a two-level model formulation.

Gill (1981) made an additional approximation to the above system known as the "long-wave" approximation which consists of ignoring the terms involving v in (1b). We have reproduced the results of Gill (1980) and Yamagata and Hayashi (1984) without this approximation and found that for those simple cases we obtain a nearly identical solution. Since the system coefficients are constant in x, it is traditional to take a discrete Fourier series representation in that direction, yielding

$$\frac{\partial u^k}{\partial t}+\bar{u}\cdot iku^k-fv^k=-ik\phi^k-\alpha u^k \quad (2a)$$

$$\frac{\partial v^k}{\partial t}+\bar{u}\cdot ikv^k+fu^k=-\frac{\partial \phi^k}{\partial y}-\alpha v^k \quad (2b)$$

$$\frac{\partial \phi^k}{\partial t}+\bar{u}\cdot ik\phi^k=-gH\left(iku^k+\frac{\partial v^k}{\partial y}\right)+Q^k-\alpha\phi^k, \quad (2c)$$

where $a(x, y, t) = \sum_k a^k(y, t)\exp(ikx)$.

Now if we use a suitable discretization in the y direction, we can write the system in schematic form as

$$\frac{\partial \mathbf{S}^k}{\partial t}=\mathbf{A}^k\mathbf{S}^k+\mathbf{Q}^k$$

where the vector $\mathbf{S}^k$ is the system state vector (u, v, ϕ) for wavenumber k and the matrix $\mathbf{A}^k$ represents the linear dynamics operator. The discretization for all of our calculations was accomplished using a Fourier representation in the meridional direction: $\mathbf{A}$ is calculated using transform techniques and is truncated at meridional wavenumber 20. In order to mask the intrinsic periodicity of the basis functions, an absorbing layer, α_{sponge}, was incorporated at the poleward limits of the domain. If the system is stable (real part of all eigenvalues of $\mathbf{A} < 0$) and the forcing $\mathbf{Q}^k \propto \exp(i\omega t)$ then we can represent the forced system by the matrix problem

$$i\omega\mathbf{R}^k=\mathbf{A}^k\mathbf{R}^k+\mathbf{Q}^k$$

or

$$(i\omega\mathbf{I}-\mathbf{A}^k)\mathbf{R}^k=\mathbf{Q}^k, \quad (3)$$

where $\mathbf{R}^k$ is the system response vector for zonal wavenumber k. The complete response is then determined by solving (3) to get $\mathbf{R}(x, y)$ for each zonal wavenumber component of the forcing and summing the results.

The parameters chosen for our calculation are given below.

$$Q=\begin{cases}\exp(i\omega t)\ \exp\left[-\left(\dfrac{y}{L_y}\right)^2\right]\ \cos(k_x x) & \text{for } |k_x x|<\pi\\ 0 \quad \text{for } |k_x x|>\pi\end{cases}$$

$$H = 400\ \text{m}$$

$$L_y = 1.25\times10^6\ \text{m}$$

$$k_x = \pi/(9.4\times10^6\ \text{m})$$

$$\alpha=\frac{1}{12\ \text{days}}+\alpha_{\text{sponge}}(\lambda)$$

$$\alpha_{\text{sponge}}=\begin{cases}\dfrac{1}{1\ \text{day}}(|y|-|y_{\max}|+y_0)/y_0, & |y|+y_0\geqslant y_{\max}\\ 0, & \text{otherwise}\end{cases}$$

$$y_{\max} = 45°\ \text{lat}$$

$$y_0 = 15°\ \text{lat}$$

$$\bar{u} = 0$$

$$\omega = 2\pi/(45\ \text{days}) \quad (4)$$

Yamagata and Hayashi made a similar calculation using the scale and dissipation parameters proposed by Gill. These parameters involve a very fast damping time (α = 1/(2.2 days)) and a large meridional scale ($L_y = 2.34\times10^6$ m). When those parameters are used, almost no propagation is observed. This result is due to the damping time being significantly shorter than the oscillatory time scale ($\alpha \gg \omega$) (cf. Geisler and Stevens, 1982). In fact, the results are nearly identical to those obtained by Gill for $\omega = 0$.

Here we have reduced the value of the model dissipation (α = 1/(12 days)) and we have adopted a smaller meridional extent for the heating ($L_y = 1.25\times10^6$ m). The heating field is shown in Fig. 1. The meridional scale was chosen in an attempt to represent the observed fluctuations in the Pacific convection. While the dissipative time scale for a model this simple is always somewhat arbitrary, we have chosen a less dissipative system so that a propagating response is at least possible.

The results of this calculation are shown in Fig. 2. The in-phase ($\omega t = 0$) field is very similar to that reported by Gill and Yamagata and Hayashi; however

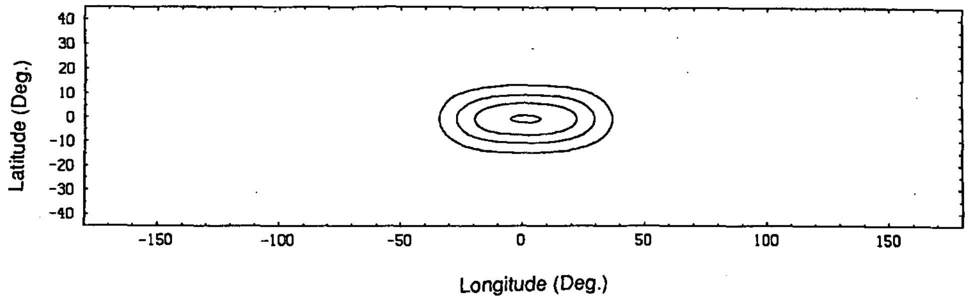

FIG. 1. Horizontal heating distribution. Contour interval is 2 gpm day^{-1}.

the $\omega t = \pi/2$ field now has significant amplitude and most of the characteristics of a westward propagating Rossby–gravity wave. This is not surprising as the eigenvalues of this problem are analytically known and the forcing period is much closer to the planetary resonance time for the $c = \sqrt{gh}/3$ Rossby wave (22 days) than the $c = \sqrt{gh}$ Kelvin wave (7 days).

The second calculation will address the effects of adding a mean zonal flow. This is the most complicated state which is possible for a model that is separable in both x and z. The basic state zonal wind field $\bar{u}(y)$ is shown in Fig. 3 and is taken from a mass weighted vertical average of the two-dimensional $\bar{U}$-field, which we use in section 3. All other parameters remain unchanged. The wind field response with the $\bar{u}(y)$ basic state is given as Fig. 4. The fields show a strong similarity to the resting case and the $\omega t = 90°$ response still indicates a general westward propagation of the disturbance. This feature is not in good agreement with the observed eastward propagation of the oscillation as described by Madden and Julian (1972). In the next section we will see if the inclusion of other physical processes can improve the agreement with observations.

3. Calculations with a fully stratified model

In this section we will now expand the basic state definition for our linear problem to include a general two-dimensional (latitude–height) description. We will present two calculations. The first will be for a basic state consisting of a $\bar{u}(y, z)$ field in thermal wind balance, and the second will also include $\bar{v}(y, z)$, $\bar{w}(y, z)$ and cumulus mass flux terms. We have chosen to limit this calculation to the effects of a zonally symmetric basic state due to significant computational difficulties associated with allowing the basic state variables to vary in the third spatial dimension. In effect, we are including the new dynamics introduced by the "Hadley cell" but have not yet included a "Walker" type circulation

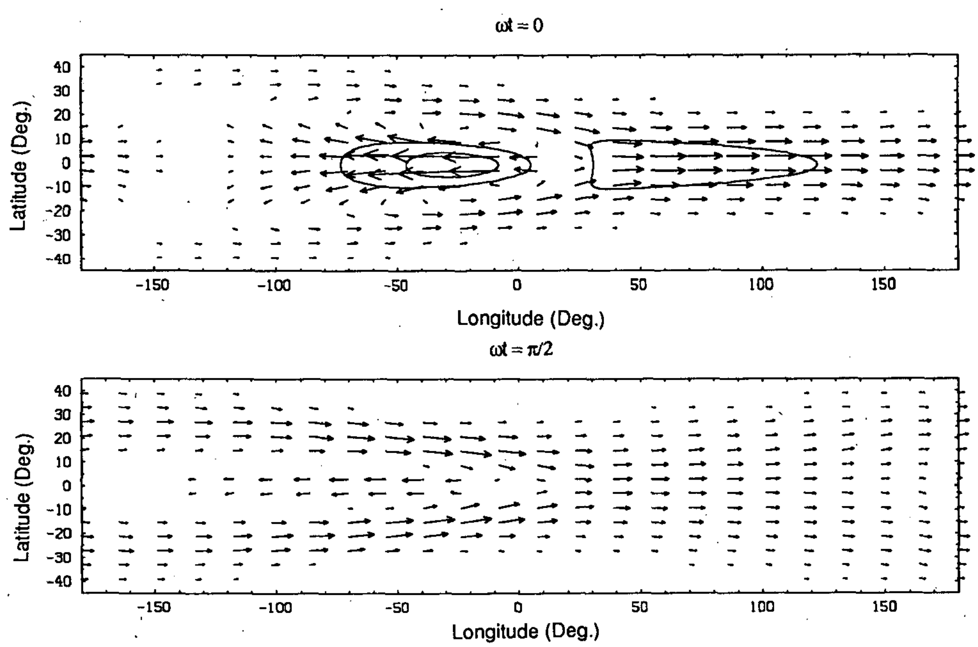

FIG. 2. Wind field response, resting basic state. Isotach contour interval is 0.5 m s^{-1}.

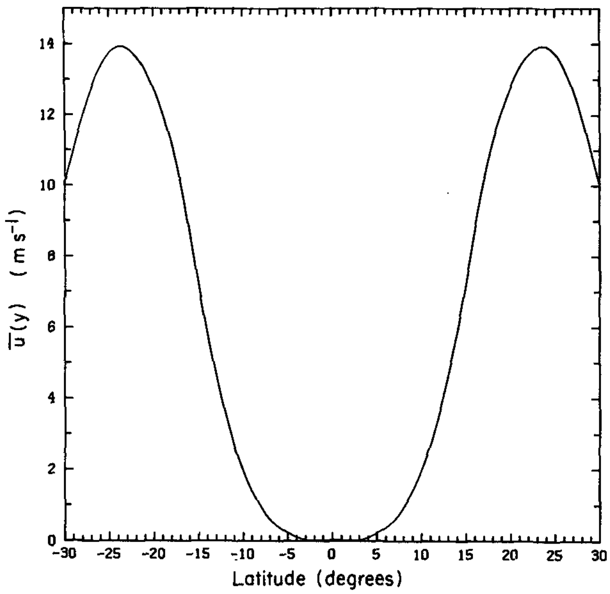

FIG. 3. $\bar{u}(y)$ for shallow water calculation.

in our model basic state. As part of an ongoing research effort we are currently examining various approaches to relax this restriction.

With a basic state that includes vertical shear of the mean zonal wind, the vertical dependence of the linearized equations is no longer separable, and therefore, the linear shallow water formulation with a specified equivalent depth is not a valid approximation to the primitive equations. Not only can a Hadley cell be incorporated into the basic state, but a more physical representation of internal dissipation—with much stronger justification in observations—can be used. The approach which we will adopt here is the so-called "cumulus friction" introduced by Schneider and Lindzen (1976) where the dissipation results from vertical momentum transport by cumulus clouds. An important aspect of this form is that the dissipation is concentrated in regions of basic state heating.

The linear perturbation equations describing hydrostatic flow on an equatorial β-plane about this basic state are given for each zonal wavenumber k by

$$\frac{Du'}{Dt}=\beta yv'-\frac{1}{\tilde{\rho}}p'_x-\alpha(y,z)u'+\mu u'_{zz}+F_{cx} \quad (5a)$$

$$\frac{Dv'}{Dt}=-\beta yu'-\frac{1}{\tilde{\rho}}p'_y-\alpha(y,z)v'+\mu v'_{zz}+F_{cy} \quad (5b)$$

$$\frac{D\theta'}{Dt}=Q'-\gamma(y,z)\theta'+\mu\theta'_{zz} \quad (5c)$$

$$-\frac{1}{\tilde{\rho}}(\tilde{\rho}w')_z=u'_x+v'_y \quad (5d)$$

$$\left(\frac{p'}{\tilde{\rho}}\right)_z=\frac{g}{\tilde{\Theta}}\theta' \quad (5e)$$

$$\frac{D(\ )'}{Dt}=(\ )'_t+u'(\bar{\ })_x+v'(\bar{\ })_y+w'(\bar{\ })_z + \bar{u}(\ )'_x+\bar{v}(\ )'_y+\bar{w}(\ )'_z \quad (5f)$$

where $\partial/\partial x = ik$ and $\tilde{\rho}(z)$ and $\tilde{\theta}(z)$ are a hydrostatically

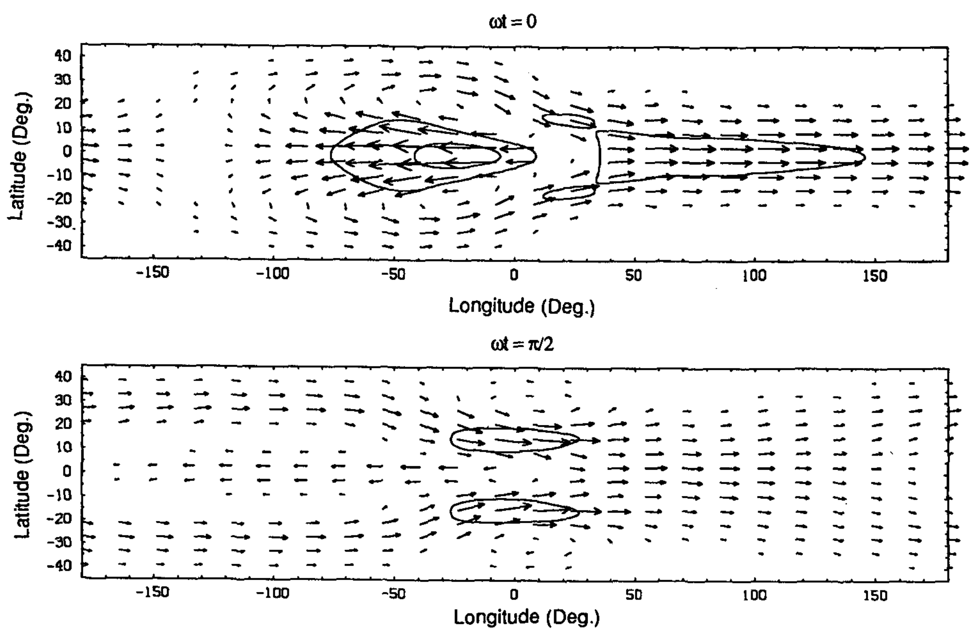

FIG. 4. Wind field response, $\bar{u}(y)$ basic state. Isotach contour interval is 0.5 m s^{-1}.

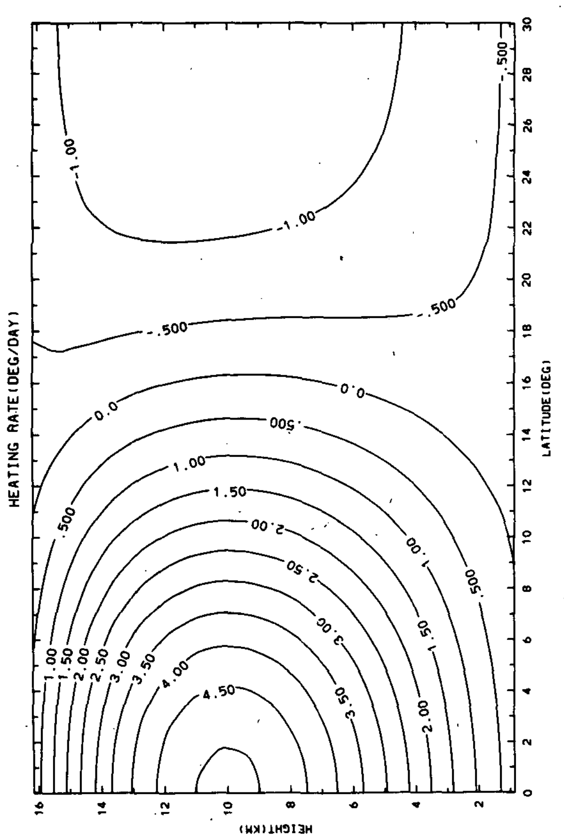

FIG. 5a. Heating field (°C day^{-1}) for calculation of Hadley cell basic state.

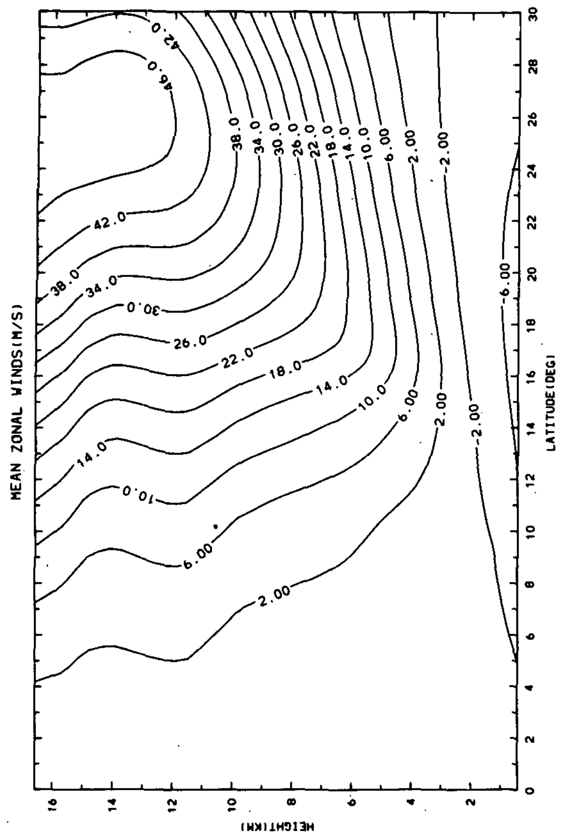

FIG. 5b. Wind field u-component (m s^{-1}) of Hadley cell basic state.

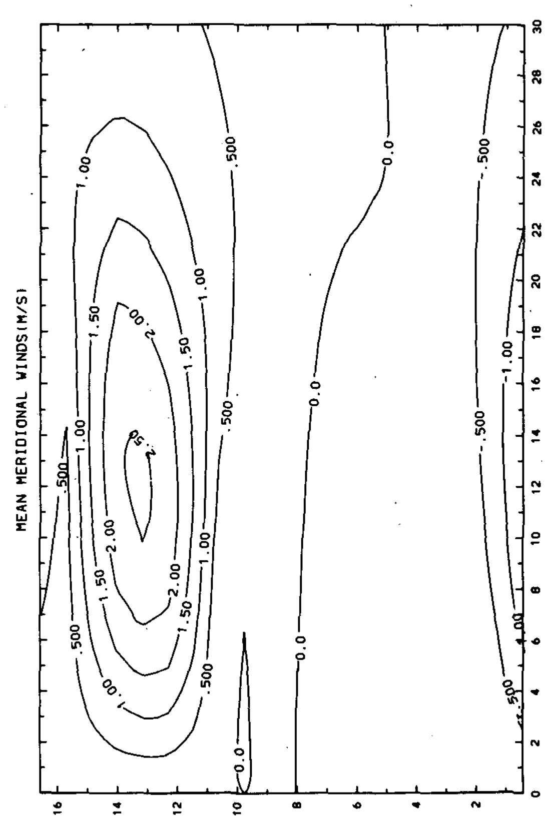

FIG. 5c. As in Fig. 5b but the v-component.

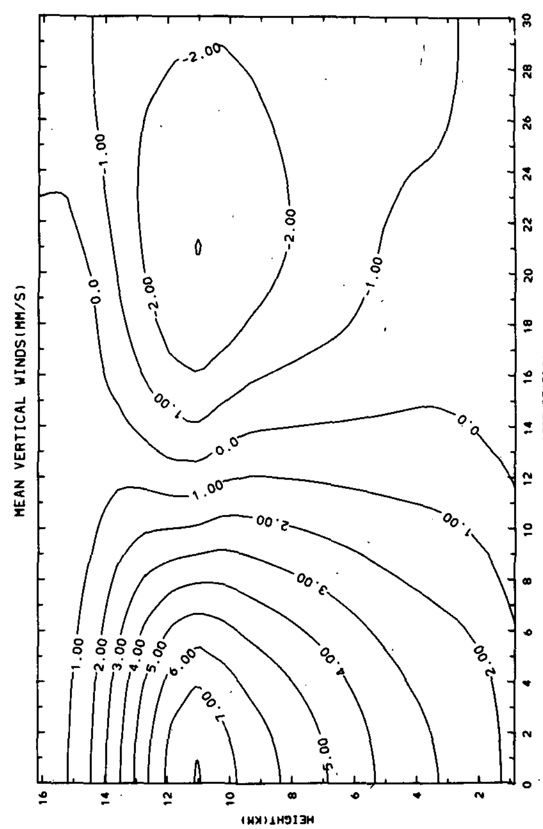

FIG. 5d. As in Fig. 5b but the w-component.

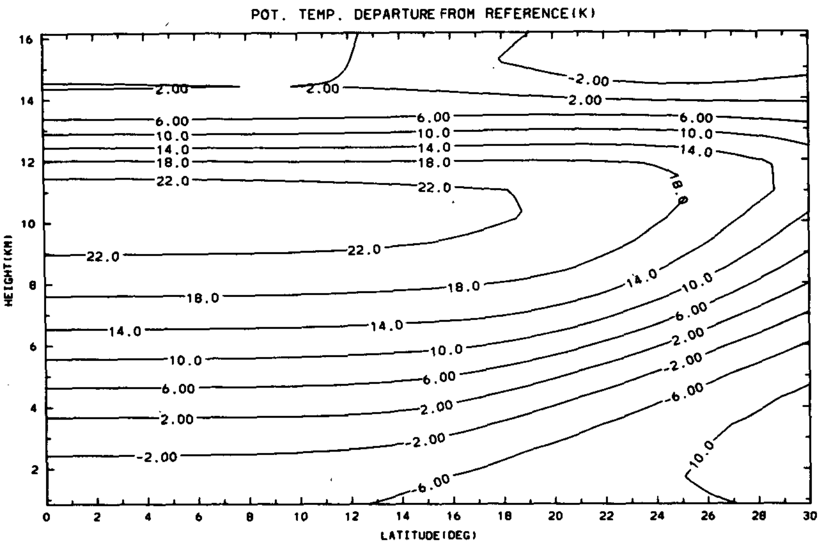

FIG. 5e. Potential temperature perturbation (°C) for Hadley cell basic state.

balanced reference stratification. The F_{cx} and F_{cy} are the cumulus momentum transport terms given by

$$F_{cx} = \frac{1}{\tilde{\rho}}\{[\bar{M}_c(u' - u'_c)]_z + [M'_c(\bar{u} - \bar{u}_c)]_z\} \qquad (6a)$$

$$F_{cy} = \frac{1}{\tilde{\rho}}\{[\bar{M}_c(v' - v'_c)]_z + [M'_c(\bar{v} - \bar{v}_c)]_z\} \qquad (6b)$$

The dissipation terms are given by

$$\alpha(y, z) = \alpha_{\text{surf}}(z) + \alpha_{\text{sponge}}(y)$$

$$\alpha_{\text{surf}} = \frac{1}{1.5\ \text{days}} \exp[-(z/1.75\ \text{km})^2]$$

$$\gamma(z) = \frac{1}{2.3\ \text{days}} \exp[-((z_{\text{top}} - z)/z_0)]^2$$

$$Z_{\text{top}} = 18\ \text{km}$$

$$Z_0 = 5\ \text{km}.$$

The cloud mass transport M_c is given by:

$$M_c(y, z) = M_c(y)\left[1 - \exp\left(\frac{p_T - p}{p_{\text{DTR}}}\right)\right], \quad p_T \leqslant \tilde{p}$$

$$\leqslant p_c p_T \approx 150\ \text{mb}, \quad p_{\text{DTR}} = 75\ \text{mb}, \quad p_c \approx 900\ \text{mb}$$

Here the basic state is given by $\bar{u}$, $\bar{v}$, $\bar{w}$, $\bar{\theta}$ and $\bar{M}_c$ which is the cumulus mass flux as defined by Schneider and Lindzen. The basic state fields which are presented in Fig. 5 are found by integrating a nonlinear version of (5) with a specified heating field, $\bar{Q}(y, z)$. The $\bar{Q}$ and Q' are related to $\bar{M}_c(y)$ and and $M'_c(y)$ by an integral constraint that requires the total heat released to be equal to the cloud moisture flux in a fashion similar to the cumulus parameterization scheme proposed by Stevens and Lindzen (1978). The dissipation and other parameters of the nonlinear model were identical to the linear calculations.

In the context of our more complete model we will now use the simple dissipation parameters α and γ solely for the purpose of imposing boundary conditions. The Rayleigh friction terms are used to impose the poleward boundary sponge and surface frictional effects while the Newtonian cooling term is used to absorb energy at the top of the domain and simulate a radiation condition; α_{sponge} is the same as described in the previous sections.

The horizontal discretization is accomplished as before using Fourier basis functions; a staggered centered difference formulation is used for vertical derivatives. A small vertical viscosity term is used to smooth the effects of the vertical differencing scheme. All of the results presented in this section were calculated with a 20 meridional wavenumber truncation on the same 45° North–45° South domain used for the shallow water calculations and 20 vertical levels up to 18 km. The three-dimensional fields were constructed by summing the results of zonal wavenumbers 0–4, which was found to give good convergence for our forcing. In principle, the system could be solved using linear algebra techniques as in the separable case; however, in practice, the large order of the matrix (1200 × 1200 complex elements) makes this approach computationally intractable. Instead, each wavenumber is solved by integrating a prognostic model with oscillating forcing until an equilibrium oscillating response is reached. On a Cray-1 computer, the solution requires approximately 200 seconds of CPU time for each wavenumber using semi-implicit time integration techniques.

The forcing function for this case was chosen to have the same horizontal structure as the second shallow water case with a vertical distribution which is given

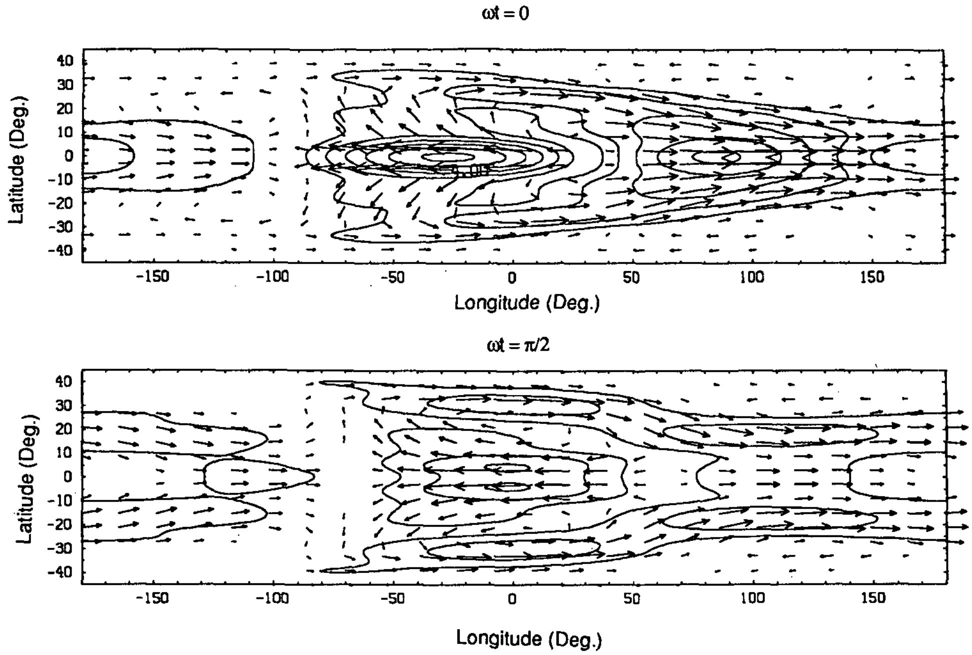

FIG. 6a. Response of the 250 mb wind field for $\bar{u}(y, z)$ basic state. Isotach contours are 1.0 m s^{-1}.

by a Gaussian centered at 400 mb and an e-folding distance of 5.1 km. Since the calculation is linear, the actual magnitude of the heating is arbitrary; however, the response fields are presented so that they are consistent with a 1°C day^{-1} heating maximum.

As previously mentioned, the first calculation performed was to generalize the $\bar{u}(y)$ fields from the shallow water calculations to one with $\bar{u}(y, z)$. Here we will use the $\bar{u}$ field from our nonlinear calculations as the basic state and calculate a $\bar{\theta}(y, z)$ field to be in thermal wind balance with it. The response u-component wind field for the model 250 mb level is presented in Fig. 6a. One striking feature of this is the presence of a strong jetlike response maximum located to the east of the heating

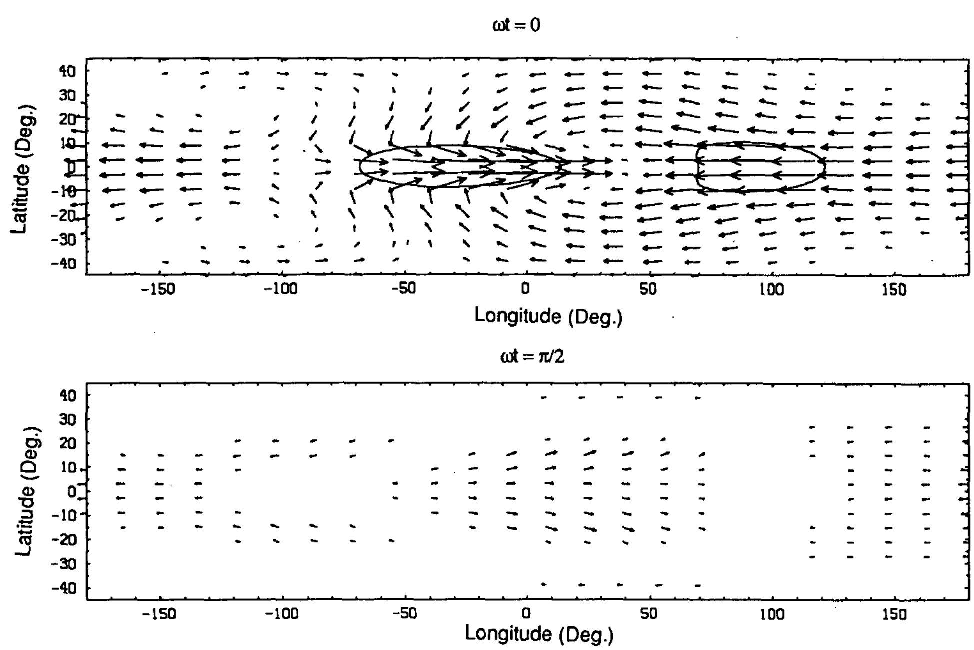

FIG. 6b. As in Fig. 6a but at 850 mb. Isotach contours are 0.5 m s^{-1}.

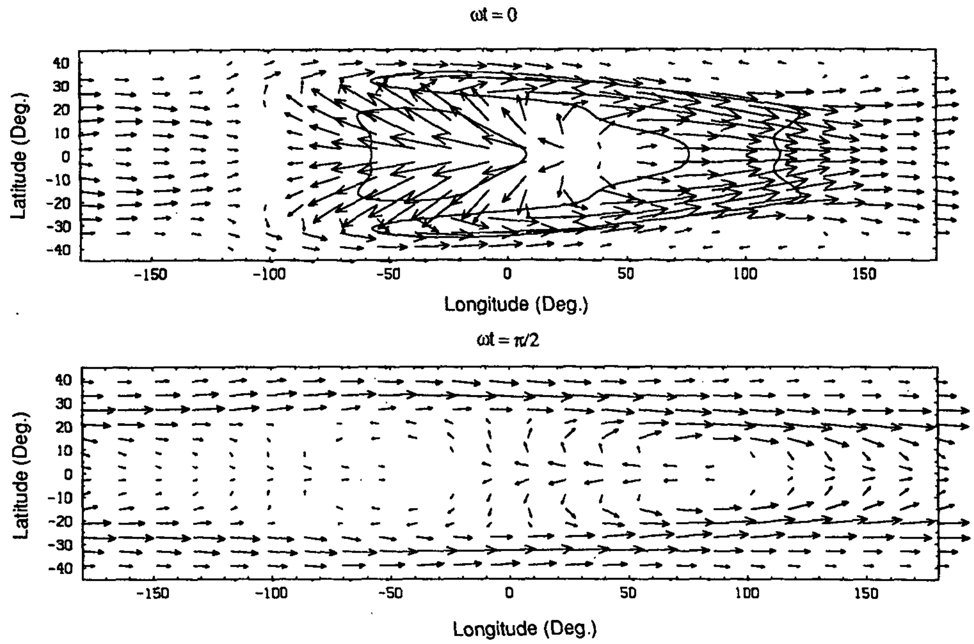

FIG. 7a. Response of the 250 mb wind field for Hadley cell basic state with cumulus momentum transport. Isotach contours are 1.0 m s^{-1}.

region at approximately 20° latitude. The presence of this feature is in good agreement with observations as well, as in agreement with the recent work by Lim and Chang (1986) who show that the inclusion of vertical shear in $\bar{u}$ can lead to excitation of deep vertical modes. In Fig. 7a we show the 250 mb wind field response when the complete Hadley cell basic state is used. One of the most notable differences is the significant reduction in the amplitude of the easterly jet located west of the heating. This is undoubtedly due to the presence of damping by the cumulus momentum terms which are large in that region. In addition, there is now clear

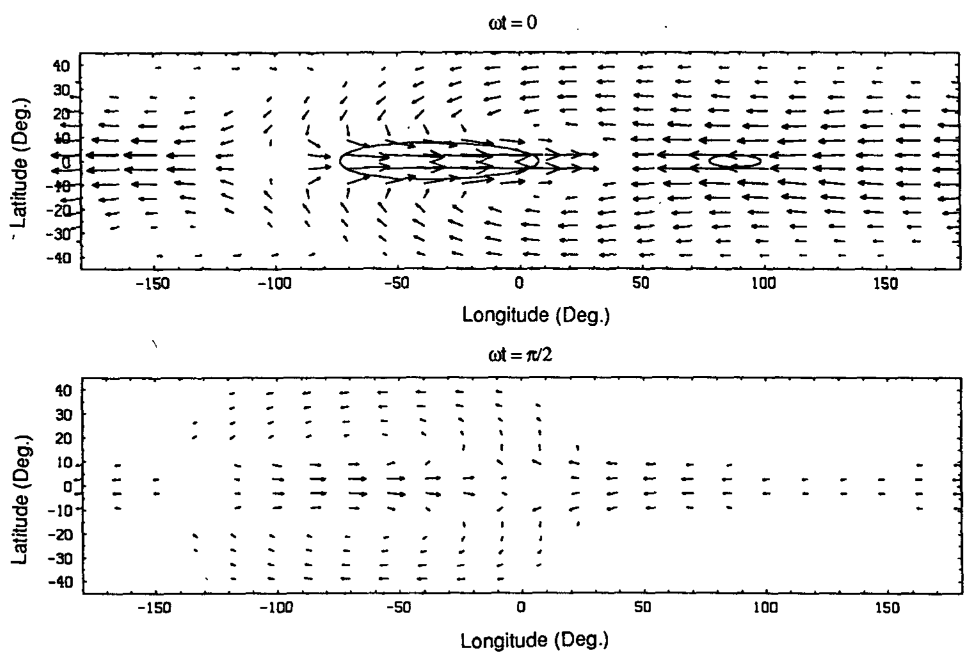

FIG. 7b. As in Fig. 7a but at 850 mb. Isotach contours are 0.5 m s^{-1}.

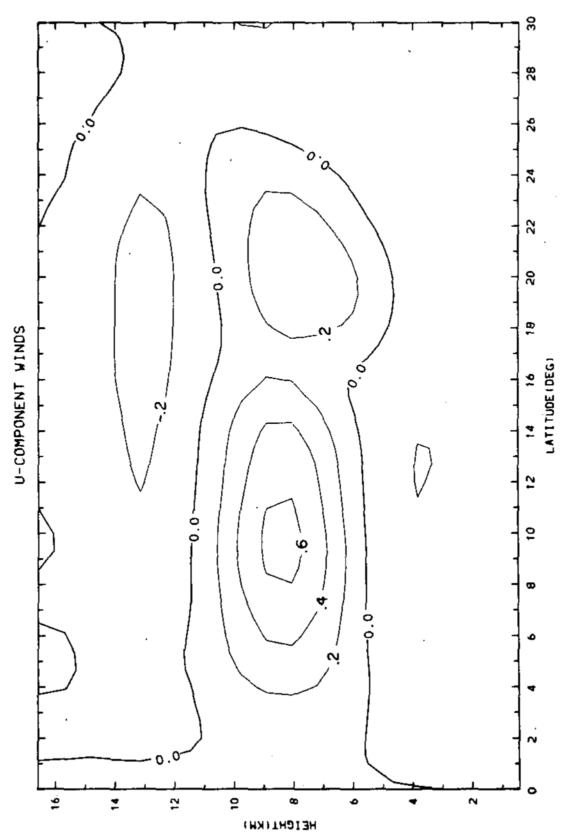

FIG. 8a. Wavenumber 0 response at $\omega t = -45$ deg for Hadley cell calculation.

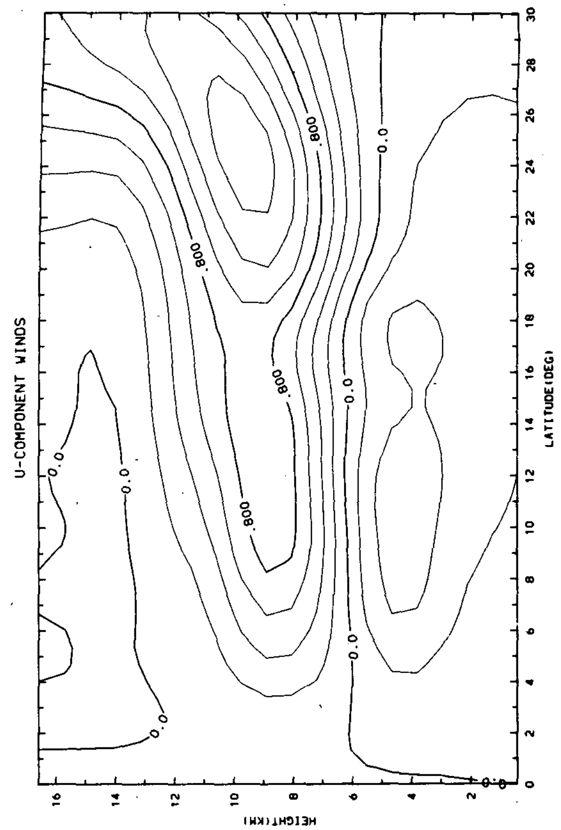

FIG. 8b. As in Fig. 8a but at $\omega t = 0$.

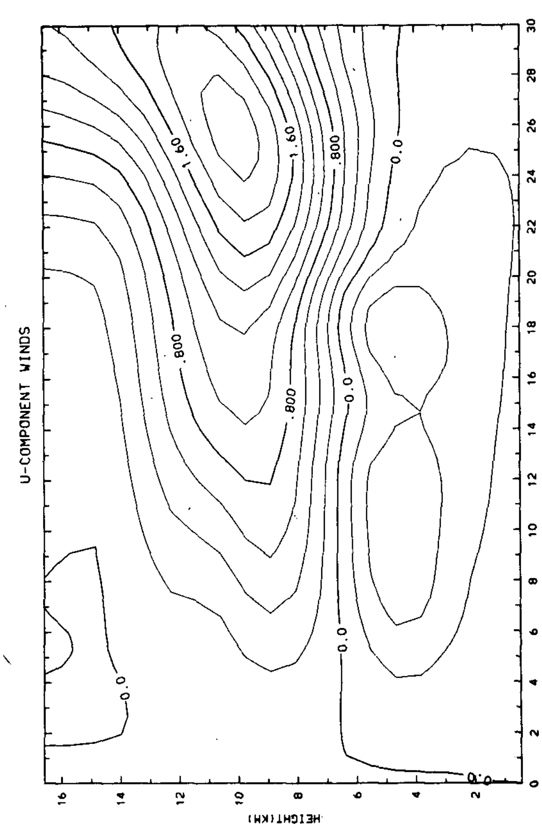

FIG. 8c. As in Fig. 8a but at $\omega t = 45$ deg.

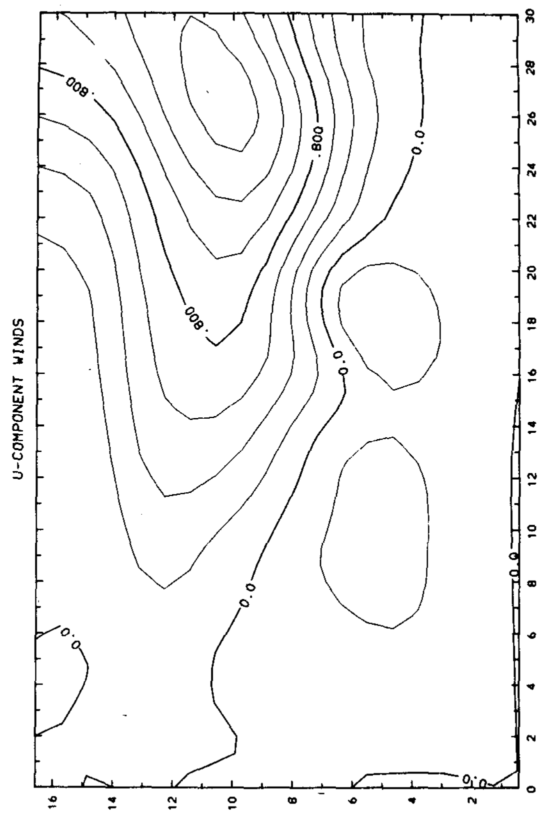

FIG. 8d. As in Fig. 8a but at $\omega t = 90$ deg.

evidence of eastward propagation along the equator in the upper troposphere. This feature resembles the observations by Madden and Julian (1972) and may be due to the excitation of the viscous Kelvin wave mechanism proposed by Chang (1977).

The 850 mb response is shown in Fig. 7b and shows no noticeable tendency toward eastward propagation. This is not in good agreement with observations and may result from the fact that the forcing in our calculations oscillates in time but remains localized at one longitude, whereas the observed cloudiness maximum and presumably the heating propagate eastward.

In order to examine the subtropical response in some detail, we have displayed latitude–height cross sections of the zonal wavenumber 0 *u*-wind response in Fig. 8. This figure shows a well-defined maximum which originates near the equator and then propagates poleward in the upper troposphere. This behavior is in agreement with the observed zonally averaged field presented by Anderson and Rosen (1983) and the partial zonal average presented by Murakami et al. (1983) which are reproduced as Figs. 9 and 10. Based on these figures, it appears that the model result may be in somewhat better agreement with the Murakami et al. fields, which are averaged over the monsoon region only, in the sense that the model displays upward propagation on the equator. It is possible that this is a result of our assumption of a zonally invariant basic state. We will pursue this topic in future research.

4. Conclusion

We have found that the linear response of the tropical atmosphere to low-frequency thermal forcing is as follows. For the case where the basic state is a zonally symmetric Hadley cell, the model produces results that differ considerably from calculations based on equivalent shallow water systems designed to represent internal atmospheric modes. The fully stratified results that include the Hadley cell effects appear to be in better agreement with observations of the tropical 40–50 day oscillation, since they reproduce upper tropospheric motions with slow eastward and poleward phase propagation.

In many ways the results of our calculations are similar to those of Lim and Chang (1983) for a deep ($c = 120$ m s^{-1}, $h \sim 15$ km) shallow water response. Lim and Chang (1986) demonstrate that high amplitude forcing of the deep mode can result from the inclusion of dissipation effects or vertical wind shear. In our calculations, we have shown that when a more complex basic state which includes both cumulus momentum transport effects and vertical wind shear is considered, high amplitude excitation of these deep motions does indeed occur. This effect has also been noted by Rosenlof et al. (1986) for steady-state forcing. In that paper, the roles of the various terms were studied and it was found that the inclusion of the cumulus momentum transport terms was of critical importance.

The presence of the poleward-propagating modes as opposed to a standing deep response is probably associated with the advection of perturbation winds by the background Hadley circulation. These effects have been shown to give rise to slowly propagating eigenmodes described in Anderson (1984a,b). These modes are the subject of a comparison paper, whereas the eastward propagating equatorial feature may be associated with the viscous Kelvin wave mechanism proposed by Chang (1977).

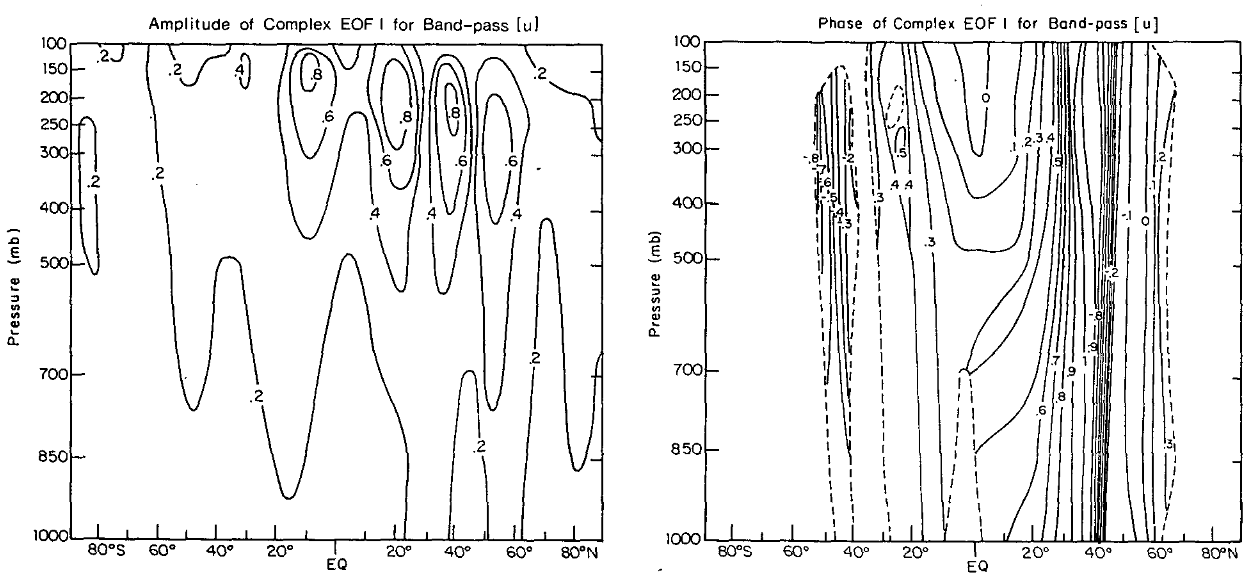

FIG. 9. (a) Amplitude and (b) phase (rad/π) plot of the observed oscillation (from Anderson and Rosen, 1983).

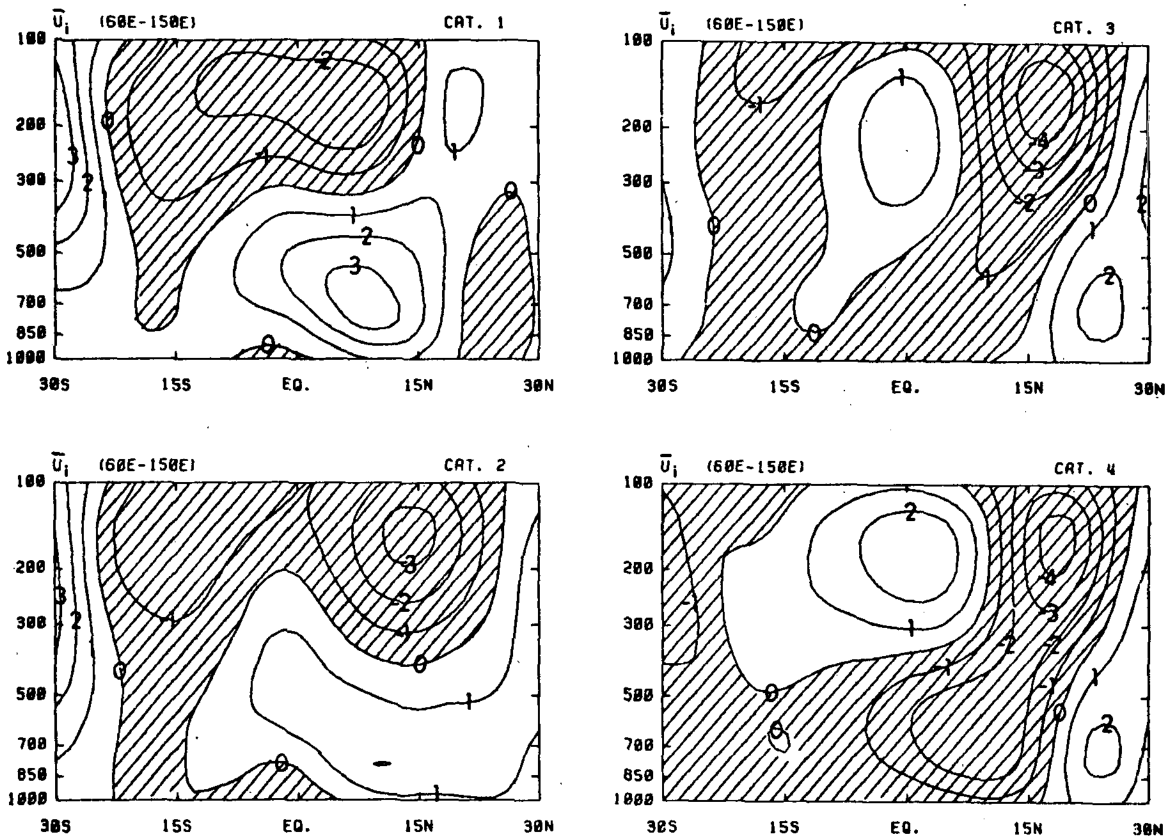

FIG. 10. Latitude–height structure of partial zonal average (60°E to 150°E) u-wind field (with interval 1 m s^{-1}) at 45° phase intervals (from Murakami et al., 1983).

It is our hope that this work will help lead to a better understanding of the processes that are important for the tropical 40–50 day oscillation and its various manifestations in the global atmosphere.

Acknowledgments. This work has been supported by NSF grants ATM-8305759, ATM-8405059 and NASA Grant NAG 5-136 to Colorado State University. The National Center for Atmospheric Research provided computing resources.

REFERENCES

Anderson, J. R., and R. D. Rosen, 1983: The latitude–height structure of 40–50 day variations in atmospheric angular momentum. *J. Atmos. Sci.*, **40**, 1584–1591.

——, 1984a: Low frequency modes of the tropical troposphere. Paper, *15th Tech. Conf. on Hurricanes and Tropical Meteorology*, Jan. 9– 13, 1984, Miami, FL.

——, 1984b: Slow motions in the tropical troposphere. Atmos. Sci. Paper 381, Colorado State University, 144 pp.

Chang, C. P., 1977: Viscous internal gravity waves and low-frequency oscillations in the tropics. *J. Atmos. Sci.*, **34**, 901–910.

Geisler, J. E., and D. E. Stevens, 1982: On the vertical structure of damped steady circulation in the tropics. *Quart. J. Roy. Meteor. Soc.*, **108**, 87–93.

Gill, A. E., 1980: Some simple solutions for heat-induced tropical circulations. *Quart. J. Roy. Meteor. Soc.*, **106**, 447–462.

Julian, P. R., and R. A. Madden, 1981: Comments on a paper by T. Yasumari. *J. Meteor. Soc. Japan*, **59**, 435–437.

Lim, H., and C.-P. Chang, 1983: Dynamics of teleconnections and Walker circulations forced by equatorial heating. *J. Atmos. Sci.*, **40**, 1897–1915.

——, and ——, 1986: Generation of internal- and external-mode motions from internal heating: Effects of vertical shear and damping. *J. Atmos. Sci.*, **43**, 948–957.

Lindzen, R. S., 1967: Planetary waves on beta-planes. *Mon. Wea. Rev.*, **95**, 441–451.

Madden, R. A., and P. R. Julian, 1971: Detection of a 40–50 day oscillation in the zonal wind in the tropical Pacific. *J. Atmos. Sci.*, **28**, 702–708.

——, and ——, 1972: Description of global-scale circulation cells in the tropics with a 40–50 day period. *J. Atmos. Sci.*, **29**, 1109–1123.

Murakami, T., T. Nakazawa, and J. He, 1983: 40–50 day oscillations during the northern hemisphere summer. University of Hawaii Meteor. Publ. UNMET 83-02.

Rosenlof, K. H., D. E. Stevens, J. A. Anderson, and P. E. Ciesielski, 1986: The Walker circulation with observed zonal winds, a mean Hadley cell, and cumulus friction. *J. Atmos. Sci.*, **43**, 449–467.

Schneider, E. K., and R. S. Lindzen, 1976: A discussion of the parameterization of momentum exchange by cumulus convection. *J. Geophys. Res.*, **81**, 3158–3160.

Stevens, D. E., 1983: On symmetric stability and instability of zonal mean flows near the equator. *J. Atmos. Sci.*, **40**, 882–893.

——, and R. S. Lindzen, 1978: Tropical wave-CISK with a moisture budget and cumulus friction. *J. Atmos. Sci.*, **35**, 940–961.

Yamagata, T., and Y. Hayashi, 1984: A simple diagnostic model for the 30–50 day oscillation in the tropics. (Unpublished manuscript.)

Yasunari, T., 1980: A quasi-stationary appearance of 30 to 40 day period in the cloudiness fluctuations during the summer monsoon over India. *J. Meteor. Soc. Japan*, **58**, 225–229.