A Physical Mechanism for the Asymmetry in Top-Down and Bottom-Up Diffusion

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ABSTRACT

Recent large-eddy simulations of the vertical diffusion of a passive, conservative scalar through the convective boundary layer (CBL) show strikingly different eddy diffusivity profiles in the "top-down" and "bottom-up" cases. These results indicate that for a given turbulent velocity field and associated scalar flux, the mean change in scalar mixing ratio across the CBL is several times larger if the flux originates at the top of the boundary layer (i.e., in top-down diffusion) rather than at the bottom. The large-eddy simulation (LES) data show that this asymmetry is due to a breakdown of the eddy-diffusion concept.

A simple updraft–downdraft model of the CBL reveals a physical mechanism that could cause this unexpected behavior. The large, positive skewness of the convectively driven vertical velocity gives an appreciably higher probability of downdrafts than updrafts; this excess probability of downdrafts, interacting with the time changes of the mean mixing ratio caused by the nonstationarity of the bottom-up and top-down diffusion processes, decreases the equilibrium value of mean mixing-ratio jump across the mixed layer in the bottom-up case and increases it in the top-down case. The resulting diffusion asymmetry agrees qualitatively with that found through LES.

1. Introduction

In treating the vertical diffusion of a (statistically) horizontally homogeneous, passive, conservative scalar field through a convective boundary layer (CBL), Wyngaard and Brost (1984) decomposed the problem into what they called its "top-down" and "bottom-up" components. From data generated through large-eddy simulation (LES) they found the eddy diffusivities for these two processes to be quite different. Moeng and Wyngaard (1984), using an improved LES model and a refined technique for calculating the properties of the top-down and bottom-up processes, corroborated this finding.

The weakness of the eddy-diffusivity parameterization in convective turbulence is well known, of course; Deardorff (1966), for example, discussed its failure for temperature flux in the CBL and proposed a simple remedy that restores proper behavior to the mean temperature profile. These LES results, however, emphasized an aspect that was not well known: that the eddy diffusivity, for the same turbulent velocity field, can depend strongly on the geometry of the diffusion problem. Lamb and Durran (1978) found this initially for point sources in the CBL, where the eddy diffusivity profiles vary with release height. In effect, the LES studies extended their results to area sources at CBL top and bottom. These findings could have much broader implications, given the prevalence of convection in the boundary layers of the atmosphere and ocean, and in clouds.

Improved but still simple closures have recently been offered. Fiedler (1984) and Fiedler and Moeng (1985) showed that the nonlocal generalization of eddy diffusivity, an idea applied to stellar convection by Spiegel (1963) and also explored by Berkowicz and Prahm (1979), can reproduce the LES results; Stull (1984) has advocated a closely related closure.

In the meantime, it seems that this striking asymmetry in top-down and bottom-up diffusion has yet to be demonstrated experimentally. In the hope of stimulating the experimental search for it, I offer one interpretation of its underlying mechanism.

2. A review of top-down and bottom-up diffusion

Figure 1 shows typical ensemble mean mixing ratio (C) and vertical flux ($\overline{cw}$) profiles for a CBL in the general case when surface exchange causes a scalar flux at the bottom and entrainment generates a scalar flux at the CBL top. As Fig. 1 also shows, we consider the CBL to have a thin surface layer extending to height h_0, a "mixed" layer between h_0 and h_1, and an interfacial layer above that (Wyngaard, 1983). The traditional CBL depth scale z_i, the distance to the first inversion, lies between h_1 and h_2. In our analysis here we will simply use z_i as the mixed-layer top and not distinguish among h_1, h_2 and z_i except where necessary.

If the diffusion process is horizontally homogeneous the scalar mixing ratio within the mixed layer satisfies

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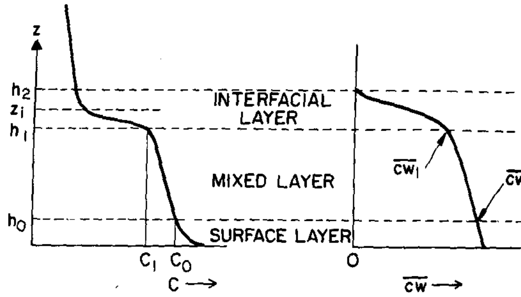

FIG. 1. A three-layer model of the convective boundary layer, showing typical profiles of mean scalar mixing ratio (left) and scalar flux (right).

the equation and boundary conditions (in the usual notation)

$$\frac{\partial C}{\partial t}+\frac{\partial}{\partial z}\overline{cw}=0;\quad \begin{cases}\overline{cw}(0)=\overline{cw}_0\\ \overline{cw}(z_i)=\overline{cw}_1.\end{cases}\tag{1}$$

Because of the linearity of (1) we can decompose the ensemble mean and turbulent scalar fields into top-down and bottom-up components,

$$C=C_t+C_b;\quad \overline{cw}=\overline{c_tw}+\overline{c_bw}\tag{2}$$

the top-down component satisfying

$$\frac{\partial C_t}{\partial t}+\frac{\partial}{\partial z}\overline{c_tw}=0;\quad \begin{cases}\overline{c_tw}(0)=0\\ \overline{c_tw}(z_i)=\overline{cw_1}\end{cases}\tag{3}$$

and the bottom-up component satisfying

$$\frac{\partial C_b}{\partial t}+\frac{\partial}{\partial z}\overline{c_bw}=0;\quad \begin{cases}\overline{c_bw}(0)=\overline{cw_0}\\ \overline{c_bw}(z_i)=0.\end{cases}\tag{4}$$

We assume that each diffusion problem [i.e., the full problem (1), the top-down problem (3), and the bottom-up problem (4)] is quasi-steady. By this we mean that the mean gradients of C, C_t and C_b and the scalar fluxes are independent of time, which from the mean conservation equations implies that the scalar flux profiles are linear. We will use the form

$$\overline{cw}=\overline{c_bw}+\overline{c_tw}=\left(1-\frac{z}{z_i}\right)\overline{cw_0}+\left(\frac{z}{z_i}\right)\overline{cw_1}.\tag{5}$$

Note that the time changes in the component diffusion problems are

$$\frac{\partial C_b}{\partial t}=-\frac{\partial}{\partial z}\overline{c_bw}=\frac{\overline{cw_0}}{z_i}\tag{6}$$

$$\frac{\partial C_t}{\partial t}=-\frac{\partial}{\partial z}\overline{c_tw}=-\frac{\overline{cw_1}}{z_i}.\tag{7}$$

Moeng and Wyngaard (1984) isolated the properties of the top-down and bottom-up processes by using a similarity hypothesis for mixed-layer structure. To illustrate their procedure, we write the mean scalar gradient as the sum of top-down and bottom-up components and make a similarity hypothesis for each:

$$\frac{\partial C}{\partial z}=\frac{\partial C_t}{\partial z}+\frac{\partial C_b}{\partial z}=-\frac{\overline{cw_1}}{w_* z_i}g_t-\frac{\overline{cw_0}}{w_* z_i}g_b.\tag{8}$$

Moeng and Wyngaard used two independent LES experiments, with different top and bottom scalar fluxes in each, to provide the two data sets needed to solve (8) for the dimensionless gradient functions g_b and g_t.

In order to provide a perspective on these LES results, consider the behavior of g_b and g_t when there is an eddy diffusivity K such that

$$(\overline{c_bw},\overline{c_tw})=-K\left(\frac{\partial C_b}{\partial z},\frac{\partial C_t}{\partial z}\right).\tag{9}$$

Using (5) it follows that

$$(1-z/z_i)\overline{cw_0}=-K\partial C_b/\partial z\tag{10}$$

$$(z/z_i)\overline{cw_1}=-K\partial C_t/\partial z\tag{11}$$

which with (8) gives

$$g_b=(1-z/z_i)w_* z_i/K\tag{12}$$

$$g_t=(z/z_i)w_* z_i/K.\tag{13}$$

If the diffusivity profile is symmetric about the midplane, i.e., if $K(z/z_i)=K(1-z/z_i)$, then it follows from (12) and (13) that the gradient functions have the analogous property

$$g_b(z/z_i)=g_t(1-z/z_i).\tag{14}$$

Since K is not symmetric in the flat-plate boundary layer (Townsend, 1976, p. 292), we might not expect a symmetric K within the CBL. If instead we simply require that K exist, we have from (12) and (13) the more general property

$$g_b=\left(\frac{1-z/z_i}{z/z_i}\right)g_t.\tag{15}$$

The cited LES results for g_b and g_t indicate that neither (14) nor (15) is satisfied because the assumption (9) fails; that is, a single K does not describe both the top-down and bottom-up processes. While the top-down K is well behaved, the bottom-up K has a singularity (Fig. 2).

A quantitative measure of the differences in the two diffusion processes is the mean change in scalar mixing ratio across the mixed layer, for fixed scalar flux and CBL velocity scale $w_*=(gQ_0z_i/T)^{1/3}$ (Deardorff, 1970) and associated scalar flux. Referring to Fig. 1, if we write (Wyngaard, 1983)

$$\Delta C_b\equiv C_b(h_1)-C_b(h_0)\equiv-\frac{\overline{cw_0}}{w_*}\Delta_b\tag{16}$$

$$\Delta C_t\equiv C_t(h_1)-C_t(h_0)\equiv-\frac{\overline{cw_1}}{w_*}\Delta_t,\tag{17}$$

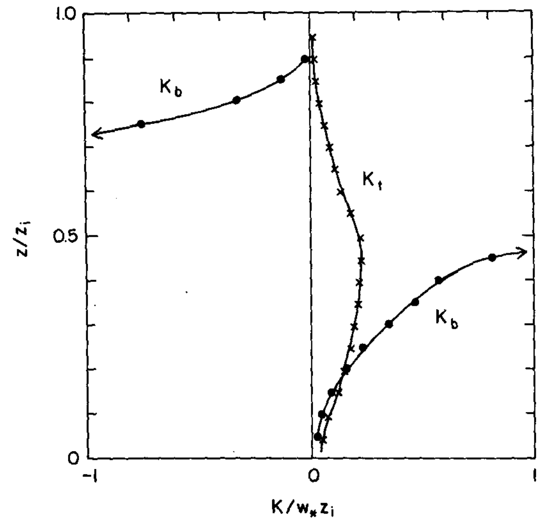

FIG. 2. The top-down and bottom-up eddy diffusivities of conservative scalars in the CBL according to the LES results of Moeng and Wyngaard (1984).

then we can calculate Δ_t and Δ_b from g_t and g_b:

$$\Delta_t = \int_{h_0/z_i}^{h_1/z_i} g_t d(z/z_i) \tag{18}$$

$$\Delta_b = \int_{h_0/z_i}^{h_1/z_i} g_b d(z/z_i). \tag{19}$$

In order to limit the contributions to these integrals from the large-gradient regions near the CBL top and bottom, where the LES resolution is the poorest, we chose $h_0 = 0.1z_i$ and $h_1 = 0.9z_i$. Using the Moeng and Wyngaard (1984) LES results (with 40^3 grid points), we find $\Delta_b \sim 1.0$, $\Delta_t \sim 4.0$. More recent LES runs with 96^3 grid points (Moeng, private communication) reproduce these values within 10%. We conclude, therefore, that the LES data indicate that

$$\Delta_t/\Delta_b \approx 4. \tag{20}$$

In the case of potential temperature, it is typically assumed that $\overline{\theta w_1} = -A\overline{\theta w_0}$, where A is an "entrainment coefficient" in the range 0.1 to 0.3 (e.g., Wilczak and Businger, 1983). If we take $A = 0.25$, then (16)–(20) imply that $\Delta\Theta$, the change in the mean potential temperature across the mixed layer, is

$$\Delta\Theta = -\frac{\overline{\theta w_0}}{w_*}\Delta_b - \frac{\overline{\theta w_1}}{w_*}\Delta_t = 0 \tag{21}$$

which agrees with CBL observations (Caughey, 1982).

In principle a K profile asymmetric about the center plane could also cause Δ_b and Δ_t to differ, as one can see from (15). The LES studies show that this difference is not due to an asymmetric K, however; instead, it is due to the breakdown of the K parameterization itself.

3. A simple physical mechanism for the asymmetry

In order to shed some light on a mechanism that could cause the asymmetry in top-down and bottom-up diffusion, and in particular the result (20), consider a simple model of the transfer process within the mixed layer. We will deliberately construct a model without vertical structure to insure that its behavior is not due an asymmetric K.

We take the mixed-layer eddies to be either updrafts (of fraction p^u and vertical velocity w^u) or downdrafts (of fraction p^d and vertical velocity w^d). We require that the probability of being in either updraft or downdraft is 1, and that the mean vertical velocity vanishes:

$$p^u + p^d = 1 \tag{22}$$

$$p^u w^u + p^d w^d = 0. \tag{23}$$

Now consider scalar diffusion through this mixed layer. We decompose this diffusion into its top-down and bottom-up components satisfying (3) and (4), respectively, each having linear scalar flux profiles. For the top-down process, we denote the characteristic c fluctuation in the updrafts by c_t^u, and that in the downdrafts by c_t^d. These maintain the top-down flux, whose value averaged over the mixed layer is $\overline{cw}_1/2$. Thus, we write

$$p^u w^u c_t^u + p^d w^d c_t^d = \overline{cw}_1/2. \tag{24}$$

The corresponding flux equation for the bottom-up process is

$$p^u w^u c_b^u + p^d w^d c_b^d = \overline{cw}_0/2. \tag{25}$$

In order to relate the parameters of (24) and (25) to ΔC_b and ΔC_t, the C changes across the mixed layer in (16) and (17), we now consider the nature of the component diffusion processes in more detail.

Consider bottom-up diffusion in this mixed layer, Fig. 3. According to (6) the C_b profile changes uniformly with time in response to the constant scalar flux divergence, as sketched in Fig. 3. The vertical change in C_b from the bottom to midlayer at any instant is some fraction of ΔC_b, say nC_b, where $0 < n < 1$. The C_b profile shape determines n; a linear profile, for example, implies $n = 1/2$. As Figure 3 suggests, the change in C_b occurs largely in the lower half of the mixed layer, implying that $1/2 < n < 1$. To first approximation the mixing ratio of an updraft parcel is constant during its vertical travel, but during the travel time to midlayer (of order $z_i/2w^u$) the background mixing ratio changes by an amount of order $(\partial C_b/\partial t)(z_i/2w^u)$. The mixing ratio fluctuation characteristic of the updrafts (i.e., at midlayer) is the negative sum of these "spatial" and "temporal" parts and is therefore of order

$$c_b^u = -n\Delta C_b - \frac{\partial C_b}{\partial t}\frac{z_i}{2w^u}. \tag{26}$$

Similar reasoning shows that the mixing-ratio fluctuation characteristic of the downdrafts is of order

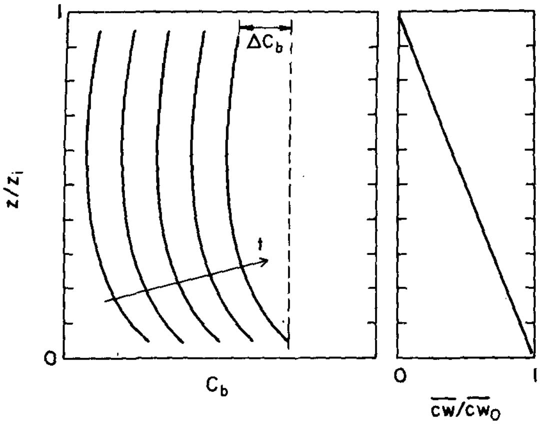

FIG. 3. Sketches of profiles of mean mixing ratio C_b (left) and scalar flux (right) in quasi-steady bottom-up diffusion. Note that the gradient of C_b changes sign near midlayer, and that the flux divergence causes a steady increase of C_b with time.

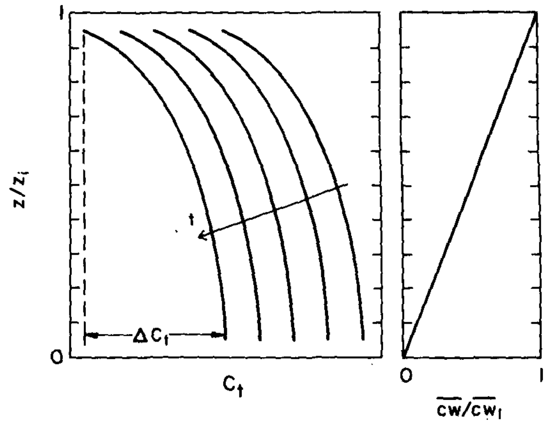

FIG. 4. Sketches of profiles of mean mixing ratio C_t (left) and scalar flux (right) in quasi-steady top-down diffusion. Note that the change in mean mixing ratio across the mixed layer is substantially larger than for the bottom-up case, Fig. 3, and that the flux divergence causes a steady decrease of C_t with time.

$$c_b{}^d = (1-n)\Delta C_b + \frac{\partial C_b}{\partial t}\frac{z_i}{2w^d}. \tag{27}$$

Using (26), (27), (22), and (23) in (24) then gives an expression for ΔC_b:

$$\Delta C_b = -\overline{cw}_0\left[1+\frac{\partial C_b}{\partial t}\frac{z_i}{\overline{cw}_0}(p^u-p^d)\right]\Big/(w^u p^u - w^d p^d). \tag{28}$$

Note that this is independent of the value of n.

The same arguments show that the expressions (26) and (27), with the subscripts b replaced by t, hold for the top-down process (Fig. 4). Thus, the expression for ΔC_t becomes

$$\Delta C_t = -\overline{cw}_1\left[1+\frac{\partial C_t}{\partial t}\frac{z_i}{\overline{cw}_1}(p^u-p^d)\right]\Big/(w^u p^u - w^d p^d). \tag{29}$$

We have written out the coefficient of the $(p^u - p^d)$ term in (28) and (29) in order to show explicitly the effect of the time change. Since (6) and (7) indicate the coefficients are +1 and −1 in the two cases, if $(p^u - p^d)$ is not zero we have the possibility of asymmetry in top-down and bottom-up diffusion,

$$\frac{\Delta C_t}{\Delta C_b} = \frac{\overline{cw}_1}{\overline{cw}_0}\frac{[1-(p^u-p^d)]}{[1+(p^u-p^d)]} \tag{30}$$

so that

$$\frac{\Delta_t}{\Delta_b} = \frac{1-(p^u-p^d)}{1+(p^u-p^d)}. \tag{31}$$

In order to proceed we need information on $(p^u - p^d)$, and we can develop this from further considerations of the kinematics of the large eddies. We require that these eddies have the observed second and third moments,

$$p^u(w^u)^2 + p^d(w^d)^2 = \sigma_w{}^2 \tag{32}$$

$$p^u(w^u)^3 + p^d(w^d)^3 = \sigma_w{}^3 S \tag{33}$$

where $\sigma_w{}^2$ is the variance of the vertical velocity and S is its skewness.

The set (22), (23) and (32), (33) represents four equations in four unknowns (p^u, p^d, w^u, w^d). Its solution can be written

$$p^d = 1 - p^u \tag{34}$$

$$w^d = w^u\left(\frac{p^u}{p^u-1}\right) \tag{35}$$

$$w^u = \frac{\sigma_w(1-p^u)^{1/2}}{(p^u)^{1/2}} \tag{36}$$

$$1-2p^u = (p^u)^{1/2}(1-p^u)^{1/2}S. \tag{37}$$

Note that (34) and (37) indicate that $p^u = p^d = 1/2$ only if the skewness S of the vertical velocity field is zero, in which case the transport asymmetry disappears according to (31). In fact, however, CBL observations clearly show that the skewness is quite pronounced. The AMTEX data (Lenschow et al., 1980), for example, give $S \sim 0.8$ in mid-CBL, as do data from the Minnesota experiment (Kaimal et al., 1976).

From (37) we find

$$p^u = \frac{4+S^2-S\sqrt{4+S^2}}{2(4+S^2)}. \tag{38}$$

If $S = 0.8$ Equation (38) gives $p^u = 0.3$, a reasonable value in light of the finding by Lenschow and Stephens (1980, 1982) that thermals covered between 20% and 30% of the area in the AMTEX convective boundary layer. If we use (38) in (31) we find

$$\frac{\Delta_t}{\Delta_b} = \frac{1-p^u}{p^u} = \frac{4+S^2+S\sqrt{4+S^2}}{4+S^2-S\sqrt{4+S^2}}. \tag{39}$$

If we take $S = 0.8$ we have from (39)

$$\frac{\Delta_t}{\Delta_b} \sim 2.2 \quad (40)$$

somewhat less than the LES result of 4; the updraft-downdraft model produces that result for $S = 1.5$. We conclude that the model does qualitatively reproduce the asymmetry observed in the LES studies of top-down and bottom-up diffusion.

4. Conclusions

A simple updraft–downdraft model of the CBL suggests the following candidate mechanism for the asymmetry in top-down and bottom-up diffusion. The nonstationarity of those processes causes significant changes in the mean mixing ratio over the vertical transit times for the large eddies. As a result the mixing ratio fluctuation in the updrafts and downdrafts is the sum of a "spatial" part due to the average mixing ratio difference across the mixed layer at any instant, and a "temporal" part due to the mean mixing-ratio evolution during a large-eddy transit time. In bottom-up diffusion the temporal part reduces the fluctuation in the updrafts, and enhances it in the downdrafts. The updraft–downdraft field is positively skewed, however, presumably due to the decrease with height of the heat flux (i.e., the buoyant production rate of vertical velocity variance). Because of this skewness the downdraft contribution dominates. Thus, in bottom-up diffusion the net enhancement of the mixing ratio fluctuation by the temporal contribution causes a compensating reduction in the spatial contribution; this reduces the mixing-ratio change across the CBL. The situation is reversed in the top-down process; there the temporal contribution decreases the mixing-ratio fluctuations in the downdrafts, which increases the change in mean mixing ratio across the CBL.

It remains to be seen whether this mechanism causes the asymmetry seen in LES studies of top-down and bottom-up diffusion in the CBL. An alternative explanation could be the vertical inhomogeneity of statistical properties of the velocity field, e.g., the variance and integral time and space scales (J. C. McWilliams, personal communication). Sawford and Guest (1986) have qualitatively reproduced the LES results with a one-dimensional Langevin equation and vertical inhomogeneity, suggesting that this alternative explanation is plausible. These candidate mechanisms are quite different, and perhaps further numerical studies could help us choose between them.

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